UNCOVERING BASIC WANTS USING THE ROTTERDAM AND AIDS MODELS: THE US HOUSEHOLD ENERGY CONSUMPTION CASE

By

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Abstract

Economists inflate the explanatory power of measurable variables such as price and income to explain demand. Using only quantifiable variables is very attractive since it makes it easy to construct mathematically consistent and well expressed models. However, since Lancaster (1971), economic awareness has increased to such a degree that latent, hardly observable and/or measurable variables may bring more insight to the demand analysis. Two approaches compete to introduce such variables: an economic approach originally developed by Lancaster and Becker (1965), and a statistical approach. For Lancaster, beyond quantities of goods demanded, the characteristics of goods are what shape consumer utility and consequently determine its choice. This approach is theoretical and largely based on economic intuition. Few empirical studies using Lancaster have been successful so far. The second, purely statistical approach, considers the possibility of transforming observed data to obtain the "basic wants" that truly affect consumer choice. This approach, known as the Preference Independence Transformation (PIT), has so far been applied only in a few studies using the Rotterdam model frame. The PIT was certainly deduced through mathematically thorough and consistent analysis to uncover the basic want, denoted as T-goods. We intend to revisit the PIT under the Rotterdam framework to uncover the basic goods. Alongside, we implement —for the first time—an independent transformation that eliminates the Slusky interdependencies from the Almost Ideal Demand System (AIDS) setting. We will refer to it as the Slutsky Matrix Independent Transformation (SMIT). Regarding our purpose to check if the two techniques identically define the basic goods, the findings were not conclusive. As a result, we further the analysis by introducing a possibility to unveil the basic wants using US household data.

To my beloved parents: My prayers will be always with you.

To my wife, Marlyatou Sow.

To my daughters, Khadidiatou R. and Aissatou.

To the youngest of the family, Aliou Soufi.

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1 Introduction

For a long time, it has been considered convenient and realistic to explain consumer demand by considering economic variables such as prices and income, whether the consumer is an individual or a set of individuals. This has been the case because such variables are amenable to measurement and quantification. There exists, however, strong research that considers the characteristics of goods to be strong candidates for explaining consumption behavior(Lancaster, K. 1971) and (Becker, G. S. 1965) are two of them. (Brooks, R. B. 1970) and (Theil, H. 1975-76) initiated an original technique to move from an existing set of commodities – that are complementary or substitutes—into a new set of goods of equal numbers and intended to meet the basic wants of the consumer. The underlying assumption is that the latent goods, that is, the goods that correspond to basic wants, are unrelated. The literature rather, cautiously refers to these latent goods as *transformed goods* or T-goods. Leading researchers have explored this technique of incorporating characteristics.

In this study, we revisit this technique by trying to uncover the basic wants behind the demand for gas, distillate fuel oil, and the liquefied petroleum gases (LPG) by US households. To give some examples, electricity may be used for many basic wants such as lighting, cooking, and cooling. Similarly, without being exhaustive, gas may be used for heating, and cleaning. We will first explore the technique under the Rotterdam model framework and then undertake its extension to the Almost Ideal Demand System (AIDS).

The Rotterdam model is a model sufficiently studied to allow the Preference Independence Technique to use it as a framework. In a recent paper, (Barnett, W. A. and A.

Serletis 2008) address the issues of how to estimate the Rotterdam model for two versions: the absolute and the relative price versions.

On the other hand, economists interested in the topic have not, to the best of our knowledge, so far applied any independent transformation technique using a theoretical framework other than the one defined by the Rotterdam model. In this work, we implement an independent transformation on the Slutsky matrix to one of the most, with the Rotterdam model, popular demand systems: The Almost Ideal Demand System (AIDS). We will refer to it as the Slutsky Matrix Independence Transformation (SMIT). The introduction of the SMIT enables us to first check its feasibility and then the sensitivity of the transformed goods to the technique and the demand system chosen.

In the next chapter, we will address the literature review and analyze the two models. More importantly, we will dissect the PIT and see how close technique could be associated with the AIDS. Chapter 3 covers the estimations with the two models. The last chapter will examine a realistic and meaningful way to establish the thread between commodities and basic wants.

2 The theoretical toolbox

In this chapter we will first conduct an in-depth presentation of the two systems of demand—Rotterdam and AIDS—that are going to serve as platforms for the Preference Independence Technique (the Rotterdam case) and the Slutsky Matrix Independence Transformation (the AIDS case). We will highlight all theoretical tools that will be needed to unfold the transformation techniques. In a second stage we will, after introducing the technique, provide the understanding on how to apply it in the two systems of demand.

2.1. The theoretical framework

2.1.1. The Rotterdam model

The Rotterdam model is one of the most popular models for the empirical estimation of the consumer demand for goods. It originated with (Theil, H. 1965) and (Barten, A. P. 1966) and, because they were both based in Rotterdam during that period, the model is referred to as the Rotterdam model. The model is based on a log linear specification. Economic literature provides substantial applications of this model to various markets. Using the Rotterdam framework, (Barnett, W. A. 1979), studied the issue of aggregation over consumers. Introducing the possibility for consumers to have different tastes, while using a convergence technique, he presents a Rotterdam model with constant coefficients.

We present in the next sections some important theoretical tools necessary to have a sufficient knowledge of the derivation as well as the formulation of the Rotterdam model. Next, we set the problem of the Preference Independence Technique and the way to apply it under the Rotterdam model framework. To be specific, we provide an overview of the Stone model, the differential system formulation through the Barten's equation, formulate the Rotterdam model and proceed to the transformation technique.

In demand analysis, two categories of models emerge: those constructed outside the utility theory—the earliest ones—and those using microeconomic theory. (Stone, R. 1954) is a pioneer in utilizing the first type.

2.1.1.1. Stone's demand function

Because of its double logarithmic expression, Stone's demand function directly captures the commodity economic interdependences, without referring to the consumer theory formalism. It can be expressed as follows

$$\ln q_i = \alpha_i + \eta_i \ln M + \sum_{j=1}^{n} \xi_{ij} \ln p_{j,} \quad i = 1,\dots,n \qquad (2.1.1)$$

Where q_i is the quantity demanded of good i, α_i is the constant for equation i, M is the income, p_j is the price of good j, η_i is the income elasticity of demand for good i, and ξ_{ij} is the Cournot elasticity of demand for good i with respect to price j. Differentiating this equation—denoted as the double-log system—allows a straight computation of price and income elasticities.

We can draw some important properties from this model when we multiply the income elasticity of good i by its corresponding budget share, $w_i = \frac{p_i q_i}{M}$

- The relation between the marginal share, the budget share, and the income elasticity

$$\eta_i w_i = \frac{p_i q_i}{M}\frac{\partial \ln q_i}{\partial \ln M} = \frac{p_i q_i}{M}\frac{\partial q_i}{\partial M}\frac{M}{q_i} = \frac{\partial (p_i q_i)}{\partial M} \qquad (2.1.2)$$

For each good, the product of the income elasticity and the budget share equals the marginal share. Let us denote the latter as μ_i :

$$\mu_i = \frac{\partial (p_i q_i)}{\partial M}.$$

Equivalently, the income elasticity is the ratio of the marginal share to the budget share

$$\frac{\mu_i}{w_i} = \eta_i$$

- The Engel aggregation or adding-up property

$$\sum_{i=1}^{n} \mu_i = \sum_{i=1}^{n} \frac{\partial (p_i q_i)}{\partial M} = \sum_{i=1}^{n} w_i \eta_i = 1 \qquad (2.1.3)$$

This relation states that the total expenditure equals income. That is,

$$\sum_{i=1}^{M} p_i q_i (M, p_1, ..., p_i, ..., p_n) = M \qquad (2.1.4)$$

The Engel aggregation is obtained by differentiating both sides of (2.1.4) with respect to M . We still have the Engel aggregation by differentiating (2.1.4) with respect to P_i on both sides.

$$\sum_j p_j \frac{\partial q_j}{\partial p_j} + q_i = 0$$

In terms of budget shares this can be translated as

$$\sum_j w_j \varepsilon_{ij} = -w_i$$

A last way of apprehending the adding-up condition is to divide both sides of (2.1.4) by by M.

$$\sum_{i=1}^{n} w_i = 1$$

- The symmetry condition

$$\omega_i \xi_{ij} = \omega_j \xi_{ji}, \text{ i,j=1,...,n} \qquad (2.1.5)$$

In some ways, this model is the predecessor of the Rotterdam model. According to (Deaton, A. and J. Muellbauer 1980), it suffices to envisage variable income and price elasticities, differentiate Stone's equation, and incorporate the Slutsky decomposition, to get the basic equation for the Rotterdam model. From this perspective, the Rotterdam model is just a differential of Stone's equation. However, a huge difference exists between the Rotterdam model and the Stone model since the latter is not supported by any utility theory. In addition, doing this way neglects many important concepts crucial for a good understanding of the Rotterdam model.

One advantage of the Rotterdam model over the Stone demand function is, on the one hand, its generation from the utility theory and, on the other hand, its thorough logical consistency. Before giving the definitive version of the Rotterdam model, it is worth reviewing all the steps leading to its definition. Knowledge of these steps is vital for its correct use in empirical studies. Barten's Fundamental Equation is an important system of equations leading to a good understanding of the Rotterdam model.

2.1.1.2. Barten's fundamental equation

The Rotterdam model is understood as a first order approximation of a demand system. Its determination begins with the differentiation of the Marshallian demand and budget constraint. For this, the Rotterdam model is a further specification of the differential approach. This requires having a "well-behaving" utility function, that is, twice differentiable, strictly quasi-concave with positive marginal utility for each good, and a second derivative matrix that is a negative definite. In addition, no specific form of the utility function is needed.

Suppose a utility function $U = U(\mathbf{q})$ where $\mathbf{q} = (q_1, q_2, \ldots, q_{n-1}, q_n) \in \mathbb{R}_+^n$ and an economic environment defined by the income M , and the price vector $\mathbf{p} = (p_1, p_2, p_3, \ldots, p_n) \in \mathbb{R}_+^n$.

Subject to the budget constraint, the first order conditions of the utility function maximization gives

$$\frac{\partial U(q_1 \ldots, q_n)}{\partial q_i} = \lambda p_i, \qquad i = 1, \ldots, n \tag{2.1.6}$$

and

$$\sum_{i=1}^{n} p_i q_i = M \tag{2.1.7}$$

The first step towards the Rotterdam model is to differentiate each of these two equations, (2.1.6) and (2.1.7), with respect to the price of each good and the income M . By doing so, we seek to assess the impact on the endogenous variables $(\mathbf{q}, \lambda)$ of changes in the explanatory variables $(\mathbf{p}, M)$.

Differentiating them with respect to M yields:

$$\begin{cases} \sum_{k=1}^{n} \frac{\partial^2 U}{\partial q_i \partial q_k} \frac{\partial q_k}{\partial M} = p_i \frac{\partial \lambda}{\partial M} \\ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial M} = 1 \end{cases} \qquad i = 1, \ldots, n$$

Next, we differentiate them with respect to p_j.

$$\begin{cases} \sum_{k=1}^{n} \frac{\partial^2 U}{\partial q_i \partial q_k} \frac{\partial q_k}{\partial p_j} = \lambda \delta_{ij} + p_i \frac{\partial \lambda}{\partial p_j} \\ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial p_j} = -q_j \end{cases} \qquad i, j = 1, \ldots, n$$

We have previously seen that the second element of each of these two systems represents two ways of expressing the Engel aggregation. Because of this adding-up property, we will always assimilate the total expenditure to the income.

Note that δ_{ij} is the Kronocker product with value 1 when $i = j$, and 0 otherwise. These four equations can be concisely written in matrix terms as follow

$$\begin{cases} \mathbf{U}_{n,n} \frac{\partial \mathbf{q}}{\partial M} = \frac{\partial \lambda}{\partial M} \mathbf{p} \\ \mathbf{p}' \frac{\partial \mathbf{q}}{\partial M} = 1 \\ \mathbf{U}_{n,n} \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{I} + \mathbf{p}' \frac{\partial \lambda}{\partial \mathbf{p}'} \\ \mathbf{p}' \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = -\mathbf{q}' \end{cases}$$

Where $\mathbf{U}$ is the matrix of the second derivatives of the utility function $\mathbf{U}_{n,n} = \left[\frac{\partial^2 U}{\partial q_i q_k} \right]$.

All these equations can be summarized in one matrix relation known as the Barten's fundamental matrix equation that can be expressed as follows

$$\begin{pmatrix} \mathbf{U}_{n,n} & \mathbf{p}' \\ \mathbf{p}' & \mathbf{0} \end{pmatrix} \begin{pmatrix} \frac{\partial \mathbf{q}}{\partial M} & \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} \\ -\frac{\partial \lambda}{\partial M} & -\frac{\partial \lambda}{\partial \mathbf{p}'} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \lambda \mathbf{I} \\ 1 & -\mathbf{q}' \end{pmatrix} \tag{2.1.8}$$

$\mathbf{U}_{n,n} = \left[\frac{\partial^2 U}{\partial q_i \partial q_j} \right]$, the matrix of second derivatives, is symmetric and negative definite as the utility function is quasi-concave.

We solve this matrix relation by taking the first element of the left-hand side—provided it is invertible— to the right.

$$\begin{pmatrix} \frac{\partial \mathbf{q}}{\partial M} & \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} \\ -\frac{\partial \lambda}{\partial M} & -\frac{\partial \lambda}{\partial \mathbf{p}'} \end{pmatrix} = \begin{pmatrix} \mathbf{U}_{n,n} & \mathbf{p}' \\ \mathbf{p}' & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0} & \lambda \mathbf{I} \\ \mathbf{1} & -\mathbf{q}' \end{pmatrix} \tag{2.1.9}$$

It can be shown that

$$\begin{pmatrix} \mathbf{U}_{n,n} & \mathbf{p} \\ \mathbf{p}' & \mathbf{0} \end{pmatrix}^{-1} = \frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \begin{bmatrix} \left(\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}\right)\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}\left(\mathbf{U}^{-1}\mathbf{p}\right)' & \mathbf{U}^{-1}\mathbf{p} \\ \left(\mathbf{U}^{-1}\mathbf{p}\right)' & -1 \end{bmatrix} \tag{2.1.10}$$

Plugging (2.1.10) into the right hand-side of (2.1.9) yields

$$\begin{pmatrix} \frac{\partial \mathbf{q}}{\partial M} & \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} \\ -\frac{\partial \lambda}{\partial M} & -\frac{\partial \lambda}{\partial \mathbf{p}'} \end{pmatrix} = \frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \begin{bmatrix} \left(\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}\right)\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}\left(\mathbf{U}^{-1}\mathbf{p}\right)' & \mathbf{U}^{-1}\mathbf{p} \\ \left(\mathbf{U}^{-1}\mathbf{p}\right)' & -1 \end{bmatrix} \begin{pmatrix} \mathbf{0} & \lambda \mathbf{I} \\ \mathbf{1} & -\mathbf{q}' \end{pmatrix} \tag{2.1.11}$$

This new relation yields the following relations:

$$\frac{\partial \mathbf{q}}{\partial M} = \frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \mathbf{U}^{-1}\mathbf{p} \tag{2.1.12}$$

$$\frac{\partial \lambda}{\partial M} = \frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \tag{2.1.13}$$

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{U}^{-1} - \frac{\lambda}{\mathbf{p}'\mathbf{U}\mathbf{p}} \mathbf{U}^{-1}\mathbf{p}\left(\mathbf{U}^{-1}\mathbf{p}\right)' - (\mathbf{U}^{-1}\mathbf{p})\mathbf{q}' \tag{2.1.14}$$

$$\frac{\partial \lambda}{\partial \mathbf{p}} = -\frac{\lambda}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \mathbf{U}^{-1}\mathbf{p} - \frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \mathbf{q} \tag{2.1.15}$$

We can plug (2.1.13) into (2.1.12) to get

$$\frac{\partial \mathbf{q}}{\partial M} = \frac{\partial \lambda}{\partial M} \mathbf{U}^{-1}\mathbf{p} \tag{2.1.16}$$

On the other hand, we also use (2.1.12) and (2.1.13) to reformulate (2.1.15) as

$$\frac{\partial \lambda}{\partial \mathbf{p}} = -\lambda \frac{\partial \mathbf{q}}{\partial M} - \frac{\partial \lambda}{\partial M} \mathbf{q} \tag{2.1.17}$$

Finally, we use (2.1.12) and (2.1.13) to rewrite (2.1.14) as

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{U}^{-1} - \frac{\lambda}{\frac{\partial \lambda}{\partial M}} \frac{\partial \mathbf{q}}{\partial M} \frac{\partial \mathbf{q}'}{\partial M} - \mathbf{q}' \frac{\partial \mathbf{q}}{\partial M} \tag{2.1.18}$$

Let u^{ij} be the i^{th} row and the j^{th} column of the U^{-1} matrix, we can rewrite this equation as:

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda u^{ij} - \frac{\lambda}{\frac{\partial \lambda}{\partial M}} \frac{\partial \mathbf{q}}{\partial M} \frac{\partial \mathbf{q}'}{\partial M} - \mathbf{q}' \frac{\partial \mathbf{q}}{\partial M} \qquad (2.1.19)$$

Equation (2.1.18) or, equivalently, (2.1.19) decomposes the *total effect* in *total substitution effect* and *income effect*. The first two terms, together, measure the total substitution effect that can be decomposed, according to Houtthaker, into a specific effect, the first term, and a general effect, the second term. The first term is called the specific effect because it is specific to each (i, j). It shows the specific interdependences between the goods i and the goods j. The second term, the general effect, indicates how all the commodities compete for the last unit of dollar. A thorough analysis of the distinction between specific effect and general effect can be found in pages 191-193 of (Theil, H. 1967).

An important observation for the subsequent developments is that when the marginal utility of each good is independent of the quantity demanded of any other good, the second derivative of the marginal utility is 0 whenever i is not equal to j. The matrix of the second derivatives becomes diagonal and so does its inverse. In both cases, the Hessian matrix of the utility function, by assumption, is a negative definite.

It is worth keeping in mind that the Rotterdam model is a specific extension of the differential approach.

In the second stage, we will connect this remark to the results found above. Let us consider a differential of the Marshalian demand function:

$$dq_i = \frac{\partial q_i}{\partial M} dM + \sum_{j=1}^{n} \frac{\partial q_i}{\partial p_j} dp_j \qquad (2.1.20)$$

Where $i = 1,..,n$.

This is equivalent to $q_i \frac{dq_i}{q_i} = \frac{\partial q_i}{\partial M} M \frac{dM}{M} + \sum_{j=1}^{n} \frac{\partial q_i}{\partial p_j} p_j \frac{dp_j}{p_j}$.

Multiplying this equation by $\frac{p_i}{M}$ yields:

$$w_i d \ln q_i = \mu_i d \ln M + \sum_{j=1} \frac{p_i p_j}{M} \frac{\partial q_i}{\partial p_j} d \ln p_j \qquad (2.1.21)$$

Incorporating (2.1.19) into (2.1.21) yields:

$$w_i d \ln q_i = (\frac{\partial p_i q_i}{\partial M}) d \ln M + \sum_{j=1}^{n} \frac{p_i p_j}{M} \left[\lambda u^{ij} - \frac{\lambda}{\frac{\partial \lambda}{\partial M}} \frac{\partial \mathbf{q}}{\partial M} \frac{\partial \mathbf{q}^{'}}{\partial M} - \mathbf{q}' \frac{\partial \mathbf{q}}{\partial M} \right] dlnp_j \qquad (2.1.22)$$

As denoted above, $\frac{\partial (p_i q_i)}{\partial M} = \mu_i$ is the marginal share.

By rewriting (2.1.22), we obtain:

$$w_i d \ln q_i = \mu_i d \ln M - \sum_{j=1}^{n} \left[\frac{\lambda u^{ij} p_i p_j}{M} - \frac{\lambda / M}{\frac{\partial \lambda}{\partial M}} \frac{\partial (p_i q_i)}{\partial M} \frac{\partial (p_j q_j)}{\partial M} \right] dlnp_j - \mu_i \sum_{j=1}^{n} w_j dlnp_j \qquad (2.1.23)$$

Next, we define the Divisia volume index, $d \ln Q$:

$$d\ln Q = \sum_{i=1}^{n} w_i d\ln q_i = d\ln M - \sum_{i=1}^{n} w_i dlnp_j \qquad (2.1.24)$$

If we define:

the income elasticity of the marginal utility of income as

$$\phi^{-1} = \frac{\partial \lambda}{\partial M}\frac{M}{\lambda} \qquad (2.1.25)$$

the normalized price coefficients as

$$\mu_{ij} = \frac{\lambda u^{ij} p_i p_j}{\phi \mathrm{M}} \quad i, j = 1, \ldots, n, \qquad (2.1.26)$$

and the Frisch price index can be expressed as

$$P' = \sum_{i=1}^{n} \mu_i d\ln p_i \qquad (2.1.27)$$

Then the equation can be written as:

$$w_i d\ln q_i = \mu_i d\ln Q + \phi \sum_{j=1}^{n} \mu_{ij} (dln \frac{p_j}{P'}) \qquad (2.1.28)$$

The left-hand side of (2.1.28) indicates the contribution of the i^{th} good to the Divisia volume index. It can be viewed as the "quantity component" of the change of the j^{th} budget share. The first term on the right-hand side is a fraction of the Divisia volume index. Two observations need to be made for μ_i and μ_{ij}: All μ_i sum to one and, given a row i, the row sum of all μ_{ij} equals μ_i. For this reason, the μ_{ij}'s are denoted as the normalized price coefficients.

From now on, we denote $[\mu_{ij}]_{n,n}$ as $\mathbf{M}_{n,n}$. Make distinction between M, the total expenditure and $\mathbf{M}_{n,n}$, the matrix of normalized price coefficients. In matrix term, $\mathbf{M}_{n,n}\boldsymbol{\iota} = [\mu_{ij}]_{n,n}\boldsymbol{\iota} = [\mu_i]_{n,1}$ and $\boldsymbol{\iota}_{1,n}'\mathbf{M}_{n,n}\boldsymbol{\iota}_{n,1} = 1$, where $\boldsymbol{\iota}$ is the n by 1 matrix of ones. It is also important to highlight the difference between the matrix of the normalized price coefficient $\mathbf{M}_{n,n}$ with the matrix of price coefficients that we will denote as $\mathbf{V}_{n,n}$

$$\mathbf{V}_{n,n} = [\phi\mu_{ij}]_{n,n} = [v_{ij}]_{n,n}$$

We can easily see from what is said above that $\sum_i^n \sum_j^n v_{ij} = \phi$. This parameter is supposed to be negative in the theory. Indeed, $\frac{\partial\lambda}{\partial M} = \frac{\partial(\frac{\partial U}{\partial M})}{\partial M} = \frac{\partial^2 U}{\partial M^2} < 0$. The sign is explained by the negative definiteness of the Hessian matrix $\mathbf{U}$ (not to be confused with the utility U which is not in bold). (Theil, H. 1975, vol. 1, p.29) reports (Frisch, R. 1959) conjectures on the income flexibility the higher $|\phi|$, the richer the consumer. Note that high *income flexibility* in absolute value is identical to *low income elasticity of the marginal utility of income* since one is the reciprocal of the other.

The fact that the sum of all price coefficients equals one allows us to express the normalized price coefficient matrix in a more explicit way

$$\mathbf{M}_{n,n} = \left[\frac{v_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij}} \right] \tag{2.1.29}$$

From (2.1.26), we see that μ_{ij} is positive since the matrix $[u^{ij}]_{n,n}$ is negative definite and ϕ is negative. In matrix terms, (2.1.26) should be written as

$$\mathbf{M}_{n,n} = \frac{\lambda \mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}{\phi M}, \tag{2.1.30}$$

where $\mathbf{P}$ is a diagonal matrix that has the prices along the diagonal. The marginal utlity of income, λ, and M are positive.

This means that, the matrix of the normalized price coefficients $\mathbf{M}_{n,n} = [\mu_{ij}]_{n,n}$, is definite positive. The estimation of the (2.1.28) system would not directly gives $\mathbf{M}_{n,n}$ but the matrix $[\phi\mu_{ij}]_{n,n}$, where $\phi\mu_{ij}$ is the coefficient of the j^{th} relative price.

$$\mathbf{V}_{n,n} = [v_{ij}]_{n,n} = [\phi\mu_{ij}]_{n,n} = \frac{\lambda \mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}{M}. \tag{2.1.31}$$

Observe that if we fix i and sum $\phi\mu_{ij}$ over the columns, we find $\phi\mu_i$. The matrix $\mathbf{V}_{n,n}$ is a symmetric and negative definite matrix.

In the next developments, we should note that if the marginal utility of each good is independent from the change in any quantity consumed of another good ($u^{ij} = 0$ whenever i is different from j), then from (2.1.26)

$$\mu_{ij} = \begin{cases} 0 \text{ if i} \neq \text{j} \\ \mu_{ii} \text{ if i=j} \end{cases} \tag{2.1.32}$$

The differential equation simply becomes:

$$w_i d \ln q_i = \mu_i d \ln Q + \phi\theta_i (dln \frac{p_j}{P^{'}}) \tag{2.1.33}$$

In that case, (2.1.31) can be explicitly written as

$$\mathbf{V}_{n,n} = \begin{bmatrix} \phi\mu_1 & 0 & . & . & . & 0 \\ 0 & \phi\mu_2 . & . & . & . & 0 \\ . & & . & & & \\ . & & & . & & \\ . & & & & . & \\ 0 & 0 & . & . & . & \phi\mu_n \end{bmatrix}$$

On the other hand, the equation (2.1.28) can be re-expressed to show the decomposition of the total effect into *general effect* and *specific effect*:

$$w_i d \ln q_i = \mu_i d \ln Q + \sum_{j=1}^{n} [\phi\mu_{ij} dlnp_j - \sum_{j=1}^{n} \phi \sum_{i=1}^{n} \mu_i dlnp_i] \tag{2.1.34}$$

While the first term inside the brackets is the specific effect, the second stands for the general effect. Moreover, as seen in(2.1.27), $\sum_{i=1}^{n} \mu_i dlnp_i$ measure corresponds to the Frisch index (1932).

At this point, it is appropriate to introduce the Slutsky equation in a more convenient way. From the expression (2.1.19), the direct substitution effect can be written as

$$\left.\frac{\partial \mathbf{q}}{\partial \mathbf{p}'}\right|_{u=\bar{u}} = \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} + \mathbf{q}\frac{\partial \mathbf{q}}{\partial M} = \lambda[u^{ij}]_{n,n} - \frac{\lambda}{\frac{\partial \lambda}{\partial M}}\frac{\partial \mathbf{q}}{\partial M}\frac{\partial \mathbf{q}'}{\partial M} \qquad (2.1.35)$$

Note also the direct substitution effect can be written as

$$\left.\frac{\partial \mathbf{q}}{\partial \mathbf{p}'}\right|_{u=\bar{u}} = \frac{w_i d\ln q_i}{d\log p_j}\frac{M}{p_i p_j} \qquad (2.1.36)$$

If we denote π_{ij} as the $(i,j)^{th}$ fraction of the direct substitution effect

$$\pi_{ij} = \frac{w_i \partial \ln q_i}{\partial \ln p_j}$$

Then equalizing (2.1.35) and (2.1.36) enables to write

$$\pi_{ij} = \lambda\frac{p_i u^{ij} p_j}{M} - \frac{\lambda/M}{\partial\lambda/\partial M}\mu_i \mu_j \qquad (2.1.37)$$

It suffices to consider (2.1.25) and (2.1.26) to express our previous expression as

$$\pi_{ij} = v_{ij} - \phi\mu_i \mu_j \qquad (2.1.38)$$

$\pi_{ij} = v_{ij} - \phi\mu_i \mu_j$. Observe that $v_{ij} = \phi\mu_{ij}$. In other words, $\pi_{ij} = \phi\mu_{ij} - \phi\mu_i \mu_j$

This allows us to re-express (2.1.34) as:

$$w_i d\ln q_i = \mu_i d\ln Q + \sum_{j=1}^{n}\pi_{ij}\, dlnp_j,\ i,j = 1,\dots,n \qquad (2.1.39)$$

$[\pi_{ij}]_{n,n}$ is a symmetric negative semi-definite with rank n-1. We can verify that $\pi_{ij} = \frac{w_i \partial \ln q_i}{\partial \ln p_j}$. We should remember that this relation gives the Slutsky coefficient. Hence, the Slutsky elasticity (compensated) of the i^{th} good with respect to the j^{th} price is

$$\tilde{\xi}_{ij}^{rot} = \frac{\pi_{ij}(Y, \mathbf{p})}{\omega_i} \tag{2.1.40}$$

The resultant uncompensate elasticity can be expressed as

$$\xi_{ij}^{rot} = \tilde{\xi}_{ij}^{rot} - w_j \eta_i \tag{2.1.41}$$

Equation (2.1.39), equivalent to equation (2.1.34), is the final step for the differential equation. However, they can only be estimated when the differentials are replaced by finite approximations.

In the last step of the formulation, we write the discrete equations. The Rotterdam model is in fact a discrete formulation of either (2.1.34) or (2.1.39).

In the first case, *the relative price version* $v_{ij} = \phi \mu_{ij} \, v_{ij}$ is the $(i, j)^{th}$ price coefficient. The parameters are the μ_i's and v_{ij}'s.

In the second case, *the absolute version*, we need to explicitly keep π_{ij} in the equation. The parameters are the μ_i's and the π_{ij}'s. In both cases, we set, for any variable x_{it}, $Dx_{it} = \ln x_{it} - \ln x_{it-1}$.

Since the approach is discrete, we consider $\overline{w}_{it}$ as the arithmetic average of the budget share between t and t-1. Hence, $\overline{w}_{it} = \frac{w_{i-1t} + w_{it}}{2}$. Also, $DQ_t = \sum_{i=1}^{n} \overline{w}_{it} Dq_{it}$.

In the first case, the following equation is obtained:

$$\overline{w}_{it} Dq_{it} = \mu_i DQ_t + \sum_{i=1}^{n} v_{ij} (Dp_{jt} - DP'_t) \qquad (2.1.42)$$

Where $DP_t' = \sum_{i=1}^{n} \mu_i Dp_{it}$ is the Frisch price index.

In the second, the equation becomes:

$$\overline{w}_{it} Dq_{it} = \mu_i DQ_t + \sum_{i=1}^{n} \pi_{ij} Dp_{jt} + \varepsilon_{it} \qquad (2.1.43)$$

Where $\pi_{ij} = v_{ij} - \phi \mu_i \mu_j$ and $i, j = 1, \ldots, n$

At this level, it is important to introduce the homogeneity condition as it constitutes one criterion that characterizes the optimizing behavior of a consumer. It translates the idea that the consumer does not react after a simultaneous and proportionate change in income and prices. In other words, the consumer is only sensitive to the real income; he is not victim of money illusion. For this to happen, the demand functions should be homogenous of degree zero.

We can then apply the Euler equation to the Marshallian demand equation, $q_i = q_i(M, p_1, ..., p_j, ...p_n)$.

First, we consider the fact that by homogeneity of the demand functions, we can write

$$\text{for } \lambda > 0,\ \lambda^n q_i = q_i(\lambda M, \lambda p_1, ..., \lambda p_j, ..., \lambda p_n) \tag{2.1.44}$$

Then, we differentiate this relation with respect of the multiplicative parameter λ.

$$n\lambda^{n-1} q_i = \frac{\partial q_i}{\partial M} M + \sum_{j=1} \frac{\partial q_i}{\partial p_j} p_j \tag{2.1.45}$$

It is then easy to see that when the demand functions are homogenous of degree zero, that is $n = 0$, the relation (2.1.45) can be expressed as

$$0 = \frac{\partial q_i}{\partial M} M + \sum_{j=1} \frac{\partial q_i}{\partial p_j} p_j \tag{2.1.46}$$

This equation can be reformulated as

$$0 = q_i \left(\frac{\partial q_i}{\partial M} \frac{M}{q_i} + \sum_{j=1} \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i}_{\ j} \right)$$

This last relation provides the demand homogeneity property in terms of the elasticities:

$$\eta_i + \sum_{j=1}^{n} \xi_{ij}^{rot} = 0, \quad i = 1, ..., n \tag{2.1.47}$$

To get the demand homogeneity in terms of the Hicksian elasticities, we first send the second term of (2.1.47) to the right hand-side. By doing so, we obtain the expression of the income elasticity in terms of the uncompensated elasticitites. Second we sum over j the relation (2.1.41) to get

$$\sum_j \xi_{ij}^{rot} = \sum_j \tilde{\xi}_{ij}^{rot} - \eta_i$$

Next, we plug the expression of the income elasticity in terms of the price elasticities into this last relation. The resulting relation is the homogeneity condition in terms of the Hicksian elasticities.

$$\sum_j \tilde{\xi}_{ij}^{rot} = 0 \tag{2.1.48}$$

We recall that this relation is equivalent to $\sum_j \frac{\pi_{ij}}{w_i} = 0$, which implies our final expression of the homogeneity condition:

$$\sum_j \pi_{ij} = 0 \tag{2.1.49}$$

We can now close the parenthesis of the homogeneity issue and come back to our Rotterdam model expression in (2.1.43).

