A Design for a Steel Concrete Arch

by E. Nelson

1905

Presented as Master’s Thesis in Civil Engineering
A DESIGN FOR

A STEEL CONCRETE ARCH

E. NELSON
A Design For
A
75 Ft. Steel Concrete Arch

Presented as Master's Thesis

Kansas University

Carl Nelson
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Civil Engineering
A Design for a Stab Concrete Arch Bridge

This design is based entirely on Walter W. Colpitts' pamphlet on the calculation of stresses and practical design of structures of steel concrete. The formulas given in this book are for railroad bridges and had to be changed whenever the live load entered into them.

The arch will have a span of 75 feet and a rise of about 15 feet. It is intended for use in a park so that live load will be assumed at 100 pounds per square foot and uniformly distributed. The earth fill will extend one foot above the crown and will be assumed to weigh 100 pounds per cubic foot. The clear width of the bridge will be 30 feet and of the roadway 20 feet, there being room left for a five-foot granitoid sidewalk on each side. The weight of the concrete steel is about 155 pounds per cubic foot.

The strength of steel in steel has not been fully developed unless a length equal to twenty-five times its side is imbedded in the concrete (Colpitts, page 5). As it is often convenient to be able to develop the full strength of the bars in a length less than twenty-five times their side and because it has not been found that vibratory loads and the continual expansion and contraction due to change of temperature do not tend to reduce the adhesion between the concrete and steel, a form of bar which does not depend entirely upon adhesion to transmit the stress into the concrete.
is to be used. Corrugated steel bars of a pattern similar to those manufactured by the Saint Louis Expanded Metal Fireproofing Company, which have an elastic limit of between 50,000 and 60,000 pounds per square inch will be used.

The concrete should be of good quality since, in places, it has to withstand intense stress. The theory of the proper proportioning of good concrete requires that the cement and water be slightly in excess of the needs in the sand and stone, respectively. With stone passing a 2-inch ring and dusted out, the proportions satisfying these conditions are: Portland cement one part; sand three parts; broken stone six parts. The following are the ultimate values for such a concrete: Tension, 200 pounds per square inch; compression, 2000 pounds per square inch; shearing, 400 pounds per square inch. (Colpitts, page 6)

The Design

Loading
Assume the linear arch parabola as 72 feet long, so as to give 75 feet clear span when the design is completed, and with a rise of 15 feet. Divide the arch into 10 panels of 8.2 feet each and number as in Fig. 1.

Take the arch ring as ring 20 inches thick and it is about 90 feet long measured on the parabolic centre line. Of the concrete steel weighs 155 pounds
for cubic foot, the total weight of such ring for a section one foot thick is: \((20 \times 90 \times 11.55) \div 2 = 23250\) pounds, which gives a panel load of 2325 pounds.

The earth load is obtained by multiplying together, the weight of a cubic foot of earth, the distance from the surface of the earth fill to the parabola and the panel length.

The concrete sidewalk will weigh about 380* per panel. Summing these three results we have the vertical loads as shown on the diagram. Use a safety factor of two.

The vertical panel live load, using a factor of safety of four and allowing 1500* pounds impact = \(150 \times 824 + 1500 = 6500\) pounds.

The horizontal dead loads are equal to one third of the vertical dead loads. This is also true for live load but no allowance is made for impact.

All live loads have a factor of safety of four and dead loads a factor of two.

It is only necessary to calculate the maximum bending moments and the horizontal thrusts at three points, the crown, panel point 3, and the abutment. (Col. 40, page 40).

Maximum Moments Due to Vertical Live Loads

Referring to column 5, table III (page 40) we see that live load on panel points 1, 2, 3, 7, 8, and 9 produce -M, while
those on 4.5 and 6 produce \( +M \) at the crown. So at the crown, maximum \( +M = (0.192 + 0.128 + 0.192) \times 37.5 \times 6500 = +368550 \text{ inch-lbs} \), and maximum \( -M = (0.120 + 0.288 + 0.240) \times 37.5 \times 6500 = -315400 \text{ inch-lbs} \).

**Panel Point 3**

Maximum \( +M = (0.312 + 0.428 + 0.576 + 0.132) \times 37.5 \times 6500 = +606700 \text{ inch-lbs} \).

Maximum \( -M = 2.256 \times 37.5 \times 6500 = -549400 \text{ inch-lbs} \).

**Adjustment**

Maximum \( +M = 4.068 \times 37.5 \times 6500 = +991575 \text{ inch-lbs} \).

Maximum \( -M = 3.264 \times 37.5 \times 6500 = -941850 \text{ inch-lbs} \).

**Maximum Bending Moments Due to Vertical Dead Loads**

Since the dead load is constantly applied we will have at each panel point a resultant positive or negative bending moment, as follows, which will be added algebraically to the maximum \( +M \)'s found above. Referring again to Table III.

