Measurement of the Ratio $\mathcal{B}(D^+ \to \pi^0 l^+\nu)/\mathcal{B}(D^+ \to \bar{K}^0 l^+\nu)$


(CLEO Collaboration)

1State University of New York at Albany, Albany, New York 12222
2Ohio State University, Columbus, Ohio, 43210
3University of Oklahoma, Norman, Oklahoma 73019
4Purdue University, West Lafayette, Indiana 47907
5University of Rochester, Rochester, New York 14627
6Southern Methodist University, Dallas, Texas 75275
7Syracuse University, Syracuse, New York 13244
8Vanderbilt University, Nashville, Tennessee 37235
9California Institute of Technology, Pasadena, California 91125
10University of California, San Diego, La Jolla, California 92093
11University of California, Santa Barbara, California 93106
12Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
13University of Colorado, Boulder, Colorado 80309-0390
14Cornell University, Ithaca, New York 14853
15University of Florida, Gainesville, Florida 32611
16Harvard University, Cambridge, Massachusetts 02138
17University of Illinois, Champaign-Urbana, Illinois, 61801
18Ithaca College, Ithaca, New York 14850
19University of Kansas, Lawrence, Kansas 66045
20University of Minnesota, Minneapolis, Minnesota 55455

(Received 16 June 1993)

Using the CLEO-II detector, the branching ratio of the Cabibbo suppressed decay $D^+ \to \pi^0 l^+\nu$, relative to the branching ratio of the Cabibbo favored decay $D^+ \to \bar{K}^0 l^+\nu$, is measured to be $\mathcal{B}(D^+ \to \pi^0 l^+\nu)/\mathcal{B}(D^+ \to \bar{K}^0 l^+\nu) = (8.5 \pm 2.7 \pm 1.4)\%$. From this ratio, the product of the ratio of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $|V_{cd}/V_{cs}|^2$, and the form factors at $q^2 = 0$, $|f^{\pi}(0)|^2/f^{\bar{K}}(0)|^2$, is determined to be $|V_{cd}/V_{cs}|^2 f^{\pi}(0)/f^{\bar{K}}(0) = 0.085 \pm 0.027 \pm 0.014$.

PACS numbers: 13.20.Fc, 14.40.Jz

$0031-9007/93/71(9)/1311(5)$S06.00
Semileptonic decays of charm mesons are theoretically straightforward to interpret. The amplitude for the decay mode $D^+ \rightarrow \pi^0 l^+ \nu$ is proportional to the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cd}$ and the form factor describing the strong interactions between the final state quarks $f_{q}^+(q^2)$. The decay rate for $D^+ \rightarrow \pi^0 l^+ \nu$ can be written as

$$\Gamma \propto \frac{1}{2} |V_{cd}|^2 |f_{q}^+(0)|^2 \int \frac{P_{l}}{(1-q^2/M_{D^+}^2)} dq^2,$$

where $q^2$ is the mass squared of the virtual $W$. We have assumed a monopole form [1] for the $q^2$ dependence of the form factor and $f_{q}^+(0)$ is the value of the form factor at $q^2 = 0$. The factor of $\frac{1}{2}$ accounts for the coupling of a $d\bar{d}$ pair to a $\pi^0$ meson [2]. For the decay $D^+ \rightarrow K^0 l^+ \nu$ the CKM matrix element, $V_{cs}$, and the form factor, $f_{K}^+(0)$, are substituted into Eq. (1) and the pole mass is changed to that of the $D^*$. Since the only pseudoscalar meson an sd quark combination can form is a $K^0$, there is no factor of $\frac{1}{2}$ for $D^+ \rightarrow K^0 l^+ \nu$.

Thus measurement of the ratio of branching ratios provides a way to measure the product $|f_{q}^+(0)/f_{K}^+(0)|^2 \times |V_{cs}/V_{cd}|^2$. Using the unitarity constraint on the CKM matrix, the CKM matrix elements, $V_{cd}$ and $V_{cs}$ [3,4], are currently known to a greater precision than are the values for the form factors [2,5-9]. Thus we will concentrate on the determination of $|f_{q}^+(0)/f_{K}^+(0)|$.