The Slutsky coefficients satisfy the symmetry conditions and add up to zero—the homogeneity condition:

- $\pi_{ij} = \pi_{ji}$ *for all i and j.* (2.1.50)

- $\sum_{j=1}^{n} \pi_{ij} = 0$ *for all i and j.* (2.1.51)

In the relative price version the adding-up property states that the marginal propensity to spend of all goods sum to one and the net effect of a price change on the budget is zero. μ_i and v_{ij} are the parameters, and the model is nonlinear in the parameters.

In the absolute version, μ_i and μ_{ij} are the parameters. The model is linear in the parameters. Note that when the preferences are independent, the model simplifies to the following system:

$$\bar{w}_{it} Dq_{it} = \mu_i DQ_t + \phi\mu_i (Dp_{jt} - DP'_t) \tag{2.1.52}$$

2.1.2 Formulation of the Almost Ideal Demand System

The AIDS, due to (Deaton, A. and J. Muellbauer 1980), specifies a system of demand the goal of which is to capture the behavior of the budget shares as dependent on the logarithms of the prices of all the commodities involved and the logarithm of the income. This way of expressing the demand system found its plausibility on the fact that it is not the quantity demanded that is of interest but, rather, the demand behavior. What matters to economists is mainly finding parameters that enable the calculation of the price and income elasticities.

AIDS is derived from a cost function that presents a Cobb Douglas structure

$$C(U, \mathbf{p}) = a(\mathbf{p})^{1-U} b(\mathbf{p})^U \tag{2.1.53}$$

where the utility index U is such that $0 \le U \le 1$. The relation $U = 0$ corresponds to a subsistence state, and $U = 1$ to a bliss state. $\mathbf{p}$ is a column vector of n unit prices. Applying the logarithm operator to this equation will give

$$\ln C(U, \mathbf{p}) = (1-U)\ln a(\mathbf{p}) + U \ln b(\mathbf{p}) \tag{2.1.54}$$

Deaton and Muellbauer (1980) assign a *Translog* structure to $\ln a(\mathbf{p})$

$$\ln a(\mathbf{p}) = a_0 + \sum_{k=1}^{n} a_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \tilde{\beta}_{kj} \ln p_k \ln p_j \tag{2.1.55}$$

The symmetry condition is imposed by the relation $\beta_{ij} = \beta_{ji} = \frac{1}{2}\left(\tilde{\beta}_{kj} + \tilde{\beta}_{jk} \right)$

$\ln b(\mathbf{p})$ has the following structure

$$\ln b(\mathbf{p}) = \ln a(\mathbf{p}) + \beta_0 \prod_{k}^{n} p_k^{\beta_k} \tag{2.1.56}$$

Combining (2.1.53), (2.1.54) and (2.1.55) we get

$$\ln C(U,\mathbf{p}) = a_0 + \sum_{k=1}^{n} a_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \tilde{\beta}_{kj} \ln p_k \ln p_j + U\beta_0 \prod_{k=1}^{n} p_k^{\beta_k} \tag{2.1.57}$$

or equivalently,

$$\ln C(U,\mathbf{p}) = \ln a(\mathbf{p}) + U\beta_0 \prod_{k=1}^{n} p_k^{\beta_k} \tag{2.1.58}$$

From the Sephard's lemma, $\frac{\partial C(U,\mathbf{p})}{\partial p_i} = q_i$, we can use the equivalent formula $\frac{\partial C(U,\mathbf{p})}{\partial \ln p_i} = \frac{p_i q_i}{C(U,\mathbf{p})} = w_i$ to get

$$w_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln p_j + \beta_i U \beta_0 \prod_{k=1}^{n} p_k^{\beta_k} \tag{2.1.59}$$

Deducing the expression of the third term in (2.1.58), we can write the final expression of the AIDS system:

$$w_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln p_j + \beta_i [\ln M - \ln P], \text{ where } \ln P = \ln a(\mathbf{p}) \text{ and } M = C(U,\mathbf{p}) \tag{2.1.60}$$

Note that:

1. The adding-up restriction is verified if $\sum_{i=1}^{n} a_i = 1$, and $\sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \beta_{ij} = 0$.

2. The homogeneity condition can be expressed as $\sum_{i=1}^{n} \beta_{ij} = 0$

3. The symmetry is imposed by $\beta_{ij} = \beta_{ji} = \frac{1}{2}\left(\tilde{\beta}_{kj} + \tilde{\beta}_{jk} \right)$

4. The semi-negative definiteness of the Slutsky matrix is usually checked and not imposed. This is the case when

$$\sum_{i=1}^{n}\sum_{j=1}^{n} v_i v_j \xi_{ij}^{AIDS} \frac{q_i}{p_j} \leq 0 \quad \forall \text{ for any column vector } v \text{ n elements.}$$

2.1.2.1. AIDS elastcities and the Slutsky matrix

First, we will derive the general formula for the elasticities. Second, we will determine the Aids elasticities. Finally, we will determine the AIDS Slutsky matrix. It is important to point out that the AIDS slutsky matrix constitutes its Hessian matrix.

2.1.2.1.1. Elasticity formula

We need to formulate the price and income elasticities in a more suitable way for calculating the AIDS elasticities and the Hessian matrix as well. We will normalize the price by setting $v_i = \frac{pi}{m}$. Since the mathematical definition of the budget share is $w_i = \frac{p_i q_i}{M}$, it can be written as $w_i = v_i q_i$, or equivalently, $q_i = \frac{w_i}{v_i}$.

By definition, the price elasticities can be expressed as $\xi_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}$. This definition is the product of two elements:

1. $\frac{\partial q_i}{\partial p_j} = \frac{\partial w_i}{\partial p_j}\frac{1}{v_i} - \frac{\partial v_i}{\partial p_j}\frac{q_i}{v_i}$

2. $\frac{p_j}{q_i} = \frac{p_j v_i}{w_i}$

Hence $\xi_{ij} = \frac{\partial w_i}{\partial p_j}\frac{p_j}{w_i} - \frac{\partial v_i}{\partial p_j}\frac{p_j}{v_i}$.

Note that for $i = j$, $\frac{\partial v_i}{\partial p_j}\frac{p_j}{v_i} = 1$ and for $i \neq j$ $\frac{\partial v_i}{\partial p_j} = 0$.

Finally,

$$\xi_{ij}(v) = \frac{\partial w_i}{\partial p_j}\frac{p_j}{w_i} - \delta_{ij}, \text{ or equivalently } \frac{\partial \ln w_i}{\partial \ln p_j} - \delta_{ij} \tag{2.1.61}$$

Where δ_{ij} is the kronocker product with $\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$

On the other hand, the income elasticity $\eta_i = \frac{\partial q_i}{\partial M}\frac{M}{q_i}$ can be expressed in terms of w, and M . In fact, from the expression of the budget share, we can obtain the following expression

$$q_i = w_i \frac{M}{p_i}$$

Deriving this expression gives $\frac{\partial q_i}{\partial M} = \frac{w_i}{p_i} + \frac{\partial w_i}{\partial M}\frac{M}{p_i}$. Hence,

$$\frac{\partial q_i}{\partial M}\frac{M}{q_i} = \frac{w_i M}{p_i q_i} + \frac{\partial w_i}{\partial M}\frac{M^2}{p_i q_i} .$$

Note that $M/p_i = 1/v_i$, and equivalently $p_i/v_i = M$. As a result,

$$\eta_i = 1 + \frac{\partial w_i}{\partial M}\frac{M}{w_i} \tag{2.1.62}$$

2.1.2.1.2. The AIDS price and income elasticities

We recall the full AIDS system of equations:

$$\begin{cases} w_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln p_j + \beta_i [\ln M - P], \text{ where} \\ P = \ln a(\mathbf{p}) = a_0 + \sum_{k=1}^{n} a_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \beta_{kj} \ln p_k \ln p_j , \text{ and} \\ i, j = 1, \ldots, n \end{cases} \tag{2.1.63}$$

The income elasticity

Using the (2.1.62) formula,

$$\eta_i^{AIDS} = 1 + \frac{\beta_i}{w_i} \tag{2.1.64}$$

Although, the AIDS system does not directly compute the marginal budget share, it can be deduced from the relation

$$\mu_i = \eta_i^{AIDS} . w_i \tag{2.1.65}$$

In other words

$$\mu_i^{AIDS} = w_i + \beta_i \tag{2.1.66}$$

Its sign is determined by the β_i sign, and its absolute value relatively to the budget share. It should also be noted that one of the adding-up condition ($\sum_{i=1}^{n} \beta_1 = 0$) guarantees the unity of the sum of the marginal shares of all the commodities involved.

The uncompensated price elasticities

The procedure is to first calculate the matrix of uncompensated elasticities (ξ_{ij}) . Then, we will use the Slutsky equation to get the matrix of compensated elasticities($\tilde{\xi}_{ij}$).

From the relation (2.1.61), we want to compute the first element of the right side:

$$\frac{\partial w_i}{\partial v_j} = M \frac{\partial w_i}{\partial p_j} = M \left[\frac{\beta_{ij}}{p_j} - \beta_i \left(\frac{\alpha_j}{p_j} + \frac{\sum_{k=1}^{n} \beta_{kj} \ln p_k}{p_j} \right) \right]$$

Plugging this relation into (2.1.61), we get

$$\xi_{ij}^{AIDS} = \frac{1}{w_i} \left[\beta_{ij} - \beta_i \left(\alpha_j + \sum_{k=1}^{n} \beta_{kj} \ln p_k \right) \right] - \delta_{ij} \tag{2.1.67}$$

It is possible to reformulate this equation by using the AIDS equation in (2.1.63).` In that case,

$$\xi_{ij}^{AIDS} = \frac{1}{w_i} \left[\beta_{ij} - \beta_i \left(w_j - \beta_j \ln \left(\frac{M}{P} \right) \right) - w_i \delta_{ij} \right] \tag{2.1.68}$$

Note that the uncompensated elasticity matrix is not symmetric since $\beta_i w_j \neq \beta_j w_i$.

The compensated price-elasticity and the Slutsky matrices

From the uncompensated elasticity matrix, we can easily derive the compensated price elasticity matrix. It suffices to use the Slutsky equation. For some given i and j, the Slutsky equation appears to be

$$\tilde{\xi}_{ij} = \xi_{ij} + w_j \eta_i \tag{2.1.69}$$

Using the income elasticity in (2.1.64), we found

$$\tilde{\xi}_{ij}^{AIDS} = \frac{1}{w_i}\left[\beta_{ij} + \beta_i \beta_j \ln\left(\frac{M}{P}\right) - w_i \delta_{ij} + w_i w_j\right] \tag{2.1.70}$$

2.1.2.1.3. The Slutsky matrix

It is derived from the formula of the compensated elasticities

$$\left.\frac{\partial q_i}{\partial p_j}\right|_{u=\tilde{u}} = \frac{q_i}{p_j}.\xi_{ij}^{AIDS} \quad .$$

Hence,

$$\left.\frac{\partial q_i}{\partial p_j}\right|_{u=\tilde{u}} = \frac{p_i p_j}{M}\left[\beta_{ij} + \beta_i \beta_j \ln\left(\frac{M}{P}\right) - w_i \delta_{ij} + w_i w_j\right] \tag{2.1.71}$$

It is clear that the Slutsky matrix obtained shows symmetry in its structure.

2.1.3. Some AIDS-related systems of demand

2.1.3.1. The LA-AIDS

2.1.3.1.1. Definition

The AIDS estimation, formulated above, is non-linearly expressed. It is usual to proceed to its linearization by using a proxy of the income deflator, which originally presents a translog structure. (Deaton, A. and J. Muellbauer 1980) advise the use of The Stone's Price index. It is a geometric means of the commodity prices weighted by the corresponding budget shares.

Applying the logarithm operator to the index, this can finally be formulated as

$$\ln P = \ln a(\mathbf{p}) \approx \sum_{i=1}^{n} w_{i,t} \ln p_{it} \tag{2.1.72}$$

(Eales, J. S. and L. J. Unnevehr 1988) propose lagging the budget share in order to avoid any simultaneity problem.

This Linearized AIDS is referred to in the literature as LA-AIDS. The LA-AIDS can be explicitly written as:

$$w_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln p_j + \beta_i [\ln M - \sum_{i=1}^{n} w_{i,t} \ln p_{it}],\ i, j = 1,\ldots,n \tag{2.1.73}$$

Though its estimation is easier because of its linearity in the parameters, the finding of the Hessian matrix can be more demanding.

2.1.3.1.2. The Hicksian elasticity matrix of the LA-AIDS

The Marshallian price elasticity matrix

We will still use our elasticity formula written above in (2.1.61).

First we consider

$$\frac{\partial \ln w_i}{\partial \ln p_j} = \frac{dw_i}{d\ln p_j}\frac{1}{w_i} = {}^{1}\!/_{w_i}\left(\beta_{ij} - \beta_i \frac{dP}{d\ln p_j} \right),\ \ln P = \sum_{i=1}^{n} w_i \ln p_i \qquad (2.1.74)$$

Note that $\frac{\partial P}{\partial \ln p_j} = w_j + \sum_{i=1}^{n} \ln p_i \frac{\partial w_k}{\partial \ln p_j}$ and $\frac{\partial w_i}{\partial \ln p_j} = w_i(\xi_{ij} + \delta_{ij})$ by (2.1.61).

Finally

$$\xi_{ij}^{LA/AIDS} = -\delta_{ij} + {}^{1}\!/_{w_i}\left[\beta_{ij} - \beta_i \left(w_j + \sum_{i=1}^{n} w_i \left(\xi_{ij}^{LA/AIDS} + \delta_{ij} \right) \ln p_i \right) \right] \qquad (2.1.75)$$

Evidently, there is a circularity issue in the sense that the elasticity is simultaneously on the left and right sides of the equation. It becomes logical that the Slutsky matrix that could result from it is not appropriate for applying the Preference Independence Transformation.

The Differential AIDS (DAIDS)

We obtain the DAIDS by differentiating the LA/AIDS equation:

$$dw_i = \beta_{ij} d\ln p_j + \beta_i d\ln M - \beta_i dP^* \qquad (2.1.76)$$

Where $P^* = \sum_{i=1}^{n} w_k \ln p_k$. From the following two observations,

- the relative change of the income can be decomposed in Divisia price and Divisia volume indices,

$$\frac{dM}{M} = \sum_{i=1}^{n} w_i d\ln p_i + \sum_{i=1}^{n} w_i d\ln q_i = DP + DQ$$

- and the Stone index can be approximated by the Divisia price index

$$dP^* = \sum_{k=1}^{n} w_k d\ln p_k + \sum_{k=1}^{n} dw_k \ln p_k \approx \sum_{k=1}^{n} w_k d\ln p_k + 0 \ ,$$

The DAIDS can be formulated as

$$dw_i = \sum_{j=1}^{n} \beta_{ij} d\ln p_j + \beta_i DQ \qquad (2.1.77)$$

(Barten, A. P. 1993) shows it is possible to give this equation the same structure as the Rotterdam model. It suffices to consider the three following relations:

1. $dw_i = w_i d\ln p_i + w_i d\ln q_i - w_i d\ln M$
2. $w_i d\ln p_i = \sum_j \delta_{ij} w_j d\ln p_j$
3. $w_i DP = \sum_j w_i w_j d\ln p_j$

The DAIDS equation can then be presented as follow

$$w_i d\ln q_i = (w_i + \beta_i) DQ + \sum_{j=1}^{n} (\beta_{ij} - \delta_{ij} w_j + w_i w_j) d\ln p_j \qquad (2.1.78)$$

Recall that we have shown above that the marginal share of the AIDS model is $\mu_i = w_i + \beta_i$. Then the expression in parentheses on the second term of the right hand side is comparable to π_{ij} of the Rotterdam model. In addition, we see easily that the income elasticity, $\eta_i^{DAIDS} = 1 + \dfrac{\beta_i}{w_i}$, and the Hicksian price elasticity is given by $\xi_{ij}^{DAIDS} = \dfrac{\beta_{ij} - \delta_{ij} w_j + w_i w_j}{w_i}$.

The DAIDS Slutsky matrix is given by

$$\left.\frac{\partial q_i}{\partial p_j}\right|_{u=\tilde{u}} = \frac{p_i p_j}{M}\left[\beta_{ij} - w_i \delta_{ij} + w_i w_j\right] \qquad (2.1.79)$$

2.2. The Preference Independence Transformation: Genesis, formulation, and procedure

At this point it is relevant to precise three core concepts we will be using all along the subsequent developments.

By commodities, we mean the goods bought in the markets at some prices. Basically, we will be dealing with four commodities consumed by the US households: electricity, distillate fuel oil, gas, and liquefied petroleum gas. By T-goods, we understand the basic wants supporting the acquisition of the goods.

The third concept is the Preference Independence Transformation process which is a linear transformation of a matrix, namely the specific effect, that describes the commodity interdependences at the preference level in such a way its matrix counterpart becomes diagonal under the assumptions that the number of goods equals the number of basic wants, and the basic wants are preference independent.

2.2.1. Genesis

In the relatively recent history of consumer demand, economists have realized that the main factors that determine consumer demands are not as apparent as many think. Behind quantities of goods demanded exists basic wants whose definition may be essential to capturing the evolution of demand. In practice, two main paths are open to researchers to incorporate basic wants: the first, primarily theoretical, was initially explored by (Barten, A. P. 1993), (Lancaster, K. 1971, Lancaster, K. J. 1966), and(Becker, G. S. 1965); the second, purely statistical, is the Preference Independence Transformation introduced by Henry (Theil, H. 1967). (Brooks, R. B. 1970), (Theil, H. 1975, 1977), and (Theil, H. and K. Laitinen 1992) make decisive contributions on how to assess consumer basic wants using this technique.

Economists are stressing more and more the empirically challenging issue of quality on demand. In his article,—"Qualities, Prices and Budget Enquiries" (1951-1952)—Theil makes a distinction between goods and commodities using the quality criteria. According to this approach, a commodity is a group of goods, a good being a quality of the commodity, which is perfectly homogenous. In other words, he defined the good as having a perfectly homogenous quality. Later, while economists used to assume that consumption of goods produced a homogenous utility, (Ironmonger, D. S. 1972) pointed out that wants are various, and multiple. He introduced the assumption that, although consumers may be considered as ultimately having one want, in practice, they pursue many wants. This raises the interest of disaggregating the utility. There is an advantage in microeconomic research to break the synthetic concept of utility into many sub-utilities. In his model, he assumed that the consumer has a consumption technology (consumption matrix) that transforms units of a commodity into units of '*personal satisfaction'* of the wants. Concurrently, (Lancaster, K. 1971) introduced a slightly similar but very innovative approach. According to Lancaster, goods are defined by characteristics, and consumers buy goods not for their quantities but for the characteristics they have. For Lancaster, all goods possess *objective* characteristics. The difference between the Lancaster model and the

Ironmonger model is that Lancaster model assumes that the characteristics are the same for everybody, though people have different reactions to them. Theoretically, this difference can be noticed in the consumption matrix. For Ironmonger, each consumer has his own consumption technology. Lancaster on the other hand considers a unique consumption technology matrix for all consumers. In the Lancaster model each good presents “measurable characteristics in fixed proportions with quantities of the characteristics directly proportional to the quantities of the goods” (Brooks, R. B. 1970). Under the Lancaster approach, the problem of the consumer can be expressed as follows:

$$\begin{cases} Max\ \mathrm{U}(\mathbf{z}) \\ s.t.: \mathbf{z} = \mathbf{Bq},\ \mathbf{pq} \leq Y \text{ and } \mathbf{q}, Y \geq \mathbf{0}. \end{cases} \tag{2.2.1}$$

Where $\mathrm{U}(\mathbf{z})$ is the utility function, $\mathbf{z}$ is the vector of characteristics, $\mathbf{q}$ is the vector of goods, $\mathbf{B}$ is the consumption technology matrix, and Y is the income. To have a clear idea of how $\mathbf{B}$ is constructed, let us consider that each commodity has K potential characteristics. Given $k \in [1, K]$, $\mathbf{b}^k = [b_{1k}\, b_{2k} \ldots b_{jk} \ldots b_{nk}]^T$ is the amount of the k^{th} characteristic possessed by the j^{th} commodity. From here, we see that $\mathbf{B} = [\mathbf{b}^1\ \mathbf{b}^2\ \ldots \mathbf{b}^K]$.

This problem raises a recurring question: How can we use the model in empirical studies if the characteristics are not measurable and, even less, observable?

A close, but different and no less important, model is the one introduced by Becker (1965). He considers the household unit as a “factory” where goods purchased on the market are combined with the time—by the mean of a household production function—to produce “commodities”. These commodities provide the direct arguments in the household’s utility function. They are denoted as “basic commodities.” If we denote Z_i the basic commodity i, T_i

the vector of time inputs used to get the i^{th} basic commodity, and X_i the vector of market goods, then the household production function can be expressed as

$$Z_i = Q(X_i, T_i) \qquad (2.2.2)$$

This research raises the necessity to find a technical way of uncovering the basic wants. Theil (1967; 1975-76; 1977) and Brook (1970) introduced a statistical technique in the analysis: the Preference Independence Transformation (PIT). Historically, two techniques were used in economics to formulate “a set of variables that are in some way ‘more basic’ than the observed variables”: Principal Component Analysis and Factor Analysis. The first technique was introduced in economics by (Stone, R. 1947). The second was used by (Gorman, W. M. 1965, 1959, 1976) to assess the consumer’s basic wants. According to (Theil, H. 1967, 1975, 1977), the PIT is between these two techniques. It defines a set of variables that are more essential than the observed variables. In practice, observed data on closed goods reveal “substitutability” or “complementarity” patterns. The PIT changes “observed consumer goods” into “transformed goods,” the latter being characterized by the independence of the marginal utility of each good relative to the consumption of all other goods. The transformed goods—denoted in the literature as T-goods— are representative of the basic wants. The main assumption made with this technique is that the consumer chooses “basic wants” independently. In other words, the “transformed goods” are neither complement, nor substitute. For this reason, (Theil, H. and K. Laitinen 1992) found it somehow unrealistic to apply the PIT technique on “narrowly defined goods.” The “transformed goods” should have an additive structure at the level of the utility function. An example of such structure is given by the Klein-Rubin (1948) utility function which can be expressed as follows

$$U(\boldsymbol{q}) = \sum_{i=1}^{n} \theta_i \ln(q_i - a_i) \tag{2.2.3}$$

Where $\sum \theta_i = 1$ and the a_i 's are constant.

2.2.2. Formulation under the Rotterdam model

The formulation of the PIT technique began with Brook's dissertation who first noticed that a change in real income modifies the proportion of luxuries and necessities in the consumer's basket. Under the Rotterdam model framework, measuring these changes is simplified if the cross-partial derivatives of the consumer's utility function are zero. From this observation, he considers Taylor's expansion of the utility function around the optimal point:

$$U(\boldsymbol{x}) = \boldsymbol{a}'\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}'U\boldsymbol{x} + \text{ remainder term} \tag{2.2.4}$$

If we suppose that the Hessian matrix is diagonal, then the utility function can explicitly be written as:

$$U(x) = \sum_{i=1}^{n} a_i x_i + \frac{1}{2}\frac{\partial^2 U}{\partial^2 x_i} x_i^2 + \text{ remainder term} \tag{2.2.5}$$

In the general case, it is not. Brook wanted to find a way to diagonalize the Hessian matrix. This problem is not different from Lancaster's, who focused on uncovering the characteristics of goods based on their quantities. In fact, diagonalizing the Hessian matrix is equivalent to the construction of a set of "new commodities" referred to as "basic goods" with the propriety of being "want or preference independent."Brooks' first step is to transform linearly the price and quantity vectors by setting:

$$\mathbf{z} = \mathbf{B}\boldsymbol{x} \;\; and \;\; \mathbf{y} = \mathbf{B}\boldsymbol{p} \tag{2.2.6}$$

Where $\mathbf{B}$ is the transformation matrix, $\mathbf{z}$ is the transformed quantity vector, and $\mathbf{y}$ is the transformed price vector.

For the budget constraint to hold in the new prices and quantities, it suffices that $\mathbf{B}'\mathbf{B} = \mathbf{I}$. In that case the budget constraint $\mathbf{p}'\mathbf{x} = Y$ is equivalent to

$$\mathbf{y}'\mathbf{B}^{-1'}\mathbf{B}^{-1}\mathbf{z} = Y. \tag{2.2.7}$$

Likewise,

$$\mathbf{y}'\mathbf{z} = \mathbf{p}'\mathbf{B}'\mathbf{B}\mathbf{x} = \mathbf{p}'\mathbf{x} = Y \tag{2.2.8}$$

As a result, the transformation matrix must be diagonal.

It becomes then possible to write Taylor's utility function expansion in terms of the transformed goods.

Let

$$U_T(\mathbf{z}) = \boldsymbol{U}\left(\mathbf{B}^{-1}\mathbf{z}\right) \tag{2.2.9}$$

Then

$$U_T(\mathbf{z}) = (\mathbf{Ba})'\,\mathbf{z} + \frac{1}{2}\mathbf{z}'\mathbf{BUB}'\mathbf{z} + \text{ remainder term} \tag{2.2.10}$$

For the basic goods to be independent, the matrix $\mathbf{BUB}'$ must be diagonal. This means that the marginal utility of each basic good is independent of the quantity consumed of any other transformed good.

From the expressions (2.2.6), we can draw the following equations:

For every good i, $z_i = b_{i1}x_1 + b_{i2}x_2 + ... + b_{in}x_n$ and $\mathrm{y}_i = b_{i1}p_1 + b_{i2}p_2 + ... + b_{in}p_n$ with the condition that $\sum_{i=1}^{n} b_{in}^2 = 1$ because of the orthogonality condition. The first equation gives the contribution of each good quantity to the transformed good. The second equation shows how each price contributes to the transformed good.

Barnett (1979) pointed out that under the Rotterdam model framework a diagonalization of the Hessian matrix $\mathbf{U}$ involves at the same time a diagonalization of $\mathbf{M}_{n,n} = [\mu_{ij}]$ —the normalized price coefficient matrix—with respect to the n by n diagonal matrix of the budget shares, $(\overline{\mathbf{w}})_\Delta = \begin{bmatrix} \overline{w}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \overline{w}_n \end{bmatrix}$. This is equivalent to writing

$$(\mu_{ij} - \lambda_i \overline{\mathbf{w}}_\Delta) x_i = 0, \; i = 1, ..., n \qquad (2.2.11)$$

Where the λ_i's are latent roots and the x_i's are characteristic vectors normalized. From now on, we will read $\mathbf{Y}_\Delta$ as the matrix $\mathbf{Y}$ in diagonal form.

As a result,

$$x_i'(\overline{\mathbf{w}})_\Delta x_j = \begin{cases} 0 \text{ if } & i \neq j \\ 1 \text{ otherwise} \end{cases} \qquad (2.2.12)$$

In matrix form, this can be concisely written as

$$\left(\mathbf{M} - \lambda_i (\bar{\mathbf{w}})_\Delta\right)\boldsymbol{x}_i = \mathbf{0} \text{ and } \boldsymbol{X}^{'}(\bar{\mathbf{w}})_\Delta \boldsymbol{X} = \mathbf{I}, \\ i = 1,\dots,n;\ \mathbf{X} = \left[\mathbf{x}_1 \quad \mathbf{x}_2 \quad \dots \quad \mathbf{x}_3\right] \tag{2.2.13}$$

In view of (2.1.30), Theil (1975) defines (2.2.12) as a "diagonalization of the inverted Hessian in expenditure terms relative to the expenditure levels". He noticed a more practical way of rewriting it by premultiplying (2.2.13) by the matrix of the square root of the budget share in diagonal form, $(\bar{w})_\Delta^{-1/2}$, and taking $(\bar{\mathbf{w}})_\Delta^{-1/2}\mathbf{X}$ out the braces:

$$[(\bar{\mathbf{w}})_\Delta^{-1/2}\mathbf{M}(\bar{\mathbf{w}})_\Delta^{-1/2} - \lambda\mathbf{I}](\bar{\mathbf{w}})_\Delta^{-1/2}\mathbf{X} = \mathbf{0} \text{ where } \mathbf{X}^{'}(\bar{\mathbf{w}})_\Delta^{1/2}[\mathbf{X}^{'}(\bar{\mathbf{w}})_\Delta^{1/2}]^{'} = \mathbf{I} \tag{2.2.14}$$

From the normalization expression in (2.2.14), we find that:

$$(\mathbf{w})_\Delta = (\mathbf{X}\mathbf{X}^{'})^{-1} \tag{2.2.15}$$

This allows us to write the normalized price coefficient matrix as function of the eigenvectors of the diagonlization:

$$\mathbf{M}_{n,n} = (\mathbf{X}^{'}_{n,n})^{-1}\mathbf{\Lambda}\mathbf{X}^{-1}_{n,n} \tag{2.2.16}$$

Theil (1975), using three axioms showed that the transformation—the move from the knowledge of the parameters of the commodity goods to this of the basic wants—denoted as T-goods in the literature—is a passage to equation (2.1.34) to a new equation involving two additional matrix operators, denoted $\mathbf{R}$ and $\mathbf{S}$.

The full understanding of the transformation requires to write the model in matrix term and a thorough explanation of the three operators involved: the transformation matrices $\mathbf{R}_{n,n}$ and $\mathbf{S}_{n,n}$, and the composition matrix $\mathbf{T}_{n,n}$.

2.2.2.1. The Rotterdam model in matrix term

Let us reconsider the Rotterdam model equation (2.1.42). To write it in a matrix format, we consider the following:

$$\mathbf{y}_t = [\,\bar{w}_{1t}Dq_{1t} \quad \bar{w}_{2t}Dq_{2t} \quad \ldots \quad \bar{w}_{nt}Dq_{nt}\,]', \quad \boldsymbol{\pi}_t = [\,Dp_{1t} \quad Dp_{2t} \quad \ldots \quad Dp_{nt}\,]', \text{ and}$$

$$\boldsymbol{\varepsilon}_t = [\,\varepsilon_{1t} \quad \varepsilon_{2t} \quad \ldots \quad \varepsilon_{nt}\,]'.$$

It is important to note that the second term of the right hand side of the Rotterdam model can be expressed as

$$\phi\sum_{j=1}^{n}\mu_{ij}\left(Dp_{jt} - \sum_{k=1}^{n}\mu_k Dp_{kt}\right) = \phi(\mathbf{M}_{n,n}\boldsymbol{\pi}_t\boldsymbol{\pi}_t - \mathbf{M}_{n,n}\boldsymbol{\iota\iota}'\mathbf{M}_{n,n}\boldsymbol{\pi}_t) = \phi\mathbf{M}_{n,n}\,(\mathbf{I} - \boldsymbol{\iota\mu}')\,\boldsymbol{\pi}_t$$

We have used $\boldsymbol{\iota}'\mathbf{M}_{n,n} = \boldsymbol{\mu}'$ obtained from the relation seen above that $\mathbf{M}_{n,n}\boldsymbol{\iota} = \boldsymbol{\mu}_{n,1}$.