**Moment at the Crown**

Load on 1 \& 9, \( M = 7640 \times -0.120 \times 37.5 \times 2 = -63360 \)

\( 2.48, M = 6350 \times -0.288 \times 37.5 \times 2 = -137160 \)

\( 3.47, M = 5860 \times -0.240 \times 37.5 \times 2 = -105480 \)

\( 4.46, M = 5720 \times +0.192 \times 37.5 \times 2 = +82368 \)

\( 5.7, M = 5600 \times +1.28 \times 37.5 \times 2 = +236320 \)

\( -306000 \times +318688 \)

Resultant \( M = +12700 \text{ inch-lbs} \).
Moment at Panel Point 3

\[
\begin{align*}
\text{Load on 1, } M &= 7040 \times 37.5 \times 0.132 = +34848 \\
\text{2, } M &= 6350 \times 37.5 \times 0.576 = +137160 \\
\text{3, } M &= 5860 \times 37.5 \times 1.428 = +313803 \\
\text{4, } M &= 5720 \times 37.5 \times 0.312 = +66924 \\
\text{5, } M &= 5600 \times 37.5 \times (-0.372) = -78120 \\
\text{6, } M &= 5720 \times 37.5 \times (-0.672) = -144144 \\
\text{7, } M &= 5760 \times 37.5 \times (-0.648) = -142397 \\
\text{8, } M &= 6350 \times 37.5 \times (-0.420) = -100000 \\
\text{9, } M &= 7040 \times 37.5 \times (-0.144) = -38016 \\
\text{Resultant Moment} &= +50060 \text{ in-lbs.}
\end{align*}
\]

Moment at Aislement

\[
\begin{align*}
\text{Load on 1, } M &= 7040 \times 37.5 \times (-1452) = -383328 \\
\text{2, } M &= 6350 \times \times (-1.536) = -365760 \\
\text{3, } M &= 5860 \times \times (-0.876) = -192501 \\
\text{4, } M &= 5720 \times \times 0.0 \\
\text{5, } M &= 5600 \times \times +0.744 = +156240 \\
\text{6, } M &= 5720 \times \times +1.152 = +247104 \\
\text{7, } M &= 5760 \times \times +1.140 = +250595 \\
\text{8, } M &= 6350 \times \times +0.768 = +188880 \\
\text{9, } M &= 7040 \times \times +0.264 = +67696 \\
\text{Resultant Moment} &= -29200 \text{ in-lbs.}
\end{align*}
\]

Bending Moments Due to Horizontal Line Loads

Referring to table IV, column 5 (Cafette, page 32) and placing horizontal line loads on the points which
when loaded with vertical live loads, produce maximum negative and positive bending moments respectively we have the following for the cross section. For -M load points 1, 2, 3, 8, 3, and 9, and for +M load points 4, 5, and 6.

Load on 1 + 9, \( M = 1640 \times 15 \times 0.22 \times 2 = -10824 \)

" " 2 + 8, \( M = 1640 \times 15 \times -0.45 \times 2 = -22140 \)

" " 3 + 7, \( M = 1640 \times 15 \times -0.42 \times 2 = -20664 \)

-53628 inch lbf.

This is to be added to the maximum negative moment produced by the vertical live loads at this point.

Load on 4 + 6, \( M = 1640 \times 0.17 \times 2 \times 15 = -8364 \) inch lbf. which is to be added to the maximum positive moment produced by the vertical live loads at this point.

Proceeding in a similar manner we will get the moments at panel point 3 and at the abutment.

**Panel Point 3**

For +M load points 1, 2, 3 and 4.
\[ M = 1640 \times 15 \times (0.22 + 0.35 + 0.89 + 0.99) = +64944 \text{ inch lbf. to be added to maximum +} Mv. \]

For -M load points 6, 7, 8 and 9.
\[ M = 1640 \times 15 \times (-0.39 - 0.37 - 0.63 - 0.25) = -15744 \text{ inch lbf. to be added to maximum -} Mv. \]

**Abutment**

For +M load points 6, 7, 8, and 9
\[ M = 1640 \times 15 \times (1.58 + 1.57 + 16 + 0.48) = +117834 \text{ inch lbf. to be added to maximum +} Mv. \]

For -M load points 1, 2, 3 and 4
\[ M = 1640 \times (15 - 2.71 - 2.83 - 2.21 - 1.66) = -231486 \text{ inch lbs. to be added to maximum } -M. \]

Moments Due to Horizontal Dead Loads

The dead load is constantly applied. We therefore find the resultant moment at each panel point (using table IV) which will be algebraically added to the maximum - and +M3.

At Course

Load on \(1+9\), \(M = 800 \times 15 \times -0.22 \times 2 = -5280 \)

\(278, M = 560 \times 15 \times -0.45 \times 2 = -7560 \)

\(347, M = 400 \times 15 \times -0.42 \times 2 = -5040 \)

\(446, M = 360 \times 15 \times -0.17 \times 2 = -1836 \)

Total = -19116 inch lbs. to be added both to the maximum -M and maximum +M.

At Panel Point 3

Load on

\(1, M = 800 \times 15 \times 0.22 = +2640 \)

\(2, M = 560 \times 15 \times 0.85 = +7140 \)

\(3, M = 400 \times 15 \times 1.69 = +10140 \)

\(4, M = 360 \times 15 \times 0.99 = +5346 \)

\(6, M = 360 \times 15 \times -0.89 = -4806 \)

\(7, M = 400 \times 15 \times -0.87 = -5220 \)

\(8, M = 560 \times 15 \times -0.63 = -5292 \)

\(9, M = 800 \times 15 \times -0.25 = -3000 \)

\(-18318 \quad +25266 \)

Resultant \(W = +7000 \text{ inch lbs.} \)
At Deflection

load on 1, $M = 800 \times 15 \times -2.71 = -32520$

" " $2, M = 560 \times 15 \times -2.83 = -23772$

" " $3, M = 400 \times 15 \times -2.21 = -13260$

" " $4, M = 360 \times 15 \times -1.66 = -8964$

" " $6, M = 360 \times 15 \times +1.58 = +8532$

" " $7, M = 400 \times 15 \times +1.57 = +9420$

" " $8, M = 560 \times 15 \times +1.16 = +9744$

" " $9, M = 800 \times 15 \times +0.48 = +5760$

$-78516 +33456$

Resultant $M = 45100$ inch lb.

