Two Measurements of $B^0\bar{B}^0$ Mixing


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(Rceived 29 April 1993)

We have measured the $B^0\bar{B}^0$ mixing probability, $\chi_{d}$, using a sample of 965,000 $B\bar{B}$ pairs from $\Upsilon(4S)$ decays. Counting dilepton events, we find $\chi_{d} = 0.157 \pm 0.016 \pm 0.018^{+0.026}_{-0.021}$. Using tagged $B^0$ events, we find $\chi_{d} = 0.149 \pm 0.023 \pm 0.019 \pm 0.010$. The first (second) error is statistical (systematic). The third error reflects a ±15% uncertainty in the assumption, made in both cases, that charged and neutral $B$ pairs contribute equally to dilepton events. We also obtain a limit on the $CP$ impurity in the $B^0_d$ system, $|Re(\epsilon^{*}_{B})| < 0.045$ at 90% C.L.

The 1987 discovery of $B^0\bar{B}^0$ mixing [1] ($B^0 \equiv \bar{b}d$) signaled a large top quark mass and gave rise to new prospects for the observation of $CP$ violation in $B$ meson decay. A precise determination of the rate of $B^0$ mixing is desirable, both to estimate the magnitude of $CP$ violating effects in $B$ meson decay, and to measure the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{td}$, once the top quark mass and the decay constant $f_B$ are better determined.

In this Letter we report two measurements of the $B^0\bar{B}^0$ mixing parameter $\chi_d$, defined by

$$\chi_d \equiv \frac{\Gamma(B^0 \to \bar{B}^0)}{\Gamma(B^0 \to B^0) + \Gamma(ar{B}^0 \to B^0)}.$$  

Previous measurements at the $\Upsilon(4S)$ [1–3] expressed mixing in terms of $r$, the ratio of mixed to unmixed widths, which is related to $\chi_d$ by $r = \chi_d/(1 - \chi_d)$. For coherent $B\bar{B}$ production at the $\Upsilon(4S)$, $\chi_d$ is given by

$$\chi_d = \frac{N(B^0\bar{B}^0) + N(\bar{B}^0\bar{B}^0)}{N(B^0\bar{B}^0) + N(B^0\bar{B}^0) + N(\bar{B}^0\bar{B}^0)}.$$ 

To measure this quantity it is necessary to have a tag of the flavor of the decaying $B^0$ mesons. The charge of leptons in semileptonic decays provides such a tag, and $\chi_d$ can be determined from the production of same-sign and opposite-sign dilepton events $[N(\ell^+\ell^+)\approx N(\ell^+\ell^-)]$ by

$$\chi_d = \frac{1}{1 - \Lambda \frac{N(\ell^+\ell^-)}{N(\ell^+\ell^+)}}.$$ 

where $\Lambda = f_+ b_+^2/(f_+ b_+^2 + f_0 b_0^2)$ is the fraction of dilepton events coming from $B^+B^-$ decays. It depends on the semileptonic branching fractions, $b_+$ and $b_0$, and the production fractions at the $\Upsilon(4S)$, $f_+$ and $f_0$, for charged and neutral $B$ mesons, respectively.

Present evidence suggests [4] equal dilepton contributions from charged and neutral $B$'s, i.e., $\Lambda = 0.5$, to an accuracy of (10–15)% . More precise information on $b_+/b_0$ and $f_+/f_00$ will be forthcoming soon from the CERN $e^+e^-$ collider LEP and from CLEO. Pending that information, the uncertainty in $\Lambda$ represents the largest systematic error in $\chi_d$ when it is obtained from dileptons. Consequently we have developed a second method for measuring $\chi_d$ in which we enrich our $B^0\bar{B}^0$ sample by tagging the decay $B_0^0 \rightarrow D^{*+}\ell^-\bar{\nu}$, $D^{*+} \rightarrow D^0\pi^+$, through correlations of $\ell^+$ and $\pi^+$ (inclusion of charge conjugate modes is implied throughout this Letter). The tagging method reduces sensitivity to $\Lambda$, but has poorer statistical accuracy.

The large sample of same-sign dileptons obtained in this study presents an opportunity to search for a $CP$ impurity in the mass eigenstates $B^0_1 \approx B^0_0 + \epsilon _{CP} B^0_2$, $B^0_2 \approx B^0_0 + \epsilon _{CP} B^0_1$, where $B^0_{1,2}$ are $CP$ eigenstates. The $CP$ impurity would appear as a charge asymmetry in same-sign dileptons [5],

$$a_{\ell\ell} \equiv \frac{N(\ell^+\ell^+)}{N(\ell^+\ell^-)} - \frac{N(\ell^-\ell^-)}{N(\ell^+\ell^-)} \approx 4 \text{Re}(\epsilon_2).$$ 

The standard model predicts [6] $a_{\ell\ell}$ to be small, $\sim 10^{-3}$. We present a measurement that is consistent with zero within large errors.

