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Text of paper:

DESIGN OF EXTENDED WARRANTIES IN SUPPLY CHAINS UNDER ADDITIVE DEMAND

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Abstract

We study the design of extended warranties in a supply chain consisting of a manufacturer and an independent retailer. The manufacturer produces a single product and sells it exclusively through the retailer. The extended warranty can be offered either by the manufacturer or by the retailer. The party offering the extended warranty decides on the terms of the policy in its best interest and incurs the repair costs of product failures. We use game theoretic models to answer the following questions. Which scenario leads to a higher supply-chain profit, the retailer offering the extended warranty or the manufacturer? How do the optimum price and extended warranty length vary under different scenarios? We find that, depending on the parameters, either party may provide better extended warranty policies and generate more system profit. We also compare these two decentralized models with a centralized system where a single party manufactures the product, sells it to the consumer and offers the extended warranty. We also consider an extension of our basic model where either the manufacturer or the retailer resells the extended warranty policies of a third party (an independent insurance company, for example), instead of offering its own policy.

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1. INTRODUCTION

Selling extended warranties on products is a highly profitable and rapidly expanding business. Extended warranties are offered on almost all consumer electronics and domestic appliances, ranging from laptop computers to washing machines and refrigerators. Berner (2004) notes that the profits from extended warranties accounted for almost half of Best Buy's operating income and all of Circuit City's operating income for the year 2003. Berner further notes that the profit margins on extended warranties are nearly 18 times higher than those on the products for these retailers.

An extended warranty is actually a service plan, under which the provider promises to repair, replace, or maintain a product for free or at a lower price over a certain period of time after the manufacturer's original warranty expires. The extended warranty may also offer additional benefits (such as return and/or exchange privileges) that are not provided by the original warranty. Extended warranties are sold separately from the products. Generally, an extended warranty can be offered by a manufacturer, a retailer, or by a third party (Publication 153, *Better Business Bureau*). The terms of a typical extended warranty specify the price and the length of time during which the product is covered. The provider of the extended warranty incurs the actual repair costs related to the warranty.

The primary focus of this research is to analyze the design of extended warranties using a supply chain framework. We study a simple supply chain involving a single manufacturer and a single retailer. The manufacturer produces a single product and sells exclusively through the retailer. The extended warranty may be offered either by the manufacturer or by the retailer. The party offering the extended warranty decides on the terms of the policy in its best interest and incurs the repair cost when the product fails. Thus, the repair cost directly influences the provider's extended warranty decisions. We use game theoretic models to answer the following questions. Which scenario leads to a higher supply chain profit, a retailer offering the extended warranty or the manufacturer? How do the optimum price and coverage length of the extended warranty vary under different scenarios? How is the total supply chain profit distributed between the parties in each scenario? We also compare these two situations with a centralized system, in which a single firm manufactures the product, sells directly to the end consumer, and offers the extended warranty. We also consider an extension of the basic model, where the manufacturer or the retailer resells the extended warranty policies of a third party (an independent insurance company, for example), instead of offering its own policy. We then compare the profits of the manufacturer/retailer between the two different ways of doing business: being an extended warranty provider or being a reseller.

The scenarios described above are indeed found in practice. Many manufacturers offer extended warranties directly to the end consumers. GE Appliances offers extended warranties on almost all the products it manufactures. Firms like Ford, GM, JVC, and Apple have devoted whole divisions solely to managing and to serving extended warranty contracts (Padmanabhan 1995). Retail stores such as Best Buy and Home Depot offer and promote third party extended warranties underwritten by insurers, and thereby act as extended warranty resellers. Sears is an example of a retailer that directly provides extended warranties[*]. An example of a centralized system selling a product as well as managing the extended warranty is Dell. A customer can choose from a menu of extended warranties while customizing a computer at Dell's website. It is difficult to find examples of a manufacturer reselling extended warranties, and we suggest some reasons in Section 5.

In this paper, we use game theoretic models to analyze four scenarios: a retailer or a manufacturer being an extended warranty *provider* (model R; model M) and a retailer or a manufacturer being an extended warranty *reseller* for a third party provider (model 3R; model 3M). We first discuss and compare the extended warranty provider (EWP) models, model R and model M. We analyze the total channel profit as well as extended warranty decisions. In addition, we consider the consumer's welfare in terms of the price per unit time of coverage. Our results show that, depending on the parameters, either model R or model M can provide higher channel profit and better coverage. We also discuss how the total channel profit is distributed between the manufacturer and the retailer in the two models. In addition, we compare the performance of the decentralized models with those of a centralized system and show that the centralized system always has the highest total profit, but does not necessarily offer the best consumer welfare. We next analyze the extended warranty reseller (EWR) models, model 3R

[*] Source: http://www.sears.com/shc/s/nb_10153_12605_NB_ProtectionAgreements?adCell=WF, retrieved on February 17, 2011.

and model 3M. Our analyses offer insights on when should a manufacturer or a retailer be a provider or a reseller of extended warranty.