Moments Due to Change of Temperature

First we must determine the moment of inertia of the crown section from the formula,

$I = \frac{Mx}{f}$ (Gillette, page 34), in which $I$ is the moment of inertia of the section, $M$ is the maximum bending moment at the section, $x$ is the distance in inches from the neutral axis to the compression face and $f$ is the intensity of compressive stress in pounds per square inch on the extreme fibers of the concrete due to bending moment only. The method of finding this last stress will be shown later.

The maximum $-M$ at the crown due to the vertical and horizontal loads are:

$+M = +368550 +12700 - 8364 - 19716 = +353170$ inch lb.

$-M = 315900 +12700 - 53628 - 19716 = -376544$ inch lb.

Since the negative moment is the greater,
we will combine with it the horizontal thrust from the vertical and horizontal loads which have produced the maximum -M, and design a section temporarily to resist those stresses. The method of determining the thrusts will be given in detail later, we shall simply note the results here:

<table>
<thead>
<tr>
<th>Source</th>
<th>Thrust (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical live loads</td>
<td>18962</td>
</tr>
<tr>
<td>Vertical dead loads</td>
<td>36850</td>
</tr>
<tr>
<td>Horizontal live loads</td>
<td>2699</td>
</tr>
<tr>
<td>Horizontal dead loads</td>
<td>1188</td>
</tr>
<tr>
<td><strong>Total thrust at crown</strong></td>
<td><strong>59699</strong></td>
</tr>
</tbody>
</table>

To design the section we use formulas 1, 2, and 3 (Sills, page 7) and Table V (page 39).

\[ M = \text{bending moment in inches} \times \text{pounds on a section of beam one foot thick} \]

\[ Y = \text{theoretical depth of beam in inches} \]

\[ a = \text{square inches of metal reinforcement required per foot width of beam} \]

\[ x = \text{distance in inches from compression face to neutral axis} \]

Then:

\[ M = 4460Y^2 \quad \text{or} \quad Y = \sqrt{\frac{M}{4460}} \quad (1) \]

\[ a = 0.086Y \quad (2) \]

\[ x = 0.367Y \quad (3) \]

When a combined bending moment and thrust are to be provided for at any point, we determine temporarily the section necessary to resist the bending moment only, by means of (1) and (2). Then we find the intensity of stress per square inch on the section which would be produced.
by the thrust. The total intensity of compressive stress on the extreme fibers must not exceed 2000 pounds per square inch, and consequently that permitted by the bending moment will be 2000 pounds, less that produced by the thrust. To facilitate the work of designing sections for various compressive stresses, due to the bending moment, Table V was calculated.

<table>
<thead>
<tr>
<th>fc</th>
<th>M</th>
<th>x</th>
<th>a</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>983Y²</td>
<td>0.225Y</td>
<td>0.044Y</td>
<td>5</td>
</tr>
<tr>
<td>1250</td>
<td>1706Y²</td>
<td>0.266Y</td>
<td>0.022Y</td>
<td>4</td>
</tr>
<tr>
<td>1500</td>
<td>2600Y²</td>
<td>0.303Y</td>
<td>0.041Y</td>
<td>3</td>
</tr>
<tr>
<td>1750</td>
<td>3505Y²</td>
<td>0.336Y</td>
<td>0.063Y</td>
<td>2</td>
</tr>
<tr>
<td>2000</td>
<td>4460Y²</td>
<td>0.367Y</td>
<td>0.086Y</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table V**

In our section we have a bending moment of 376544 inch-lbs and a thrust of 59705 lbs.

\[ Y = \sqrt{376544 \div 4460} = 9.2 \text{, add } 3'' \text{ and embed beam } = 12'' \]

The area of the section is then 144 square inches.

\[ 59705 \div 144 = 415 \text{ lbs. per square inch due to thrust.} \]

The compressive stress on the extreme fibers due to bending moment must not then exceed 2000 - 415 = 1585 pounds per square inch.

Interpolating in Table V between (2) and (3)

\[ Y = \sqrt{376544 \div 2907} = 11.4 \text{ ins., add } 3'' \text{ to embed beam } = 144'' \]

The area of the section is then 173 square inches.

\[ 59700 \div 173 = 343 \text{ pounds per square inch due to thrust.} \]

The compression on outer fibers must not then exceed 2000 - 343 = 1657 pounds per square inch.
This is a safer section because the actual compression due to bending moment is only 1585 pounds while it will stand 1657 pounds per square inch.

Repeating the process again, we obtain Y = 11" and adding 3" makes the total depth 14 inches.

Interpolating between (2) and (3),
\[ x = \frac{0.303 + \frac{1657-1585}{1657-1500}(0.336-0.303)}{0.324} = 0.324\times11 = 3.56\] " call it 4"

Now substituting in the formula \( I = \frac{Mx}{E} \), we have
\[ I = \frac{376660\times4}{1657} = 909 \]

To get the bending moment due to change of temperature we use formula 31. (Colpitts' page 33)

Moment at crown, \( M_c = 387\frac{3}{6} \)

\( \kappa = \text{rise of arch in feet} \).