The Rotterdam can then be expressed as

$$\mathbf{y}_t = (DQ_t)\mathbf{M}\boldsymbol{\iota} + \phi\mathbf{M}\left(\boldsymbol{I} - \boldsymbol{\iota\iota}'\mathbf{M}\right)\boldsymbol{\pi}_t + \boldsymbol{\varepsilon}_t \qquad (2.2.17)$$

2.2.2.2. The transformation matrix R

By definition, $\mathbf{R}_{n,n} = [r_{ij}]\ (i,\ j = 1,\ldots,n)$ is the square matrix for which the components r_{ij} measures the quantity of dollars on the i^{th} T-good (*basic want*) when one dollar is spent on the j^{th} commodity. The total expenditure on the i^{th} T-good is then

$$\sum_{j=1}^{n} r_{ij} p_j q_j$$

Consequently, the total expenditure *(full income)* on all the basic wants is

$$\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij} p_j q_j$$

Theil's first axiom states that the T-goods satisfy the budget constraint. Consequently,

$$\text{Full income}=\sum_{i=1}^{n}\sum_{j=1}^{n} p_j q_j = \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij} p_j q_j \tag{2.2.18}$$

This implies that

$$\boldsymbol{\iota}'\mathbf{R}_{n,n} = \boldsymbol{\iota}' \text{ that is } \mathrm{r}_{1j} + r_{2j} + \ldots + r_{nj} = 1 \text{ for any } j = 1,\cdots,n. \tag{2.2.19}$$

This constraint, very important for subsequent developments, is the condition under which the full income is invariant under the transformation. It suffices to divide on both side of the equality in (2.2.18) to get the definition of the i^{th} T-good budget share in terms of the commodity budget shares:

$$(\overline{w}_{it})_T = \sum_{j=1}^{n} r_{ij} \overline{w}_{jt} \tag{2.2.20}$$

In matrix terms this relation can be expressed as $\mathbf{w}_T = \mathbf{R}_{n,n}\mathbf{w}$

To summarize the three equations (2.2.18), (2.2.19), and (2.2.20) express somehow the same fact that the full income is invariant under the transformation.

2.2.2.3. The transformation matrix S

The operator $\mathbf{S}$ is determined after a decomposition of the T-good budget share change, first using the discrete time, second, the continuous time, and finally proceeding by identification between the two expressions.

In general, the decomposition of the budget share in discrete time can be expressed as follows

$$\Delta\overline{w}_{jt} \approx \overline{w}_{jt} Dp_{jt} + \overline{w}_{jt} Dq_{jt} - \overline{w}_{jt} DM_t$$

The same decomposition when the changes are infinitesimal is:

$$dw_{jt} = w_{jt} d\ln p_{jt} + w_{jt} d\ln q_{jt} - w_{jt} d\ln M_t$$

Moving these two equations from the commodity space to the T-good space requires applying $\mathbf{R}$ and summing over j in the first equation. In the second, it requires just to add the index T (which design time in the T-good space). It is important to note that in both equations, we deal with the i^{th} T-good. This explains why we should have the index i, and not j in the second equation. We get the following two equations:

$$\begin{cases} - \quad \sum_j r_{ij} \Delta\overline{w}_{jt} \approx \sum_j r_{ij} \overline{w}_{jt} Dp_{jt} + \sum_j r_{ij} \overline{w}_{jt} Dq_{jt} - \sum_j r_{ij} \overline{w}_{jt} DM_t \\ - \quad d(\overline{w}_{it})_T = (\overline{w}_{it})_T (Dp_{it})_T + (\overline{w}_{it})_T (Dq_{it})_T - (\overline{w}_{it})_T (DM_{it})_T \end{cases} \tag{2.2.21}$$

Recall that Dp_{jt}, and Dq_{jt} are the commodity price and quantity log-change. By contrast, $(Dp_{jt})_T$, and $(Dq_{jt})_T$ are the T-good price and quantity log-change. Insofar as these two equations express the same reality, we can approximate the corresponding terms of the two equations.

Hence, equalizing the two first terms of the right hand side leads to the following relation

$$(Dp_{it})_T = \frac{\sum_j r_{ij}\bar{w}_{jt} Dp_{jt}}{(\bar{w}_{it})_T} = \frac{\sum_j r_{ij}\bar{w}_{jt} Dp_{jt}}{\sum_{j=1}^{n} r_{ij}\bar{w}_{jt}} \qquad (2.2.22)$$

The definition of the price log-change of the i^{th} T-good corresponds to the second Theil axiom. This definition enables us to identify the $(i, j)^{th}$ component of our operator $S_{n,n} = [s_{ij}]$ as

$$s_{ij} = \frac{r_{ij}\bar{w}_{jt}}{(\bar{w}_{it})_T} \qquad (2.2.23)$$

The price log-change of the the i^{th} T-good (2.2.22) can be simply expressed as

$$(Dp_{it})_T = \sum_{j=1} s_{ij} Dp_{jt}, \qquad \text{where } \sum_{j=1} s_{ij} = 1$$

Likewise, $(Dq_{it})_T = \sum_{j=1} s_{ij} Dq_{jt}$. The transformation matrix (2.2.23) can be written in matrix terms as

$$\mathbf{S} = (\mathbf{w}_T)_{\Delta}^{-1} \mathbf{R} (\mathbf{w})_{\Delta} \qquad (2.2.24)$$

The constraint on the s_{ij} is:

$$\mathbf{S}\boldsymbol{\iota} = \boldsymbol{\iota}$$

2.2.2.4. The composition matrix T

Applying the operator $\mathbf{R}_{n,n}$ to the Rotterdam dependent variable $\mathbf{y}_t$ proceeds to moving from the j[th] component of the Divisia ($\overline{w}_{jt} Dq_{jt}$) to the Divisia quantity component of the i^{th} T-good ($\sum_{j=1}^{n} r_{ij} \overline{w}_{jt} Dq_{jt}$).

Hence,

$$\mathbf{R}\mathbf{y}_{\boldsymbol{t}} = \left(DQ_t\right)\mathbf{RM}\boldsymbol{\iota} + \phi\mathbf{RM}[\boldsymbol{I} - \boldsymbol{\iota\iota}'\mathbf{M}]\boldsymbol{\pi}_t + \mathbf{R}\boldsymbol{\varepsilon}_t \qquad (2.2.25)$$

This transformation is not yet complete since it contains the vector of price log-changes of commodities. We need to move from $\boldsymbol{\pi}_t$ to $\mathbf{S}\boldsymbol{\pi}_t$. This is the reason why the previous equation is reformulated as

$$\mathbf{R}\mathbf{y}_{\boldsymbol{t}} = \left(DQ_t\right)(\mathbf{RMS}^{-1})\boldsymbol{\iota} + \phi(\mathbf{RMS}^{-1})[\boldsymbol{I} - \boldsymbol{\iota\iota}'(\mathbf{RMS}^{-1})]\mathbf{S}\boldsymbol{\pi}_t + \mathbf{R}\boldsymbol{\varepsilon}_t \qquad (2.2.26)$$

We obtain this result using the two main properties of the transformation matrices $\mathbf{R}$ and $\mathbf{S}$, namely, $\boldsymbol{\iota}'\mathbf{R} = \boldsymbol{\iota}'$ and $\mathbf{S}\boldsymbol{\iota} = \boldsymbol{\iota}$. We infer then that the independent transformation technique is the fact from moving from $\mathbf{M}$ to $\mathbf{RMS}^{-1}$. (Theil, H. 1975, chapter 12) has shown that this expression is equivalent to solving the diagonalizaton of the normalized price coefficient matrix with respect to the budget share – see relation(2.2.11).

In fact using the definition of $\mathbf{S}_{n,n}$ in (2.2.24), we can write that:

$$\mathbf{RMS}^{-1} = \mathbf{RM}[(\mathbf{w}_T)_{\Delta}^{-1}\mathbf{R}(\mathbf{w})_{\Delta}]^{-1} = \mathbf{RM}(\mathbf{w})_{\Delta}^{-1}\mathbf{R}^{-1}(\mathbf{w}_T)_{\Delta} \qquad (2.2.27)$$

Using (2.2.15) , and (2.2.16) the expression can be written as:

$$\mathbf{RMS}^{-1} = \mathbf{R}(\mathbf{X}')^{-1}\mathbf{\Lambda X}^{-1}(\mathbf{XX}')\mathbf{R}^{-1}(\mathbf{w}_T)_\Delta \tag{2.2.28}$$

If we take the T-good budget shares in diagonal form to the left hand-side, the relation implies:

$$(\mathbf{RMS}^{-1})(\mathbf{w}_T)_\Delta^{-1} = \mathbf{R}(\mathbf{X}')^{-1}\mathbf{\Lambda X}'\mathbf{R}^{-1} = [\mathbf{X}'\mathbf{R}^{-1}]^{-1}\mathbf{\Lambda X}'\mathbf{R}^{-1} \tag{2.2.29}$$

We remark that the left hand side is the product of two matrices in diagonal form. As a result, the right hand-side should be diagonal.

Hence, the problem is to choose the transformation matrix $\mathbf{R}$ in such a way $(\mathbf{X}'\mathbf{R}^{-1})^{-1}\mathbf{\Lambda}(\mathbf{X}'\mathbf{R}^{-1})$ is diagonal under the constraint $\boldsymbol{\iota}'\mathbf{R} = \boldsymbol{\iota}'$. $\mathbf{X}$ and $\mathbf{\Lambda}$ are the eigenvector and eigenvalue matrices.

(Theil, H. 1975) has proved that the solution of such a problem is $\mathbf{R} = \mathbf{B}\left(\mathbf{X}^{-1}\boldsymbol{\iota}\right)_\Delta \mathbf{X}'$, where $\mathbf{B}_{n,n}$ represents a permutation matrix with exactly one in each row and each column. Without loss of generality, we may choose $\mathbf{B} = \mathbf{I}$.

According to Theil third axioms, $\mathbf{RMS}^{-1}$ is diagonal with positive elements. This is the case because the matrix of normalized price coefficients has positive roots—see (Theil, H. 1975) on pages 221 and 236. In addition, from the formulas above two important relations can be derived:

$$\mathbf{RMS}^{-1} = (\boldsymbol{\mu}_T)_\Delta = (\mathbf{X}^{-1})_\Delta \Lambda (\mathbf{X}^{-1})_\Delta \tag{2.2.30}$$

$$(\mathbf{w}_T)_\Delta = (\mathbf{T}\iota)_\Delta = (\mathbf{X}^{-1}\iota)_\Delta^2 \tag{2.2.31}$$

The last member of the relation (2.2.30) is obtained from the expression of the transformation matrices, the normalized price coefficients, and the budget shares in diagonalized form, all expressed in terms of the eigenvector matrix **X.** The second member of the same relation is directly read from the after transformation Rotterdam model (2.2.26). We in fact observe that $(\mathbf{RMS}^{-1})\boldsymbol{\iota} = \boldsymbol{\mu}_T$. Dividing (2.2.30) by (2.2.31) yields the important relation that:

$$\boldsymbol{\Lambda} = \frac{(\boldsymbol{\mu}_T)_\Delta}{(\mathbf{w}_T)_\Delta} \tag{2.2.32}$$

Consequently, the eigenvalues constitutes the transformed good expenditure elasticities.

2.2.2.5. Important formulas

In practice, five formulas are of special interest:

- The composition matrix definition $\mathbf{T} = \mathbf{R}(\mathbf{w})_\Delta$
- The transformation of the commodity budget share into the T-good budget share via the operator $\mathbf{R} : \mathbf{w}_T = \mathbf{R}_{n,n}\mathbf{w}$
- The definition of the Transformation matrix $\mathbf{S}_{n,n}$: $\mathbf{S}_{n,n} = (\mathbf{w}_T)_\Delta^{-1} \mathbf{R}_{n,n} (\mathbf{w})_\Delta$
- The property of the $\mathbf{T}$ matrix that its row sums yield the T-good budget shares: $\mathbf{T}\boldsymbol{\iota} = \mathbf{w}_T$
- The property of the $\mathbf{T}$ matrix that its column sums yield the commodity budget shares: $\boldsymbol{\iota}'\mathbf{T} = \mathbf{w}'$

To visualize the two last properties, we may want to observe it on a general structure of a T matrix of four commodities and four T-goods.

$$
\begin{array}{c}
\left[\begin{array}{ccccc}
r_{1,1}\overline{w}_1 & r_{1,2}\overline{w}_2 & \vdots & r_{1,n-1}\overline{w}_{n-1} & r_{1,n}\overline{w}_n \\
r_{2,1}\overline{w}_1 & r_{2,2}\overline{w}_2 & \vdots & r_{2,n-1}\overline{w}_{n-1} & r_{2,n}\overline{w}_n \\
\cdots & \cdots & & \cdots & \cdots \\
r_{n-1,1}\overline{w}_1 & r_{n-1,2}\overline{w}_2 & \vdots & r_{n-1,n-1}\overline{w}_{n-1} & r_{n-1,n}\overline{w}_n \\
r_{n,1}\overline{w}_1 & r_{n,2}\overline{w}_2 & \vdots & r_{n,n-1}\overline{w}_{n-1} & r_{n,n}\overline{w}_n
\end{array}\right]
\begin{array}{c}
\overline{w}_{1T} \\ \overline{w}_{2T} \\ \vdots \\ \overline{w}_{n-1T} \\ \overline{w}_{nT}
\end{array} \\
\hline
\begin{array}{ccccc}
\overline{w}_1 & \overline{w}_2 & & \overline{w}_{n-1} & \overline{w}_n \quad 1
\end{array}
\end{array}
$$

The column outside the matrix is the T-good budget share. It is the row sum of the $\mathbf{T}_{n,n}$ matrix. This comes from our second formula above. Likewise, the row below the matrix is the column sum that gives the commodity budget shares. This is possible because of the invariance of the income. See (2.2.19). It is easy to see from this matrix that for a given observation period, each element of $\mathbf{R}_{n,n}$ is obtained by the corresponding element in column of $\mathbf{T}_{n,n}$ divided by the column sum. Likewise, each element of $\mathbf{S}_{n,n}$ is obtained by the corresponding element in row of $\mathbf{T}_{n,n}$ divided by the row sum.

2.2.2.6. Sequential procedure to solving the problem

Step 1: we solve the diagonalization problem (2.2.14) to get the eigenvalues and eigenvectors.

Step 2: We need to remember that our original problem is a diagonalization with respect to the budget share. Hence from the solution of step 1 we deduce the solution for the problem (2.2.13). That is the reason we consider

$$k_i = (\overline{w})_{\Delta}^{1/2} x_i \tag{2.2.33}$$

as the characteristic vector for (2.2.14). From it, we deduce the characteristic vector of our initial problem, the relation(2.2.13).

$$x_i = (\overline{w})_{\Delta}^{-1/2} k_i \tag{2.2.34}$$

Step 3: From the knowledge of $\mathbf{X}_{n,n}$, we find the transformation matrices $\mathbf{R}_{n,n}$ and $\mathbf{S}_{n,n}$, and the transformation matrix $\mathbf{T}_{n,n}$. We will expressed all these matrices in terms of $\mathbf{X}$. These additional formulas can be deduced from the main formulas above.

- $\mathbf{R}_{n,n} = \mathbf{B}_{n,n}(\mathbf{X}^{-1}\iota)_{\Delta}\mathbf{X}'_{n,n}$ (2.2.35)

Where $\mathbf{B}_{n,n}$ is any permutation matrix with exactly one in each row and each column, and 0's elsewhere. If we specify $\mathbf{B}_{n,n}$ to be $\mathbf{I}_{n,n}$, then (2.2.35) becomes:

- $\mathbf{R}_{n,n} = (\mathbf{X}^{-1}\iota)_{\Delta}\mathbf{X}'_{n,n}$ (2.2.36)

- $\mathbf{S}_{n,n} = (\mathbf{X}^{-1}\iota)_{\Delta}^{-1}\mathbf{X}_{n,n}^{-1}$ (2.2.37)

- $\mathbf{T}_{n,n} = (\mathbf{X}^{-1}\iota)_{\Delta}\mathbf{X}_{n,n}^{-1}$ (2.2.38)

The next relations lead to the determination of the outputs of the diagonalization. The relation (2.2.39) defines the T-goods budget share in diagonal form. (2.2.40) is commodity income elasticity. (2.2.41) is the T-good marginal share in diagonal form. (2.2.42) is the T-good income elasticity in diagonal form.

- $(\mathbf{w}_T)_{\Delta} = (\mathbf{X}^{-1}\iota)^2$ (2.2.39)

- $\mathbf{\Lambda} = (\mathbf{w})_{\Delta}^{-1}\mathbf{M}_{n,n}\boldsymbol{\iota}$ (2.2.40)

- $(\boldsymbol{\mu}_T)_{\Delta} = \mathbf{R}_{n,n}\mathbf{M}_{n,n}\mathbf{S}_{n,n}^{-1} = (\mathbf{X}^{-1}\iota)_{\Delta}\mathbf{\Lambda}(\mathbf{X}^{-1}\boldsymbol{\iota})_{\Delta}$ (2.2.41)

- $\mathbf{\Lambda}_T = (\mathbf{w}_T)_{\Delta}^{-1}(\boldsymbol{\mu}_T)_{\Delta}$ (2.2.42)

A consequence of these results is that

$$- \quad \mathbf{R}_{n,n}{}^{'}\mathbf{S}_{n,n} = \mathbf{I}_{n,n} \tag{2.2.43}$$

(2.2.43) can be used for verification of the validity of the econometric results as well the constraints on $\mathbf{R}$ and $\mathbf{S}$.

2.2.3. Formulation under the AIDS model

In this part, we explore the possibility of implementing the independent transformation technique to the Almost Ideal Demand System. We recall that the Rotterdam model does not require specifying the utility function, and is in addition determined from welfare maximization under the budget constraint. This will not be the case with the AIDS. The AIDS originated from *the logarithm subclass of price-independent generalized-linear preferences* denoted as PIGLOG. With PIGLOG preferences, the expenditure function is expressed as $e(\mathbf{p},U) = a(\mathbf{p}) + Ub(\mathbf{p})$ and results from an expenditure minimization for some level of utility. $a(\mathbf{p})$ and $b(\mathbf{p})$ are supposed to be positive and linearly homogeneous. We have seen previously that the main AIDS equations are deduced by application of the Shepard's lemma. This is the main difference with the Rotterdam model. As a result, we apply an independent transformation on the Slutsky matrix. For this reason, we refer to the technique as the Slutsky Matrix Independent Transformation. As it was done with the Rotterdam model, it is conceivable to proceed to its diagonalization.

2.2.3.1. Transformation of the AIDS in matrix form

It is important to point out here that we keep all the notations and definitions we used while applying the transformation to the Rotterdam model. In the following developments we will need to write the AIDS in matrix notation. We have seen that the full AIDS can be written as:

$$w_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln p_j + \beta_i [\ln M - \ln P], \text{ where } \ln P = \ln a(\mathbf{p}) \text{ , } i = 1, \ldots, n \text{.}$$

$$\ln a(\mathbf{p}) = a_0 + \sum_{k=1}^{n} a_k \ln p_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \tilde{\beta}_{kj} \ln p_k \ln p_j$$

In matrix notation, it can be expressed as

$$\mathbf{Z}_t = \boldsymbol{\alpha}_{n,1} + \mathbf{B}_{n,n} \boldsymbol{\Pi}_t + \boldsymbol{\Theta}_{n,1} m_t - \boldsymbol{\Theta}_{n,1} (a_0 + \boldsymbol{\alpha}'_{1,n} \boldsymbol{\Pi}_t + \frac{1}{2} (\boldsymbol{\Pi}_t' \mathbf{B}_{n,n} \boldsymbol{\Pi}_t)) + \boldsymbol{\varepsilon}_t \qquad (2.2.44)$$

Where variable matrices are

$$\mathbf{Z}_t = [\, w_{1t} \quad w_{2t} \quad \ldots \quad w_{n-1t} \quad w_{nt} \,]',$$

$$\boldsymbol{\Pi}_t = [\, \ln p_{1t} \quad \ln p_{2t} \quad \ldots \quad \ln p_{n-1t} \quad \ln p_{nt} \,]',$$

and $\boldsymbol{\varepsilon}_t = [\, \varepsilon_{1t} \quad \varepsilon_{2t} \quad \ldots \quad \varepsilon_{n-1t} \quad \varepsilon_{nt} \,]'$.

We express the logarithm of the total expenditure as:

$$m_t = \ln M_t,$$

The parameter matrices are:

$$\boldsymbol{\alpha}_{n,1} = [\, \alpha_{1t} \quad \alpha_{2t} \quad \ldots \quad \alpha_{n-1t} \quad \alpha_{nt} \,]',$$

$$\boldsymbol{\Theta}_{n,1} = [\, \beta_{1t} \quad \beta_{2t} \quad \ldots \quad \beta_{n-1t} \quad \beta_{nt} \,]'$$

$$\mathrm{B}_{n,n} = [\beta_{ij}] \text{.}$$

a_0 is the constant.

As we have seen it in the previous sub-section, the transformation consists of applying the matrix $\mathbf{R}_{n,n}$ to the system of demand and rewriting it in such a way that we will have in the equation only variables in the T-good space. For example, we would like to have in the equation $\mathbf{S}_{n,n}\mathbf{\Pi}_t$ instead of $\mathbf{\Pi}_t$.

First, we apply the matrix $\mathbf{R}_{n,n}$ to (2.2.44) to get:

$$\begin{aligned}\mathbf{R}_{n,n}\mathbf{Z}_t &= \mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1} + \mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{\Pi}_t + \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}m_t - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}a_0 \\ &\quad - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\boldsymbol{\alpha}'_{1,n}\mathbf{\Pi}_t - \frac{1}{2}\mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\mathbf{\Pi}_t{}'\mathbf{B}_{n,n}\mathbf{\Pi}_t) + \mathbf{R}_{n,n}\boldsymbol{\varepsilon}_t\end{aligned} \tag{2.2.45}$$

Next, we apply $\mathbf{S}_{n,n}$ on $\mathbf{\Pi}_t$, using the fact that we does not change the equation when we introduce $\mathbf{S}_{n,n}^{-1}\mathbf{S}_{n,n}$:

$$\begin{aligned}\mathbf{R}_{n,n}\mathbf{Z}_t &= \mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1} + \mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) + \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}m_t \\ &\quad - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}a_0 - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\boldsymbol{\alpha}'_{1,n}\mathbf{S}_{n,n}^{-1}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) \\ &\quad - \frac{1}{2}\mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\mathbf{\Pi}_t{}'\mathbf{S}'_{n,n}(\mathbf{S}'_{n,n})^{-1}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) + \mathbf{R}_{n,n}\boldsymbol{\varepsilon}_t\end{aligned} \tag{2.2.46}$$

Since The two transformation matrices are linked by the relation (2.2.43), we use $\mathbf{S}_{n,n}^{-1} = \mathbf{R}_{n,n}$ on the fifth term of the right-hand side, and $(\mathbf{S}'_{n,n})^{-1} = \mathbf{R}_{n,n}$ on the sixth term of the right-hand side. This yields the final expression of the transformed AIDS:

$$\begin{aligned}\mathbf{R}_{n,n}\mathbf{Z}_t &= \mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1} + \mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) + \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}m_t \\ &\quad - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}a_0 - \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\boldsymbol{\alpha}'_{1,n}\mathbf{R}'_{n,n}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) \\ &\quad - \frac{1}{2}\mathbf{R}_{n,n}\mathbf{\Theta}_{n,1}\mathbf{\Pi}_t{}'\mathbf{S}'_{n,n}\mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1}(\mathbf{S}_{n,n}\mathbf{\Pi}_t) + \mathbf{R}_{n,n}\boldsymbol{\varepsilon}_t\end{aligned}$$

Reformulating the fifth and sixth term of the right hand-side of the equation give the final expression of the AIDS after an independent transformation:

$$\begin{aligned}\mathbf{R}_{n,n}\mathbf{Z}_t &= (\mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1}) + (\mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1})(\mathbf{S}_{n,n}\boldsymbol{\Pi}_t) + (\mathbf{R}_{n,n}\boldsymbol{\Theta}_{n,1})m_t \\ &- (\mathbf{R}_{n,n}\boldsymbol{\Theta}_{n,1})a_0 - (\mathbf{R}_{n,n}\boldsymbol{\Theta}_{n,1})(\mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1})'(\mathbf{S}_{n,n}\boldsymbol{\Pi}_t) \\ &- \frac{1}{2}(\mathbf{R}_{n,n}\boldsymbol{\Theta}_{n,1})(\mathbf{S}_{n,n}\boldsymbol{\Pi}_t)'(\mathbf{R}_{n,n}\mathbf{B}_{n,n}\mathbf{S}_{n,n}^{-1})(\mathbf{S}_{n,n}\boldsymbol{\Pi}_t) + \mathbf{R}_{n,n}\boldsymbol{\varepsilon}_t\end{aligned} \tag{2.2.47}$$

Comparing the before-transformation AIDS in matrix form (2.2.44)to the after-transformation AIDS in matrix notation (2.2.47) allows us to draw the following remarks.

1. Applying the transformation matrix $\mathbf{R}_{n,n}$ to the budget share column matrix of the commodities directly yields the basic want budget share column matrix: $\mathbf{w}_T = \mathbf{R}_{n,n}\mathbf{Z}_t$
2. The SMIT is essentially the move from:

$\mathbf{B}$ to $\mathbf{RBS}^{-1}$,

$\mathbf{S}$ to $\mathbf{S\Pi}$,

$\boldsymbol{\Theta}$ to $\mathbf{R\Theta}$,

$\boldsymbol{\alpha}_{n,1}$ to $\mathbf{R}_{n,n}\boldsymbol{\alpha}_{n,1}$

It should be noted that because of the transformation, $\mathbf{RBS}^{-1}$ is diagonal.

It is also easy to see in view of the AIDS income elasticity that the T-good expenditure elasticitiy matrix for each period t can be expressed as:

$$\mathbf{H}_t = \frac{(\mathbf{R}_{n,n}\mathbf{Z}_t + \mathbf{R}_{n,n}\boldsymbol{\Theta}_{n,1})_\Delta}{(\mathbf{R}_{n,n}\mathbf{Z}_t)_\Delta} \ . \tag{2.2.48}$$

3 Applications of the PIT and SMIT technique

The Preference Independence Transformation is the transformation that only applies on the *specific substitution effect.* Indeed, we will associate it with the Rotterdam model. Concerning the SMIT, the transformation is implemented at the level of the expenditure function Hessian matrix.

3.1 Data description

According to the Energy Information Administration (EIA), the types of energy used in the home are natural gas (45%), electricity (41%), heating oil (8%), and propane (5%). We have obtained the following residential sector data to conduct our analysis:

1. The Electricity Retail Sales to the Residential Sector in million kilowatt-hours and the Average Retail Price of Electricity in Nominal Cents per kilowatt-hour (Taxes included). After cleaning and harmonizing all the data, the quantity for electricity is expressed in million kilowatt hours, the price in dollars per kilowatt hours.
2. The Natural Gas Consumed by the Residential Sector in Billion Cubic Feet and the Natural Gas Price, Delivered to Residential Consumers expressed in Nominal Dollars per Thousand Cubic Feet. For reason of consistency, the quantity is expressed in million cubic feet, and the price in dollars per cubic feet.

3. The raw Distillate Fuel Oil Consumed by the Residential Sector expressed in Thousand Barrels (42 gallons) per Day is the number 2. Distillate Price to Residences, U.S. Average, expressed in Nominal Cents per Gallon Excluding Taxes. Some technical explanations need to be made. Heating oil is refined from oil; it is what Americans use to heat their homes. Note that at refineries, crude oil is separated into different fuels such as gasoline, kerosene, lubricating oil, heating oil, and diesel. Heating oil and the diesel fuel are denoted as distillates. The difference between the two is that the heating oil contains more sulfur. It should, however, be stated that the number 2 fuel oil is the main heating oil in US residences. The quantity and price of the final data we use on Distillate Fuel Oil are respectively in million gallon and dollar per gallon.
4. The Liquefied Petroleum Gases (LPG) are mixtures of propane, ethane, normal butane, and isobutane. However, American homes basically use propane. We express the quantity of LPG in Million gallons and the price in dollar per gallon.

All data start on January 1995 and end on August 2010, which corresponds to 187 observations.

3.2 The Preference Independence Transformation under the Rotterdam model

We have previously seen that the Rotterdam model is a discrete formulation of the differential system. As such, two estimation procedures are available: the absolute version estimation, linear in the parameters, and the relative version estimation which involves a nonlinear estimation. The application of the Preference Independence Transformation under the Rotterdam model requires a nonlinear estimation as it gives the possibility to extract the normalized price coefficient v_{ij}, which gives the *specific* nature—in the Houthakker's sense—of the relation between the commodities involved. We recall that when v_{ij} is positive the commodities i and j are *specific complements*, and vice versa. Brooks, Theil, Barnett, and followers have used it are the core of the transformation. This does not mean that we should omit the absolute version. In practice, the Absolute version estimation enables us to control the validity of the relative version estimation. It is also very useful, when we will proceed to the conditional demand estimation that will be necessary for the relative version estimation.

In the following developments, we will respectively proceed to the absolute version estimation, the relative version estimation, and the diagonalization of the normalized price coefficient matrix.

3.2.1. The absolute version estimation

As a first approach, the model of the absolute version can be expressed as follow

$$\bar{w}_{it} Dq_{it} = \mu_i DQ_t + \sum_{i=1}^{4} \pi_{ij} Dp_{jt} + \varepsilon_{it}, \qquad i = 1,..4 \qquad (3.1.1)$$

Where the parameters are the marginal budget share, μ_i , and the $(i, j)^{th}$ (Slutsky, E. 1915) coefficient. The latter can be explicitly written as $\pi_{ij} = v_{ij} - \phi \mu_i \mu_j$ with $i, j = 1,...,4$. Four restrictions are of paramount importance:

- The adding-up restriction: $\sum_{i=1}^{n} \mu_i = 1$
- The homogeneity condition:
- This is an attribute of a rational consumer that is not veiled by the nominal change of income. It precisely means that he is insensitive to any simultaneous and proportionate change in income and prices. In other words, consumption is

only sensitive to real income change. In the Rotterdam model, this is translated by $\sum_{j=1}^{4} \pi_{ij} = 0$.

- The Slutsky symmetry condition:

In essence, this is an economic translation of the mathematic Young's theorem that stipulates that when a valued function of n variables is twice continuously differentiable on its domain, then, on the interior of its domain, the n x n matrix of second-order partial derivatives is symmetric. This is the case when

$$\pi_{ij} = \pi_{ji}, \qquad i, j = 1, \ldots, 4$$

In practice, real data always show discrete patterns; their changes are not infinitesimal. Hence, it is always an approximation to suppose that demands for goods are symmetric in their interactions. Still, for the sake of the theory elegance, this condition is widely accepted as one of the regularity condition of utility theory.

- The negative semi-definiteness of the Slutsky matrix of rank 3.

Our study takes into account four goods: electricity, distillate fuel oil, gas, and liquefied petroleum gas. The negative semi-definiteness could be translated by the three following inequalities:

$$\pi_{11} \leq 0, \quad \begin{vmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} \geq 0, \quad \begin{vmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{vmatrix} \leq 0,$$

3.2.1.1. Redundancy issue and last equation parameter derivation

3.2.1.1.1. Redundancy issue

To show that one of the equations is redundant, it suffices to add all the equations to see that

$$\sum_{i=1}^{4} \varepsilon_{it} = 0, \qquad\qquad t=1,...,T.$$

In fact, this is the case from the following summation

$$\sum_{i=1}^{4} \bar{w}_{it} Dq_{it} = DQ_t \sum_{i=1}^{4} \mu_i + \sum_{i=1}^{4} \left(\sum_{j=1}^{4} \pi_{ij} \right) Dp_{jt} + \sum_{i=1}^{4} \varepsilon_{it} \; .$$

By invoking the Divisia formula, the adding-up restriction, and the homogeneity condition, the equation is equivalent to

$$DQ_t = DQ_t + \sum_{i=1}^{4} \varepsilon_{it}$$

In conclusion, the disturbances are linearly dependent and the resulting variance-covariance matrix is singular. This is a reason why we said previously that the Slutsky matrix is of rank 3, i.e. n-1. One equation has to be dropped, no matter which one.