Most of the recent experimental work on charm semileptonic decays has been in the Cabibbo favored modes, $D \rightarrow K l \nu$ and $D \rightarrow K^* l \nu$ [10-14]. Mark III [15] has reported on one Cabibbo suppressed channel, $D^0 \rightarrow \pi^- e^+ \nu$; they have observed seven events.

The CLEO-II detector with its excellent photon detection is ideally suited to detect the decay chain $D^{++} \rightarrow D^+ \pi^0, D^+ \rightarrow \pi^0 l^+ \nu$. The initial $D^{++} \rightarrow D^+ \pi^0$ decay is used to improve the signal to background ratio of the event sample. In this paper the $D^+ \rightarrow \pi^0 l^+ \nu$ branching ratio is normalized to that of $D^+ \rightarrow K^0 l^+ \nu$, which is detected via $K^0_s \rightarrow \pi^+ \pi^-$, also using the $D^{++} \rightarrow D^+ \pi^0$ decay chain.

The data sample for this analysis was recorded with the CLEO-II detector operating at the CESR storage ring at Cornell University. A total luminosity of 1.8 fb$^{-1}$ of $e^+e^-$ collisions was recorded at the $\Upsilon(4S)$ and in the nearby continuum, at energies below and above the $\Upsilon(4S)$ resonance. The CLEO-II detector has been described elsewhere [16]; relevant features of the detector are summarized below.

Electrons with momentum above 0.7 GeV/c are identified by requiring that the ratio of the energy ($E$) deposited in the CsI calorimeter and the momentum ($p$) measured in the tracking system, $E/p$, be close to unity and that the energy loss measured by the tracking system be consistent with the electron hypothesis. Electrons within the fiducial volume are identified with an efficiency of 94%; the probability that a charged track will be misidentified as an electron is (0.3 ± 0.1)% per charged track. Muons are identified by their ability to penetrate five nuclear interaction lengths of iron. Muons with momentum above 1.4 GeV/c, and within the fiducial volume, are identified with an efficiency of 93%; the fake rate is (1.4 ± 0.2)% per charged track. For both electrons and muons we require that the absolute value of the cosine of the angle between the lepton momentum and beam direction be less than 0.81 [17].

Isolated photons, detected by the CsI calorimeter, with minimum energies of 30 MeV, are paired to form $\pi^0$ candidates. Combinations within 2.5$\sigma$ of the nominal $\pi^0$ mass ($\sigma \sim 5$ MeV/c$^2$) are selected as $\pi^0$ candidates. The $\gamma$ momentum vectors are then kinematically constrained to the nominal $\pi^0$ mass. The $K^0_s$ sample is reconstructed by combining two oppositely charged tracks that fit to a common vertex, displaced from the primary vertex. The invariant mass of the two charged tracks, assumed to be $\pi^+\pi^-$, is required to be within 3.0$\sigma$ ($\sigma \sim 4$ MeV/c$^2$) of the nominal $K^0_s$ mass.

For fully reconstructed hadronic $D^{**} \rightarrow D^{+}\pi^{0}$ decays the mass difference, $\Delta M = M_{D^{**}} - M_{D^{++}}$, is sharply peaked where the mass difference has a typical sigma of 1 MeV/c$^2$. In semileptonic decays the neutrino is undetected. We define a mass difference where we use only the observable information as $\Delta M = M_{\pi\nu} - M_{\pi\nu}$. Because of the missing momentum information of the neutrino the $\Delta M$ distribution is broadened, but a definite peak remains. We use the $\Delta M$ distribution to extract the yield of $D^{+} \rightarrow \pi^0 l^+ \nu$ and $D^{+} \rightarrow K^0 l^+ \nu$ events. As $M_{\pi\nu}$ increases the momentum of the neutrino decreases in the $D^+$ center of mass, and the $\Delta M$ distribution becomes more sharply peaked. We require $M_{\pi\nu} > 1.3$ GeV/c$^2$ in order to select combinations with a narrow $\Delta M$ distribution. We have denoted the slow $\pi^0$ from the initial $D^{**}$ decay as $\pi^0_s$, since production and decay kinematics constrain its momentum to be less than 0.4 GeV/c. To reduce background from random diphoton combinations we require that $p_\gamma > 0.2$ GeV/c and that the $\pi^0_s$ from the $D^+$ decay have a momentum greater than 0.5 GeV/c; these $\pi^0_s$ are denoted as $\pi^0_f$. For a $\pi^0_f l^+$ combination to be consistent with a charm parent hypothesis, we require that $M_{\pi\nu} < 1.8$ GeV/c$^2$. To reduce combinatoric background we require that $p_\gamma > 2.5$ GeV/c. Combinatoric background from the $\Upsilon(4S)$ decays is suppressed by requiring that the ratio of Fox-Wolfram moments [18], $R_2 = H_2/H_0 \geq 0.2$.