Substituting our value of \( I \) and \( \kappa \) in this formula,
\[ M_o = 387\times9.09/65 = 23518 \text{ inch-pounds} \]

To get the value of the temperature moment at other points we multiply this moment at the crown by the factors given on page 84 of Colpitts' pamphlet.

At Panel Point 3

Temperature moment = 23518 \times 0.52 = 12230 \text{ inch-lbs}.

At Abutment

Temperature moment = 23518 \times 2.0 = 47036 \text{ inch-lbs}.

We have now calculated all the bending moments necessary for the design of the arch ring and they are summarized in the following table:
Summary of Moments

<table>
<thead>
<tr>
<th></th>
<th>Vert. Live</th>
<th>Vert. Dead</th>
<th>Hor. Live</th>
<th>Hor. Dead</th>
<th>Temp.</th>
<th>Total in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Crown</td>
<td>-315900</td>
<td>+12700</td>
<td>-53628</td>
<td>-19716</td>
<td>-23518</td>
<td>-400062</td>
</tr>
<tr>
<td></td>
<td>+368550</td>
<td>+12700</td>
<td>-8364</td>
<td>-19716</td>
<td>+23518</td>
<td>+37688</td>
</tr>
<tr>
<td>At Point 3</td>
<td>-549900</td>
<td>+50060</td>
<td>-15744</td>
<td>+7000</td>
<td>-12230</td>
<td>-574986</td>
</tr>
<tr>
<td></td>
<td>+606700</td>
<td>+50060</td>
<td>+64944</td>
<td>+7000</td>
<td>+12230</td>
<td>+740934</td>
</tr>
<tr>
<td>At Abutment</td>
<td>-981875</td>
<td>-29200</td>
<td>-231436</td>
<td>-45100</td>
<td>-47036</td>
<td>-1294672</td>
</tr>
<tr>
<td></td>
<td>+91575</td>
<td>-29200</td>
<td>+117334</td>
<td>+45100</td>
<td>+47036</td>
<td>+1082145</td>
</tr>
</tbody>
</table>

Thrusts

Thrust at Crown

Placing vertical live loads on the points which produce a maximum negative moment at the crown, i.e., 1, 2, 3, 7, 8, and 9 and referring to table I (page 27 Aljetti)

Load on 1 + 9, \( H = 0.607 \times \frac{375}{12} \times 6500 \times 2 \)

\[ \frac{2 + 8, H = 0.192 \times \frac{375}{12} \times x \times 2}{2 + 7, H = 0.3308 \times \frac{375}{12} \times x \times 2} = -18962 \text{ pounds} \]

Load points 4, 5 and 6 to find the thrust to combine with the maximum +M

Load on 4 & 6, \( H = 0.432 \times \frac{375}{12} \times 6500 \times 2 \)

\[ \frac{5 \times H = 0.4637 \times \frac{375}{12} \times 6500}{x} = +21655 \text{ pounds} \]

Placing the vertical dead loads on all points because it acts constantly and again referring to table I we have:
Load on $1+9, H = 0.0607 \times \frac{375}{12} \times 7040 \times 2$

\[ H = 0.1420 \times \frac{375}{12} \times 6380 \times 2 \]

\[ H = 0.3308 \times \frac{375}{12} \times 5860 \times 2 \], \(= 3685.0 \) pounds which is uniform over the whole arch.

Place horizontal line loads on points 1, 2, 3, 7, 8 and 9 to find thrust to combine with maximum $M$ and referring to Table II (Colfitt, page 28) we have:

Load on $1+9, H_2 = 0.166 \times 1640 \times 2$

\[ H_2 = 0.289 \times 1640 \times 2 \], \(= 2699 \) pounds.

Load points 4 and 6 to find the thrust to combine with maximum $+M$

Load on $4+6, H_2 = 0.480 \times 1640 \times 2 \), \(= 1604 \) pounds.

Place horizontal dead loads on all points and again referring to Table II:

Load on $1+9, H_2 = 0.166 \times 800 \times 2$

\[ H_2 = 0.289 \times 560 \times 2 \], \(= 1188 \) pounds.

This thrust is not uniform throughout the arch like that due to vertical dead loads, but since this is the largest value it can have, and since it is small, the horizontal thrust at each point will be taken as 1200 pounds.

Thrust at Point 3.

Place vertical line loads on points 5, 6, 7, 8, and 9 to
find the thrust to combine with maximum -M

\[
\begin{align*}
\text{Load on 5, } H &= 6500 \times \frac{375}{19} \times 0.4687 \\
\text{Load on 6, } H &= 6500 \times \frac{375}{19} \times 0.4320 \\
\text{Load on 7, } H &= 6500 \times \frac{375}{19} \times 0.3308 \\
\text{Load on 8, } H &= 6500 \times \frac{375}{19} \times 0.1920 \\
\text{Load on 9, } H &= 6500 \times \frac{375}{19} \times 0.0602
\end{align*}
\]

Load points 1, 2, 3, and 4 to find the thrust to combine with the maximum +M

\[
\begin{align*}
\text{Load on 1, } H &= 6500 \times \frac{375}{19} \times 0.0602 \\
\text{Load on 2, } H &= 6500 \times \frac{375}{19} \times 0.1920 \\
\text{Load on 3, } H &= 6500 \times \frac{375}{19} \times 0.3308 \\
\text{Load on 4, } H &= 6500 \times \frac{375}{19} \times 0.4320
\end{align*}
\]

Place horizontal line loads on the same points respectively to find the respective thrusts.