3.2.1.1.2. Parameter derivation of the last equation

By simplification, let us suppose we deleted the last equation as recommended above. It could be any equation. After the estimation we need to recover its parmeters and corresponding standard deviations. To do this, let us add the three undeleted equations.

$$\sum_{i=1}^{3} \bar{w}_{it} Dq_{it} = DQ_t \sum_{i=1}^{3} \mu_i + \sum_{i=1}^{3} \left(\sum_{j=1}^{4} \pi_{ij} \right) Dp_{jt} + \sum_{i=1}^{3} \varepsilon_{it}$$

We can observe the followings: $\sum_{i=1}^{3}\bar{w}_{it}Dq_{it} = DQ_t - \bar{w}_{4t}Dq_{4t}$, $\sum_{i=1}^{3}\mu_i = 1-\mu_4$,

$\sum_{i=1}^{3}\varepsilon_{it} = -\varepsilon_{4t}$, and $\sum_{i=1}^{3}\pi_{ij} = -\pi_{4j}$ (by homogeneity) . Plugging these expressions into the equation, multiplying both sides by -1, and adding on both sides DQ_t enables us to get the last equation:

$$w_{4t} = \mu_4 DQ_t + \sum_{j=1}^{4}\pi_{4j}DP_{jt} + \varepsilon_{4t} \tag{3.1.2}$$

3.2.1.1.3. Derivation of the standard deviations

From now on, we suppose that equation 3 is the dropped one. This assumption is taken for the sake of consistency with the next developments. We will provide the reason on subsections 3.2.1.2.

The following eight equations enter into the last equation parameter derivation:

$$1-\mu_3 = \mu_1 + \mu_2 + \mu_4 \tag{3.1.3}$$

$$\pi_{13} = -\pi_{11} - \pi_{12} - \pi_{14} \tag{3.1.4}$$

$$\pi_{23} = -\pi_{21} - \pi_{22} - \pi_{24} \tag{3.1.5}$$

$$\begin{aligned}\pi_{33} &= -\pi_{13} - \pi_{23} - \pi_{43} \\ &= (\pi_{11} + \pi_{12} + \pi_{14}) + (\pi_{21} + \pi_{22} + \pi_{24}) + (\pi_{14} + \pi_{24} + \pi_{44})\end{aligned} \tag{3.1.6}$$

$$\pi_{34} = -\pi_{14} - \pi_{24} - \pi_{44} \tag{3.1.7}$$

$$\pi_{31} = -\pi_{11} - \pi_{21} - \pi_{41} \tag{3.1.8}$$

$$\pi_{32} = -\pi_{12} - \pi_{22} - \pi_{42} \quad (3.1.9)$$

$$\pi_{43} = -\pi_{41} - \pi_{42} - \pi_{44} \quad (3.1.10)$$

We can write this system in matrix form as

$$\mathbf{A}_{8,1} = \mathbf{L}_{8,12}\mathbf{C}_{12,1} \quad (3.1.11)$$

where

$$\mathbf{A}_{8,1} = [1-\mu_3 \quad \pi_{13} \quad \pi_{23} \quad \pi_{33} \quad \pi_{34} \quad \pi_{31} \quad \pi_{32} \quad \pi_{43}]',$$

$$\mathbf{C}_{12,1} = [\mu_1 \quad \pi_{11} \quad \pi_{12} \quad \pi_{14} \quad \mu_2 \quad \pi_{21} \quad \pi_{22} \quad \pi_{24} \quad \mu_4 \quad \pi_{41} \quad \pi_{42} \quad \pi_{44}]', \text{ and}$$

$$\mathbf{L}_{8,1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

It is noteworthy that $\mathbf{L}_{8,12}$ is the matrix that enables the passage from the matrix of the estimates to the parameter matrix of the dropped equation. Since the relation (3.1.11) is linear, we can derive the variance by considering

$$Var(\mathbf{A}_{8,1}) = \mathbf{L}_{8,12}Var(\mathbf{C})\mathbf{L}'_{12,8} \quad (3.1.12)$$

The 12×12 square matrix $Var(\mathbf{C})$ can be provided by Stata after finishing the estimation.

3.2.1.2. Econometric estimation and results

In the above developments, we pointed out the importance of dropping one equation to circumvent the singularity effect of the variance-covariance matrix. We first conducted preliminaries estimation using the seemingly unrelated regression and dropping the third equation. Obviously, it could be easier to drop the last equation. However, our first estimations have yielded a Slutsky coefficient matrix in which one of the diagonal entries (coefficient of LPG) is positive. A closer examination shows that the coefficient is not significant at 5 %. Consequently, it could be considered as 0, which is consistent with the negative definiteness of the Hessian matrix which requires that all the diagonal elements be less or equal to 0 ($\pi_{ii} \leq 0$ for all $i \in [1,4]$). However we found embarrassing for the PIT unfolding that the entry π_{44} vanishes. After reconsidering the data, we found that lagging the prices of DFO (commodity 2) and LPG (good 4) by three periods solves the problem when all the regularity conditions (adding-up, symmetry, homogeneity) are incorporated. In that case, all the diagonal elements become negative. This is comprehensible if we remember the fact that the two goods (DFO, LPG) are not consumed all the year. Households use them basically for heating, which only occurs during the span October to March. Hence, it seems plausible that their current consumptions depend on lagged prices by three months. This explains why we have chosen not to drop one of the goods for which the prices are lagged. Instead, we have chosen to lag the third commodity (gas).Thereby we end up having 184 observations instead of 187.

3.2.1.2.1. Estimation technique

With the third equation dropped, the absolute version can be finally expressed as:

$$\bar{w}_{it} Dq_{it} = \mu_i DQ_t + \sum_{j \neq 3} \pi_{ij} (Dp_{jt} - Dp_{3t}) + \varepsilon_{it}, \quad i = 1,2,4. \tag{3.1.13}$$

From the initial absolute version equation, we have dropped the third equation and used the expression $\pi_{i3} = -(\pi_{i1} + \pi_{i2} + \pi_{i4}) = \sum_{j \neq 3} \pi_{ij}$ to get this ready-to-use equation.

Zellner's Seemingly Unrelated Technique (*SUR*) is appropriate for estimating this system. It is a joint estimation of several regressions. It is presumed that the contemporaneous errors associated with the dependent variables are correlated. SUR on a set of regressions presenting the same explanatory variables on the right hand side give identical results of individual OLS estimations of the models. However we would still gain from the SUR if we have to perform some joint tests.

Let $\mathbf{y}_i = [\,\bar{w}_{i1}Dq_{i1} \quad \bar{w}_{i2}Dq_2 \quad \ldots \quad \bar{w}_{i184}Dq_{i184}\,]'$, $\boldsymbol{\beta}_i = [\,\mu_i \quad \pi_{i1} \quad \pi_{i2} \quad \pi_{i4}\,]'$,

$\boldsymbol{\varepsilon}_i = [\,\varepsilon_{i1} \quad \varepsilon_{i2} \quad \ldots \quad \varepsilon_{i184}\,]'$, and

$$\mathbf{X}_{T,n} = \begin{bmatrix} DQ_1 & Dpx_{1,1} & \ldots & Dpx_{1,n-1} \\ \vdots & & & \vdots \\ DQ_T & Dpx_{T,1} & \ldots & Dpx_{T,n-1} \end{bmatrix}, \text{ where } Dpx_{t,j} = Dp_{jt} - Dp3t.$$

The absolute version model can be reformulated as:

$$\mathbf{y}_i = \mathbf{X}\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i \qquad \text{where } i = 1,2,4 \qquad (3.1.14)$$

Or more explicitly,

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{X} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_4 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_4 \end{bmatrix} \qquad (3.1.15)$$

This last equation can be succinctly written as

$$\mathbf{y} = (\mathbf{I}_{n-1} \otimes \mathbf{X})\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{3.1.16}$$

$\mathbf{I}_{n-1}$ is the identity matrix of order n-1. $\otimes$ is the Kronecker product.

The following assumptions apply to the disturbances:

$$E(\varepsilon_{it}) = 0$$

$$E(\varepsilon_{is}\varepsilon_{jt}) = \begin{cases} 0 \text{ if } s \neq t \\ \omega_{ij} \text{ otherwise} \end{cases}$$

Let us consider the contemporaneous variance-covariance matrix $\mathbf{\Omega}_{n-1,n-1} = [\omega_{ij}],\ \text{i,j} \neq 3$. Not eliminating the third equation—in our case—or anyone of the four equations—in the general case—would make $\mathbf{\Omega}_{n,n}$ singular.

Hence, we have

$$E(\boldsymbol{\varepsilon\varepsilon}') = \begin{bmatrix} \omega_{11}\mathbf{I}_{T,T} & \omega_{12}\mathbf{I}_{T,T} & \omega_{14}\mathbf{I}_{T,T} \\ \omega_{21}\mathbf{I}_{T,T} & \omega_{22}\mathbf{I}_{T,T} & \omega_{24}\mathbf{I}_{T,T} \\ \omega_{41}\mathbf{I}_{T,T} & \omega_{42}\mathbf{I}_{T,T} & \omega_{44}\mathbf{I}_{T,T} \end{bmatrix} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{14} \\ \omega_{21} & \omega_{22} & \omega_{24} \\ \omega_{41} & \omega_{42} & \omega_{44} \end{bmatrix} \oplus \mathbf{I}_{T,T} = \mathbf{\Omega}_{3,3} \otimes \mathbf{I}_{T,T}$$

We can show that the SUR will give the following results:

$$\hat{\boldsymbol{\beta}} = \left[\mathbf{I} \otimes (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \right]\mathbf{y} \tag{3.1.17}$$

$$Var(\hat{\boldsymbol{\beta}}) = \left[\mathbf{\Omega} \otimes (\mathbf{X}'\mathbf{X})^{-1} \right] \tag{3.1.18}$$

3.2.1.2.2. Results

Two preliminary estimations have been conducted. First (see Appendix A), we estimate the absolute version without any restrictions. It turns out that the constants were not significant. At this level, only marginal shares are of interest. The marginal share is higher for gas (47%). In second position, we have the marginal share electricity (45%). DFO, and gas are expected presents the same marginal share. Consequently, without any restriction on the estimation, gas is more sensitive to the total expenditure. 45% of the increase of the total expenditure is due to gas consumption. Dropping the constants (see appendix B) slightly improve the estimation.

Table 1: **Absolute version with all regularity conditions (adding-up, symmetry, homogeneity)**

Equation	Observation	Parameter	RMSE	R^2	χ^2	p-value
Equation 1	184	4	.0487502	0.8003	811.51	0.0000
Equation 2	184	4	.0062405	0.5932	129.26	0.0000
Equation 4	184	4	.0055228	0.7638	591.28	0.0000

	Coefficients	Std. error	t-student	p-value	Confidence intervals	
Equation 1						
DQt	.4271871	.0201586	21.19	0.000	.3876769	.4666973
Elec.	-.857377	.0485713	-17.65	0.000	-.9525751	-.7621789
Dfo	.0214555	.0093793	2.29	0.022	.0030724	.0398387
Lpg	.1049314	.005809	18.06	0.000	.0935459	.1163168
Equation 2						
DQt	.0421955	.0039182	10.77	0.000	.034516	.0498749
Elec.	.0214555	.0093793	2.29	0.022	.0030724	.0398387
Dfo	-.0120974	.0079147	-1.53	0.126	-.0276099	.0034151
Lpg	-.0062495	.0036314	-1.72	0.085	-.0133669	.0008678
Equation 4						
DQt	.0394701	.0023984	16.46	0.000	.0347693	.0441708
Elec.	.1049314	.005809	18.06	0.000	.0935459	.1163168
Dfo	-.0062495	.0036314	-1.72	0.085	-.0133669	.0008678
Lpg	-.0059645	.0034273	-1.74	0.082	-.0126818	.0007528

The results contained in the above table excludes the third equation parmaters. The two subsequent tables gives respectively the matrix **L** --see equation (3.1.12)—that allows to uncover the missing estimates related to the third equation.

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Table 2: **The matrix $\mathbf{L}_{8,12}$ enabling the moves from** Table 1 **estimates to last equation parameter estimates**

	μ_1	π_{11}	π_{12}	π_{14}	μ_2	π_{21}	π_{22}	π_{24}	μ_4	π_{41}	π_{42}	π_{44}
μ_3	1	0	0	0	1	0	0	0	1	0	0	0
π_{13}	0	-1	-1	-1	0	0	0	0	0	0	0	0
π_{23}	0	0	0	0	0	-1	-1	-1	0	0	0	0
π_{33}	0	1	1	1	0	1	1	1	0	1	1	1
π_{34}	0	0	0	-1	0	0	0	-1	0	0	0	-1
π_{31}	0	-1	0	0	0	-1	0	0	0	-1	0	0
π_{32}	0	0	-1	0	0	0	-1	0	0	0	-1	0
π_{43}	0	0	0	0	0	0	0	0	0	-1	-1	-1

Table 3: **The asymptotically estimated variance-covariance of the last equation**

	μ_3	π_{13}	π_{23}	π_{33}	π_{34}	π_{31}	π_{32}	π_{43}
μ_3	**.00030613**							
π_{13}	-8.841e-06	**.00184058**						
π_{23}	.0000129	.00002653	**.00014105**					
π_{33}	-3.563e-06	-.00168514	-.0001528	**.00167652**				
π_{34}	-4.915e-07	-.00018197	-.00001478	.00016142	**.00003533**			
π_{31}	-8.841e-06	.00184058	.00002653	-.00168514	-.00018197	**.00184058**		
π_{32}	.0000129	.00002653	.00014105	-.0001528	-.00001478	.00002653	**.00014105**	

π_{43}	-4.915e-07	-.00018197	-.00001478	.00016142	.00003533	-.00018197	-.00001478	**.00003533**

Finally the complete estimates of marginal shares and Slutsky coefficients are displayed in the subsequent table. Among all the four goods Gas is the most sensitive to its price. Table 5 shows that an increase of the gas price by 1% decreases Gas consumption by 2.6%.

Table 4: **Marginal share and Slutsky coefficient estimates**

Marginal share	Slutsky coefficients			
.4271871	**-.857377**	.0214555	.7309901	.1049314
(.02016)	*(.04857)*	*(.00938)*	*(.04290)*	*(.00581)*
.0421955	.0214555	**-.0120974**	-.0031086	-.0062495
(.00392)	*(.00938)*	*(.00791)*	*(.01188)*	*(.00363)*
.4911473	.7309901	-.0031086	**-.6351641**	-.0927174
(.01750)	*(.04290)*	*(.01188)*	*(.04095)*	*(.00594)*
.0394701	.1049314	-.0062495	-.0927174	**-.0059645**
(.00240)	*(.00581)*	*(.00363)*	*(.00594)*	*(.00343)*

After getting all the estimates, It is insightful to check the overall income and price elasticities and their evolutions over the time as well to get a complete picture of the issue.

Expenditure and price elasticitites

Table 5 displays the overall expenditure and price elasticities. At this level we should insist on the use of expenditure instead of income. It is somehow unreasonable to mistake the expenditure elasticity for the income elasticity. For example, gas expenditure elasticity over the period is 2 %. We may be tempted to infer that gas is a luxury. It is however hard to believe that any of these four goods could acceptably be viewed as a luxury. To clarify the paradox, we need to remember that the Rotterdam model—this will be the case also for the AIDS model—does not consider the income in its construction but rather the total expenditure. The US household energy total expenditure is in fact a portion of the US household income. From this perspective our four goods could be viewed as a block of commodities detachable from the set all goods purchased by the US households.

Suppose that R is the income of a US household and M the portion of that income allocated to the residential energy consumption as a block of commodities (electricity, dfo,gas,and lpg). Let ε_{q_iR} the income elasticity for good i with respect to q_i; and η_{q_iM} the total expenditure for good i with respect to q_i. We show:

$$\begin{aligned}\varepsilon_{q_iR} &= \frac{\partial q_i}{\partial R}\frac{R}{q_i} = \left(\frac{\partial q_i}{\partial M}\frac{\partial M}{\partial R}\right)\left(\frac{R}{M}\frac{M}{q_i}\right) \\ &= \left(\frac{\partial M}{\partial R}\frac{R}{M}\right)\left(\frac{\partial q_i}{\partial M}\frac{M}{q_i}\right) \\ &= \left(\frac{\partial M}{\partial R}\frac{R}{M}\right).\eta_{q_iM}\end{aligned} \tag{3.1.19}$$

We identify the first element of the last equality member as the *elasticity of energy total expenditure* with respect to the income. It gives the percentage of increase in the total expenditure whenever the income increases by 1%. In the US, this percentage may range, depending on the income interval in which belongs the household. According to American Coalition for Clean Electricity (ACCE), in 2012, US families with gross annual income below $ 50,000 were expected to spend 8% of their after-tax income of 22,390 on residential energy.

These families represent 50% of the US households. The average US household who has an estimated after-tax income of $53,229 spend 4% of their after tax income on residential energy.

Finally, having a gas expenditure elasticity of 2% does not mean at all that gas is a luxury good. All that can be inferred is that gas consumption is the most sensitive to the expenditure change. Indeed when total expenditure increases, US households tend to first restrict the Gas consumption. We note also that DFO, LPG, and gas are complement. Electricity and gas appear as substitute. Over the time the elasticities are not very flexible.

Table 5: **The average total expenditure and price elasticities during the 184 months**

	EXPENDITURE ELASTICITIES	**ELE**	**DFO**	**GAS**	**LPG**
ELEC	.64079947	-1.2861033	.03218419	1.0965173	.15740172
DFO	.86350242	.43907232	-.24756512	-.0636154	-.1278918
GAS	2.0189012	3.0047947	-.01277815	-2.6108941	-.38112247
LPG	1.568138	4.1689003	-.24829119	-3.6836409	-.23696821

Table 6 : **Dynamics of the compensated price and income elasticities**

OBS.	Commodities	Income eslaticities	Cross and direct price elasticities ELE	DFO	GAS	LPG
1	Ele	0.74331628	-1.4918575	0.03733311	1.2719411	0.18258327
1	Dfo	0.87059921	0.44268089	-0.24959977	-0.06413823	-0.12894289
1	Gas	1.3618764	2.0269239	-0.00861967	-1.761213	-0.25709118
1	Lpg	2.4381794	6.4819085	-0.38604924	-5.7274152	-0.36844398
2	Ele	0.72458275	-1.4542588	0.03639222	1.2398849	0.17798169
2	Dfo	0.83426493	0.42420569	-0.23918277	-0.06146143	-0.12356149
2	Gas	1.4300387	2.1283719	-0.00905109	-1.8493621	-0.26995866
2	Lpg	2.4054417	6.3948751	-0.38086571	-5.6505126	-0.36349684
3	Ele	0.68357668	-1.3719584	0.03433268	1.1697165	0.16790923
3	Dfo	0.83174176	0.42292271	-0.23845938	-0.06127555	-0.12318778
3	Gas	1.6062157	2.3905818	-0.01016616	-2.0771988	-0.30321687
3	Lpg	2.1266162	5.6536168	-0.33671788	-4.9955366	-0.32136231
4	Ele	0.62373511	-1.2518546	0.03132714	1.0673173	0.15321015
4	Dfo	0.85557338	0.43504058	-0.24529188	-0.06303126	-0.12671744
4	Gas	2.0038268	2.9823589	-0.01268274	-2.5913994	-0.37827676
4	Lpg	1.9075506	5.0712301	-0.30203211	-4.4809396	-0.28825835
5	Ele	0.55662891	-1.1171705	0.02795672	0.95248715	0.13672663
5	Dfo	0.95133369	0.48373263	-0.27274625	-0.07008605	-0.14090033
5	Gas	2.9076897	4.3276067	-0.01840353	-3.7602978	-0.54890544
5	Lpg	2.0473415	5.4428645	-0.3241659	-4.8093158	-0.30938275
6	Ele	0.51202666	-1.0276525	0.02571657	0.87616508	0.12577083
6	Dfo	1.1604564	0.59006701	-0.33270148	-0.08549241	-0.17187312
6	Gas	4.3247357	6.4366413	-0.02737239	-5.5928575	-0.81641139
6	Lpg	2.5035747	6.6557621	-0.3964036	-5.8810323	-0.37832615

OBS.	Commodities	Income eslaticities	Cross and direct price elasticities ELE	DFO	GAS	LPG
91	Ele	0.49820438	-0.99991075	0.02502235	0.85251279	0.12237561
91	Dfo	1.340946	0.68184206	-0.38444763	-0.09878932	-0.19860511
91	Gas	5.3927456	8.0261943	-0.0341321	-6.9740349	-1.0180273
91	Lpg	1.9731149	5.245533	-0.31241324	-4.6349537	-0.29816605
92	Ele	0.50489301	-1.013335	0.02535828	0.86395818	0.12401856
92	Dfo	1.1777134	0.59884182	-0.33764904	-0.08676375	-0.17442903
92	Gas	5.0453877	7.5092105	-0.03193358	-6.5248229	-0.95245404
92	Lpg	1.9038645	5.0614304	-0.30144846	-4.4722806	-0.28770131
93	Ele	0.5484372	-1.1007295	0.02754529	0.93846973	0.13471447
93	Dfo	0.91268505	0.46408063	-0.26166573	-0.06723875	-0.13517615
93	Gas	3.2585075	4.84974	-0.02062395	-4.2139842	-0.61513183
93	Lpg	1.6362034	4.3498524	-0.25906833	-3.8435302	-0.24725387
94	Ele	0.64166193	-1.2878343	0.03222751	1.0979932	0.15761357
94	Dfo	0.80383517	0.40873282	-0.23045859	-0.05921963	-0.11905459
94	Gas	1.916427	2.8522791	-0.01212957	-2.4783719	-0.36177768
94	Lpg	1.5494626	4.1192518	-0.24533423	-3.6397715	-0.2341461
95	Ele	0.74052941	-1.4862642	0.03719314	1.2671723	0.18189872
95	Dfo	0.77957438	0.39639673	-0.22350305	-0.05743231	-0.11546137
95	Gas	1.4161226	2.1076601	-0.00896301	-1.8313655	-0.26733162
95	Lpg	1.7794471	4.7306665	-0.28174884	-4.1800176	-0.26890006
96	Ele	0.80263687	-1.6109157	0.04031249	1.3734488	0.19715439
96	Dfo	0.73221011	0.37231302	-0.20992377	-0.05394292	-0.10844633
96	Gas	1.2610026	1.8767901	-0.00798122	-1.6307604	-0.2380485
96	Lpg	1.9110813	5.0806164	-0.30259114	-4.4892334	-0.28879188

OBS.	Commodities	Income eslaticities	Cross and direct price elasticities			
			ELE	DFO	GAS	LPG
182	Ele	0.73220667	-1.4695602	0.03677513	1.2529307	0.17985438
182	Dfo	0.99648959	0.50669342	-0.28569239	-0.07341275	-0.14758829
182	Gas	1.4444757	2.1498589	-0.00914247	-1.8680324	-0.27268403
182	Lpg	1.1536271	3.0669217	-0.1826596	-2.7099325	-0.17432965
183	Ele	0.66659979	-1.3378853	0.03348002	1.1406661	0.16373914
183	Dfo	1.1749154	0.59741909	-0.33684685	-0.08655762	-0.17401462
183	Gas	1.7344816	2.581484	-0.01097799	-2.2430755	-0.32743055
183	Lpg	0.9849111	2.6183896	-0.15594594	-2.3136095	-0.14883424
184	Ele	0.6001841	-1.204587	0.03014429	1.0270175	0.14742523
184	Dfo	1.1547472	0.58716398	-0.33106465	-0.0850718	-0.17102754
184	Gas	2.3410714	3.4842908	-0.01481725	-3.0275327	-0.44194084
184	Lpg	0.94193904	2.5041483	-0.14914196	-2.2126658	-0.14234054
185	Ele	0.54197701	-1.0877637	0.02722083	0.92741525	0.13312763
185	Dfo	1.1624136	0.59106219	-0.3332626	-0.08563659	-0.17216299
185	Gas	3.5385892	5.2665945	-0.02239666	-4.5761929	-0.66800487
185	Lpg	1.0754474	2.8590807	-0.17028101	-2.5262841	-0.16251557
186	Ele	0.50454706	-1.0126407	0.02534091	0.8633662	0.12393359
186	Dfo	1.4627226	0.74376286	-0.41936085	-0.10776077	-0.21664123
186	Gas	5.207839	7.7509919	-0.03296178	-6.734909	-0.98312113
186	Lpg	1.3082955	3.4781082	-0.20714903	-3.0732569	-0.19770228
187	Ele	0.49211523	-0.98768966	0.02471652	0.84209323	0.12087991
187	Dfo	1.9152667	0.97387171	-0.54910469	-0.14110031	-0.28366672
187	Gas	6.0465285	8.9992401	-0.03827006	-7.8195235	-1.1414466
187	Lpg	1.3763323	3.6589844	-0.21792164	-3.2330791	-0.20798362

3.2.2. The relative version estimation

The relative version system can be formulated as follows:

$$\bar{w}_i Dq_{it} = \mu_i DQ_t + \sum_{j=1}^{n} v_{ij} (DP_{jt} - \sum_{j=1}^{4} v_{ij} \left(Dp_{jt} - \sum_{k=1}^{4} \mu_k Dp_{kt} \right) + \varepsilon_{it}, \quad (3.1.20)$$

$$i=1,\ldots,4$$

The Slutsky equation, aboved determined, $\pi_{ij} = v_{ij} - \phi\mu_i\mu_j$, links the previous model and the relative version. Note that the Slutsky coefficients are replaced with their expressions in terms of the price coefficients, the income flexibility, and the marginal shares. This makes the new model nonlinear in the parameters. As an estimation technique, we will need to use the feasible generalized nonlinear least square which is a nonlinear version of Zellner's Seemingly Unrelated regression model. The literature denotes it as nonlinear SUR (*NLSUR*). *NLSUR* becomes relevant when the conditional means of the dependent variables given the in dependent variables are nonlinear. It can be expressed as

$$\mathbf{Y}_t = H(\boldsymbol{\beta}, \mathbf{X}_t) + \mathbf{U}_t,$$

where $t = 1,\ldots,T$ observations ;

$n = 1,\ldots,N$ equations, and

$$H(\boldsymbol{\beta}, \mathbf{X}_t) = (h_1(\beta, x_{1t}),\ldots,h_N(\beta, x_{Nt}))$$

In practice, the relative version estimation is the appropriate model for applying the Preference Independence Transformation technique as it enables to directly obtain the price coefficient matrix. Under the assumption that basic wants should have positive basic wants, some identification problem may be raised. Barnett (981) suggests to circumventing the issue by introducing the assumption of block independence. Depending on the number of goods in the models, at least one good should be assumed block-independent from the others. Obviously, some test is required to check which good is more likely to be block-independent from the others.

As in the absolute version, it is necessary to drop one equation to avoid the singularity issue. Theil (1975) has shown how we can delete one equation by reformulating the relative version in a more convenient way.

In the next sections, before conducting the estimation, strictly speaking, we will follow Theil by showing how to reformulate the relative version by deleting the last equation. We will also define some concepts that are related to the block independence notion. The estimation will be conducted gradually. We will first estimate a full version of the model without imposing the regularity conditions. Next we will proceed to the estimation of the full version with all the regularity conditions. Finally, under the block independence assumption, we will estimate the conditional demand before deriving the normalized price coefficient matrix. The last part of this chapter will present the diagonalization process.

3.2.2.1. On dropping the last equation

Dropping the last equation ($i = 4$) or any other equation from the previous formulation (3.1.20) of the relative version is not enough. This is the case because we still need to drop the parameter μ_4 inside the second term of the right hand side.

Theil observed that, since the row sums of the normalized price coefficients yields the product of the income flexibility and the marginal shares, $v_{ii} = \phi\mu_i - \sum_{i \neq j} v_{ij}$. Therefore the second term can be written as

$$\sum_{j=1}^{4} v_{ij} (Dp_{jt} - \sum_{k=1}^{4} \mu_4 Dp_{4t}) = v_{ii} (Dp_{it} - \sum_{k=1}^{4} \mu_k Dp_{kt}) + \sum_{i \neq j} v_{ij} (Dp_{jt} - \sum_{k=1}^{4} \mu_k Dp_{kt})$$

$$= (\phi\mu_i - \sum_{i \neq j} v_{ij})(Dp_{it} - \sum_{k=1}^{4} \mu_k Dp_{kt}) + \sum_{i \neq j} v_{ij} (Dp_{jt} - \sum_{k=1}^{4} \mu_k Dp_{kt})$$

Using the adding-up condition, $\mu_4 = 1 - \sum_{k=1}^{3} \mu_k$, we have

$$\sum_{j=1}^{4} v_{ij} (Dp_{jt} - \sum_{k=1}^{4} \mu_4 Dp_{4t}) = \phi\mu_i \left((Dp_{it} - Dp_{4t}) - \sum_{k=1}^{3} \mu_k (Dp_{kt} - Dp_{4t}) \right)$$
$$+ \sum_{i \neq j} v_{ij} (Dp_{jt} - Dp_{it})$$

Hence the relative version can be formulated as

$$\bar{w}_i Dq_{it} = \mu_i DQ_t + \phi\mu_i \left(Dpx_{it} - \sum_{k=1}^{3} \mu_k (Dp_{kt} - Dp_{4t}) \right), \text{ where } i = 1,...,3 \quad (3.1.21)$$
$$\text{and } Dpx_{it} = Dp_{it} - Dp_{4t}$$

3.2.2.2. Preference independence, block-independence, blockwise dependence

Preference independence

Preference independence occurs when all the two-by-two commodities are independent. There are no two goods that are *specific substitutes* or *specific complements*. This is the case when

$$v_{ij} = \begin{cases} 0 \text{ for } i \neq j \\ \phi\mu_i \text{ for } i = j \end{cases} \quad i, j = 1, \ldots, n$$

The preference system can then be represented by a utility function expressed as the sum of n sub-utility functions corresponding to the n goods. Such a utility function presents a diagonal Hessian matrix. Its Slutsky coefficients are

$$\pi_{ij} = \begin{cases} \phi\mu_i - \phi\mu_i^2 \text{ for } i = j \\ -\phi\mu_i\mu_j, \text{ for } i \neq j \end{cases}$$

We observe that because of the negativity of the income flexibility (ϕ), the off-diagonals are positive.

Block independence

Whereas the preference independence assumption sets the independence on the commodities, the block independence puts the additivity-assumption on the groups of goods. Suppose that we have G groups of goods ($G < n$) that are mutually exclusive and exhaustive blocks: $S_1,..,S_G$. The utility is henceforth the sum of G group sub-utilities,

$$U(\mathbf{q}) = \sum_{g=1}^{G} U_g(\mathbf{q}_g).$$

In this formula, $\mathbf{q} = (\mathbf{q}_{1,}\mathbf{q}_{2,},...,\mathbf{q}_g,...,\mathbf{q}_G)$ and $\mathbf{q}_g$ is the column vector of quantities available to S_g. In that case, the marginal utility of a good is a function of the consumption of only the goods belonging to the same groups, not those belonging to other groups. As a consequence, the Hessian matrix of such a utility function becomes block-diagonal. This means that for $i \in g, j \in g', g \neq g'$, $\frac{\partial^2 U}{\partial q_i \partial q_j} = 0$ and for $i, j \in g$, $\frac{\partial^2 U}{\partial q_i \partial q_j} \neq 0$.

(Barnett, W. A. 1981, p.53) strongly recommended using this approach to avoid "identification problems." It consists of setting, from the matrix of price coefficients, $v_{ij} = 0$ whenever i and j are in different blocks. In practice, when the number of goods is very limited (four goods in our case), it is enough to find one good that is block-independent from the others. Obviously, some tests are indicated to uncover which commodity qualifies to be block-independent from the others.

Blockwise dependence

This assumption is often denoted in the literature as the weak separability hypothesis. It supposes the existence of an increasing function by which the utility is a function of some sub-utility functions under the assumption that there exist G groups of commodities as defined above. We have Blockwise dependence if

$$U(\mathbf{q}) = U(u_1(\mathbf{q}_1),...,u_G(\mathbf{q}_G))$$

Blockwise dependence is a weaker version of block independence. Consequently, (Barnett, W. A. 1981) rightly pointed out that a rejection of weak separability induces a rejection of block independence. It has to be noted that acceptance of weak separability does not necessarily mean acceptance of the block independence assumption.