The extended warranties directly sold by third parties to consumers, though common in practice, are not a focus of this work. Such policies can be modeled as insurance policies, the design, pricing, and analysis of which have been well studied in the economics, insurance and risk management literatures (e.g., Lutz and Padmanabhan 1998, Manove 1983, Schlesinger 1983, and Taylor 1995). The extended warranty provider in our paper not only sells and administers the policy, but also influences the product retail price directly (in model R) or indirectly (through the product wholesale price in model M, and the extended warranty wholesale price in model 3R and model 3M). Our model thus allows us to study the interactions of the product decisions and the warranty decisions in a supply chain.

The remainder of the paper is organized as follows. The next section reviews the related literature. We present our basic models (models R and M) and the solution procedures in Section 3. In Section 4, we analyze the results and develop insights. Model 3R and model 3M are discussed in Section 5, while Section 6 summarizes and concludes the paper.

2. LITERATURE REVIEW

The research on design and analysis of extended warranty policies is limited. However, theories of product failure and warranties have received extensive attention in both the economics and operations management literatures.

2.1 Economics/Marketing Literature

Three distinct theories have been proposed in the economics literature to explain the existence of product warranties: the insurance theory, the signaling theory, and the incentive theory. Our paper relates closely to the insurance theory (first addressed by Heal, 1977), which assumes that consumers are more risk-averse than sellers, therefore warranties are provided to consumers as a form of insurance against product failure.

The economics and marketing literature typically treats warranties as cash compensation paid to consumers in case of a product failure. This also applies to the sparse literature on extended warranties. The "insurance" view of warranty may be the result of consumer heterogeneity, as mentioned by Hollis (1999) and as observed by Emons (1989) and Padmanabhan (1995). Self-selection is usually adopted to deal with the consumer heterogeneity. Another example of such work is Lutz and Padmanabhan (1998). Warranties are also treated as monetary compensation in the empirical study by Padmanabhan and Rao (1993).

Although warranties are a form of insurance, in practice, warranty contracts are usually specified by the time duration of coverage rather than by the amount of monetary payment. Our paper captures this feature of warranties and models the extended warranty as duration of time. This distinguishes our work from most extended warranty literature. Although Hollis (1999) models warranty as time duration, unlike our work, his work deals with consumer heterogeneity and self selection.

The paper that comes closest to our work is Desai and Padmanabhan (2004). They discuss the role of extended warranties in channel price coordination for a manufacturer, who sells a durable product through a retailer. They model the extended warranty as a reduced consumer liability when a product fails. The extended warranty is offered to the consumers directly either by the manufacturer or by an independent provider. They also consider a case where the manufacturer sells the extended warranty through the retailer. Our research differs from theirs as follows.

- They only model scenarios where either the manufacturer or the independent party is the extended warranty provider. Our model allows the possibility of a retailer offering the extended warranty (model R), as often observed in practice.
- Our paper focuses on the design of the extended warranties. Both the price and the length of the warranty are decision variables. Their work considers price only.
- As indicated above, consistent with actual practice, we model the extended warranty as duration of time, and consider the number of product failures as well as the corresponding repair costs. Their paper follows the insurance approach and models the extended warranty as reduced monetary loss at a one-time product failure.

2.2 Operations Management Literature

The operations management literature on warranties considers four broad types of problems: (a) characterization of failure rates and warranty costs, (b) lifecycle models for products, where warranty cost is a component of the product lifecycle cost, (c) inventory-warranty models, where the influence of warranties on inventory policies is considered, and (d) supply chain models that consider the impact of warranties on supply chain profits.

Stream (a) focuses on mathematical models with considerable scope and details in the description of warranty costs. The product quality is usually modeled in conjunction with the product failure. Product life-time distribution and failure rate function are important components of the formulation of the warranty cost model. Mamer (1987) and Balcer and Sahin (1986) are good examples of such work. In addition, interested readers may refer to Sahin and Polatogu (1998) for an excellent review of various warranty policies and product failure models.

In stream (b), Cohen and Whang (1997) are among the first to develop a normative model for entire life-cycle of a product. They incorporate warranty costs in the profit function of a firm seeking to maximize total lifecycle profit from a product. They assume that the warranty will run for a fixed interval and that the warranty cost is linear with the manufacturer's quality of after-sales service. However, the design or the management of the warranty is not the focus of their work.