For maximum -M load 5, 6, 7, 8, and 9.

Load on 5, H = 1640 x 0.490

\[
\begin{align*}
\text{Load on 6, } H &= 1640 \times 0.428 \\
\text{Load on 7, } H &= 1640 \times 0.289 \\
\text{Load on 8, } H &= 1640 \times 0.106
\end{align*}
\]

For maximum +M load 1, 2, 3, and 4.

Load on 1, H = 1640 x 0.106

\[
\begin{align*}
\text{Load on 2, } H &= 1640 \times 0.289 \\
\text{Load on 3, } H &= 1640 \times 0.428 \\
\text{Load on 4, } H &= 1640 \times 0.490
\end{align*}
\]

Thrust at the Abutment.

Place vertical line loads on points 1, 2, 3, and 4 to find the thrust to combine with maximum -M
Load on \(1, H = 6500 \times \frac{375}{12} \times 0.607\) 
\[\begin{align*}
\text{" "} & 2, H = 6500 \times \frac{375}{12} \times 0.192 \\
\text{" "} & 3, H = 6500 \times \frac{375}{12} \times 0.3308 \\
\text{" "} & 4, H = 6500 \times \frac{375}{12} \times 0.432
\end{align*}\]
\[= -25380 \text{ pounds}\]

Load on \(4, H = 6500 \times \frac{375}{12} \times 0.432\) 
\[\begin{align*}
\text{" "} & 5, H = 6500 \times \frac{375}{12} \times 0.4687 \\
\text{" "} & 6, H = 6500 \times \frac{375}{12} \times 0.432 \\
\text{" "} & 7, H = 6500 \times \frac{375}{12} \times 0.3308 \\
\text{" "} & 8, H = 6500 \times \frac{375}{12} \times 0.1920 \\
\text{" "} & 9, H = 6500 \times \frac{375}{12} \times 0.0607
\end{align*}\]
\[= +40015 \text{ pounds}\]

Place horizontal line loads on points 1, 2, 3, and 4 to find the thrust to combine with maximum \(-M\)

Load on \(1, H = 1640 \times 0.106\) 
\[\begin{align*}
\text{" "} & 2, H = 1640 \times 0.289 \\
\text{" "} & 3, H = 1640 \times 0.428 \\
\text{" "} & 4, H = 1640 \times 0.490
\end{align*}\]
\[= -2153 \text{ pounds}\]

Load \(6, H = 1640 \times 0.490\) 
\[\begin{align*}
\text{" "} & 7, H = 1640 \times 0.490 \\
\text{" "} & 8, H = 1640 \times 0.490 \\
\text{" "} & 9, H = 1640 \times 0.490
\end{align*}\]
\[= +2153 \text{ pounds}\]

We have now determined all the thrusts necessary for the design of the arch ring and they are sum-
The table below summarizes the results:

<table>
<thead>
<tr>
<th></th>
<th>Vert.</th>
<th>Hor.</th>
<th>Temp.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Crown</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. -M, thrust</td>
<td>55812</td>
<td>3887</td>
<td>390</td>
<td>60690</td>
</tr>
<tr>
<td>Max. +M, thrust</td>
<td>58500</td>
<td>2792</td>
<td>390</td>
<td>61690</td>
</tr>
<tr>
<td>At Point 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. -M, thrust</td>
<td>60970</td>
<td>3350</td>
<td>390</td>
<td>64710</td>
</tr>
<tr>
<td>Max. +M, thrust</td>
<td>63350</td>
<td>3350</td>
<td>390</td>
<td>57090</td>
</tr>
<tr>
<td>At Abutment</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Max. -M, thrust</td>
<td>62230</td>
<td>3350</td>
<td>390</td>
<td>65970</td>
</tr>
<tr>
<td>Max. +M, thrust</td>
<td>76865</td>
<td>3350</td>
<td>390</td>
<td>80600</td>
</tr>
</tbody>
</table>

The &thrust; was determined by formula 30 (Collett's page 33), \( H_0 = 96 \frac{F}{L} \), using the I of the crown sections as found above (page 11) with:

\[ H_0 = 96 \times 909 \div 15^2 = 390 \text{ lbs.} \]

which is constant throughout the arch.

**Abutment Reaction**

The resultant which produces the maximum negative and positive bending moments at the abutments must now be determined without the factor of safety applied.

To find the vertical component of the resultant producing maximum -M at the abutment:

For vertical live loads place loads on points 1, 2 and 3, and refering to table I (Collett's page 27) and
using actual loads we have:

\[ P_1 = 0.972 \times 1625 \]
\[ P_2 = 0.896 \times 1625 = 4310 \text{ pounds} \]
\[ P_3 = 0.784 \times 1625 \]

\[ P \text{ for vertical dead load} = \frac{1}{2} \text{ total dead load} = 15300 \text{ pounds} \]

For horizontal line loads use table II (or first page 28) and load points 1, 2 and 3.

\[ P_1 = \frac{15}{375} \times 410 \times 0.049 \]
\[ P_2 = \frac{15}{375} \times 410 \times 0.154 = -80 \text{ pounds} \]
\[ P_3 = \frac{15}{375} \times 410 \times 0.265 \]

The vertical component of the resultant producing maximum \(-M\) is \[ 4310 + 15300 - 80 = 19500 \text{ pounds} \]

To find the vertical component of the resultant producing maximum \(+M\) at the abutments:

For vertical line loads referring to table I and load points 5, 6, 7, 8 and 9.