3.2.2.3. Unconditional, composite and conditional demand equations

Under the block independence assumption, three types of equations can be considered: the within group demand equation (unconditional demand equation), the between group demand equation (composite demand equation), and the conditional demand equation. To correctly apply the relative version estimation, it is important to find one good that is separable, in the block independence sense, from the rest of the goods. Upon finding a good for which the test of block independence is conclusive, the estimation of the conditional demand equation on the rest of the goods guarantees the elimination of any identification issue.

It is instructive to specify the three types of equation, and to see how a test may work before displaying the results for the full version estimation.

The unconditional demand equation

It is the individual equation which describes what is happening inside the groups. As such, it can be referred to as the within group demand equation.

$$\bar{w}_{it} Dq_{it} = \mu_i DQ_t + \sum_{jeSg} v_{ij} \left(Dp_{jt} - DP' \right) + \varepsilon_{it}, \quad i \in S_g \tag{3.1.22}$$

These equations describe only the demand for goods in the same group. As (Selvanathan, S. and E. A. Selvanathan 2005) observed it, the equations only take into account the prices of the same group.

The demand for the groups

The demand for groups can be obtained by adding over $i \in S_g$ the unconditional demand equation. This requires defining the group budget and marginal shares as well as the conditional budget and marginal shares. We define them in order as:

$$\bar{W}_{gt} = \sum_{i \in S_g} \bar{w}_{it} \tag{3.1.23}$$

$$\mathrm{M}_g = \sum_{i \in Sg} \mu_i \tag{3.1.24}$$

$$\bar{w}_{it}^{'} = \bar{w}_{it} \big/ \bar{W}_{gt} , \qquad i \in S_g \tag{3.1.25}$$

$$\mu_i^{'} = \mu_i \big/ \mathrm{M}_g , \qquad i \in S_g \tag{3.1.26}$$

These new concepts verify the adding-up property:

$$\sum_{g}^{G} \bar{W}_g = \sum_{g}^{G} \mathrm{M}_g = 1 \text{ and } \sum_{i \in S_g} \bar{w}_i^{'} = \sum_{i \in S_g} \mu_i^{'} = 1$$

We define in addition:

1. The group divisia index $DQ_{gt} = \sum_{i \in g} \bar{w}_i^{'} d \ln q_{it}$
2. The group price index $DP_{gt} = \sum_{i \in g} \bar{w}_{it}^{'} d \ln p_{it}$
3. The Frisch price index $DP_{gt}^{F} = \sum_{i \in S_g} \mu_i^{'} Dp_{it}$

We can show that the weighted average of the group price and volume Divisia indices verify the volume and price indices of the set of n goods.

$$DQ_t = \sum_{gt}^{G} W_{gt} DQ_{gt} \text{ and } DP = \sum_{g}^{G} W_{gt} DP_{gt}$$

Similarly, the weighted average of the group Frisch index equals the complete Frisch price index.

$$DP_t^{'} = \sum_{g=1}^{G} M_{gt} DP_{gt}^{F}$$

Using the above formula and adding over all the unconditional equations inside the same group, we define the group demand equation as

$$W_{gt} DQ_{gt} = \mathrm{M}_g DQ_t + \phi \mathrm{M}_g (DP_{gt}^F - DP') \tag{3.1.27}$$

We observe that the parentheses on the second term contain the difference between two Frisch indices. The first is the group Frisch price index and the second is the overall Frisch price index. The right-hand side expresses the idea that the demand for a group of goods as a whole is explained by the real income and the relative price of the group (the group Frisch price index deflated by the overall Frisch price index).

The conditional demand equations

As their names suggest, these individual equations are conditional to the group demand. They specify the group expenditure allocation among commodities within the group.

We can obtain the conditional demand equation in three steps:

Step 1: we get an expression of DQ_t from the group demand equation

$$DQ_t = \frac{W_{gt}}{\mathrm{M}_g} DQ_{gt} - \phi(DP_{gt}^F - DP') \tag{3.1.28}$$

Step 2: we plug this expression into the unconditional demand equation (3.1.22) to obtain

$$w_{it} Dq_{it} = \mu' W_{gt} DQ_{gt} - \phi\mu_i (DP_{gt}^F - DP') + \sum_{j \in S_g} v_{ij} (DP_{jt} - DP') \tag{3.1.29}$$

Step 3: we remark that if the block independence is true, then for any row of the price coefficient matrix, the sum of the entries is the product of the row marginal share and the income flexibility: $\sum_{j \in S_g} v_{ij} = \phi\mu_i$.

This finally allows us to write the final expression of the conditional demand equation

$$w_{it} Dq_{it} = \mu_i^{'} W_{gt} DQ_{gt} + \sum_{j \in S_g} v_{ij} (DP_{jt} - DP_{gt}^F), \ i \in S_g \tag{3.1.30}$$

As mentioned before, estimating this equation requires eliminating one equation.

The explicit expression of the conditional demand equation

Suppose we decide to eliminate the equation k of block one that has three goods, the equation becomes:

$$\begin{gathered} w_{it} Dq_{it} = \mu_i^{'} W_{gt} DQ_{gt} + \sum_{j \in S_g} v_{ij} (DP_{jt} - DP_{gt}^F), \text{ with } i \neq k, \\ i \in S_g \text{ and } DP_{gt}^F = \sum_{i \in S_g} \mu_i^{'} Dp_{it}. \end{gathered} \tag{3.1.31}$$

As seen before, the fact of eliminating the equation k does not eliminate all the $\mu_i^{'}$.we need to reformulate it in such a way that all the parameters belonging to equation k are eliminated.

Let us denote the second term of the right hand-side A. Then,

$$A = v_{ii} (DP_{it} - DP_{gt}^F) + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{gt}^F) \tag{3.1.32}$$

We now consider the fact that

$\sum_{j \in S_g} v_{ij} = \phi_g \mu_i^{'}$, where ϕ_g is the income flexibility obtained from the block S_g estimation. We can rewrite the expression as $v_{ii} = \phi_g \mu_i^{'} - \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij}$. This expression allows us to reformulate

A.

$$
\begin{aligned}
A &= (\phi_g \mu_i^{'} - \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij})\left(DP_{it} - DP_{gt}^{F} \right) + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{gt}^{F}) \\
&= \phi_g \mu_i^{'} \left(DP_{it} - DP_{gt}^{F} \right) + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{it})
\end{aligned}
$$

Now we use the expression of the Frisch price index $DP_{gt}^{F} = \sum_{i \in S_g} \mu_i^{'} Dp_{it}$.

$$
\begin{aligned}
A &= \phi_g \mu_i^{'} \left(DP_{it} - \sum_{j \in S_g} \mu_i^{'} Dp_{jt} \right) + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{it}) \\
&= \phi_g \mu_i^{'} [DP_{it} - (\sum_{\substack{j \in S_g, \\ j \neq k}} \mu_i^{'} Dp_{jt} + \mu_k^{'} Dp_{kt})] + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{it}).
\end{aligned}
$$

Note that $\mu_k^{'} = 1 - \sum_{j \neq k} \mu_j^{'}$. Then,

$$
A = \phi_g \mu_i^{'} [(DP_{it} - Dp_{kt}) + \sum_{\substack{j \in S_g, \\ j \neq k}} \mu_j^{'} (Dp_{jt} - Dp_{kt}) + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{it}).
$$

Finally the elaborated model of the conditional demand equation can be expressed as:

$$
\begin{aligned}
w_{it} Dq_{it} &= \mu_i^{'} W_{gt} DQ_{gt} + \phi_g \mu_i^{'} [(DP_{it} - Dp_{kt}) + \sum_{\substack{j \in S_g, \\ j \neq k}} \mu_j^{'} (Dp_{jt} - Dp_{kt}) \\
&\quad + \sum_{\substack{j \in S_g, \\ j \neq i}} v_{ij} (DP_{jt} - DP_{it}), \text{ where } i \in S_g \text{ and } i \neq k.
\end{aligned}
\tag{3.1.33}
$$

3.2.2.4. The relative version estimation

Table 7: **The Full version estimation without symmetry**

Equation	Observation	Parameters	RMSE	R^2	Constants
w1tdq1t	184	6	.0492983	0.7958*	(none)
w2tdq2t	184	6	.0074195	0.4250*	(none)
w4tdq4t	184	6	.0081426	0.4866*	(none)

	Coefficients	Std. errors	t-student	p-value	95 %Confidence intervals	
mu1	.5163949	.0190412	27.12	0.000	.4790748	.5537149
Phi	-2.50476	.1743041	-14.37	0.000	-2.84639	-2.16313
mu2	.0190418	.0022529	8.45	0.000	.0146262	.0234574
mu4	.020909	.002286	9.15	0.000	.0164285	.0253895
v12	-.4339414	.0668528	-6.49	0.000	-.5649704	-.3029124
v14	.083956	.0764421	1.10	0.272	-.0658677	.2337798
v21	-.0749307	.0172387	-4.35	0.000	-.1087179	-.0411435
v24	-.000289	.0135831	-0.02	0.983	-.0269114	.0263334
v41	-.0873149	.018545	-4.71	0.000	-.1236625	-.0509672
v42	.0430541	.0123956	3.47	0.001	.0187593	.067349

In the absence of the regularity conditions, the marginal share is higher than the one obtained under the same conditions when we used the absolute version method. It was 45% in the absolute version, here it is 51%. We can remark that v_{13}, v_{14}, and v_{24} are the only coefficients that are not significant. However, we have, as expected, a negative income flexibility. Without the symmetry condition, the third equation, and the diagonal elements we have ten parameters (six v_{ij}'s, three μ_i's, and ϕ). Imposing the symmetry conditions would bring the number of parameters to seven.

Table 8: **The Full version estimation with symmetry constraints imposed**

Equation	Observation	Parameters	RMSE	R^2	Constants
w1tdq1t	184	6	.0501013	0.7890*	(none)
w2tdq2t	184	6	.0075786	0.4001*	(none)
w4tdq4t	184	6	.0083203	0.4639*	(none)

Parameters	Coefficients	Std. errors	t-student	p-value	95 %Confidence intervals	
mu1	.4810664	.0180915	26.59	0.000	.4456077	.5165252
Phi	-2.721291	.1787509	-15.22	0.000	-3.071636	-2.370946
mu2	.0197855	.0026731	7.40	0.000	.0145463	.0250248
mu4	.0213345	.0022672	9.41	0.000	.0168909	.025778
v12	-.1039904	.0190024	-5.47	0.000	-.1412345	-.0667464
v14	-.0846439	.0192149	-4.41	0.000	-.1223044	-.0469835
v24	.0201714	.0099983	2.02	0.044	.0005752	.0397676

Table 8 brings back the electricity marginal share to a lower level (48%). It was (42 %) in the absolute version. The DFO and LPG marginal shares are also lower than before. However, as before, they are in the same range. The income flexibility in absolute value went up. It is now -2.7%.

Derivation of the standard deviations of the dropped equation

(Barnett, W. A. 1981, p. 54-55) concisely presents the general procedure to derive the standard deviations of the dropped equation. He originates the procedure to (Theil, H.1971, p.598-602).

Let β be the vector of parameters to be deduced and γ the vector of parameters within the estimated model. Obviously, $\hat{\gamma}$ and $\hat{\beta}$ are the resulting estimators. In our case, we will see in the next paragraph that

$$\boldsymbol{\beta} = (\mu_3 \quad v_{11} \quad v_{13} \quad v_{22} \quad v_{23} \quad v_{44} \quad v_{43} \quad v_{33})' \text{ and}$$

$$\boldsymbol{\gamma} = (\mu_1 \quad \phi \quad \mu_2 \quad \mu_4 \quad v_{12} \quad v_{14} \quad v_{24})'$$

We can observe that γ contains all the parameters in table 8 (relative version estimation with symmetry) and $\boldsymbol{\beta}$ contains the remaining parameters not in table 8. The model has in fact 15 parameters. 10 parameters define the coefficient price matrix. They would have been 16 in the absence of the symmetry restriction. The remaining parameters are the four marginal shares and the income flexibility.

It is permissible to write $\boldsymbol{\beta} = \beta(\boldsymbol{\gamma})$ since the parameters not in the relative version estimation table can be deduced by γ. In fact, the missing parameters can be deduced using the following system of equations:

$$\begin{aligned}
\mu_3 &= 1 - \mu_1 - \mu_2 - \mu_4 \\
v_{11} &= \pi_{11} + \phi\mu_1^2 \\
v_{13} &= \phi\mu_1 - (v_{11} + v_{12} + v_{14}) = \phi\mu_1 - \phi\mu_1^2 - \pi_{11} - v_{12} - v_{14} \\
v_{22} &= \pi_{22} + \phi\mu_2^2 \\
v_{23} &= \phi\mu_2 - (v_{12} + v_{22} + v_{24}) = \phi\mu_2 - \phi\mu_2^2 - \pi_{22} - (v_{12} + v_{14}) \\
v_{44} &= \pi_{44} + \phi\mu_4^2 \\
v_{43} &= \phi\mu_3 - (v_{14} + v_{24} + v_{44}) = \phi\mu_3 - \phi\mu_4^2 - \pi_{44} - (v_{14} + v_{24}) \\
v_{33} &= \phi\mu_3 - (v_{13} + v_{23} + v_{34}) \\
&= \phi\mu_1 - \phi\mu_2 + \phi\mu_1^2 + \phi\mu_2^2 + \phi\mu_4^2 + \sum_i \pi_{ii} + 2v_{12} + 2v_{14} + 2v_{24}
\end{aligned} \tag{3.1.34}$$

Let us now set that $\mathbf{D}(\boldsymbol{\gamma}) = \dfrac{\partial \beta(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}'}$ as the derivative of $\boldsymbol{\beta}$ with respect to $\boldsymbol{\gamma}$ and $\mathbf{V}$ the covariance matrix of the limiting distribution of $\sqrt{T}(\hat{\gamma} - \gamma)$. One can show that

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{\infty} N(\mathbf{0}, \mathbf{D}(\boldsymbol{\gamma})\mathbf{V}\mathbf{D}(\boldsymbol{\gamma})'). \tag{3.1.35}$$

Consequently, a consistent estimator of the limiting variance-covariance matrix is

$$\frac{1}{T}\mathbf{D}(\boldsymbol{\gamma})\hat{\mathbf{V}}\mathbf{D}(\boldsymbol{\gamma})' = \mathbf{D}(\boldsymbol{\gamma})\left[\frac{1}{T}\hat{\mathbf{V}}\right]\mathbf{D}(\boldsymbol{\gamma})'$$

In Stata, the estimate, $\frac{1}{T}\hat{\mathbf{V}}$, of the asymptotic variance-covariance matrix of $\hat{\boldsymbol{\gamma}}$ is denoted as $e(\mathbf{V})$. If we denote the variance-covariance of the deduced parameters as $\mathbf{H}_{8,8}$, then

$$\mathbf{H}_{8,8} = \mathbf{D}(\boldsymbol{\gamma})e(\mathbf{V})\mathbf{D}(\boldsymbol{\gamma})' .$$

The matrix $\mathbf{D}(\boldsymbol{\gamma})$ is given by the subsequent table

Table 9: **Derivatives of eliminated parameters**

	μ_1	ϕ	μ_2	μ_4	v_{12}	v_{14}	v_{24}
μ_3	-1	0	-1	-1	0	0	0
v_{11}	$2\pi\mu_1$	μ_1^2	0	0	0	0	0
v_{13}	$\phi(1-2\mu_1)$	$\mu_1 - \mu_1^2$	0	0	-1	-1	0
v_{22}	0	μ_2^2	$2\phi\mu_2$	0	0	0	0
v_{23}	0	ϕ	μ_2	0	-1	-1	0
v_{44}	0	μ_4^2	0	$2\phi\mu_4$	0	0	0
v_{43}	0	$\phi(\mu_3 - \mu_4^2)$	0	$-2\phi\mu_4$	0	-1	-1
v_{33}	$\phi(1+2\mu_1)$	$\mu_1 - \mu_2 + \mu_1^2 + \mu_2^2 + \mu_4^2$	$\phi(1+2\mu_2)$	$2\phi\mu_4$	2	2	2

The corresponding valued $\mathbf{D}(\hat{\boldsymbol{\gamma}})$ is given by

Table 10: **Values of the derivatives of the eliminated parameters**

	μ_1	ϕ	μ_2	μ_4	v_{12}	v_{14}	v_{24}
μ_3	-1	0	-1	-1	0	0	0
v_{11}	-2.6182433	.23142488	0	0	0	0	0
v_{13}	-.10304767	.24964152	0	0	-1	-1	0
v_{22}	0	.00039147	-.10768421	0	0	0	0
v_{23}	0	-2.721291	.0197855	0	-1	-1	0
v_{44}	0	.00045516	0	-.11611477	0	0	0
v_{43}	0	-1.2990312	0	.11611477	0	-1	-1
v_{33}	-5.3395343	.69355241	-2.8289752	-.11611477	2	2	2

Table 11: **The estimated variance-covariance of the missing parameters**

	μ_3	v_{11}	v_{13}	v_{22}	v_{23}	v_{44}	v_{43}	v_{33}
μ_3	**.00031032**							
v_{11}	.0007822	**.00410267**						
v_{13}	-1.323e-07	.00180231	**.00249164**					
v_{22}	3.468e-07	-2.144e-06	-2.344e-06	**7.579e-08**				
v_{23}	.00035765	-.02120435	-.01784581	5.953e-06	**.24389948**			
v_{44}	-4.923e-07	-2.876e-06	-1.184e-06	2.270e-08	-5.928e-06	**6.417e-08**		
v_{43}	.0001888	-.01003115	-.0087025	3.119e-06	.11599694	-4.148e-06	**.05538576**	
v_{33}	.00157812	.01028679	.00391035	1.075e-07	-.06882259	-1.014e-06	-.03244668	**.03010824**

Finally, displays the complete marginal share and price coefficient estimates. Under the Rotterdam model, the **V** matrix has to be normalized. Indeed, the assumption that the basic wants should have nonnegative income elasticities requires the roots of the candidate-matrix for diagonalization to be positive. Note that we make some nuance on the assumption. In fact, it is improbable to have a basic want with negative income elasticity. Such a basic want would not be one. On the other hand, it is not impossible to have a basic want with null income elasticity. It is the case whenever we reach the needed consumption amount on that basic want. In that case, whenever the income increases, the household increases the other basic want consumption.

Table 12: **Marginal shares and relative version price coefficients**

	MARGINAL SHARES	V MATRIX			
		ELE	DFO	GAS	LPG
ELE	.4810664	-1.48715144	-.1039904	.3666640817	-.0846439
	(.01809)	(.06405)	*(.01900)*	*(.06405)*	*(.01921)*
DFO	.0197855	-.1039904	-.013162692	.0431395898	.0201714
	(.00267)	*(.01900)*	(.00028)	*(.49386)*	*(.01000)*
GAS	.4778136	.3666640817	.0431395898	-1.72369176	.0136182424
	(.017616)	*(.06405)*	*(.49386)*	*(.17352)*	*(.23534)*
LPG	.0213345	-.0846439	.0201714	.0136182424	-.0072031252
	(.00227)	*(.01921)*	*(.01000)*	*(.23534)*	*(.00025)*

The test of block independence

All it takes to solve the identification issues is to find one good that is strongly separable from the others. Since we have four goods, we have four possibilities, provided the block independence assumption is true, to have two blocks of one and three goods, respectively. The previous results on the full version estimation provide an income flexibility $\phi_{full} = -2.721291$. This indicator is important as it represents the sum of all the price coefficients. Suppose that good $k \in [1, 4]$ is the strongly separable good. Removing temporarily this good and estimating the block of three goods would give a new income flexibility specific to that block. Let us call it $\phi_{\bar{k}}$. Under the validity that good k is strongly separable, our new estimation relative to the former one would verify

$$\phi_{full} = \phi_{\bar{k}} + v_{kk}$$

Where v_{kk} is the diagonal entry corresponding to good k.

Consequently to check which good is strongly separable, we will test the null hypothesis

$$H_o : \phi_{full} = \phi_{\bar{k}} + v_{kk} \text{ versus } H_1 : \phi_{full} \neq \phi_{\bar{k}} + v_{kk}$$

After estimating the conditional demand equations, we found that only good four does not reject the null hypothesis. We cannot reject the coefficient -2.714088 to be equal to ϕ_{full} with $\chi_1^2 = 3.11$ and a p-value of 0.0780. Hence, if we impose the strong separability on good four, the two following price coefficient matrices should provide identical income flexibility.

$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} \\ v_{12} & v_{22} & v_{23} & v_{24} \\ v_{13} & v_{23} & v_{33} & v_{34} \\ v_{14} & v_{24} & v_{34} & v_{44} \end{bmatrix} \begin{matrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{matrix} ; \quad \begin{bmatrix} v_{11} & v_{12} & v_{13} & 0 \\ v_{12} & v_{22} & v_{23} & 0 \\ v_{13} & v_{23} & v_{33} & 0 \\ 0 & 0 & 0 & v_{44} \end{bmatrix} \begin{matrix} \mu_1' \\ \mu_2' \\ \mu_3' \\ \end{matrix}$$

We observe that in the first matrix corresponding to the full version estimation with symmetry, the marginal share sum equals one. In the second we use the conditional demand estimation. As such, the sum of the conditional marginal shares (μ_i' not μ_i) of the first three goods equals one.

Results of the conditional demand estimation

The conditional demand estimation concerns the first three goods: electricity, DFO and gas. As we did before, one of the equations has to be dropped. As shown Table 13, we drop the third equation.

Table 13: **Estimates of the conditional demand**

Equation	Observation	Parameters	RMSE	R^2	Constants
w1tdq1t	184	4	.0483831	0.8033*	(none)
w2tdq2t	184	4	.0086054	0.2265*	(none)

	Coefficients	Std. errors	t-student	p-value	95 %Confidence intervals	
mup1	.5314166	.0196842	27.00	0.000	.4928362	.569997
Phi	-3.058302	.1953032	-15.66	0.000	-3.441089	-2.675515
mup2	.0109806	.0018258	6.01	0.000	.007402	.0145591
v12	-.0196034	.0108008	-1.82	0.070	-.0407725	.0015657

The next step is to derive the matrix of price coefficients From Table 13 and Table 8 under the block independence assumption on the first three goods. Then, we normalize the price coefficient matrix. To do that, we divide each price coefficient by the sum of all the price coefficients. As we have seen before, the sum of all price coefficients equals one. The matrix below displays the results. We can easily check that the sum of all coefficients equals one. In addition, the matrix presents positive eigenvalues, which attests that the normalized price coefficient matrix is positive definite. This is the desirable effect in order to have basic want income elasticities that are positive.

Matrix to be diagonalized

$$\begin{array}{c} \textit{Matrix } \mathbf{M}_{4,4} \qquad\qquad\qquad \textit{Roots} \\ \left[\begin{array}{cccc|c} .54648747 & & & & .73323895 \\ .03821363 & .00483693 & & & .44940884 \\ -.13473902 & -.01585262 & .63340957 & & .00264695 \\ 0 & 0 & 0 & .00264695 & .00208618 \end{array}\right] \end{array}$$

3.2.2.5. Composition matrices, Transformation matrices R and S under Rotterdam model

By definition, the composition matrix is a square table whose column sums yield the commodity budget shares and the row sums the T-goods. The table below shows that for the second T-good all commodities positively contribute into its budget shares. Logically, the second T-good is the T-good with the highest budget share (89%). The product that contributes the most on this T-good is the electricity. This excludes heating form being the second T-good. We suspect it to be cooling, since it has the second largest income elasticity.

The first T-good is the most difficult to interpret. The reason is that electricity and DFO negatively contribute to it. It has however the highest contribution of gas and the highest income elasticity. We may legitimately view it as heating. The fact that electricity and DFO contribute negatively to it may be due to the overwhelming gas contribution.

We may think that T-good 1 is a luxury for being the only basic want with the highest income elasticity. It ranges from 2% to up to 7%. (it is 2.16% in the first month, 3.81% in the fifth month and 1.77% in the last month of our observations running from February 1995 to August 2010). In fact, none of the T-goods should be qualified as luxury. As we have seen in (3.1.19), applying *the elasticity of energy total expenditure with respect to income* to its elasticity in some period yields result less than one.

Table 14: **Rotterdam compostion matrices and transformation matrices**

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
1	-0.0726	-0.0068	0.1648	0.0000	0.0854	1.87			
1	0.6565	0.0442	0.1950	0.0000	0.8958	0.00	0.89		
1	0.0000	0.0000	0.0000	0.0162	0.0162	0.00	0.00	0.16	
1	-0.0092	0.0110	0.0008	0.0000	0.0027	0.00	0.00	0.00	0.04
SUM	0.5747	0.0485	0.3606	0.0162	1.0000				
	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
2	-0.0700	-0.0067	0.1698	0.0000	0.0932	1.95			
2	0.6693	0.0453	0.1728	0.0000	0.8875	0.00	0.88		
2	0.0000	0.0000	0.0000	0.0164	0.0164	0.00	0.00	0.16	
2	-0.0098	0.0119	0.0008	0.0000	0.0030	0.00	0.00	0.00	0.04
SUM	0.5896	0.0506	0.3435	0.0164	1.0000				
	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
3	-0.0623	-0.0061	0.1713	0.0000	0.1029	2.16			
3	0.6952	0.0477	0.1339	0.0000	0.8768	0.00	0.84		
3	0.0000	0.0000	0.0000	0.0186	0.0186	0.00	0.00	0.14	
3	-0.0080	0.0092	0.0006	0.0000	0.0018	0.00	0.00	0.00	0.04
SUM	0.6249	0.0507	0.3058	0.0186	1.0000				
	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
4	-0.0480	-0.0050	0.1553	0.0000	0.1023	2.66			
4	0.7359	0.0513	0.0896	0.0000	0.8768	0.00	0.79		
4	0.0000	0.0000	0.0000	0.0207	0.0207	0.00	0.00	0.13	
4	-0.0029	0.0030	0.0001	0.0000	0.0002	0.00	0.00	0.00	0.04
SUM	0.6849	0.0493	0.2451	0.0207	1.0000				
	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
5	-0.0311	-0.0034	0.1169	0.0000	0.0825	3.81			
5	0.7881	0.0561	0.0523	0.0000	0.8965	0.00	0.72		
5	0.0000	0.0000	0.0000	0.0193	0.0193	0.00	0.00	0.14	
5	0.0105	-0.0084	-0.0003	0.0000	0.0017	0.00	0.00	0.00	0.04
SUM	0.7675	0.0444	0.1689	0.0193	1.0000				
	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
6	-0.0200	-0.0022	0.0818	0.0000	0.0596	5.63			
6	0.8187	0.0598	0.0323	0.0000	0.9108	0.00	0.68		
6	0.0000	0.0000	0.0000	0.0158	0.0158	0.00	0.00	0.17	
6	0.0356	-0.0212	-0.0006	0.0000	0.0138	0.00	0.00	0.00	0.05
SUM	0.8343	0.0364	0.1136	0.0158	1.0000				

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	ROW SUMS
1	-0.1263	-0.1404	0.4569	0.0000	-0.8498	-0.0797	1.9295	0	1
1	1.1423	0.9124	0.5408	0.0000	0.7329	0.0494	0.2177	0	1
1	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
1	-0.0161	0.2280	0.0023	0.0000	-3.4569	4.1400	0.3169	0	1
COLUMN SUMS	1	1	1	1					
	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
2	-0.1187	-0.1319	0.4944	0.0000	-0.7514	-0.0716	1.8230	0	1
2	1.1353	0.8962	0.5031	0.0000	0.7542	0.0511	0.1947	0	1
2	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
2	-0.0166	0.2357	0.0024	0.0000	-3.2877	4.0080	0.2797	0	1
COLUMN SUMS	1	1	1	1					
	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
3	-0.0996	-0.1211	0.5601	0.0000	-0.6052	-0.0597	1.6649	0	1
3	1.1124	0.9399	0.4380	0.0000	0.7929	0.0544	0.1528	0	1
3	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
3	-0.0128	0.1812	0.0019	0.0000	-4.5169	5.1970	0.3199	0	1
COLUMN SUMS	1	1	1	1					
	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
4	-0.0702	-0.1009	0.6337	0.0000	-0.4696	-0.0486	1.5183	0	1
4	1.0744	1.0408	0.3657	0.0000	0.8392	0.0585	0.1022	0	1
4	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
4	-0.0043	0.0601	0.0006	0.0000	-15.315	15.5368	0.7786	0	1
COLUMN SUMS	1	1	1	1					
	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
5	-0.0405	-0.0763	0.6923	0.0000	-0.3769	-0.0411	1.4180	0	1
5	1.0269	1.2659	0.3096	0.0000	0.8790	0.0626	0.0583	0	1
5	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
5	0.0136	-0.1895	-0.0019	0.0000	6.0228	-4.8412	-0.1816	0	1
COLUMN SUMS	1	1	1	1					
	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
6	-0.0240	-0.0618	0.7207	0.0000	-0.3355	-0.0377	1.3732	0	1
6	0.9813	1.6437	0.2849	0.0000	0.8989	0.0656	0.0355	0	1
6	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
6	0.0427	-0.5820	-0.0055	0.0000	2.5744	-1.5290	-0.0454	0	1
COLUMN SUMS	1	1	1	1					

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
91	-0.0158	-0.0018	0.0665	0.0000	0.0489	7.01			
91	0.8176	0.0607	0.0254	0.0000	0.9036	0.00	0.68		
91	0.0000	0.0000	0.0000	0.0200	0.0200	0.00	0.00	0.13	
91	0.0556	-0.0274	-0.0007	0.0000	0.0275	0.00	0.00	0.00	0.06
SUM	0.8575	0.0315	0.0911	0.0200	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
92	-0.0169	-0.0019	0.0708	0.0000	0.0520	6.56			
92	0.8238	0.0603	0.0271	0.0000	0.9112	0.00	0.68		
92	0.0000	0.0000	0.0000	0.0207	0.0207	0.00	0.00	0.13	
92	0.0393	-0.0226	-0.0006	0.0000	0.0161	0.00	0.00	0.00	0.05
SUM	0.8461	0.0358	0.0973	0.0207	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
93	-0.0273	-0.0030	0.1059	0.0000	0.0756	4.26			
93	0.7971	0.0568	0.0451	0.0000	0.8990	0.00	0.71		
93	0.0000	0.0000	0.0000	0.0241	0.0241	0.00	0.00	0.11	
93	0.0091	-0.0075	-0.0002	0.0000	0.0013	0.00	0.00	0.00	0.04
SUM	0.7789	0.0462	0.1507	0.0241	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
94	-0.0507	-0.0052	0.1592	0.0000	0.1033	2.55			
94	0.7233	0.0501	0.0967	0.0000	0.8701	0.00	0.80		
94	0.0000	0.0000	0.0000	0.0255	0.0255	0.00	0.00	0.10	
94	-0.0068	0.0076	0.0004	0.0000	0.0012	0.00	0.00	0.00	0.04
SUM	0.6658	0.0525	0.2563	0.0255	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
95	-0.0705	-0.0067	0.1673	0.0000	0.0902	1.94			
95	0.6596	0.0444	0.1784	0.0000	0.8824	0.00	0.89		
95	0.0000	0.0000	0.0000	0.0222	0.0222	0.00	0.00	0.12	
95	-0.0122	0.0164	0.0011	0.0000	0.0052	0.00	0.00	0.00	0.04
SUM	0.5769	0.0541	0.3468	0.0222	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
96	-0.0734	-0.0066	0.1444	0.0000	0.0644	1.77			
96	0.6207	0.0407	0.2434	0.0000	0.9049	0.00	0.93		
96	0.0000	0.0000	0.0000	0.0207	0.0207	0.00	0.00	0.13	
96	-0.0151	0.0235	0.0017	0.0000	0.0100	0.00	0.00	0.00	0.03
SUM	0.5322	0.0576	0.3895	0.0207	1.0000				