The use of the $D^{**}$ tag allows an estimate of the $D^+$ momentum. Continuum production of charm events produces two well defined jets. This jet or thrust axis approximates the $D^+$ direction; Monte Carlo simulations show that the distribution of the angle between the thrust axis and the $D^+$ direction has a width of 0.150 rad (FWHM). Assuming the $D^+$ direction is along the thrust axis, the momentum of the $D^+$ can be estimated from the $\pi^0_s$ momentum vector and the thrust axis direc-
tion [19]. We require that this momentum estimate, $P_{\text{est}}$, be greater than 2.5 GeV/c.

In semileptonic $D$ decays involving a pseudoscalar, the energy spectrum of the pseudoscalar is dominated by phase space and is peaked strongly at high energy. If the momentum of the slow pion from the $D^{*+}$ decay is neglected then $M(\pi^0_{\pm}\pi^0_{\pm})^2 \sim 2M_{\pi^0}^2 + M_{D^0}^2 E_{\pi^0}^2$, where $E_{\pi^0}^*$ is the energy of the $\pi^0$ in the $D^+$ center of mass. Thus $M(\pi^0_{\pm}\pi^0_{\pm})$ is a measure of the $\pi^0$ energy in the center of mass of the $D^+$ decay and allows one to select events from the appropriate range of $E_{\pi^0}^*$. The Monte Carlo $M(\pi^0_{\pm}\pi^0_{\pm})$ distribution is described well by a Gaussian function, and we require that the data events be within 1.5$\sigma$ ($\sigma = 0.065$ GeV/c$^2$) of the Monte Carlo mean.

Backgrounds are divided into two classes: correlated backgrounds which peak in the signal region, $\Delta M < 0.15$ GeV/c$^2$, and uncorrelated backgrounds which do not peak in the signal region. The correlated backgrounds originate from a true $D^* \rightarrow D\pi^0$ decay. The $D^0$ or $D^+$ meson subsequently decays either semileptonically, with a $\pi^0$ in its final state, or hadronically, with one of the hadronic daughters faking a lepton. To reduce the correlated backgrounds we take all tracks in the event with charge opposite to that of the lepton, $X^-$, and calculate $M(\pi^0_{\pm}X^-)$, assigning $X^-$ the pion mass. If any $\pi^0_{\pm}X^-$ combination has a mass below 1.9 GeV/c$^2$, we reject that $\pi^0_{\pm}X^-$ combination. To study and estimate the amount of correlated background from semileptonic feeddown we generated a large sample of $c\bar{c}$ Monte Carlo events with the decay chain under study removed. Correlated backgrounds from lepton fakes are studied and estimated by using the measured, momentum dependent fake rates to label measured hadronic tracks as leptons. The same analysis code as for the data is then used to extract the shape and normalization of the $\Delta M$ distribution for combinations which include fake leptons. From these studies we find that the $M(\pi^0_{\pm}X^-)$ cut removes over 60% of the semileptonic feeddown and over 50% of the fake lepton background while retaining over 85% of the signal.

To limit systematic effects of our selection criteria, the selection criteria for the $D^+ \rightarrow K^0_{\pm}\pi^0_{\pm}$ decay mode are identical to those of the $D^+ \rightarrow \pi^0\pi^0\nu$ mode: $1.3 < M_{K^0_{\pm}} < 1.8$ GeV/c$^2$, $p_{K^0_{\pm}} > 2.5$ GeV/c, $p_{\pi^0_{\pm}} > 0.5$ GeV/c, $P_{\text{est}} > 2.5$ GeV/c, $M(\pi^0_{\pm}K^0_{\pm})$ within 1.5$\sigma$ of the Monte Carlo mean, and $M(\pi^0_{\pm}K^0_{\pm}) > 1.9$ GeV/c$^2$.