\[ P_1 = 0.50 \times 1625 \]
\[ P_2 = 0.352 \times 1625 \]
\[ P_3 = 0.216 \times 1625 = 2000 \text{ pounds} \]
\[ P_4 = 0.104 \times 1625 \]
\[ P_5 = 0.028 \times 1625 \]

\[ P \text{ for vertical dead load} = \frac{1}{2} \text{ total load} = 15300 \text{ pounds} \]

For horizontal line loads use table II and load points 5, 6, 7, 8 and 9, we have.
Load on 5. \( P = \frac{15}{37.5} \times 0.375 \times 410 \)

6. \( P = \frac{15}{37.5} \times 0.346 \times 410 \)

7. \( P = \frac{15}{37.5} \times 0.265 \times 410 \) = -200 pounds

8. \( P = \frac{15}{37.5} \times 0.154 \times 410 \)

9. \( P = \frac{15}{37.5} \times 0.049 \times 410 \)

Therefore, the vertical component of the resultant producing maximum +\( M \) at the abutment is equal to 2000 + 15300 - 200 = +17100 pounds

The horizontal component of the abutment reaction is equal to the horizontal thrust at that joint.

This thrust for maximum -\( M \), without the safety factor = \( 36850 \div 2 + 25380 \div 4 + 1200 \div 2 + 2153 \div 4 = 25908 \) called 26000 pounds

The thrust for maximum +\( M \) without a factor of safety = \( 36850 \div 2 + 40015 \div 4 + 1200 \div 2 + 2153 \div 4 = 29560 \) pounds; called 30000 pounds

The resultant abutment reaction which produces maximum -\( M \) = \( \sqrt{19500^2 + 26000^2} = 32500 \) pounds

The resultant reaction which produces maximum +\( M \) = \( \sqrt{7100^2 + 30000^2} = 34000 \) pounds.

We also find the maximum - and +\( M \) at the abutment without a factor of safety applied. Refering to the summary of moments, page 12,

Maximum -\( M \) = \( 961825 \div 4 + 29200 \div 2 + 23846 \div 4 + 45100 \div 47036 \div 2 = -352000 \) inch pounds

Maximum +\( M \) = \( 991575 \div 4 + 29200 \div 2 + 117834 \div 4 + 45100 \div 2 + 47036 \div 2 = +263000 \) inch pounds
Formula 35, page 36 (Colfitt) is \( d = \frac{N}{R} \)

d = distance in inches from the springing normal to R

\( M \) = bending moment in inch lbs. at the section without a factor of safety.

\( R \) = resultant abutment reaction.

For maximum -\( M \); \( d = 352000 \div 32500 = 10.8 \) inches.
For maximum +\( M \); \( d = 263000 \div 34500 = 7.6 \) inches.

Shear

The investigation of shearing stresses in the arching will be similar to that of a beam.

The shear due to the vertical dead load will be found in the ordinary way.

The maximum shear at any point due to the vertical live load will occur when the load covers the greater segment up to the point, and is equal to the abutment reaction at the near abutment. The abutment reactions however are not proportional to the lengths in which the load divides the arch, but are given in Table I (Colfitt, page 27), for loads in any position. It is frequently found that the area necessary to resist the shears due to live and dead loads, and to change of temperature will determine the dimensions of the section.

Dead Load Shears.

At Abutment = Dead Load over all = 30570 pounds
At Panel Point 3.

Shear = 30570 - 7040 - 6350 = 17180 pounds

At Crown

Shear = 30570 - 7040 - 6350 - 5780 - 5720 = 5600 pounds

Live Load Shears

At Abutment (Use Table I)

Shear = (0.972 + 0.896 + 0.784 + 0.648 + 0.5 + 0.322 + 0.216 + 0.104 + 0.028) 6500 = 29250 pounds

At Panel Point 3

Shear = 2.632 x 6500 = 17100 pounds

At Crown

Shear = 1.2 x 6500 = 7800 pounds

Temperature Shears

The vertical shear due to a change in temperature may be considered to be equal to that of a uniform load which produces the horizontal thrust H0, the equilibrium polygon of which is a parabola, and the shear diagram similar to that of a beam uniformly loaded. The shear is therefore zero at the crown, increasing uniformly to a maximum at each abutment.

The dead load W which will produce this value of H for an arch of two spans is:

\[ W = \frac{H_0 k}{2.49972} \] (From Formula 34, page 34, Ceyte's)

The temperature thrust was found to be 390 pounds (page 16), \( k = 15 \) feet, the rise, and the half span \( C = 37.5 \) ft.

\[ W = (390 \times 15) \div 2.49972 \times 37.5 = 62 \frac{1}{8} \text{ pounds} \]
This W is treated as a dead panel load and the values of the shears are shown in the following table:

To find the section required to resist this shear divide it by 300, i.e. 300 pounds per square inch is taken as a safe shearing value for the arching.

<table>
<thead>
<tr>
<th>Point</th>
<th>Live</th>
<th>Dead</th>
<th>Temp.</th>
<th>Total</th>
<th>Sq. ins. Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abut.</td>
<td>29250</td>
<td>30570</td>
<td>310</td>
<td>60130</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>17100</td>
<td>17180</td>
<td>186</td>
<td>34485</td>
<td>115</td>
</tr>
<tr>
<td>Crown</td>
<td>7800</td>
<td>5600</td>
<td>62</td>
<td>13462</td>
<td>45</td>
</tr>
</tbody>
</table>

In this design all the sections as determined with reference to bending moment and thrust had shears greatly in excess of that required to resist the shears so that shears had no influence on the design.