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	ROW SUMS
91	-0.0184	-0.0569	0.7297	0.0000	-0.3228	-0.0366	1.3594	0	1
91	0.9535	1.9280	0.2784	0.0000	0.9048	0.0671	0.0281	0	1
91	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
91	0.0649	-0.8711	-0.0081	0.0000	2.0237	-0.9970	-0.0267	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
92	-0.0200	-0.0535	0.7277	0.0000	-0.3257	-0.0368	1.3626	0	1
92	0.9736	1.6841	0.2783	0.0000	0.9041	0.0662	0.0297	0	1
92	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
92	0.0464	-0.6307	-0.0060	0.0000	2.4418	-1.4057	-0.0361	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
93	-0.0351	-0.0650	0.7027	0.0000	-0.3616	-0.0398	1.4014	0	1
93	1.0234	1.2281	0.2989	0.0000	0.8867	0.0632	0.0501	0	1
93	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
93	0.0117	-0.1631	-0.0016	0.0000	6.8222	-5.6409	-0.1813	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
94	-0.0762	-0.0991	0.6213	0.0000	-0.4910	-0.0503	1.5414	0	1
94	1.0864	0.9549	0.3773	0.0000	0.8313	0.0576	0.1111	0	1
94	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
94	-0.0102	0.1442	0.0015	0.0000	-5.8440	6.5209	0.3232	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
95	-0.1222	-0.1232	0.4825	0.0000	-0.7817	-0.0740	1.8556	0	1
95	1.1434	0.8203	0.5144	0.0000	0.7475	0.0503	0.2022	0	1
95	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
95	-0.0212	0.3029	0.0031	0.0000	-2.3426	3.1353	0.2074	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
96	-0.1379	-0.1149	0.3708	0.0000	-1.1390	-0.1028	2.2418	0	1
96	1.1663	0.7070	0.6250	0.0000	0.6860	0.0450	0.2690	0	1
96	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
96	-0.0284	0.4079	0.0042	0.0000	-1.5098	2.3447	0.1651	0	1
COLUMN SUMS	1	1	1	1					

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
182	-0.0691	-0.0066	0.1660	0.0000	0.0902	1.97			
182	0.6566	0.0448	0.1737	0.0000	0.8751	0.00	0.89		
182	0.0000	0.0000	0.0000	0.0342	0.0342	0.00	0.00	0.08	
182	-0.0040	0.0041	0.0003	0.0000	0.0004	0.00	0.00	0.00	0.05
SUM	0.5834	0.0423	0.3400	0.0342	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
183	-0.0572	-0.0058	0.1642	0.0000	0.1012	2.33			
183	0.6893	0.0484	0.1195	0.0000	0.8573	0.00	0.85		
183	0.0000	0.0000	0.0000	0.0401	0.0401	0.00	0.00	0.07	
183	0.0087	-0.0068	-0.0005	0.0000	0.0014	0.00	0.00	0.00	0.05
SUM	0.6408	0.0359	0.2832	0.0401	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
184	-0.0402	-0.0043	0.1381	0.0000	0.0937	3.09			
184	0.7353	0.0526	0.0723	0.0000	0.8602	0.00	0.78		
184	0.0000	0.0000	0.0000	0.0419	0.0419	0.00	0.00	0.06	
184	0.0166	-0.0118	-0.0007	0.0000	0.0042	0.00	0.00	0.00	0.05
SUM	0.7118	0.0365	0.2098	0.0419	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
185	-0.0250	-0.0028	0.0981	0.0000	0.0703	4.62			
185	0.7848	0.0569	0.0414	0.0000	0.8831	0.00	0.72		
185	0.0000	0.0000	0.0000	0.0367	0.0367	0.00	0.00	0.07	
185	0.0284	-0.0179	-0.0007	0.0000	0.0098	0.00	0.00	0.00	0.05
SUM	0.7882	0.0363	0.1388	0.0367	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
186	-0.0164	-0.0019	0.0686	0.0000	0.0503	6.77			
186	0.7992	0.0597	0.0266	0.0000	0.8855	0.00	0.70		
186	0.0000	0.0000	0.0000	0.0302	0.0302	0.00	0.00	0.09	
186	0.0639	-0.0290	-0.0009	0.0000	0.0340	0.00	0.00	0.00	0.06
SUM	0.8467	0.0288	0.0943	0.0302	1.0000				

	COMPOSITION MATRIX				SUM	T-EXPENDITURE ELASTICITTIES			
	ELE	DFO	GAS	LPG		T1	T2	T3	T4
187	-0.0140	-0.0016	0.0594	0.0000	0.0438	7.85			
187	0.7721	0.0596	0.0230	0.0000	0.8547	0.00	0.71		
187	0.0000	0.0000	0.0000	0.0287	0.0287	0.00	0.00	0.09	
187	0.1100	-0.0360	-0.0011	0.0000	0.0728	0.00	0.00	0.00	0.08
SUM	0.8681	0.0220	0.0812	0.0287	1.0000				

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	ROW SUMS
182	-0.1185	-0.1558	0.4881	0.0000	-0.7663	-0.0731	1.8394	0	1
182	1.1254	1.0580	0.5109	0.0000	0.7503	0.0512	0.1985	0	1
182	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
182	-0.0069	0.0978	0.0010	0.0000	-9.3406	9.5582	0.7824	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
183	-0.0893	-0.1602	0.5799	0.0000	-0.5650	-0.0568	1.6218	0	1
183	1.0757	1.3490	0.4220	0.0000	0.8041	0.0565	0.1394	0	1
183	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
183	0.0136	-0.1888	-0.0019	0.0000	6.2247	-4.8446	-0.3800	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
184	-0.0564	-0.1166	0.6584	0.0000	-0.4286	-0.0455	1.4741	0	1
184	1.0331	1.4386	0.3448	0.0000	0.8548	0.0611	0.0841	0	1
184	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
184	0.0233	-0.3220	-0.0032	0.0000	3.9742	-2.8160	-0.1582	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
185	-0.0317	-0.0762	0.7067	0.0000	-0.3554	-0.0394	1.3948	0	1
185	0.9957	1.5686	0.2980	0.0000	0.8887	0.0645	0.0468	0	1
185	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
185	0.0360	-0.4924	-0.0047	0.0000	2.8863	-1.8195	-0.0668	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
186	-0.0194	-0.0644	0.7274	0.0000	-0.3260	-0.0369	1.3629	0	1
186	0.9439	2.0700	0.2818	0.0000	0.9025	0.0674	0.0300	0	1
186	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
186	0.0754	-1.0056	-0.0092	0.0000	1.8788	-0.8533	-0.0255	0	1
COLUMN SUMS	1	1	1	1					

	R-MATRICES				S-MATRICES				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
187	-0.0162	-0.0726	0.7312	0.0000	-0.3203	-0.0365	1.3568	0	1
187	0.8894	2.7074	0.2829	0.0000	0.9033	0.0698	0.0269	0	1
187	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1	1
187	0.1267	-1.6349	-0.0141	0.0000	1.5102	-0.4945	-0.0157	0	1
COLUMN SUMS	1	1	1	1					

3.2.2.6. Commodity price and income elasticities

Table 15: **Price and Income elasticities**

	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
1	-3.8052	-0.2661	0.9382	0.0000	0.7829			
1	-3.1551	-0.3994	1.3089	0.0000	0.0000	0.5612		
1	1.4951	0.1759	-7.0283	0.0000	0.0000	0.0000	1.3388	
1	0.0000	0.0000	0.0000	-0.6543	0.0000	0.0000	0.0000	0.1635
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
2	-3.7093	-0.2594	0.9145	0.0000	0.7632			
2	-3.0234	-0.3827	1.2542	0.0000	0.0000	0.5377		
2	1.5699	0.1847	-7.3801	0.0000	0.0000	0.0000	1.4058	
2	0.0000	0.0000	0.0000	-0.6455	0.0000	0.0000	0.0000	0.1613
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
3	-3.4994	-0.2447	0.8628	0.0000	0.7200			
3	-3.0143	-0.3815	1.2504	0.0000	0.0000	0.5361		
3	1.7633	0.2075	-8.2893	0.0000	0.0000	0.0000	1.5790	
3	0.0000	0.0000	0.0000	-0.5707	0.0000	0.0000	0.0000	0.1426
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
4	-3.1930	-0.2233	0.7873	0.0000	0.6570			
4	-3.1006	-0.3925	1.2863	0.0000	0.0000	0.5515		
4	2.1998	0.2588	-10.3413	0.0000	0.0000	0.0000	1.9698	
4	0.0000	0.0000	0.0000	-0.5119	0.0000	0.0000	0.0000	0.1279
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
5	-2.8495	-0.1993	0.7026	0.0000	0.5863			
5	-3.4477	-0.4364	1.4302	0.0000	0.0000	0.6132		
5	3.1921	0.3756	-15.0059	0.0000	0.0000	0.0000	2.8584	
5	0.0000	0.0000	0.0000	-0.5494	0.0000	0.0000	0.0000	0.1373
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
6	-2.6212	-0.1833	0.6463	0.0000	0.5393			
6	-4.2055	-0.5323	1.7446	0.0000	0.0000	0.7480		
6	4.7477	0.5586	-22.3189	0.0000	0.0000	0.0000	4.2514	
6	0.0000	0.0000	0.0000	-0.6719	0.0000	0.0000	0.0000	0.1679

	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
91	-2.5504	-0.1783	0.6288	0.0000	0.5248			
91	-4.8596	-0.6151	2.0160	0.0000	0.0000	0.8643		
91	5.9201	0.6965	-27.8306	0.0000	0.0000	0.0000	5.3013	
91	0.0000	0.0000	0.0000	-0.5295	0.0000	0.0000	0.0000	0.1323
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
92	-2.5847	-0.1807	0.6373	0.0000	0.5318			
92	-4.2681	-0.5402	1.7706	0.0000	0.0000	0.7591		
92	5.5388	0.6517	-26.0380	0.0000	0.0000	0.0000	4.9598	
92	0.0000	0.0000	0.0000	-0.5109	0.0000	0.0000	0.0000	0.1277
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
93	-2.8076	-0.1963	0.6922	0.0000	0.5777			
93	-3.3076	-0.4187	1.3721	0.0000	0.0000	0.5883		
93	3.5772	0.4209	-16.8164	0.0000	0.0000	0.0000	3.2032	
93	0.0000	0.0000	0.0000	-0.4391	0.0000	0.0000	0.0000	0.1097
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
94	-3.2848	-0.2297	0.8099	0.0000	0.6759			
94	-2.9131	-0.3687	1.2085	0.0000	0.0000	0.5181		
94	2.1038	0.2475	-9.8902	0.0000	0.0000	0.0000	1.8839	
94	0.0000	0.0000	0.0000	-0.4158	0.0000	0.0000	0.0000	0.1039
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
95	-3.7909	-0.2651	0.9347	0.0000	0.7800			
95	-2.8252	-0.3576	1.1720	0.0000	0.0000	0.5025		
95	1.5546	0.1829	-7.3083	0.0000	0.0000	0.0000	1.3921	
95	0.0000	0.0000	0.0000	-0.4775	0.0000	0.0000	0.0000	0.1193
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
96	-4.1089	-0.2873	1.0131	0.0000	0.8454			
96	-2.6536	-0.3359	1.1008	0.0000	0.0000	0.4720		
96	1.3843	0.1629	-6.5077	0.0000	0.0000	0.0000	1.2396	
96	0.0000	0.0000	0.0000	-0.5129	0.0000	0.0000	0.0000	0.1282

	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
182	-3.7483	-0.2621	0.9242	0.0000	0.7712			
182	-3.6113	-0.4571	1.4981	0.0000	0.0000	0.6423		
182	1.5857	0.1866	-7.4546	0.0000	0.0000	0.0000	1.4200	
182	0.0000	0.0000	0.0000	-0.3096	0.0000	0.0000	0.0000	0.0774
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
183	-3.4125	-0.2386	0.8414	0.0000	0.7021			
183	-4.2579	-0.5390	1.7664	0.0000	0.0000	0.7573		
183	1.9041	0.2240	-8.9512	0.0000	0.0000	0.0000	1.7051	
183	0.0000	0.0000	0.0000	-0.2643	0.0000	0.0000	0.0000	0.0661
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
184	-3.0725	-0.2148	0.7575	0.0000	0.6322			
184	-4.1848	-0.5297	1.7361	0.0000	0.0000	0.7443		
184	2.5700	0.3024	-12.0817	0.0000	0.0000	0.0000	2.3014	
184	0.0000	0.0000	0.0000	-0.2528	0.0000	0.0000	0.0000	0.0632
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
185	-2.7745	-0.1940	0.6841	0.0000	0.5709			
185	-4.2126	-0.5332	1.7476	0.0000	0.0000	0.7493		
185	3.8847	0.4570	-18.2618	0.0000	0.0000	0.0000	3.4786	
185	0.0000	0.0000	0.0000	-0.2886	0.0000	0.0000	0.0000	0.0721
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
186	-2.5829	-0.1806	0.6368	0.0000	0.5314			
186	-5.3010	-0.6710	2.1991	0.0000	0.0000	0.9428		
186	5.7172	0.6726	-26.8764	0.0000	0.0000	0.0000	5.1195	
186	0.0000	0.0000	0.0000	-0.3511	0.0000	0.0000	0.0000	0.0877
	COMMODITIES PRICE ELASTICITIES				COMMODITIES INCOME ELASTICITIES			
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
187	-2.5192	-0.1762	0.6211	0.0000	0.5184			
187	-6.9410	-0.8786	2.8794	0.0000	0.0000	1.2345		
187	6.6379	0.7810	-31.2047	0.0000	0.0000	0.0000	5.9440	
187	0.0000	0.0000	0.0000	-0.3694	0.0000	0.0000	0.0000	0.0923

The table above requires some comments:

1. All the own-price elasticities are negative. Gas is the commodity with the highest own-price elasticity. The households are more sensitive to the change in gas price. This means that an increase in gas price pushes them to significantly adjust their gas consumption.
2. Gas and electricity appear to be substitute. This may explain why electricity enters negatively the T-good 1 budget share.
3. DFO and gas are also substitute.

3.3. The SMIT Under the AIDS Framework

So far, the Preference Independent Transformation technique has been exclusively associated with the Rotterdam model. Two reasons can be highlighted for this situation. Firstly, the Rotterdam model is one of the most thoroughly elaborated microeconomic models. It comes out after the *Barten Fundamental Equation*. Secondly, the Preference Independence Transformation is born out of the Rotterdam model setting. There is an umbilical-like cord tying the PIT to the Rotterdam model. It may be possible to apply it in its original form to the AIDS if we could identify a specific effect whose diagonalization concomitantly implies a diagonalization of the Hessian matrix of the utility function.

In this chapter, we apply the SMIT technique to the AIDS with the purpose of conducting a comparative analysis of the results obtained with the two frameworks. We have seen that the AIDS model is rival to the Rotterdam model in the literature. As in the Rotterdam model, we may have many levels of formulations. In its primary specification, the AIDS is nonlinear in the parameters as the level of price in the main equation presents a *Translog* structure. The AIDS can, however, be linearly expressed relatively to the parameters if we approximate, by assumption, the level of price by the Stone's index. We have also seen that a third possibility is to differentiate the AIDS in order to have what is commonly denoted as the Differential AIDS (DAIDS.)

The choice of which formulation (full AIDS, LA/AIDS or DAIDS) is used depends on how well the Hessian matrix lends itself to the diagonalization.

3.3.1. The matrix to be diagonalized

The choice is between the DAIDS and the full AIDS Slutsky matrices. From the very start, we eliminate the LA/AIDS Slutsky matrix because of a circularity issue raised on page 29.

$$\mathbf{H}^{DAIDS} = \left.\frac{\partial q_i}{\partial p_j}\right|_{u=\tilde{u}} = \frac{p_i p_j}{M}\left[\beta_{ij} - w_i\delta_{ij} + w_i w_j\right] \tag{3.1.36}$$

$$\mathbf{H}^{AIDS} = \left.\frac{\partial q_i}{\partial p_j}\right|_{u=\tilde{u}} = \frac{p_i p_j}{M}\left[\beta_{ij} + \beta_i\beta_j \ln\left(\frac{M}{P}\right) - w_i\delta_{ij} + w_i w_j\right] \tag{3.1.37}$$

We observe that the Hessian of the AIDS encompasses the LA/AIDS Hessian. In addition, it contains a quantity, $w_i\delta_{ij}$, that does not reflect any preference interdependencies between the goods. The quantity is however important since it scales down the diagonal of the Hessian matrix. For this reason, we may legitimately think that the quantity is very crucial for the semi-negative definiteness of the Hessian matrix. We choose to make the independent transformation on the full AIDS since it capture more the interaction between the commodities.

Our goal is first to conduct the full version estimation. As in the Rotterdam model there is necessity to address the redundancy issue and to find the standard deviations of the dropped equations. Concerning the procedure to drop the last equation, we will show it while questioning the possibility to use conditional demand equation. Naturally, all the estimations we perform below first drops one equation. Then we will show how to uncover the dropped equation parameters and their standard deviations. Second, we will respectively display the dynamics of uncompensated price ealsticities, the income elasticities, and the Hicksian price elasticities. The order is important in view of the fact that AIDS is derived from a cost function. There is precedence of the uncompensated elasticities over the compensated elasticities computations.

3.3.2. The full AIDS estimation with all regularity conditions imposed

3.3.2.1. The results under equation 4 dropped

Table 16: **Full version estimation**

	Equations	Obs	Parms	RMSE	R-sq	Constants
1	w1t	184	11	.0951748	0.3968	a0
2	w2t	184	11	.0089544	0.3992	a0
3	w3t	184	11	.0904787	0.3863	a0

	Coef.	Std. Err.	z	P>z	[95% Conf. Interval]	
/a1	-1.572743	.3067773	-5.13	0.000	-2.174015	-.97147
/b11	.0926476	.0352658	2.63	0.009	.0235278	.1617674
/b12	.0034667	.0112238	0.31	0.757	-.0185316	.0254649
/b13	-.1024101	.0391904	-2.61	0.009	-.1792218	-.0255983
/q1	-.1562817	.0237467	-6.58	0.000	-.2028244	-.109739
/a0	44.20367	5.243765	8.43	0.000	33.92608	54.48126
/a2	-.3293933	.1723623	-1.91	0.056	-.6672171	.0084305
/a3	3.181137	.5780746	5.50	0.000	2.048131	4.314142
/b22	.0011453	.0113568	0.10	0.920	-.0211137	.0234044
/b23	-.0410222	.0200825	-2.04	0.041	-.0803832	-.0016612
/b33	.1904219	.0613573	3.10	0.002	.0701637	.31068
/q2	-.0163288	.0066383	-2.46	0.014	-.0293396	-.0033181
/q3	.1789558	.0214131	8.36	0.000	.1369869	.2209246

3.3.2.2. Parameter derivations and corresponding standard deviations

Parameter derivation

The table above shows 13 parameters:

- Six parameters are β_{ij} 's. They would be nine without the symmetry conditions.
- Three parameters are α_i 's and three others are β_i 's.
- One parameter is α_0

The derivation of the fourth equation parameters will use the following equations:

$$\begin{aligned}
\beta_{14} &= -(\beta_{11} + \beta_{12} + \beta_{13}) \\
\beta_{24} &= -(\beta_{12} + \beta_{22} + \beta_{23}) \\
\beta_{34} &= -(\beta_{13} + \beta_{23} + \beta_{33}) \\
\beta_{44} &= -(\beta_{14} + \beta_{24} + \beta_{34}) = \beta_{11} + \beta_{22} + \beta_{33} + 2\beta_{12} + 2\beta_{13} + 2\beta_{23} \\
q_4 &= -(q_1 + q_2 + q_3) \\
a_4 &= 1 - a_1 - a_2 - a_3
\end{aligned} \tag{3.1.38}$$

The complete estimation is summarized by the following table.

Table 17: **Complete results**

	Matrix β_{ij}				MATRIX	MATRIX
	lprice ele	lprice dfo	lprice gas	lprice lpg	β_i	α_i
lprice ele	0.09265				-0.15628	-1.57274
lprice dfo	0.00347	0.00115			-0.01633	-0.32939
lprice gas	-0.10241	-0.04102	0.19042		0.17896	3.18114
lprice lpg	0.00630	0.03641	-0.04699	0.00428	-0.00635	-0.27900

Standard deviation

We use the formula (3.1.12) to derive the variance-covariance of the last equation and, consequently, the standard deviations.

The resulting matrix $\mathbf{L}_{6,6}$ of the system (3.1.38) is given by the following table:

	α_1	β_{11}	β_{12}	β_{13}	β_1	α_0	α_2	α_3	β_{22}	β_{23}	β_{33}	β_2	β_3
β_{14}	0	-1	-1	-1	0	0	0	0	0	0	0	0	0
β_{24}	0	0	-1	0	0	0	0	0	-1	-1	0	0	0
β_{34}	0	0	0	-1	0	0	0	0	0	-1	-1	0	0
β_{44}	0	1	2	2	0	0	0	0	1	2	1	0	0
β_4	0	0	0	0	-1	0	0	0	0	0	0	-1	-1
α_4	-1	0	0	0	0	0	-1	-1	0	0	0	0	0

The asymptotic variance-covariance of the last equation is given by the STATA software can be found in appendix D.The estimated variance-covariance of the dropped equation parameters is given by Table 18.

Table 18: **Estimated variance-covariance for equation 4**

	β_{14}	β_{24}	β_{34}	β_{44}	β_4	α_4
β_{14}	*0.000454*					
β_{24}	-4.7E-05	*0.000106*				
β_{34}	-0.00081	6.72E-05	*0.001587*			
β_{44}	0.000399	-0.00013	-0.00085	*0.000574*		
β_4	-0.00026	3.93E-05	0.000543	-0.00032	*0.00021*	
α_4	-0.00597	0.000614	0.012294	-0.00693	0.004545	*0.100835*

The resulting standard deviations are given by Table 19.

Table 19: **Standard deviations for the last equation parameters**

$\sigma_{\beta_{14}}$	$\sigma_{\beta_{24}}$	$\sigma_{\beta_{34}}$	$\sigma_{\beta_{44}}$	σ_{β_4}	σ_{α_4}
.0213150182	.0102727796	.0398351603	.0239486952	.0144941367	.3175451779

3.3.3. The matrix to be diagonalized

Table 20: **Matrix $\mathbf{H}_{4,4}$ and corresponding roots**

	ELE	DFO	GAS	LPG	ROOTS
ELE	.85957598				1.0588
DFO	.06946257	.0699234			0.4576
GAS	-.27635391	-.06435829	.64678228		0.0606
LPG	.02388973	.00214189	-.00970955	.0507193	0.0500

Before we proceed to the diagonalization, it is informative to check the dynamics of the Hicksian price elasticities. To the contrary of the Rotterdam model, with the AIDS model we first assess the dynamic of the uncompensated price elasticities. The Hicksian price elasticity matrices are deduced through the Channel of the Slutsky equation matrices. We in fact have shown the procedure while we were computing the Hessian matrix.

The table below shows that all uncompensated own price elasticities are negative. The magnitude order of the income elasticities ranks the commodities as follows: Gas, electricity, DFO, and LPG.

The table Table 22 displays the dynamics of the Hicksian elasticities. As expected, all the direct price elasticities are negative. The highest direct price elasticities in absolute value are related to gas. Demand for gas is very sensitive to its price. The results show that electricity and gas are substitute. DFO and gas are also substitute. Only Gas and LPG are complement.

Table 23 is related to the transformation, strictly speaking according to the SMIT. It reveals surprising results. Firstly, the eigenvalues of the transformation do no longer correspond to the transformed good expenditure elasticity. This could be understood if we consider the expression of the T-good expenditure elasticity matrices given by $\mathbf{H}_t = \frac{(\mathbf{R}_{n,n}\mathbf{Z}_t + \mathbf{R}_{n,n}\mathbf{\Theta}_{n,1})_\Delta}{(\mathbf{R}_{n,n}\mathbf{Z}_t)_\Delta}$ and the fact that this equation cannot be related to the eigenvalues as in the Rotterdam model. Secondly, it becomes possible to see basic wants that are inferior. To be precise, T-2 is an inferior good. We may see this as the most basic T-good. For this good, gas and LPG negatively enter it. This excludes heating to be the T-good. Gas contributes most on T3 and T4. Since the T good expenditure elasticity is higher in T3 than in T4, we may think that T3 is heating.

3.3.4. The dynamics of the uncompensated elasticities and income elasticities

Table 21**: uncompensated price and income elasticities**

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
1	ELE	-1.1864	-0.0340	0.4963	-0.0057	0.7299			
1	DFO	-0.3724	-1.0271	-0.0069	0.7543	0.0000	0.6521		
1	GAS	0.3503	-0.0400	-1.7041	-0.0992	0.0000	0.0000	1.4930	
1	LPG	-0.1600	3.0776	-2.7049	-0.6622	0.0000	0.0000	0.0000	0.4494
Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
2	ELE	-1.1817	-0.0336	0.4843	-0.0058	0.7368			
2	DFO	-0.3562	-1.0265	-0.0057	0.7211	0.0000	0.6673		
2	GAS	0.3677	-0.0411	-1.7404	-0.1037	0.0000	0.0000	1.5174	
2	LPG	-0.1621	3.1151	-2.7372	-0.6584	0.0000	0.0000	0.0000	0.4426
Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
3	ELE	-1.1715	-0.0333	0.4588	-0.0061	0.7521			
3	DFO	-0.3539	-1.0285	-0.0020	0.7144	0.0000	0.6700		
3	GAS	0.4134	-0.0422	-1.8367	-0.1147	0.0000	0.0000	1.5802	
3	LPG	-0.1591	3.0356	-2.6645	-0.6681	0.0000	0.0000	0.0000	0.4561
Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
4	ELE	-1.1577	-0.0317	0.4214	-0.0065	0.7744			
4	DFO	-0.3689	-1.0316	0.0037	0.7385	0.0000	0.6582		
4	GAS	0.5195	-0.0482	-2.0536	-0.1398	0.0000	0.0000	1.7221	
4	LPG	-0.1621	3.0179	-2.6426	-0.6720	0.0000	0.0000	0.0000	0.4587
Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
5	ELE	-1.1432	-0.0294	0.3812	-0.0070	0.7984			
5	DFO	-0.4133	-1.0371	0.0130	0.8162	0.0000	0.6212		
5	GAS	0.7665	-0.0641	-2.5538	-0.1967	0.0000	0.0000	2.0481	
5	LPG	-0.1835	3.2826	-2.8635	-0.6463	0.0000	0.0000	0.0000	0.4107
Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
6	ELE	-1.1328	-0.0276	0.3533	-0.0069	0.8140			
6	DFO	-0.4952	-1.0454	0.0194	0.9734	0.0000	0.5478		
6	GAS	1.1494	-0.0919	-3.3318	-0.2896	0.0000	0.0000	2.5639	
6	LPG	-0.2217	3.9171	-3.4124	-0.5795	0.0000	0.0000	0.0000	0.2965

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
116	ELE	-1.1456	-0.0276	0.3724	-0.0065	0.8074			
116	DFO	-0.3602	-1.0302	0.0242	0.6831	0.0000	0.6831		
116	GAS	1.1635	-0.0956	-3.2838	-0.2822	0.0000	0.0000	2.4982	
116	LPG	-0.1275	1.9925	-1.7223	-0.7852	0.0000	0.0000	0.0000	0.6425

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
117	ELE	-1.1557	-0.0278	0.3973	-0.0048	0.7910			
117	DFO	-0.2899	-1.0219	0.0113	0.5581	0.0000	0.7424		
117	GAS	0.8209	-0.0794	-2.6003	-0.2137	0.0000	0.0000	2.0724	
117	LPG	-0.0996	1.6096	-1.3981	-0.8236	0.0000	0.0000	0.0000	0.7117

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
118	ELE	-1.1835	-0.0321	0.4678	-0.0054	0.7532			
118	DFO	-0.2542	-1.0186	0.0087	0.4902	0.0000	0.7738		
118	GAS	0.5064	-0.0509	-1.9857	-0.1328	0.0000	0.0000	1.6630	
118	LPG	-0.0884	1.4360	-1.2480	-0.8425	0.0000	0.0000	0.0000	0.7429

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
119	ELE	-1.2171	-0.0380	0.5534	-0.0068	0.7084			
119	DFO	-0.2556	-1.0187	0.0093	0.4922	0.0000	0.7728		
119	GAS	0.3686	-0.0370	-1.7175	-0.0959	0.0000	0.0000	1.4818	
119	LPG	-0.1059	1.7137	-1.4886	-0.8124	0.0000	0.0000	0.0000	0.6932

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
120	ELE	-1.2362	-0.0411	0.6024	-0.0073	0.6822			
120	DFO	-0.2877	-1.0209	0.0099	0.5546	0.0000	0.7440		
120	GAS	0.3208	-0.0326	-1.6244	-0.0837	0.0000	0.0000	1.4199	
120	LPG	-0.1200	1.9481	-1.6928	-0.7866	0.0000	0.0000	0.0000	0.6513

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
121	ELE	-1.2374	-0.0415	0.6061	-0.0067	0.6794			
121	DFO	-0.3219	-1.0234	0.0097	0.6228	0.0000	0.7128		
121	GAS	0.3114	-0.0318	-1.6064	-0.0824	0.0000	0.0000	1.4092	
121	LPG	-0.1180	1.9334	-1.6817	-0.7875	0.0000	0.0000	0.0000	0.6539

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
182	ELE	-1.1906	-0.0295	0.4873	0.0002	0.7326			
182	DFO	-0.4019	-1.0229	-0.0127	0.8051	0.0000	0.6324		
182	GAS	0.3855	-0.0509	-1.7431	-0.1172	0.0000	0.0000	1.5257	
182	LPG	-0.0649	1.1612	-1.0207	-0.8684	0.0000	0.0000	0.0000	0.7929

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
183	ELE	-1.1732	-0.0292	0.4442	-0.0007	0.7588			
183	DFO	-0.4685	-1.0311	-0.0066	0.9327	0.0000	0.5735		
183	GAS	0.4619	-0.0539	-1.8962	-0.1373	0.0000	0.0000	1.6255	
183	LPG	-0.0728	1.2793	-1.1223	-0.8555	0.0000	0.0000	0.0000	0.7714

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
184	ELE	-1.1570	-0.0273	0.4023	-0.0014	0.7834			
184	DFO	-0.4530	-1.0319	0.0000	0.8949	0.0000	0.5901		
184	GAS	0.6270	-0.0684	-2.2186	-0.1816	0.0000	0.0000	1.8415	
184	LPG	-0.0789	1.3565	-1.1874	-0.8475	0.0000	0.0000	0.0000	0.7573

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
185	ELE	-1.1446	-0.0259	0.3693	-0.0025	0.8038			
185	DFO	-0.4547	-1.0343	0.0101	0.8858	0.0000	0.5931		
185	GAS	0.9658	-0.0954	-2.8786	-0.2670	0.0000	0.0000	2.2753	
185	LPG	-0.0923	1.5271	-1.3314	-0.8299	0.0000	0.0000	0.0000	0.7265

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
186	ELE	-1.1364	-0.0251	0.3477	-0.0031	0.8169			
186	DFO	-0.5709	-1.0451	0.0213	1.1021	0.0000	0.4927		
186	GAS	1.4409	-0.1320	-3.8050	-0.3862	0.0000	0.0000	2.8823	
186	LPG	-0.1144	1.8463	-1.6054	-0.7956	0.0000	0.0000	0.0000	0.6691

Obs.	Commodities	Uncompensated elasticities				Commodity income elasticity			
		ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG
187	ELE	-1.1335	-0.0244	0.3398	-0.0032	0.8213			
187	DFO	-0.7437	-1.0583	0.0291	1.4325	0.0000	0.3404		
187	GAS	1.6786	-0.1545	-4.2637	-0.4473	0.0000	0.0000	3.1869	
187	LPG	-0.1165	1.8696	-1.6248	-0.7932	0.0000	0.0000	0.0000	0.6649