The data sample used in these studies was collected with the CLEO-II detector [7] at the Cornell Electron Storage Ring (CESR). It consists of 951 pb$^{-1}$ on the $\Upsilon(4S)$ resonance and 445 pb$^{-1}$ at a center-of-mass energy $\sim 55$ MeV below the resonance. Events are required to contain at least 5 charged particles and to have a visible energy of at least 30% of the center-of-mass energy.

To obtain the yield of dilepton events from semileptonic decays of $B\bar{B}$ pairs, several other sources must be subtracted from the total yield. These include dileptons from the nonresonant processes at the $\Upsilon(4S)$ ("continuum" processes), hadrons misidentified as leptons ("fake leptons"), leptons from $\psi$ or $\psi'$ decay, and events in which one lepton arises from $B$ semileptonic decay $B \rightarrow \ell^+\nu\chi$ (primary lepton) and another from the semileptonic decay of a daughter $D$ meson, $B \rightarrow DX, D \rightarrow \ell^+\nu\chi$ (secondary lepton). In order to reduce the background from these sources, each dilepton event is required to contain exactly two identified leptons with momenta between 1.5 and 2.4 GeV/c, and their opening angle, $\theta_{\ell\ell}$, must satisfy $-0.8 < \cos \theta_{\ell\ell} < 0.9$. In addition, if either lepton in the pair, when combined with an oppositely charged track satisfying less restrictive lepton identification criteria, has a pair invariant mass consistent with a $\psi$ or $\psi'$, the event is rejected.

The observed dilepton yields in the off-resonance data sample, scaled for the difference in luminosity and beam energy, provide a direct measurement of real and fake continuum dileptons. The contribution of pairs consisting of one real and one fake lepton is calculated by multiplying hadron track yields in single lepton events by the experimentally determined probability that a hadron is misidentified as a lepton ("fake probability"). Leakage through the veto of $\psi$ or $\psi'$ dileptons, as well as false vetoes of $B\bar{B}$ dilepton events, are calculated from a Monte Carlo simulation tuned to match the observed $\psi$ momentum spectrum.

The same-sign dileptons that arise from primary-secondary pairs are the most severe background in this experiment. We determine this charm background from data by fitting the measured inclusive electron spectrum from $B\bar{B}$ events with the momentum distributions expected for $b \rightarrow c\ell\bar{\nu}$ and $b \rightarrow c \rightarrow s\ell\bar{\nu}$. The semileptonic $B$ decay distribution used in the fit is obtained from a variety of theoretical models [8]. The lepton spectrum from secondary charm decays is obtained [9] by convoluting the lepton spectrum from $D \rightarrow X\ell\nu$ with the $D_0^0$ and $D^+$ momentum spectra from $B \rightarrow DX$ measured by CLEO [10] and weighted with the product of their production and semileptonic decay branching ratios. For the
lepton spectrum from $D \rightarrow X \bar{\nu}$, we use a parametrization consistent with the DELCO measurement [11] of the lepton spectrum from $\psi'' \rightarrow DD^*$, taking the motion of the $D$ into account. The fit determines the fraction, $f_\ell$, of leptons above the minimum momentum cutoff that come from charm decays. We determine the error in $f_\ell$ by varying the secondary lepton spectrum fitting function within the errors of the DELCO and CLEO measurements, by varying the $b \rightarrow c\ell\bar{\nu}$ model, and by considering errors in the measured inclusive electron spectrum. We find $f_\ell = 0.020 \pm 0.006$. Aside from the uncertainty in $\Lambda$, the uncertainty in the charm background dominates the systematic error of the mixing measurement. The opposite-sign dilepton yield also requires a small correction due to primary-secondary pairs.

Table I lists the raw dilepton yields, the corrections just described, and the corrected yields (dilepton events in which both $B$ mesons decayed semileptonically). We observe a large mixing signal of $184.5 \pm 18.9 \pm 23.5$ same-sign dileptons, compared to $2169 \pm 51 \pm 19$ opposite-sign events. We calculate $\chi^2$ from these numbers, using $\Lambda = 0.5$ in Eq. (3), obtaining

$$\chi^2 = 0.157 \pm 0.016 \pm 0.018 \pm 0.029 \pm 0.029,$$  

where the first error is statistical, the second error is systematic excluding the contribution from $\Lambda$, and the third, asymmetric error shows the effect of a $\pm 15\%$ variation in $\Lambda$ about 0.5. This value of $\chi^2$ corresponds to $r = 0.187 \pm 0.022 \pm 0.025 \pm 0.039$.