Figure 1(a) shows the $\Delta M$ distribution for $D^+ \rightarrow \pi^0\pi^0\nu$ candidates. The histogram is the distribution for combinations which pass the above cuts. We call this the raw distribution. The points are the estimated fake lepton background. Figure 1(b) shows the raw $\Delta M$ distribution for the $D^+ \rightarrow K^0_{\pm}\pi^0_{\pm}$ mode and its estimated fake lepton background. Since the fake lepton backgrounds peak in the signal region, we subtract them bin by bin from the raw $\Delta M$ distributions. The background subtracted distributions are shown in Fig. 2.

We study uncorrelated backgrounds by using sidebands in the $\pi^0_{\pm}$, $\pi^0_{\pm}$, and $K^0_{\pm}$ mass distributions. The $\Delta M$ distribution for these uncorrelated backgrounds is well described by a simple polynomial. Since these backgrounds do not peak in the signal region, they are not

<table>
<thead>
<tr>
<th>TABLE I. Yields from fits to the raw and background subtracted data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
</tr>
<tr>
<td>$\pi^0_{\pm}\pi^0_{\pm}$</td>
</tr>
<tr>
<td>$K^0_{\pm}\pi^0_{\pm}$</td>
</tr>
</tbody>
</table>
TABLE II. Detection efficiencies for $D^+ \rightarrow \pi^0 l^+ \nu$ and $D^+ \rightarrow \bar{K}^0 l^+ \nu$ and for feedings into these channels. The errors are due to Monte Carlo statistics and upper limits are at the 90% confidence level.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Decay channel</th>
<th>$\epsilon$ (%)</th>
<th>Expected contribution (events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 l^+ \nu$</td>
<td>$D^+ \rightarrow \pi^0 l^+ \nu$</td>
<td>0.51 ± 0.02</td>
<td>na</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}^0 l^+ \nu$</td>
<td>0.032 ± 0.003</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>$K^* l^+ \nu$</td>
<td>&lt; 0.002</td>
<td>&lt; 0.7</td>
</tr>
<tr>
<td></td>
<td>$\eta l^+ \nu$</td>
<td>0.020 ± 0.002</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>$\eta' l^+ \nu$</td>
<td>&lt; 0.002</td>
<td>&lt; 0.05</td>
</tr>
<tr>
<td></td>
<td>$\omega l^+ \nu$</td>
<td>0.005 ± 0.002</td>
<td>0.3</td>
</tr>
<tr>
<td>$\bar{K}^0 l^+ \nu$</td>
<td>$D^+ \rightarrow \bar{K}^0 l^+ \nu$</td>
<td>0.24 ± 0.01</td>
<td>na</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}^* l^+ \nu$</td>
<td>0.007 ± 0.003</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Subtracted before fitting the $\Delta M$ distribution.

The yields are determined by fitting the background subtracted $\Delta M$ distributions. The Monte Carlo $\Delta M$ distribution fits well to a function where the left side of the distribution is described by a Gaussian function with mean $X_0$ and sigma $\sigma_l$, and where the right side of the distribution is described by a Gaussian function with mean $X_0$ and sigma $\sigma_R$. While fitting the data the mean and sigmas are fixed to values determined from the Monte Carlo sample [20]. In addition to the signal function, a slowly varying background function is also included in the fit to account for the uncorrelated backgrounds. The yields from fits to the raw and background subtracted $\Delta M$ distributions are in Table I.

Table II contains the efficiencies for the signal and feeding channels for the two modes. The dominant feed-down in the $\pi^0 l^+ \nu$ signal is from $D^+ \rightarrow \bar{K}^0 l^+ \nu$ where the $\bar{K}^0$ subsequently decays into $\pi^0 \pi^0$. This feeding is statistically independent of the normalizing signal and allows us to write

$$B(D^+ \rightarrow \pi^0 l^+ \nu) = \frac{N(\pi^0 l^+ \nu) \epsilon_{K^0 l^+ \nu}(K^0 l^+ \nu)}{B(D^+ \rightarrow \bar{K}^0 l^+ \nu)}$$

where $N(\pi^0 l^+ \nu)$ and $N(K^0 l^+ \nu)$ are the two signal yields, $\epsilon_{K^0 l^+ \nu}(K^0 l^+ \nu)$ is the efficiency for reconstructing a $K^0 l^+ \nu$ decay as $K^0 l^+ \nu$, and $\epsilon_{K^0 l^+ \nu}(\pi^0 l^+ \nu)$ is the efficiency for reconstructing a $K^0 l^+ \nu$ decay as $\pi^0 l^+ \nu$. We do not include any other feedings from Table II in the calculation of the ratio of branching ratios, as their branching ratio times efficiency is small and negligible [21].