**Design of Sections**

We have now found the maximum moments and the corresponding thrusts at the three critical points on the arch ring and we can now proceed to design the sections after the method shown on page 10.

**Crown Section:**

The maximum moment is 2,400,000 inch pounds to be combined with a thrust of 60100 pounds.
\[ Y = \sqrt{\frac{400000}{4460}} = 9.47, \text{ add } 3'' \text{ to embed hole} = 12'' \]

The area of the section is thus \(150 \text{ sq. ins.} \)

\[ 60100 \div 150 = 400 \text{ pounds per sq. in. due to thrust} \]

The compressive stress on the outer fibers due to bending moment must not then exceed \(2000\) - \(400\) = 1600 pounds per sq. in.

\[ Y = \sqrt{\frac{400000 + 2960}{11.62}} \text{, add } 3'' \text{ to embed hole} = 15'' \]

The area of the section is thus \(180 \text{ sq. in.} \)

\[ 60100 \div 180 = 337 \text{ pounds per sq. in. due to thrust} \]

The compressive stress must not then exceed \(2000 - 337 = 1663 \text{ pounds per sq. in.} \). This value is greater than 1600 pounds for which the section is designed, so it is safe. Make \( Y = 12'' \)

The area of metal required from table \( V \), interpolating between (2) and (3) = \(12 \times 0.050 = 0.642 \text{ sq. inches.} \)

At Panel Point 3.

Proceed in the same manner as above.

The maximum moment is \(+740900\text{ inch lbs.}\) to be combined with a thrust of \(57100\) pounds.

\[ Y = 15\text{ inches}, \text{ area of metal required} = 0.885 \text{ sq. inches.} \]

At Abutment

The maximum moment is \(-1244672\text{ inch lbs.}\) to be combined with a thrust of \(65970\) pounds.

\[ Y = 18\text{ inches}, \text{ area of metal required} = 1.3 \text{ sq. inches.} \]

The \( Y \) is the theoretical depth of the beam or arching and as two sets of bars will be used in the arch it will be the distance...
between the centers of the faces. Six inches must be added to this depth to obtain the total thickness of the arch ring out to cut. This will give a thickness of 12 0" at the crown, 21 inches at panel point 3, and 24 inches at the abutment.

Laying out the Arch Ring.

In laying out the arch ring the linear arch parabola is first drawn to scale and divided into 10 panels. Then on normals to the parabola lay off one half of the total thickness of the rings as determined above, on either side of the parabola at the respective points for which the thicknesses were calculated. Two circular curves compounded at panel point 3 were then calculated which would pass through the points laid off.

For the intrados the circle from the crown to point 3 had a radius of 46 1/3" while that from point 3 to the abutment had a radius of 65 1/2".

For the extrados the circles had radii respectively of 51 1/2" and 66 1/2"; the latter circle however only runs to a point 30 ft. from the center where the extrados becomes tangent to it. This was done in order to distribute the load at the abutment over a wider area.

To get the number and position of the
Longitudinal bars in the arch ring use the following table of sizes and weights of the corrugated bars:

<table>
<thead>
<tr>
<th>Size</th>
<th>Section</th>
<th>Wt. per ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2&quot;</td>
<td>0.18</td>
<td>0.64</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>0.37</td>
<td>1.45</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>0.55</td>
<td>2.05</td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.70</td>
<td>2.70</td>
</tr>
<tr>
<td>1 1/4&quot;</td>
<td>1.07</td>
<td>4.20</td>
</tr>
</tbody>
</table>

The total width of the arch ring will be at least 32 ft. :: total area of steel required at the abutment = 32 x 1.3 = 41.6 sq. ins.

63; 1" bars spaced 6" c.c. gives an area = 63 x 0.7 = 44.1 sq. in.

At panel point 3 the area of steel required

= 32 x 0.885 = 28.32 sq. in.

63; 3/8" bars spaced 6" c.c. gives an area = 63 x 0.55 = 34.65 sq. in.

At the crown the area of steel required

= 32 x 0.442 = 14.144 sq. in.

63; 3/8" bars spaced 6" c.c. gives an area = 63 x 0.37 = 23.31 sq. in.

The horizontal bars and shear bars were put in by inspecting drawings as no literature on this subject could be found; the only case taken was to have at least 0.6 square inch of steel for square foot of section in order to prevent temporary cracks.
Parapet Walls

These walls will have to resist an earth load about 15 feet in depth at the abutment, and this will be the greatest depth.

Vertical live load pressure = $4 \times 150 + 2 \times 150 = 900$ pounds per square foot (using a factor of safety of 4 and impact = 50%)

Vertical dead load pressure = $2 \times 150 \times 15 = 3000$ pounds (using a safety factor of 2)

Total horizontal pressure = $(900 + 3000) \div 3 = 1300$ lbs. per ft.

To find the thickness of wall necessary to resist this pressure, take Fig. 5 (page 12 of paper), and using curves F and f, we get $y = 14"$ and adding 3" to embed bars gives a total thickness of 17". This was changed to 18" in the design.

The area of metal required per linear foot from the same Fig. 5, $t = 1.2$ sq. inches.

1" bars spaced 6" apart give 1.33 sq. inches per linear foot and will be used. These are vertical bars. Bars $\frac{1}{2}"$ o and 18" centers were introduced longitudinally into the wall to resist the effect of temperature change.