3.3.5. The dynamics of the Hicksian elasticities

Table 22: **Hicksian elasticities**

Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
1	ELE	-0.76417	0.00022	0.76125	0.00270
1	DFO	0.00485	-0.99654	0.22984	0.76184
1	GAS	1.21396	0.03012	-1.16213	-0.08194
1	LPG	0.10001	3.09866	-2.54169	-0.65698
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
2	ELE	-0.74427	0.00257	0.73910	0.00260
2	DFO	0.04003	-0.99375	0.22506	0.72866
2	GAS	1.26862	0.03339	-1.21563	-0.08638
2	LPG	0.10067	3.13685	-2.58417	-0.65335
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
3	ELE	-0.69736	0.00391	0.69077	0.00268
3	DFO	0.06851	-0.99536	0.20461	0.72224
3	GAS	1.40960	0.03601	-1.34934	-0.09627
3	LPG	0.12850	3.05817	-2.52385	-0.66282
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
4	ELE	-0.62130	0.00535	0.61336	0.00259
4	DFO	0.08706	-1.00010	0.16687	0.74617
4	GAS	1.71228	0.03407	-1.62678	-0.11957
4	LPG	0.15565	3.03981	-2.52887	-0.66659
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
5	ELE	-0.52415	0.00501	0.51752	0.00161
5	DFO	0.06837	-1.01033	0.11907	0.82289
5	GAS	2.35452	0.02423	-2.20412	-0.17463
5	LPG	0.13494	3.30031	-2.79337	-0.64188
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
6	ELE	-0.44868	0.00182	0.44643	0.00044
6	DFO	-0.03480	-1.02559	0.08206	0.97833
6	GAS	3.30426	0.00067	-3.03846	-0.26646
6	LPG	0.02750	3.92783	-3.37849	-0.57683

Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
116	ELE	-0.49065	0.01401	0.46884	0.00779
116	DFO	0.19399	-0.99499	0.10575	0.69525
116	GAS	3.19020	0.03308	-2.98543	-0.23785
116	LPG	0.39368	2.02560	-1.64552	-0.77377
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
117	ELE	-0.56424	0.02234	0.52930	0.01259
117	DFO	0.26522	-0.97485	0.13521	0.57441
117	GAS	2.37054	0.05197	-2.25443	-0.16808
117	LPG	0.43253	1.65474	-1.27931	-0.80796
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
118	ELE	-0.70654	0.02224	0.67114	0.01316
118	DFO	0.23580	-0.96274	0.21759	0.50934
118	GAS	1.55943	0.06912	-1.53681	-0.09173
118	LPG	0.38199	1.48965	-1.04752	-0.82413
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
119	ELE	-0.83734	0.01295	0.81653	0.00786
119	DFO	0.15858	-0.96315	0.29637	0.50821
119	GAS	1.16288	0.06946	-1.16711	-0.06523
119	LPG	0.26568	1.76349	-1.23111	-0.79806
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
120	ELE	-0.90068	0.00238	0.89315	0.00514
120	DFO	0.07827	-0.97340	0.32703	0.56810
120	GAS	1.01917	0.05801	-1.01928	-0.05790
120	LPG	0.20034	1.98966	-1.41524	-0.77476
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
121	ELE	-0.90616	-0.00283	0.90321	0.00578
121	DFO	0.02555	-0.98284	0.32138	0.63592
121	GAS	0.99839	0.04832	-0.99010	-0.05661
121	LPG	0.20075	1.97054	-1.39576	-0.77553

Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
182	ELE	-0.76237	0.00305	0.73670	0.02262
182	DFO	-0.03230	-0.99483	0.20261	0.82452
182	GAS	1.27728	0.01692	-1.22373	-0.07047
182	LPG	0.39850	1.19638	-0.75080	-0.84408
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
183	ELE	-0.68160	-0.00012	0.66132	0.02040
183	DFO	-0.09693	-1.00916	0.15751	0.94858
183	GAS	1.51501	0.00836	-1.43114	-0.09223
183	LPG	0.42690	1.30881	-0.90164	-0.83407
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
184	ELE	-0.59189	0.00395	0.56887	0.01907
184	DFO	-0.02736	-1.00840	0.12544	0.91033
184	GAS	1.95543	0.00497	-1.82699	-0.13341
184	LPG	0.46741	1.38669	-1.02636	-0.82774
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
185	ELE	-0.50453	0.00630	0.48211	0.01612
185	DFO	0.01761	-1.01049	0.09337	0.89951
185	GAS	2.77770	-0.00412	-2.55934	-0.21423
185	LPG	0.48628	1.55621	-1.22942	-0.81307
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
186	ELE	-0.43911	0.00122	0.42536	0.01253
186	DFO	-0.15040	-1.02928	0.06817	1.11151
186	GAS	3.90108	-0.03918	-3.53098	-0.33091
186	LPG	0.45673	1.86781	-1.54178	-0.78276
Obs.	Commodities	Compensated elasticities			
		ELE	DFO	GAS	LPG
187	ELE	-0.41530	-0.00405	0.40697	0.01238
187	DFO	-0.44607	-1.04991	0.05699	1.43899
187	GAS	4.46549	-0.07562	-4.00293	-0.38695
187	LPG	0.46497	1.88609	-1.57042	-0.78065

3.3.6. Composition matrices T , transformation matrices R and S , and T-good income elasticities

Table 23: **AIDS composItion matrices and transformation matrices**

SUM		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	1	0.0150	0.0022	-0.0091	0.0167	0.0249	4.4584	0.2568
	1	0.0318	0.0086	-0.0326	-0.0014	0.0064	2.5739	-3.1782
	1	-0.0386	0.0735	0.1259	0.0005	0.1613	1.1242	1.2892
	1	0.5702	-0.0374	0.2788	-0.0042	0.8074	1.0013	0.9984
SUM		0.5785	0.0469	0.3630	0.0115	1.0000		
		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	2	0.0144	0.0021	-0.0088	0.0163	0.0239	4.5121	0.2429
	2	0.0218	0.0058	-0.0235	-0.0010	0.0032	2.5906	-5.1063
	2	0.0305	0.1032	0.1956	-0.0001	0.3292	1.1062	1.1789
	2	0.5270	-0.0620	0.1825	-0.0039	0.6437	0.9893	0.9666
SUM		0.5937	0.0491	0.3458	0.0114	1.0000		
		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	3	0.0149	0.0022	-0.0100	0.0163	0.0235	4.4071	0.1879
	3	0.0012	0.0003	-0.0015	-0.0001	0.0000	2.7148	-100.3316
	3	0.1067	0.1160	0.2063	-0.0008	0.4282	1.1421	1.1294
	3	0.5076	-0.0691	0.1136	-0.0037	0.5483	0.9568	0.9360
SUM		0.6304	0.0495	0.3084	0.0117	1.0000		
		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	4	0.0153	0.0026	-0.0123	0.0152	0.0207	4.4007	-0.0370
	4	-0.0208	-0.0059	0.0302	0.0018	0.0053	3.0982	6.2109
	4	0.1248	0.1147	0.1480	-0.0012	0.3863	1.2357	1.1039
	4	0.5733	-0.0636	0.0820	-0.0040	0.5877	0.9059	0.9213
SUM		0.6927	0.0478	0.2478	0.0117	1.0000		
		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	5	0.0131	0.0027	-0.0156	0.0090	0.0093	4.8696	-1.7254
	5	-0.0277	-0.0084	0.0523	0.0069	0.0231	4.0976	3.5757
	5	0.1034	0.0997	0.0779	-0.0011	0.2798	1.4180	1.0848
	5	0.6865	-0.0509	0.0561	-0.0040	0.6878	0.8441	0.9157
SUM		0.7754	0.0431	0.1707	0.0108	1.0000		
		Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
	6	-0.0041	-0.0010	0.0070	-0.0012	0.0006	6.2926	21.8365
	6	-0.0082	-0.0037	0.0279	0.0145	0.0304	5.4284	2.2055
	6	0.0707	0.0779	0.0409	-0.0008	0.1888	1.7136	1.0859
	6	0.7821	-0.0371	0.0386	-0.0035	0.7802	0.8006	0.9156
SUM		0.8404	0.0361	0.1144	0.0090	1.0000		

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
1	0.0260	0.0467	-0.0249	1.4472	0.6051	0.0882	-0.3644	0.6710	1.0000
1	0.0550	0.1836	-0.0898	-0.1226	4.9466	1.3385	-5.0656	-0.2194	1.0000
1	-0.0667	1.5655	0.3467	0.0425	-0.2392	0.4557	0.7805	0.0030	1.0000
1	0.9856	-0.7958	0.7681	-0.3671	0.7062	-0.0463	0.3453	-0.0052	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
2	0.0243	0.0421	-0.0255	1.4296	0.6019	0.0865	-0.3686	0.6802	1.0000
2	0.0367	0.1178	-0.0679	-0.0843	6.9091	1.8309	-7.4362	-0.3038	1.0000
2	0.0513	2.1026	0.5656	-0.0062	0.0925	0.3135	0.5942	-0.0002	1.0000
2	0.8877	-1.2625	0.5278	-0.3391	0.8187	-0.0963	0.2836	-0.0060	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
3	0.0237	0.0453	-0.0323	1.3947	0.6353	0.0954	-0.4241	0.6933	1.0000
3	0.0020	0.0068	-0.0048	-0.0057	0.4979	27.1899	-1.2130	-5.3934	1.0000
3	0.1692	2.3450	0.6689	-0.0687	0.2491	0.2710	0.4818	-0.0019	1.0000
3	0.8052	-1.3971	0.3682	-0.3202	0.9258	-0.1261	0.2071	-0.0068	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
4	0.0221	0.0535	-0.0498	1.2924	0.7402	0.1236	-0.5968	0.7330	1.0000
4	-0.0300	-0.1229	0.1218	0.1502	-3.9333	-1.1113	5.7114	0.3332	1.0000
4	0.1802	2.4006	0.5971	-0.1007	0.3231	0.2969	0.3830	-0.0031	1.0000
4	0.8277	-1.3312	0.3309	-0.3419	0.9755	-0.1082	0.1396	-0.0068	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
5	0.0170	0.0630	-0.0911	0.8322	1.4181	0.2931	-1.6781	0.9669	1.0000
5	-0.0357	-0.1950	0.3063	0.6431	-1.1996	-0.3640	2.2638	0.2998	1.0000
5	0.1334	2.3118	0.4562	-0.1046	0.3695	0.3561	0.2784	-0.0040	1.0000
5	0.8854	-1.1797	0.3286	-0.3708	0.9982	-0.0739	0.0816	-0.0058	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
6	-0.0049	-0.0288	0.0608	-0.1277	-6.6810	-1.6753	11.2105	-1.8542	1.0000
6	-0.0098	-0.1027	0.2439	1.6024	-0.2704	-0.1218	0.9172	0.4750	1.0000
6	0.0842	2.1577	0.3578	-0.0896	0.3747	0.4127	0.2169	-0.0043	1.0000
6	0.9306	-1.0262	0.3375	-0.3852	1.0024	-0.0475	0.0495	-0.0045	1.0000
SUM	1	1	1	1					

SUM	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
116	-0.0173	-0.0043	0.0322	-0.0011	0.0095	5.7833	6.6329
116	0.0120	0.0002	0.0080	0.0295	0.0497	2.8482	0.9805
116	0.1962	0.1236	0.0527	-0.0041	0.3684	1.2592	1.0094
116	0.6204	-0.0680	0.0266	-0.0064	0.5725	0.7985	0.9026
SUM	0.8113	0.0515	0.1194	0.0177	1.0000		

	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
117	-0.0191	-0.0045	0.0320	-0.0013	0.0070	4.2585	6.6670
117	0.0155	-0.0002	0.0141	0.0381	0.0674	2.2958	1.0143
117	0.4340	0.1499	0.1102	-0.0102	0.6839	1.0817	0.9881
117	0.3174	-0.0817	0.0105	-0.0045	0.2417	0.7972	0.8646
SUM	0.7477	0.0634	0.1669	0.0220	1.0000		

	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
118	0.0084	0.0018	-0.0104	0.0009	0.0006	2.9311	-15.3566
118	0.0009	-0.0042	0.0338	0.0405	0.0710	2.0228	1.1786
118	0.6122	0.0832	0.2482	-0.0165	0.9271	1.0471	0.9988
118	0.0117	-0.0086	-0.0016	-0.0002	0.0013	0.7762	-0.5729
SUM	0.6332	0.0722	0.2699	0.0247	1.0000		

	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
119	0.0792	0.0138	-0.0656	0.0220	0.0494	2.7092	-0.3080
119	-0.0206	-0.0047	0.0229	0.0075	0.0052	2.2316	4.0234
119	0.5167	0.0034	0.3795	-0.0094	0.8903	1.0427	1.0385
119	-0.0394	0.0593	0.0346	0.0005	0.0551	0.7867	1.2638
SUM	0.5360	0.0719	0.3714	0.0207	1.0000		

	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
120	0.0647	0.0104	-0.0436	0.0288	0.0603	2.9431	0.1441
120	0.0328	0.0068	-0.0280	-0.0052	0.0062	2.3175	-2.5378
120	0.4240	-0.0547	0.3671	-0.0059	0.7305	1.0275	1.0486
120	-0.0296	0.1014	0.1307	0.0005	0.2030	0.8606	1.1881
SUM	0.4918	0.0638	0.4262	0.0182	1.0000		

	Composition matrices				Sum	Roots	T-Exp. Elast.
	ELE	DFO	GAS	LPG			
121	0.0704	0.0120	-0.0473	0.0292	0.0642	2.9358	0.1368
121	0.0320	0.0071	-0.0270	-0.0057	0.0064	2.3371	-2.3332
121	0.3393	-0.0835	0.2167	-0.0050	0.4674	1.0398	1.0120
121	0.0459	0.1213	0.2950	-0.0001	0.4619	0.9257	1.1542
SUM	0.4875	0.0569	0.4373	0.0183	1.0000		

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
116	-0.0213	-0.0831	0.2693	-0.0636	-1.8286	-0.4526	3.4005	-0.1193	1.0000
116	0.0148	0.0038	0.0667	1.6607	0.2420	0.0039	0.1606	0.5935	1.0000
116	0.2418	2.3997	0.4412	-0.2338	0.5326	0.3356	0.1431	-0.0113	1.0000
116	0.7647	-1.3204	0.2227	-0.3633	1.0836	-0.1188	0.0465	-0.0113	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
117	-0.0256	-0.0713	0.1919	-0.0611	-2.7164	-0.6419	4.5493	-0.1910	1.0000
117	0.0207	-0.0039	0.0844	1.7302	0.2295	-0.0036	0.2091	0.5650	1.0000
117	0.5804	2.3645	0.6607	-0.4650	0.6346	0.2192	0.1612	-0.0150	1.0000
117	0.4245	-1.2893	0.0630	-0.2040	1.3133	-0.3382	0.0435	-0.0186	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
118	0.0133	0.0246	-0.0387	0.0344	14.2919	3.0205	-1.7759	1.4469	1.0000
118	0.0015	-0.0582	0.1251	1.6421	0.0132	-0.0592	0.4754	0.5707	1.0000
118	0.9668	1.1529	0.9196	-0.6688	0.6603	0.0898	0.2677	-0.0178	1.0000
118	0.0185	-0.1193	-0.0061	-0.0077	9.3208	-6.8648	-1.3035	-0.1524	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
119	0.1478	0.1917	-0.1767	1.0647	1.6040	0.2789	-1.3288	0.4459	1.0000
119	-0.0384	-0.0648	0.0618	0.3627	-3.9325	-0.8911	4.3888	1.4347	1.0000
119	0.9641	0.0479	1.0217	-0.4532	0.5804	0.0039	0.4262	-0.0105	1.0000
119	-0.0735	0.8252	0.0932	0.0259	-0.7153	1.0768	0.6287	0.0097	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
120	0.1316	0.1623	-0.1024	1.5830	1.0741	0.1718	-0.7239	0.4780	1.0000
120	0.0666	0.1058	-0.0658	-0.2873	5.2472	1.0812	-4.4910	-0.8375	1.0000
120	0.8621	-0.8570	0.8614	-0.3246	0.5804	-0.0748	0.5025	-0.0081	1.0000
120	-0.0603	1.5889	0.3067	0.0289	-0.1461	0.4994	0.6440	0.0026	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
121	0.1443	0.2102	-0.1081	1.5915	1.0956	0.1861	-0.7361	0.4544	1.0000
121	0.0657	0.1251	-0.0618	-0.3097	4.9862	1.1078	-4.2095	-0.8845	1.0000
121	0.6959	-1.4684	0.4954	-0.2739	0.7258	-0.1786	0.4635	-0.0107	1.0000
121	0.0941	2.1331	0.6745	-0.0079	0.0993	0.2625	0.6385	-0.0003	1.0000
SUM	1	1	1	1					

SUM	Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
182	0.0499	0.0132	-0.0506	0.0037	0.0162	2.7669	-1.8196
182	0.0094	-0.0085	0.0259	0.0460	0.0728	1.6392	1.0641
182	-0.0236	0.0790	0.1468	0.0029	0.2051	1.1899	1.2626
182	0.5488	-0.0393	0.2184	-0.0220	0.7059	1.0044	0.9817
SUM	0.5845	0.0444	0.3404	0.0306	1.0000		
	Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
183	0.0223	0.0067	-0.0262	0.0017	0.0045	3.0768	-4.6198
183	0.0115	-0.0065	0.0196	0.0422	0.0669	1.8133	1.0389
183	-0.0056	0.0735	0.1402	0.0019	0.2101	1.3913	1.2724
183	0.6196	-0.0355	0.1525	-0.0181	0.7186	0.9601	0.9515
SUM	0.6479	0.0383	0.2861	0.0278	1.0000		
	Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
184	-0.0072	-0.0021	0.0105	-0.0005	0.0007	3.6417	17.0301
184	0.0185	-0.0017	0.0192	0.0456	0.0816	1.9319	1.0218
184	0.0573	0.0864	0.1013	-0.0040	0.2411	1.4608	1.1592
184	0.6527	-0.0429	0.0816	-0.0149	0.6766	0.8937	0.9238
SUM	0.7214	0.0398	0.2127	0.0261	1.0000		
	Composition matrices ELE	DFO	GAS	LPG	Sum		
185	-0.0170	-0.0046	0.0299	-0.0010	0.0073	5.0670	6.9702
185	0.0208	0.0013	0.0110	0.0411	0.0742	2.1845	0.9744
185	0.0845	0.0872	0.0543	-0.0053	0.2207	1.5358	1.0846
185	0.7081	-0.0438	0.0451	-0.0116	0.6978	0.8298	0.9135
SUM	0.7963	0.0401	0.1403	0.0232	1.0000		
	Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
186	-0.0141	-0.0039	0.0272	-0.0007	0.0085	7.2680	7.5868
186	0.0165	0.0018	0.0062	0.0329	0.0574	2.6438	0.9436
186	0.0533	0.0648	0.0297	-0.0038	0.1440	1.9226	1.1009
186	0.7978	-0.0306	0.0320	-0.0092	0.7900	0.7932	0.9149
SUM	0.8536	0.0322	0.0951	0.0192	1.0000		
	Composition matrices ELE	DFO	GAS	LPG	Sum	Roots	T-Exp. Elast.
187	-0.0120	-0.0035	0.0236	-0.0006	0.0075	8.4613	8.5310
187	0.0230	0.0082	0.0091	0.0372	0.0775	2.6852	0.9739
187	0.0245	0.0410	0.0202	-0.0084	0.0774	2.4612	1.2016
187	0.8390	-0.0210	0.0289	-0.0093	0.8376	0.7824	0.9167
SUM	0.8745	0.0248	0.0818	0.0189	1.0000		

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
182	0.0853	0.2974	-0.1487	0.1210	3.0856	0.8174	-3.1324	0.2294	1.0000
182	0.0162	-0.1912	0.0760	1.5019	0.1297	-0.1166	0.3552	0.6316	1.0000
182	-0.0404	1.7788	0.4313	0.0941	-0.1152	0.3853	0.7159	0.0141	1.0000
182	0.9389	-0.8850	0.6415	-0.7170	0.7774	-0.0557	0.3094	-0.0311	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
183	0.0344	0.1758	-0.0917	0.0602	5.0015	1.5113	-5.8881	0.3753	1.0000
183	0.0178	-0.1697	0.0685	1.5211	0.1726	-0.0972	0.2932	0.6313	1.0000
183	-0.0086	1.9205	0.4901	0.0703	-0.0265	0.3499	0.6673	0.0093	1.0000
183	0.9564	-0.9267	0.5330	-0.6516	0.8623	-0.0494	0.2122	-0.0252	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
184	-0.0100	-0.0522	0.0495	-0.0187	-10.1895	-2.9250	14.8023	-0.6878	1.0000
184	0.0257	-0.0421	0.0903	1.7420	0.2272	-0.0205	0.2353	0.5580	1.0000
184	0.0794	2.1704	0.4764	-0.1534	0.2377	0.3586	0.4203	-0.0166	1.0000
184	0.9049	-1.0761	0.3838	-0.5699	0.9648	-0.0634	0.1206	-0.0220	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
185	-0.0214	-0.1146	0.2131	-0.0423	-2.3296	-0.6296	4.0934	-0.1342	1.0000
185	0.0262	0.0331	0.0781	1.7711	0.2809	0.0179	0.1476	0.5536	1.0000
185	0.1060	2.1733	0.3872	-0.2273	0.3826	0.3951	0.2462	-0.0239	1.0000
185	0.8891	-1.0918	0.3217	-0.5015	1.0148	-0.0628	0.0647	-0.0167	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
186	-0.0165	-0.1197	0.2859	-0.0387	-1.6595	-0.4536	3.2004	-0.0873	1.0000
186	0.0193	0.0561	0.0648	1.7182	0.2875	0.0314	0.1073	0.5737	1.0000
186	0.0625	2.0148	0.3125	-0.2002	0.3702	0.4502	0.2063	-0.0266	1.0000
186	0.9347	-0.9512	0.3368	-0.4794	1.0099	-0.0388	0.0405	-0.0116	1.0000
SUM	1	1	1	1					

	R-matrices				S-matrices				
	ELE	DFO	GAS	LPG	ELE	DFO	GAS	LPG	
187	-0.0138	-0.1402	0.2881	-0.0313	-1.6146	-0.4652	3.1592	-0.0794	1.0000
187	0.0263	0.3305	0.1115	1.9658	0.2964	0.1056	0.1177	0.4803	1.0000
187	0.0280	1.6576	0.2473	-0.4422	0.3167	0.5301	0.2614	-0.1082	1.0000
187	0.9595	-0.8479	0.3532	-0.4923	1.0017	-0.0251	0.0345	-0.0111	1.0000
SUM	1	1	1	1					

3.3.7. Comparative analysis between the two model results

It is logical to check if we could have close results between the two models. The differences can be directly observed comparing the composition matrices of the two models for each period. It should be noted that the results of the two models will be even closer than the two matrices to be diagonalized are identical.

Table 24 shows that for both models T2 has the highest budget share. They also classify the t-goods in the same order. Except that, they produce different results. The T-good expenditure elasticities are higher with the AIDS model framework. The analysis of these juxtaposed results makes us state that choosing a specific demand system is equivalent to putting additional assumptions on the household behavior.

The gap in the results of the two models is a stimulus for looking for a third approach of uncovering the basic wants. One question that arises is to wonder if it is possible to predefine the basic wants and a *standard composition matrix* that enables us to make the arbitrage between different models serving as Preference Independent Transformation platform.

In conclusion, imposing a demand system appears to have a similar effect as making additional restrictions.

Table 24: **AIDS versus Rotterdam**

OBS	ROTTER. COMPOSITION MATRIX				SUM	AIDS COMPOSITION MATRICES					SUM	EIGEN	E
	ELE	DFO	GAS	LPG		E	ELE	DFO	GAS	LPG			
1	-0.073	-0.007	0.165	0.000	**0.085**	1.870	0.015	0.002	-0.009	0.017	**0.025**	4.458	0.257
1	0.657	0.044	0.195	0.000	**0.896**	0.890	0.032	0.009	-0.033	-0.001	**0.006**	2.574	-3.178
1	0.000	0.000	0.000	0.016	**0.016**	0.160	-0.039	0.074	0.126	0.001	**0.161**	1.124	1.289
1	-0.009	0.011	0.001	0.000	**0.003**	0.040	0.570	-0.037	0.279	-0.004	**0.807**	1.001	0.998
SUM	**0.575**	**0.049**	**0.361**	**0.016**	**1.000**		**0.579**	**0.047**	**0.363**	**0.012**	**1.000**		
OBS	COMPOSITION MATRIX						Composition matrices				**SUM**	EIGEN	
	ELE	DFO	GAS	LPG			ELE	DFO	GAS	LPG			
2	-0.070	-0.007	0.170	0.000	**0.093**	1.950	0.014	0.002	-0.009	0.016	**0.024**	4.512	0.243
2	0.669	0.045	0.173	0.000	**0.888**	0.880	0.022	0.006	-0.024	-0.001	**0.003**	2.591	-5.106
2	0.000	0.000	0.000	0.016	**0.016**	0.160	0.031	0.103	0.196	0.000	**0.329**	1.106	1.179
2	-0.010	0.012	0.001	0.000	**0.003**	0.040	0.527	-0.062	0.183	-0.004	**0.644**	0.989	0.967
SUM	**0.590**	**0.051**	**0.344**	**0.016**	**1.000**		**0.594**	**0.049**	**0.346**	**0.011**	**1.000**		
OBS	COMPOSITION MATRIX						Composition matrices				**SUM**	EIGEN	
	ELE	DFO	GAS	LPG			ELE	DFO	GAS	LPG			
3	-0.062	-0.006	0.171	0.000	**0.103**	2.160	0.015	0.002	-0.010	0.016	**0.024**	4.407	0.188
3	0.695	0.048	0.134	0.000	**0.877**	0.840	0.001	0.000	-0.002	0.000	**0.000**	2.715	-100.3
3	0.000	0.000	0.000	0.019	**0.019**	0.140	0.107	0.116	0.206	-0.001	**0.428**	1.142	1.129
3	-0.008	0.009	0.001	0.000	**0.002**	0.040	0.508	-0.069	0.114	-0.004	**0.548**	0.957	0.936
SUM	**0.625**	**0.051**	**0.306**	**0.019**	**1.000**		**0.630**	**0.050**	**0.308**	**0.012**	**1.000**		
OBS	COMPOSITION MATRIX						Composition matrices				**SUM**	EIGEN	
	ELE	DFO	GAS	LPG			ELE	DFO	GAS	LPG			
4	-0.048	-0.005	0.155	0.000	**0.102**	2.660	0.015	0.003	-0.012	0.015	**0.021**	4.401	-0.037
4	0.736	0.051	0.090	0.000	**0.877**	0.790	-0.021	-0.006	0.030	0.002	**0.005**	3.098	6.211
4	0.000	0.000	0.000	0.021	**0.021**	0.130	0.125	0.115	0.148	-0.001	**0.386**	1.236	1.104
4	-0.003	0.003	0.000	0.000	**0.000**	0.040	0.573	-0.064	0.082	-0.004	**0.588**	0.906	0.921
SUM	**0.685**	**0.049**	**0.245**	**0.021**	**1.000**		**0.693**	**0.048**	**0.248**	**0.012**	**1.000**		
OBS	COMPOSITION MATRIX						Composition matrices				**SUM**	EIGEN	
	ELE	DFO	GAS	LPG			ELE	DFO	GAS	LPG			
5	-0.031	-0.003	0.117	0.000	**0.083**	3.810	0.013	0.003	-0.016	0.009	**0.009**	4.870	-1.725
5	0.788	0.056	0.052	0.000	**0.897**	0.720	-0.028	-0.008	0.052	0.007	**0.023**	4.098	3.576
5	0.000	0.000	0.000	0.019	**0.019**	0.140	0.103	0.100	0.078	-0.001	**0.280**	1.418	1.085
5	0.011	-0.008	0.000	0.000	**0.002**	0.040	0.687	-0.051	0.056	-0.004	**0.688**	0.844	0.916
SUM	**0.768**	**0.044**	**0.169**	**0.019**	**1.000**		**0.775**	**0.043**	**0.171**	**0.011**	**1.000**		
OBS	COMPOSITION MATRIX						Composition matrices				**SUM**	EIGEN	
	ELE	DFO	GAS	LPG			ELE	DFO	GAS	LPG			
6	-0.020	-0.002	0.082	0.000	**0.060**	5.630	-0.004	-0.001	0.007	-0.001	**0.001**	6.293	21.84
6	0.819	0.060	0.032	0.000	**0.911**	0.680	-0.008	-0.004	0.028	0.015	**0.030**	5.428	2.206
6	0.000	0.000	0.000	0.016	**0.016**	0.170	0.071	0.078	0.041	-0.001	**0.189**	1.714	1.086
6	0.036	-0.021	-0.001	0.000	**0.014**	0.050	0.782	-0.037	0.039	-0.004	**0.780**	0.801	0.916
SUM	**0.834**	**0.036**	**0.114**	**0.016**	**1.000**		**0.841**	**0.036**	**0.114**	**0.009**	**1.000**		

4. Essay on identifying the T-goods

Though the Preference Independence Technique is deduced with a rare and noticeable mathematical rigor, its originators (Brooks, and Theil) and followers did not take much risk to clearly identify the T goods with some economically meaningful basic wants. At most, they could use the income elasticity of the T goods and how the commodities enter to the T goods budget share to give them an economic meaning. The current state of the optimization software progress and data availability may allow the exploration of more ways to identify the goods.

We have seen that depending on the system of demand model we use, the results we obtain are different. This last chapter responds to a need to find a more decisive way of uncovering the basic wants.

We strive to push one step ahead in the research on the possibility of identifying the T-goods. We will mainly use the definition and the properties of the transformation matrix as stated previously. We will show that by pre-defininig the basic wants, the stock of data available enables to directly compute *empirical T matrices,* and hence resulting *empirical transformation matrices*. This computation is a noticeable advance in gaining certainty and precision while identifying the T-goods. Even if the data could not fully recover the transformation matrices, we will display a second order solution when only the knowledge of the budget shares of the commodities and the budget shares of the basic wants—statistically found—are available. In that case, we show that it is possible to approximate the T matrix and, hence, the composition matrices $\mathbf{R}_{n,n}$ and $\mathbf{S}_{n,n}$.

4.1. Preliminary discussions

The Energy Information Administration reports an Annual Energy Outlook that provides "delivered energy consumption by fuel" every year, at least since 2008." This can easily be computed as *empirical T-matrices*. We will denote them as $\mathbf{T}^{*}_{n,n}$ .Before we proceed to their computation, some clarifications on the assumptions that found the Preference Independence Transformation are important:

- The number of commodities equals the number of basic wants.
- The basic wants are independent.

- The basic wants are normal goods in the sense that increasing the consumer purchasing power increases its consumption of these goods. This is true for the PIT, not for the SMIT.

In practice, the number of basic wants is in most of the cases different from the number of commodities. In our case The Energy Information Administration refers to them as end uses although end uses and basic wants do not exactly correspond. The EIA takes into account, to name the most important, space heating, water heating, space cooling, refrigeration, cooking, and lighting. It is clear that some end uses should not be assimilated to basic wants. If we do, then the basic want would not be independent. For example, refrigeration is not preference independent from cooking, and cooking is not preference independent from heating. And yet, a Basic want should not have another basic want as substitute or complement. Therefore, the pre-definition of basic want should strictly observe the preference independence assumption.

Another aspect of the problem is the difficulty of reconciling the first two assumptions above mentioned. In order to attain this, we should consider one of the T-good as a composite of basic wants that are not too relevant to the analysis. Hence, if we consider three goods—for example, electricity, liquefied petroleum, and gas—we may consider three basic wants (heating, cooling, and T-3). We call the third basic want T-3 as it is a composite of negligible basic wants. In this example, it is obvious that heating and cooling are preference independent. Let us take the example of four goods (electicity, dfo, gas, lpg) as in our previous chapters. In that case we should consider three of our most relevant basic wants plus T-4, the composite of residual basic wants. The three most important basic wants are heating (space heating and water heating), cooling and lighting. One may raise the issue however that lighting and heating may not be completely preference independent. From this issue we infer that the more goods we add, the less plausible is the basic want preference independence assumption.

From an empirical point of view, two cases may arise:

The data are available and allow us to statistically compute the empirical composition matrix.

1. Table 25 reports these matrices from 2009 to 2012. In those cases, the results obtained provide valuable information in identifying the T-goods. The resulting transformation matrices can be easily computed. We use for that the following two equations:

$$\mathbf{R}^* = \mathbf{T}^*(\mathbf{w})_\Delta^{-1} \tag{4.1.1}$$

$$\mathbf{S}^* = (\mathbf{R}^{*'})^{-1} \tag{4.1.2}$$

We assign a star to refer to the *empirical* aspect of the matrices. This is the case whenever the composition or transformation matrices are statistically determined.