To test the robustness of the result, we calculate $\chi^2$ separately for $ee, e\mu$, and $\mu\mu$; we vary the lower momentum cut from 1.2 to 1.7 GeV/c, and we use an alternative procedure for eliminating $\psi$, in which we fit mass peaks and subtract instead of vetoing. All of these analysis methods give consistent results.

The tagging method for measuring $\chi^2$ exploits the exceptionally low energies released in the two decays, (a) $\Upsilon(4S) \rightarrow BB$ and (b) $D^{*+} \rightarrow D^0\pi^+$. The low $Q$ value of (b) enables one to tag without reconstructing the $D^0$; given a $D^{*+}$ lab energy $E_{D^*} = \gamma M_{D^*}$, the mean $\pi^+$ lab energy $E_{\pi^+}$ is $\gamma E_{\pi^+}$, where $E_{\pi^+}$ is the $\pi^+$ energy in the $D^{*+}$ frame. This relation is inverted to estimate the $D^{*+}$ energy:

$$E_{D^*} \simeq \frac{E_{\pi^+}}{\gamma} M_{D^*} \equiv \tilde{E}_{D^*}. \quad (6)$$

Taking the direction of motion of the $D^{*+}$ to be that of the $\pi^+$ gives a momentum $\tilde{p}_{D^*}$. The low $Q$ value of (a) enables one to obtain an approximate reconstruction of a three-body decay of the $B$, in this case to $D^{*+} \ell^- \bar{\nu}_\ell$, with only two final products measured [12]. Given a lepton and $\pi^+$, we examine the “missing mass squared,”

$$M_{m}^2 \equiv (E_B - E_\ell - \tilde{E}_{D^*})^2 - |p_\ell + \tilde{p}_{D^*}|^2, \quad (7)$$

which, for Monte Carlo simulation for $B \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$ events, accumulates near $M_{m}^2 = 0$ with an rms width $\sim 0.9$ (GeV/c$^2$)$^2$.

An event is rejected if it has a ratio of Fox-Wolfram event-shape parameters [13] $H_2/H_0 > 0.4$. Tags are formed by pairing identified leptons with pions of opposite (“right”) sign charge. The lepton momentum must be between 1.4 and 2.5 GeV/c, while the pion momentum must be less than 0.19 GeV/c. The pion is also required to have specific ionization ($dE/dx$) within two standard deviations of the expected value. The distribution in $M_m^2$ of all such lepton-pion pairs is shown in Fig. 1. Any candidate in the “signal” region $M_m^2 > -2$ (GeV/c$^2$)$^2$ is accepted as a tag.

The excess (“peak”) in the signal region has three main components, $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$, $D^{*+} \pi^0\ell^- \bar{\nu}_\ell$, and $B^- \rightarrow D^{*+} \pi^- \ell^- \bar{\nu}_\ell$. The contribution from the latter two is (14\%\%) of the total peak [12]. The peak area is determined by subtracting the contributions from random combinations in continuum and $\Upsilon(4S)$ events from the raw number. The shape of the “random” contribution from $\Upsilon(4S)$ events is estimated from both Monte Carlo simulation and lepton-pion pairs in the data with the same- (“wrong”) sign charges. The $M_m^2$ distributions of these samples are normalized to the right-sign distribution in the region $-20$ (GeV/c$^2$)$^2 < M_m^2 < -4$ (GeV/c$^2$)$^2$ and extrapolated to the signal region. Half the difference between these two results is taken to be the systematic error on the “random” contribution.

The leptons contributing to random background may come from $B^0$ or $B^-$, primary or secondary decays. The efficiency per event for primary decays to populate the
FIG. 1. \(M^2_{\pi}\) distribution in data (open circles; error bars are typically smaller than circles), with random components overlaid as estimated by Monte Carlo simulation from \(B^-B^+\) events (dashed histogram) and total (solid histogram).

background is estimated via Monte Carlo simulation, separately for \(B^0\bar{B}^0\), \(B^0\bar{B}^0\), and \(B^-B^+\) events. The fraction of leptons from secondary decay is found to be approximately the same as in the inclusive lepton sample, and we determine it using \(f_c\), described earlier. The effect of hadrons misidentified as leptons is studied using tracks which fail the lepton identification requirements.