The ratio $B(D^+ \rightarrow \pi^0 l^+ \nu)/B(D^+ \rightarrow \bar{K}^0 l^+ \nu)$ is measured to be $(8.5 \pm 2.7 \pm 1.4)%$, where the first error is statistical and the second systematic. Most of the systematic effects of our cuts cancel in the ratio of branching ratios since we use the same cuts for both decay modes. The systematic error in the ratio due to Monte Carlo simulations of $K^0 \rightarrow \pi^+ \pi^-$ and $K^0 \rightarrow \gamma \gamma$ reconstruction is determined to be 13% by studying well measured $D^0$ hadronic decays. Systematic error due to incorrectly subtracting the lepton fake background is estimated to be less than 5% of the measured branching ratio. We determine this by varying the lepton fake rate by 3x the measured fake rate. The effect of incorrectly determining the lepton fake rate partially cancels in the ratio. We add these systematics in quadrature along with a 7% error due to Monte Carlo statistics to obtain a total systematic error of 16%.

We have measured the branching ratio of the Cabibbo suppressed decay $D^+ \rightarrow \pi^0 l^+ \nu$ relative to the Cabibbo favored decay $D^+ \rightarrow \bar{K}^0 l^+ \nu$. Using our measurement of the ratio of branching ratios and Eq. 1 we find $|f_K^\pm(0)/f_K^\mp(0)|^2 = 0.085 \pm 0.027 \pm 0.014$ [22]. Unitarity constraints on the CKM matrix yield a value of 0.051 ± 0.002 for $|V_{cd}/V_{cs}|^2$ [3]; using this value we obtain $|f_K^\pm(0)/f_K^\mp(0)| = 1.29 \pm 0.21 \pm 0.11$. The model predictions [2,5-9] for the ratio of form factors, which range from 0.7 to 1.4, are in agreement with our measurement.

We gratefully acknowledge the effort of the CESR staff in providing us with excellent luminosity and running conditions. This work was supported by the National Science Foundation, the U.S. Department of Energy, the SSC Fellowship program of TNRLC, the Heisenberg Foundation, and the A.P. Sloan Foundation.

* Permanent address: Institute of Nuclear Physics, Novosibirsk, Russia.
† Permanent address: Carleton University, Ottawa, Canada K1S 5B6.

[4] The current measured value for $V_{cd}$ is from dimuon events in $\nu$ interactions. These measurements are in agreement with the unitarity value. The most recent measurement can be found in S. A. Rabinowitz et al., Phys. Rev. Lett. 70, 134 (1993).
[17] Throughout this paper the symbol l is used to refer to either electrons or muons.
[19] The momentum of the $D^+$ is determined from the following equations: $P(D^+) = \gamma \beta M(D^+)$, $\gamma = [P_\parallel(\pi^0) + M^2(\pi^0)]^{1/2}/M(\pi^0)$, $\beta = (1 - 1/\gamma^2)^{1/2}$, where $P_\parallel(\pi^0)$ is the momentum of $\pi^0$ projected onto the thrust axis.
[20] We have checked this method of extracting yields by comparing the $D^+ \rightarrow K^0 l^+ \nu$ yield with the yield for the hadronic decay $D^+ \rightarrow K^0 \pi^+$. Our measurement is consistent with the previous measurements of these branching ratios.
[21] The expected feddown was calculated using the efficiencies in Table II and Eq. (2) for the $K^0 l^+ \nu$ contribution to the $\pi^0 l^+ \nu$ signal, and the measured $D^+ \rightarrow K^* l^+ \nu$ branching fraction for $K^* l^+ \nu$ contributions. The predicted branching ratios were used for the Cabibbo suppressed modes [2].
[22] We do not include any systematic error due to assumption of a monopole $q^2$ dependence for the form factor in this result.