The Abutments

The abutment will consist of a base, face wall, four buttresses, and a curb resting on the face wall and buttresses and which supports the arch ring. The abutment will extend 12 feet below the springing line of the arch. This places the
bottom of the abutment nearly 30 feet below the level of the roadway and the depth will be used in the following calculations.

Base of Abutment.
The foundation is assumed to be clay which can support about 4000 pounds per square foot.
Assume base to be 24 feet long and 32 feet wide.
The supporting power of the base will thus be
\[ 24 \times 34 \times 4000 = 32,640,000 \text{ pounds} \]
The load on the abutment per foot width is
\[ \text{Wt. of arch and loads} = 35,000 \text{ pounds} \]
\[ \text{Wt. of earth above abutment} = 22 \times 100 \times 30 = 66,000 \]
\[ \text{Total load on base} = 150 \times 22 = 3,300 \]
\[ \text{Total load per foot width} = 102,300 \]
The total load on the abutment base assuming the arch ring 32 feet wide is 32 \times 102,300 = 3,273,600 pounds. This gives a pressure of 4010 pounds per square foot, which is allowable.

To obtain the thickness of the base figure it as a beam continuous over four supports and loaded with 4010 pounds per square foot. Consider a section one foot in width. The supports will be assumed 10 feet apart since the bridge is 30 feet wide in the clear.

Maximum moment = \( \frac{4010 \times 1}{12} \) (Muirhead's Mechanics)
\[ W_{max} = \frac{4010 \times 10 \times 40 \times 10 \times 10 \times 12}{12} = 192,480,000 \text{ inch lbs} \]
The 4 is the safety factor and is made so
large because the load will probably not be uniformly distributed.

To get the section, \( Y = \sqrt{\frac{M}{4460}} \)

\( Y = \sqrt{\frac{924800}{4460}} = 21'' \)

1.3 in. of steel required = 0.086 \times 21 = 1.8 sq. inches per lin. ft.

Total square inches of steel required = 24 \times 1.8 = 43.2

47-1/4" long spaced 6" cc lines 47 \times 1.07 = 50.24 sq. ins.

These bars will be placed in the upper part of the base and will run perpendicular to the longitudinal axis of the bridge. Another set of bars will be placed in the lower part of the base running at right angles to those in the upper part.

Cover for Abutment

This is also to be designed as a beam continuous over four supports, the abutments spaced 10 feet apart.

The load per foot is equal to one-half the total weight of the arch and its loads and is equal to 34500 pounds.

The top of the cover is to be 6 feet wide, but we will consider the load as uniformly distributed over only 5-1/2 feet.

\[ \text{Load per linear foot of cover in foot width} = \frac{34500}{5.5} = 6273 \text{ pounds} \]

Maximum moment = \( \frac{1}{16}WL \)

= \( \frac{1}{16} (6273 \times 10 \times 10 \times 12) = 752760 \text{ inch pounds} \)

To get the section necessary to resist this:

\( Y = \sqrt{\frac{752760}{4460}} = 13 \text{ inches} \)
Area of steel required = 13 × 0.086 = 1.12 sq. ins. per foot width.

The cover is about 6 1/2 ft. wide so:

112 × 6.5 = 728 sq. ins. = total area required
14 - 7/8" bars spaced 5" centers gives an area of
14 × 0.55 = 7.7 sq. ins.

Face Wall of Abutment.

This is also to be designed as a beam on four supports 10 feet apart.

The live load pressure will be 150 ÷ 3 = 50 pounds per square foot.

The dead load pressure will be 100 × 30 ÷ 3 = 1000 pounds per square foot.

Total load = 1050 pounds per square foot.

Consider a section one foot wide

\[ M = \frac{1}{2} W L = \frac{1}{2} (105 \times 2 \times 10 \times 10 \times 12) = 252000 \text{ in-lb} \]

Thus

\[ Y = \sqrt{252000 \div 4460} = 7\frac{1}{2} \text{ inches} \]

Area of steel required = \(0.086 \times 7\frac{1}{2} = 0.645\) sq. ins. per foot.

The width or height of wall as designed is 8'10" X 10.

Total area of steel required = \(8.83 \times 0.645 = 5.69\) sq. ins.

14 bars 7\% spaced 9" centers gives 14 \times 0.37 = 5.18 sq. ins.

This size of bar will be used because the wall will be made thicker in order to properly support the cover.

The maximum shear is at the two center supports and is equal to

\[ \frac{1}{2} W L \text{ (Kuiman's Mechanics)} = \frac{1}{2} (10 \times 2 \times 1050) = 23100 \text{ pounds} \]

To resist this stress, horizontal bars must be placed in the buttresses and hooked over the bars.
in the face wall.

To determine the size of these bars:
The whole shear on the 18'10" = 23100 \times 8.83 = 203973 lbs.
This is distributed among 14 bars.
The stress per bar = \frac{203973}{14} = 14600 pounds.
If we assume 15000 pounds per square inch as the allowable tensile strength for steel, one 14" bar having an area of 7.07 square inches will be used for the buttress for every bar in the face wall.

Buttresses.
The two center buttresses will be made 2 ft.
wide in order to form a good support for the core and near the rear edge will be embedded eight 3/4" bars as shown on the drawing in order to resist any overturning moment. These bars were not figured but were put in after inspecting other designs. The bars will extend through the base and be hooked over the longitudinal bars in the bottom.

The parapet wall extended and carried down will form the two outer supports and will be in the same manner as the buttresses.