The flavor of the “other” \(B\)'s in the tagged event sample is studied by searching for other leptons in the events. The numbers of same and opposite flavors, \(N(\ell^-\ell^-\pi^+)\) and \(N(\ell^-\ell^+\pi^-)\), are obtained from the inclusive spectra of additional leptons, separated according to charge relative to the tag lepton and fitted with distributions for primary and secondary decays. To eliminate dileptons created from double reconstruction of a single track, a lepton is discarded if its angle \(\theta_{\ell\ell}\) with the tag lepton gives \(\cos\theta_{\ell\ell} > 0.95\). The contribution to the spectra of leptons from \(\psi\) decay is evaluated using Monte Carlo simulation, scaled to the number of \(\psi\)'s found in the data. The spectra are corrected for contributions from continuum, fakes, and \(\psi\)'s, adjusted for detection efficiency, and averaged over muons and electrons. The fitted spectra are shown in Fig. 2. We find [14] \(N(\ell^-\ell^-\pi^+)/N(\ell^-\ell^+\pi^-) = 0.162 \pm 0.025\).

The various peak and random sources of tags contribute differently to the primary component of the fitted spectra. Random tags occur preferentially among events in which the second \(B\) decays hadronically, due to higher low-momentum track multiplicity. The probabilities, \(\alpha_i\), for finding a primary lepton in a random tag event, relative to that for a peak tag, are evaluated via Monte Carlo simulation for tags in \(B^0\bar{B}^0\), \(B^0\bar{B}^0\), and \(B^-B^+\) events. Each type of tag contributes to \(N(\ell^-\ell^-\pi^+)\) and \(N(\ell^-\ell^+\pi^-)\) in proportion to its probability \(\alpha_i\) and depends linearly on \(\chi_d\).

The major systematic uncertainties affecting the dilepton event counting method (\(\Lambda\) and \(f_c\)) have a reduced influence here because, allowing for the effects of \(\alpha_i\), 60% of the tag sample consists of the peak. This 60% of the tag sample has little dependence on \(\Lambda\) and contains no secondary or fake leptons. Nonetheless, the largest systematic errors still come from \(\Lambda\) and \(f_c\). Other systematic errors arise from uncertainties in the fraction of \(D^+\pi\ell\bar{\nu}\) decays, the fake probability, the composition of tagged events, \(\alpha_i\)'s, and the number of random tags. The lepton spectrum from \(B\) decay is estimated using several models, and the systematic uncertainty is taken to be the rms spread among the results. The net contribution of these sources to the systematic error is \(\pm 0.019\). The result from the tagging method is

\[
\chi_d = 0.149 \pm 0.023 \pm 0.019 \pm 0.010,
\]

where the first error is statistical, the second error is systematic excluding the contribution of \(\Lambda\), and the third error shows the effect of a \(\pm 15\%\) variation in \(\Lambda\) [15].

The results of the dilepton event counting measurement and the tagging measurement are in excellent agreement with the previous \(Y(4S)\) dilepton mixing results [16] of \(\chi_d = 0.142 \pm 0.035 \pm 0.034\) from CLEO [2] and \(\chi_d = 0.173 \pm 0.038 \pm 0.044\) from ARGUS [3]. The errors of these two new measurements are significantly smaller than those of the earlier results. It should be stressed that the two measurements reported here used the same data sample, are not independent, and should not be combined. Similarly, the systematic errors from the earlier measurements [2,3] are correlated with those reported here, through \(f_c\).

Finally, our dilepton sample from semileptonic \(B^0\) decay may be used to establish an upper limit on |Re(\(\epsilon_{B^0}\))| via a measurement of the \(CP\)-violating asymmetry parameter defined in Eq. (4). We find \(a_{\ell\ell} = 0.031 \pm 0.031\).
0.096 ± 0.032, corresponding to a 90% C.L. upper limit \(|a_{3d}| < 0.18\), which yields \(|\text{Re}(\epsilon_0)| < 0.045\).

We gratefully acknowledge the effort of the CESR staff for providing us with excellent luminosity and running conditions. J.P.A. and P.S.D. thank the PYI program of the NSF, I.P.J.S. thanks the YI program of the NSF, K.H. thanks the Alexander von Humboldt Stiftung, G.E. thanks the Heisenberg Foundation, K.K.G. and A.J.W. thank the SSC Fellowship program of TNRLC, K.K.G., H.N.N., J.D.R., and H.Y. thank the OJI program of DOE and R.P. and P.R. thank the A.P. Sloan Foundation for support. This work was supported by the National Science Foundation and the U.S. Department of Energy.

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[14] Two lepton-pion tags are found in 17% of the events containing a tag and a lepton, and we increase the statistical error by 10% to account for this redundancy.
[15] The dependence of \(\chi_d\) on \(A\) is \([2(1-A)]^{-0.4}\).
[16] Values for \(\chi_d\) from previous measurements have been reevaluated using \(A = 0.5\). Errors have been recalculated in a manner consistent with those of this measurement.