2. The data are not available. This is the case in most poor countries. In that case a statistical survey may help find the budget shares of the T-goods as well as the budget shares of the commodities. However, determining a plausible empirical $\mathbf{T}$ matrix may be more challenging as it may require a minimax optimization.

In the next section, we will expound some US *empirical composition matrices* spanning from 2009 to 2012. From these $\mathbf{T}^*$, we will deduce the transformation matrices $\mathbf{R}^*$ and $\mathbf{S}^*$. In section 04.3, we will introduce the possibility of approaching the empirical composition matrices when only the budget shares are available.

4.2. On uncovering the transformation matrices when $\mathbf{T}^*$ is available

In this section, we construct the empirical composition matrices from 2009 to 2012. We only consider three goods and, accordingly, three basic wants from which, for the sake of feasibility, one is a composite basic want. Heating comprises space and water heating. The composite basic want includes cooking and lighting as we are bound by the assumption that the number of goods equals the number of basic wants.

In consideration of Table 25, we see that for 2009, and even the other years, the first entry, 7.7%, represents the ratio of the electricity consumption due to heating to the total expenditure allocated to the three commodities. In other words, if multiplied by the total expenditure—M—the coefficient gives the electricity consumption due to heating. More visibly, it just represents the fraction of the electricity budget share entering in the heating budget share. It is noticeable to see that 45% of gas consumption is due to heating. Over all, heating (space and water heating) represents 59 % of all energy consumption. Note that DFO, LPG, and Gas do not bring any cooling. Another important remark is the absence of negative numbers inside the matrix, unlike the composition matrix obtained from the system of demand estimation. To put it more clearly, no commodity consumption will negatively enter a basic want budget share.

Table 25: US EMPIRICAL T from 2009 to 2012 (we include lighting in heating)

Years	2009				2010			
Basic wants	ELEC	DFO/LPG	GAS	$\mathbf{w}_T^*$	ELEC	DFO/LPG	GAS	$\mathbf{w}_T^*$
Heating	0.0772	0.0900	0.4500	0.6172	0.0783	0.0900	0.4500	0.6183
Cooling	0.1337	0.0000	0.0000	0.1337	0.1461	0.0000	0.0000	0.1461
Others	0.2391	0.0100	0.0000	0.2491	0.2256	0.0100	0.0000	0.2356
$\mathbf{w}^*$	0.4500	0.1000	0.4500	1	0.4500	0.1000	0.4500	1
years	2011				2012			
Basic Wants	ELEC	DFO/LPG	GAS	$\mathbf{w}_T^*$	ELEC	DFO/LPG	GAS	$\mathbf{w}_T^*$
Heating	0.0780	0.0900	0.4500	0.618	0.0793	0.0900	0.4500	0.6193
Cooling	0.1211	0.0000	0.0000	0.1211	0.1189	0.0000	0.0000	0.1189
Others	0.2399	0.0200	0.0010	0.2609	0.2408	0.0200	0.0010	0.2618
$\mathbf{w}^*$	0.4390	0.1100	0.4510	1	0.4390	0.1100	0.4510	1

In view of Table 26, the empirical transformation matrix states that 17% and 30% of the electricity consumption respectively goes to heating and cooling. These statistics do not vary a lot from one year to another. LPG, DFO or gas consumption has 0% contribution in the cooling activity. We in fact read $r_{22} = r_{23} = 0$.

Table 26: **The US empirical transformation matrices**

Years	Empirical R			Empirical S			Row sums
	0.1716	0.9000	1.0000	0.0000	0.0000	1.0000	**1**
	0.2971	0.0000	0.0000	3.3657	-17.8833	15.5176	**1**
2009	0.5313	0.1000	0.0000	0.0000	10.0000	-9.0000	**1**
Column Sums	**1**	**1**	**1**				
	0.1740	0.9000	1.0000	0.0000	0.0000	1.0000	**1**
	0.3247	0.0000	0.0000	3.0801	-15.4415	13.3614	**1**
2010	0.5013	0.1000	0.0000	0.0000	10.0000	-9.0000	**1**
Column Sums	**1**	**1**	**1**				
	0.1777	0.8182	0.9978	0.0000	-0.0123	1.0123	**1**
	0.2759	0.0000	0.0000	3.6251	-10.9976	8.3725	**1**
2011	0.5465	0.1818	0.0022	0.0000	5.5556	-4.5556	**1**
Column Sums	**1**	**1**	**1**				
	0.1806	0.8182	0.9978	0.0000	-0.0123	1.0123	**1**
	0.2708	0.0000	0.0000	3.6922	-11.2431	8.5509	**1**
2012	0.5485	0.1818	0.0022	0.0000	5.5556	-4.5556	**1**
Column Sums	**1**	**1**	**1**				

4.3. On uncovering the empirical T matrices when only the budgets shares (commodities and basic wants) are available

In most countries, the stock and quality of data available do not allow an ease of computation of the *empirical composition matrices.* In the United Sates the adequate data seem to be only available since 2008. In countries endowed with poor data, a survey is necessary to assess the basic want budget shares. It is possible that providing all data for estimating $\mathbf{T}^*$ is costly. In that case finding a way of constructing the composition matrix from the budget shares data may be considered as an option.

In the following development, we raise the possibility of approximating the composition matrix from only the knowledge of commodity and basic wants budget shares.

The procedure is to statistically assess the budget shares of the basic wants and the commodities. Then using a minimax optimization problem, we compute the *empirical transformation matrix*, which can be plausibly viewed as an approximation of the true composition matrix.

4.3.1. Context setting

As stated previously, the mathematical definition of the transformation matrix is

$$\mathbf{T}_{n,n} = \mathbf{R}_{n,n}(\mathbf{w})_{\Delta} \tag{4.1.3}$$

Three properties accompany this definition:

- The row sums of the $\mathbf{T}_{n,n}$ matrix yield the T-good budget shares

$$\mathbf{T}_{n,n}\boldsymbol{\iota} = \mathbf{w}_T \tag{4.1.4}$$

- The column sums of the $\mathbf{T}_{n,n}$ matrix yield the commodity budget shares

$$\boldsymbol{\iota}'\mathbf{T}_{n,n} = \mathbf{w}' \tag{4.1.5}$$

- The components of the $\mathbf{T}_{n,n}$ sum to one.

$$\boldsymbol{\iota}'\mathbf{T}_{n,n}\iota = \boldsymbol{\iota}'\mathbf{w}_T = \mathbf{w}'\boldsymbol{\iota} = 1 \tag{4.1.6}$$

This property is always verified since the sum of the budget shares, whether for the commodities or the goods, equals one.

Note that from (4.1.4) and (4.1.5) we can write

$$\mathbf{T}_{n,n}\boldsymbol{\iota\iota}'\mathbf{T}_{n,n} = \mathbf{w}_T\mathbf{w}' \tag{4.1.7}$$

Using the expression of $\mathbf{T}_{n,n}$ in(4.1.3), (4.1.7) can be written as

$$\mathbf{R}_{n,n}\boldsymbol{\iota\iota}'\mathbf{R}_{n,n} = \mathbf{w}_T\mathbf{w}' \tag{4.1.8}$$

Since $(\mathbf{w})_\Delta \boldsymbol{\iota} = \mathbf{w}$; $\boldsymbol{\iota}'\mathbf{R}_{n,n} = \boldsymbol{\iota}'$ and $\boldsymbol{\iota}'(\mathbf{w})_\Delta = \mathbf{w}'$, we get from (4.1.8)

$$\mathbf{R}_{n,n}\mathbf{w}\mathbf{w}' = \mathbf{w}_T\mathbf{w} \tag{4.1.9}$$

This equation would lead to the determination of the transformation matrix $\mathbf{R}_{n,n}$ if $\mathbf{w}\mathbf{w}'$ were invertible. It is not. In fact, we can explicitly write $\mathbf{w}\mathbf{w}'$ as

$$\begin{bmatrix} \mathbf{w}_1\mathbf{w}_1 & \mathbf{w}_1\mathbf{w}_2 & \cdots & \mathbf{w}_1\mathbf{w}_n \\ \mathbf{w}_2\mathbf{w}_1 & \mathbf{w}_2\mathbf{w}_2 & \cdots & \mathbf{w}_2\mathbf{w}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w}_n\mathbf{w}_1 & \mathbf{w}_n\mathbf{w}_2 & \cdots & \mathbf{w}_n\mathbf{w}_n \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1\begin{pmatrix}\mathbf{w}_1\\ \mathbf{w}_2\\ \mathbf{w}_3\\ \mathbf{w}_4\end{pmatrix} & \mathbf{w}_2\begin{pmatrix}\mathbf{w}_1\\ \mathbf{w}_2\\ \mathbf{w}_3\\ \mathbf{w}_4\end{pmatrix} & \cdots & \mathbf{w}_n\begin{pmatrix}\mathbf{w}_1\\ \mathbf{w}_2\\ \mathbf{w}_3\\ \mathbf{w}_4\end{pmatrix} \end{bmatrix}$$

This expression highlights the singularity issue.

4.3.2. The optimization problem

We can circumvent the issue by solving the following program

$$\begin{cases} \min\limits_{(r_{11},r_{12},\ldots,r_{nn})} \left\| \mathbf{R}_{n,n}\mathbf{w}\mathbf{w}' - \mathbf{w}_T\mathbf{w}' \right\|_1 \\ \text{subject to :} \\ \sum\limits_{i=1}^{n} r_{ij} = 1 \qquad j = 1,\ldots,n \\ r_{ij} \geq 0 \qquad i,j = 1,\ldots,n \end{cases} \tag{4.1.10}$$

Where $\mathbf{R}_{n.n} = (r_{ij})_{1\le i,j\le n}$.

The first constraint is the expression of the relation: $\boldsymbol{\iota}'\mathbf{R}_{n,n} = \boldsymbol{\iota}'$. This condition states that the T-goods satisfy the original constraint. We can show that the constraint is equivalent to (4.1.5), the second property of the $\mathbf{T}_{n,n}$ matrix.

In fact, premultiplying the mathematical definition— (4.1.3)— of the $\mathbf{T}_{n,n}$ by $\boldsymbol{\iota}'$ matrix we get:

$$\mathbf{w}' = \boldsymbol{\iota}'\mathbf{R}(\mathbf{w})_{\Delta} \tag{4.1.11}$$

Note that $\mathbf{w}' = \boldsymbol{\iota}'(\mathbf{w})_{\Delta}$. Replacing the left hand side of (4.1.11) by this expression reconstitutes the constraint as follows:

$$\boldsymbol{\iota}'\mathbf{R}_{n,n} = \boldsymbol{\iota}'$$

Hence, the constraint is equivalent to the second property of the $\mathbf{T}_{n,n}$ matrix.

4.3.3. Computational procedure

For simplicity we set:

- $\mathbf{K}_{n,n} = \mathbf{w}\mathbf{w}' = (k_{ij})_{1\le i,j\le n}$
- $\mathbf{P}_{n,n} = \mathbf{w}_T\mathbf{w}' = (p_{ij})_{1\le i,j\le n}$
- $\mathbf{R}_{n,n}\mathbf{K}_{n,n} = (c_{ij})_{1\le i,j\le n}$
- $b_{ij} = c_{ij} - p_{ij}, \qquad i, j = 1,\dots,n$

Under n commodities, $\mathbf{K}_{n,n}$ and $\mathbf{P}_{n,n}$ are provided by the data. On the contrary, the matrix $\mathbf{R}_{n,n}$ components remain unknown.

Using these notations, we can rewrite the objective function to be minimized in (4.1.10) as

$$J(r_{11}, r_{12}, ..., r_{nn}) = \left\| \mathbf{R}_{n,n}\mathbf{K}_{n,n} - \mathbf{P}_{n,n} \right\|_1 = \max_{1\le j\le n} \sum_{i=1}^{n} \left| b_{ij} \right|$$
$$= \max_{1\le j\le n} \sum_{i=1}^{n} \left| \sum_{m=1}^{n} r_{im}k_{mj} - p_{ij} \right|$$

We can now reformulate the optimization problem (4.1.10) as:

$$\begin{cases} \min\limits_{(r_{11}, r_{12}, ..., r_{nn})} \left\{ \max_{1\le j\le n} \sum_{i=1}^{n} \left| \sum_{m=1}^{n} r_{im}k_{mj} - p_{ij} \right| \right\} \\ \text{subject to :} \\ \sum_{i=1}^{n} r_{ij} = 1 \qquad j = 1, ..., n \\ r_{ij} \ge 0 \qquad i, j = 1, \ldots, n \end{cases} \tag{4.1.12}$$

The problem is linear and does not need the Kuhn-Tucker approach.

If we denote the objective function as

$$J_{(j)} = \max_{1\le j\le n} \sum_{i=1}^{n} \left| \sum_{m=1}^{n} r_{im}k_{mj} - p_{ij} \right|$$

The problem becomes at its last stage

$$\min \left\{ J_{(1)}, J_{(2)}, ..., J_{(l)}, ..., J_{(n)}, \right\}$$

When $j = l$,

$$J_{(l)} = \max \sum_{i=1}^{n} \left| \sum_{m=1}^{n} r_{im}k_{ml} - p_{il} \right|$$

To get rid of the absolute value in the numerical simulation, we consider

$$\left|\sum_{m=1}^{n} r_{im}k_{ml} - p_{il}\right| = \begin{cases} \alpha_i^+ \text{ if } \sum_{m=1}^{n} r_{im}k_{ml} - p_{il} \geq 0 \\ \alpha_i^- \text{ if } \sum_{m=1}^{n} r_{im}k_{ml} - p_{il} \leq 0 \end{cases}$$

Where $\alpha_i^+ \geq 0$ and $\alpha_i^- \geq 0$. We can then write $\left|\sum_{m=1}^{n} r_{im}k_{ml} - p_{il}\right| = \alpha_i^+ + a_i^-$.

This comes from the fact that $|x| = x^+ + x^-$ with $x^+ = \max(x,0)$ and $x^- = \max(-x,0)$ We should observe also that $x = x^+ - x^-$.

By setting, $x_i = \alpha_i^+$ and $\text{y}_i = a_i^-$, the optimization problem becomes

$$\begin{cases} \quad \min J_{(l)} = \min \sum_{i=1}^{n} (x_i + y_i) \\ \quad \text{subject to:} \\ \sum_{i=1}^{n} r_{ij} = 1 & j = 1,\dots,n \\ \sum_{m=1}^{n} r_{im}k_{ml} - p_{il} = x_i - y_i & i = 1,\dots,n \\ r_{ij} \geq 0 & i, j = 1,\dots,n \\ x_i \geq 0 & i = 1,\dots,n \\ y_i \geq 0 & i = 1,\dots,n \end{cases} \tag{4.1.13}$$

In order to optimally solve the problem, we use the commercial MILP, namely IBM-CPLEX V12.3. We restrict n to be four as this is our number of commodities.

Table 27: **Empirical T matrices deduced from budget shares**

Years	2009				2010			
Basic wants	Elec	Dfo_lpg	Gas	$\hat{\mathbf{w}}_T$	Elec	Dfo_lpg	Gas	$\hat{\mathbf{w}}_T$
HEATING	0.0861	0.1000	0.4500	0.6361	0.0523	0.1000	0.4500	0.6023
COOLING	0.1280	0.0000	0.0000	0.1280	0.1461	0.0000	0.0000	0.1461
OTHERS	0.2359	0.0000	0.0000	0.2359	0.2516	0.0000	0.0000	0.2516
$\hat{\mathbf{w}}$	0.4500	0.1000	0.4500	1.0000	0.4500	0.1000	0.4500	1.0000
Years	2011				2012			
Basic wants	Elec	Dfo_lpg	Gas	$\hat{\mathbf{w}}_T$	Elec	Dfo_lpg	Gas	$\hat{\mathbf{w}}_T$
HEATING	0.0619	0.1100	0.4510	0.6229	0.0598	0.1100	0.4510	0.6208
COOLING	0.1162	0.0000	0.0000	0.1162	0.1174	0.0000	0.0000	0.1174
OTHERS	0.2609	0.0000	0.0000	0.2609	0.2618	0.0000	0.0000	0.2618
$\hat{\mathbf{w}}$	0.4390	0.1100	0.4510	1.0000	0.4390	0.1100	0.4510	1.0000

We see that the table pretty well approximates Table 25. Indeed this program will always give a solution that verifies all the constraints. However, it has to be signaled that the solution is not unique. When we move from three goods to four goods, it becomes necessary to obtain additional restrictions. Hopefully, we will often get them. In our case, we have seen in section 04.2 that $r_{22} = r_{23} = 0$. We can use these two constraints to narrow down the solutions.

In conlusion, by getting the *empirical transformation matrices*, we can match them with the transformation matrices found by the Rotterdam or Aids demand system. This allows us with greater certainty to say which T-good corresponds to which basic want. It will be desirable to use both the T-good income elasticities and the empirical T matrix to identify which row in our T matrix corresponds to which basic want. The simultaneous use of these two tools enables us to better understand how the commodity consumptions contribute to the basic want satisfaction. Beyond this result, the fact of getting the empirical transformation matrix $\mathbf{S}$ offers the possibility of defining some shadow prices and quantities for the basic want. We recall that the definition of these shadow prices is:

$$(Dp_{it})_T = \sum_{j=1} s_{ij} Dp_{jt}$$

$$(Dq_{it})_T = \sum_{j=1} s_{ij} Dq_{jt}.$$

4.4. General approach to a statistical survey for the empirical T matrix elaboration

So far, this study is based on the conviction that in order to fully apprehend the economic determinants of household energy consumption, it is necessary to take the analysis at the level of basic wants which are implicit, unlike prices and quantities. Whether we use the Rotterdam model, the AIDS, or any other demand system, the results obtained by the Preference Independence Transformation would be more effective and credible if we use an empirical transformation matrix as a barometer to interpret our findings. In poor countries, and even in some rich countries, there is a need to construct data to obtain these tables. In this last section, we show that this is easy and possible if the survey directly provides the empirical **R** matrix entries. The analyses we have conducted so far suggest focusing on the key matrix: the transformation matrix $\mathbf{R}^*$. Obtaining this matrix before the composition matrix is statistically more straightforward.

For illustration, let us give some imaginary but realistic numbers. Suppose a survey establishes that 25%, 55% and 15% of electricity consumed are due to heating, cooling and lighting, respectively. The remaining is attributable to negligible basic wants. Similarly, we suppose that 75%, 0% and 0% of DFO consumption are due to heating, cooling, and lighting. The remaining is imputable to the composite basic want. Concerning gas, we suppose that 60%, 0% and 0% are due to heating, cooling, and lighting, respectively. Finally, let us consider that 90% of LPG consumption is related to heating and nothing for cooling and lighting. Of course the 10% is captured by the composite basic want. It is easy to see that having these kind of data directly enables us to define the empirical transformation matrix $\mathbf{R}^*$.

	Empirical R			
Commodities Basic wants	Elec.	Dfo	Gas	Lpg
Heating	.25	.75	.60	.90
Cooling	.55	.00	.00	.00
Lighting	.15	.00	.00	.00
Others	.05	.25	.40	.10
SUMS	**1**	**1**	**1**	**1**

This table verifies the condition $\boldsymbol{\iota}'\mathbf{R}_{n,n} = \boldsymbol{\iota}'$. All it needs is collecting data showing how the goods are allocated between basic wants. It directly defines the matrix $\mathbf{R}^*$. From this matrix we can derive the corresponding $\mathbf{T}^*$ and $\mathbf{S}^*$.

In conclusion, if a survey has to be conducted on basic wants and commodities, it should incorporate the quantification of how commodity consumptions are allocated between basic wants.

5 CONCLUSION

In this work, we humbly attempted to reopen the chapter of the Preference Independence Transformation technique in economic analysis. Since it was introduced by Brooks in his dissertation and extensively constructed by Theil, few authors have attempted to bring their contributions to it. It is definitely not due to lack of interest in the field. Research on basic wants or characteristics as explanatory variables of demand has never been so important. Nowadays, expandable possibilities of production make the love for varieties more realizable. Technological advances are fluid and continuous.

Throughout our analysis, our aim was twofold:

- To find a way to better identify the basic needs after an independent transformation.

- To explore the possibility implement and additional independent transformation technique in addition to the PIT technique.

Our line of attack sought to better unveil the basic wants as key-variables in defining US households' energy consumption. We first explored what has been done so far in the field. The Preference Independence Transformation technique was implemented only using the Rotterdam model setting. We tried to implement a new technique, related to the PIT, with the Almost Ideal Demand System framework. Doing this, there was a need to modify it by considering all the elements of the Slutsky matrix. We referred to this modified Preference Independence Transformation as the Slutsky Matrix Independence Transformation (SMIT). We found the two models did not uncover the basic wants in the same way. The results are different. On the one hand, the diagonalization under the Rotterdam model is done on the specific effect, the diagonalization of which implies the diagonalization of the Hessian matrix $\mathbf{U}$. On the other hand, the AIDS diagonalization is performed on a wider portion of the expenditure function Hessian matrix. The results will be even more different the further the candidate-matrices of the two systems to be diagonalized are from one another.

This observation raises the need to look for an additional way of uncovering the basic wants with more confidence. We asked the question whether the results provided by the transformation were reversible. More clearly, if given an *empirical transformation matrix*—a concept that we have introduced in the analysis, is it possible to get back the empirical transformation matrices? The answer is yes. We can use the following two relations:

$$\mathbf{R}^* = \mathbf{T}^*(\mathbf{w})_\Delta^{-1}$$

$$\mathbf{S}^* = (\mathbf{R}^{*'})^{-1}$$

The knowledge of $\mathbf{R}^*$ will help us see the real restrictions on the r_{ij}. That is, it will inform us of which good is not contributing to which basic want and of which good is the highest provider of a given basic want. When facing results provided by two systems of demand, the transformation matrix $\mathbf{R}^*$ is a good criterion to check which one performs better in association with the PIT or the SMIT.

The knowledge of $\mathbf{S}^*$ enables us to compute some shadow price and volume log-changes of the basic wants through the two following formulas:

$$(Dp_{it})_T = \sum_{j=1} s_{ij} Dp_{jt}$$

$$(Dq_{it})_T = \sum_{j=1} s_{ij} Dq_{jt}.$$

We asked a yet more specific question: If only the commodity and basic want budget shares are available, is it possible to uncover the *empirical T matrix*, and get back to the first question? The answer is less conclusive. If the number of goods is very limited, the answer is yes. In that case, we need to use a $l1-\text{norm}$ minimax optimization. With more than three goods, the solution exists, but it is necessary to add more restrictions.

It appears that the more realistic outcome is obtained when we conduct a survey on the empirical transformation matrix $\mathbf{R}^*$. Indeed, the transformation matrix $\mathbf{R}^*$ concentrates key information for the PIT.

In conclusion, having $\mathbf{R}^*$ or $\mathbf{T}^*$ is interesting insofar as they allow us to have more realistic basic want expenditure elasticities. It suffices to consider the two following relations:

$$(\boldsymbol{\mu}_T^*)_\Delta = \mathbf{R}_{n,n}^* \mathbf{M}_{n,n} \mathbf{S}_{n,n}^{*-1}$$

$$\boldsymbol{\Lambda}_T^* = (\mathbf{w}_T^*)_\Delta^{-1} (\boldsymbol{\mu}_T^*)_\Delta$$

These formulas provide basic want expenditure elasticities that are more accurate, since they combine the parameters estimated from one of the models and the empirical budget shares.

A last contribution of this work is showing that an approximation of the income by the total expenditure on a microeconomic scale may overvalue the income elasiticicities of the commodities and the basic wants as well. In fact, all system-wide approaches of demand lie on the adding-up property that total expenditure equals income. There is need to multiply these values by the elasticity of the total expenditure with respect to income to fully give these found elasticities their real scope.

While the preference independence transformation can be applied directly to the relative price version of the Rotterdam model, direct application to the AIDs model would be extremely difficult. The absolute price version of the Rotterdam model contains marginal budget shares and Slutsky coefficients as parameters. Those coefficients, if derived for the AIDS model, would be nonlinear functions. In principle those functions could be substituted into the absolute price version of the Rotterdam model, and then the absolute price version of the Rotterdam model, with AIDS coefficients, could be transformed into the analog of the relative price version. The resulting price coefficient matrix could be diagonalized to produce the preference independent transformation. But the elements of the resulting price coefficient matrix would be extremely complicated nonlinear functions, producing an unreasonably difficult preference independence transformation.

Since the preference independence transformation would thereby be unrealistically difficult to apply to the AIDS model, I have defined a transformation more suitable to the parameterization and form of the AIDS model. The result is transformed goods characterized by Slutsky independence among them, in the sense that Hicks-Slutsky interactions are removed. While this transformation is practical with the AIDS model, it should be emphasized that the interpretation of the resulting T-goods is more complicated than the interpretation of the basic wants produced by the preference independence transformation with the Rotterdam model.

The original preference independence transformation is a direct transformation of the utility function into a new utility function containing basic wants as arguments. That utility function has the normal properties of a utility function with nonseparable interactions removed, thereby rationalizing the interpretation of transformed goods as “basic wants.” In contrast, my transformation of AIDS produces derived demands for transformed goods having Hicks-Slutsky interactions removed. Caution should be used in imputing conventional properties to demand for those transformed goods without a derived utility function containing them. Further research would be justified on the interpretation and use of the Slutsky independent goods proposed in this dissertation.

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Appendix A Absolute version estimation without symmetry, and with constant

Equation	Observation	Parameter	RMSE	R^2	χ^2	p-value
Equation 1	184	4	.0466053	0.8172	822.35	0.0000
Equation 2	184	4	.0061578	0.6037	280.30	0.0000
Equation 3	184	4	.040203	0.8609	1138.56	0.0000
Equation 4	184	4	.0053249	0.7801	652.73	0.0000

	Coefficients	Std. error	t-student	p-value	Confidence intervals	
Equation 1						
DQt	.4500254	.0207568	21.68	0.000	.4093427	.490708
ELE	-.8907213	.0523408	-17.02	0.000	-.9933074	-.7881351
DFO	-.2112631	.059649	-3.54	0.000	-.328173	-.0943531
LPG	.2329829	.0573705	4.06	0.000	.1205388	.345427
C	.0010363	.0034405	0.30	0.763	-.0057069	.0077795
Equation 2						
DQt	.0409534	.0027425	14.93	0.000	.0355781	.0463287
ELE	.0307495	.0069156	4.45	0.000	.0171951	.0443038
DFO	.0011675	.0078812	0.15	0.882	-.0142795	.0166144
LPG	-.013639	.0075802	-1.80	0.072	-.0284959	.0012178
C	-.0001779	.0004546	-0.39	0.696	-.0010689	.0007131
Equation 3						
DQt	.4718923	.0179054	26.35	0.000	.4367983	.5069862
ELE	.7520093	.0451506	16.66	0.000	.6635156	.8405029
DFO	.1925055	.0514549	3.74	0.000	.0916557	.2933552
LPG	-.200264	.0494894	-4.05	0.000	-.2972614	-.1032666
C	-.0007207	.0029678	-0.24	0.808	-.0065376	.0050962
Equation 4						
DQt	.037129	.0023716	15.66	0.000	.0324807	.0417772
ELE	.1079626	.0059803	18.05	0.000	.0962414	.1196837
DFO	.0175901	.0068153	2.58	0.010	.0042324	.0309478
LPG	-.0190798	.0065549	-2.91	0.004	-.0319273	-.0062324
C	-.0001377	.0003931	-0.35	0.726	-.0009082	.0006327

Appendix B Absolute version without symmetry without constant

Equations	Observation	Parameter	RMSE	R^2	χ^2	p-value
Equation 1	184	4	.0466168	0.8174	823.49	0.0000
Equation 2	184	4	.0061604	0.6036	280.19	0.0000
Equation 3	184	4	.0402095	0.8609	1138.82	0.0000
Equation 4	184	4	.0053267	0.7803	653.39	0.0000

	Coefficients	Std. error	t-student	p-value	Confidence intervals	
Equation 1						
DQt	.4500314	.020762	21.68	0.000	.4093387	.4907241
ELE	-.8914686	.0522949	-17.05	0.000	-.9939647	-.7889725
DFO	-.2109469	.0596545	-3.54	0.000	-.3278676	-.0940263
GAS	.2327506	.0573794	4.06	0.000	.120289	.3452123
Equation 2						
DQt	.0409524	.0027437	14.93	0.000	.0355749	.0463299
ELE	.0308777	.0069107	4.47	0.000	.017333	.0444225
DFO	.0011132	.0078833	0.14	0.888	-.0143378	.0165642
GAS	-.0135992	.0075826	-1.79	0.073	-.0284609	.0012626
Equation 3						
DQt	.471888	.0179083	26.35	0.000	.4367884	.5069876
ELE	.752529	.0451071	16.68	0.000	.6641206	.8409373
DFO	.1922856	.0514552	3.74	0.000	.0914353	.2931359
GAS	-.2001025	.0494928	-4.04	0.000	-.2971066	-.1030983
Equation 4						
DQt	.0371282	.0023724	15.65	0.000	.0324784	.041778
ELE	.1080619	.0059755	18.08	0.000	.09635	.1197737
DFO	.0175481	.0068165	2.57	0.010	.0041881	.0309082
GAS	-.019049	.0065565	-2.91	0.004	-.0318995	-.0061984

Appendix C Absolute version estimation, symmetry and homogeneity imposed

Equation	Observation	Parameter	RMSE	R^2	χ^2	p-value
Equation 1	184	4	.0487502	0.8003	811.51	0.0000
Equation 2	184	4	.0062405	0.5932	129.26	0.0000
Equation 4	184	4	.0055228	0.7638	591.28	0.0000

	Coefficients	Std. error	t-student	p-value	Confidence intervals	
Equation 1						
DQt	.4271871	.0201586	21.19	0.000	.3876769	.4666973
ELE	-.857377	.0485713	-17.65	0.000	-.9525751	-.7621789
DFO	.0214555	.0093793	2.29	0.022	.0030724	.0398387
LPG	.1049314	.005809	18.06	0.000	.0935459	.1163168
Equation 2						
DQt	.0421955	.0039182	10.77	0.000	.034516	.0498749
ELE	.0214555	.0093793	2.29	0.022	.0030724	.0398387
DFO	-.0120974	.0079147	-1.53	0.126	-.0276099	.0034151
LPG	-.0062495	.0036314	-1.72	0.085	-.0133669	.0008678
Equation 4						
DQt	.0394701	.0023984	16.46	0.000	.0347693	.0441708
ELE	.1049314	.005809	18.06	0.000	.0935459	.1163168
DFO	-.0062495	.0036314	-1.72	0.085	-.0133669	.0008678
LPG	-.0059645	.0034273	-1.74	0.082	-.0126818	.0007528

Appendix D Asymptotic variance-covariance provided by Stata

	α_1	β_{11}	β_{12}	β_{13}	β_1	α_0	α_2	α_3	β_{22}	β_{23}	β_{33}	β_2	β_3
α_1	0.0941												
β_{11}	0.0028	0.0012											
β_{12}	-0.0016	0.0000	0.0001										
β_{13}	0.0015	-0.0012	0.0000	0.0015									
β_1	-0.0042	-0.0005	0.0001	0.0003	0.0006								
α_0	-1.3445	-0.1162	0.0223	0.0601	0.1118	27.4971							
α_2	0.0223	0.0033	-0.0015	-0.0029	-0.0022	-0.5382	0.0297						
α_3	-0.1613	-0.0065	0.0021	-0.0031	0.0091	2.6472	-0.0396	0.3342					
β_{22}	-0.0008	-0.0003	0.0000	0.0003	0.0001	0.0284	-0.0008	0.0016	0.0001				
β_{23}	0.0026	0.0003	-0.0002	-0.0003	-0.0002	-0.0547	0.0033	-0.0044	-0.0001	0.0004			
β_{33}	-0.0093	0.0008	0.0000	-0.0019	0.0001	0.0743	0.0013	0.0233	-0.0002	0.0001	0.0038		
β_2	0.0000	0.0000	-0.0001	-0.0001	0.0000	-0.0030	0.0009	0.0001	0.0000	0.0001	0.0001	0.0000	
β_3	0.0024	0.0004	-0.0001	-0.0005	-0.0004	-0.0795	0.0021	-0.0038	-0.0001	0.0002	0.0005	0.0000	0.0005