There are two research focuses in the inventory-warranty models of stream (c). One emphasizes repairs and recoveries of warranty returns in inventory systems. Khawam et al. (2007) and Mabini et al. (1992) investigate such warranty inventory systems. The other focuses on how incorporating warranty demand affects inventory planning. Huang et al. (2007) study a base-stock policy that is dependent on the number of units under warranty. Under such a policy, big cost improvements are shown over policies without considering warranty demand. Huang et al. (2008), includes age information about items under warranty in inventory planning, and shows even larger cost improvements.

The supply chain models of stream (d) are relatively new. Heese (2010) explores the supply chain conflicts in the interactions between original product warranties and extended warranties. He considers a scenario where two competing manufacturers sell their products through a common retailer. If the retailer sells its own extended warranty, the manufacturers face a dilemma in setting their base warranties. While the manufacturers have incentives to provide a better warranty to make their product attractive to the customer, the retailer might prefer selling the products with lower warranty to enhance the sales of extended warranties. Heese develops a model to determine and analyze optimal manufacturer and retailer strategies in this setting and shows that this dynamics exerts downward pressure on manufacturer warranties. He also explores extended warranty sales strategies and shows that a retailer can benefit from inducing customers to simultaneously consider product and extended warranties rather than pitching the extended warranties at checkout.

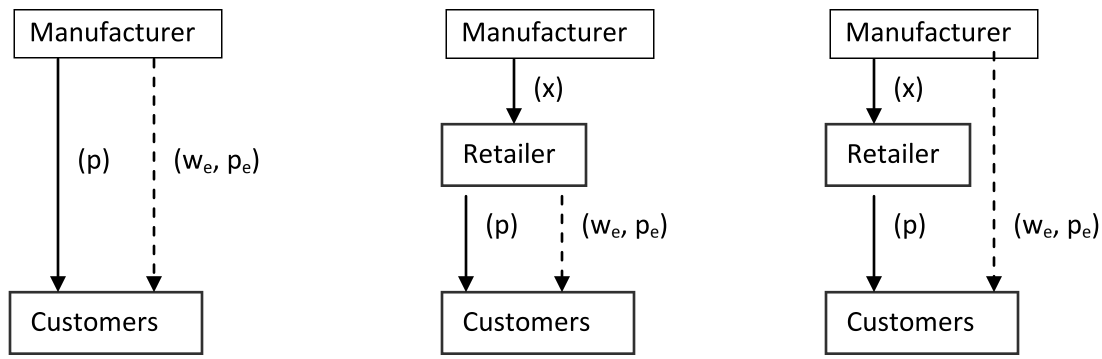
Our work also falls in this last category of supply chain models. We study the design of extended warranties in a supply chain setting, which allows us to study product pricing and extended warranty design decisions within a unified context. To the best of our knowledge, the design of the extended warranties in a supply chain context has not been studied before.

3. EXTENDED WARRANTY PROVIDER MODELS

In this section, we introduce the extended warranty provider (EWP) models, where either the manufacturer (model M) or the retailer (model R) offers the extended warranty directly to the consumers and incurs all associated costs. We are mostly interested in comparing these two models in terms of the total supply chain profit and its division between the manufacturer and the retailer. We also compare the optimal extended warranty lengths and prices between the two models.

Consider a supply chain consisting of a single manufacturer and a single retailer. The manufacturer produces a single product and sells it exclusively through the retailer at a wholesale price x. The retailer resells the product to the end consumer at a retail price p. The manufacturer offers the original product warranty, whose length is normalized to zero. This assumption enables us to focus exclusively on the analysis of the extended warranty. In either model, the specification of the extended warranty involves two components, which are decision variables for the party offering it: the length of the extended warranty in units of time, denoted by w_e; and its price, denoted by p_e. During the lifetime of the extended warranty, the provider commits to offer free repair service for a failed product and incurs the associated repair costs. We will compare the two decentralized models (models R and M) with a centralized system, model C, where the manufacturer produces the product, sells directly to the end consumer, and offers the extended warranty.

Figure 1 schematically describes the three models along with their decision variables. The solid lines depict product decisions, while the dotted lines are for extended warranty decisions.



(a) Model C **(b) Model R** **(c) Model M**

Figure 1: The EWP Models

The Costs

We assume that the manufacturing cost for the product is constant and is normalized to zero, so that the wholesale price is also the manufacturer's product profit margin in the decentralized models. We assume that the number of product failures increases quadratically with time. The literature (Anderson 1977, Menke 1969, and Patankar and Worm 1981) often assumes that the number of failure increases exponentially with time. Our quadratic failure assumption captures the increasing rate of product failure with time, while retaining analytical tractability. A similar assumption has also been used in Heese (2010). Specifically, when an extended warranty of length w_e is offered, the number of product failures is kw_e^2, where k is a constant. The cost of repair may differ from failure to failure. We use average of these repair costs. Let γ_i ($i = r, m$ representing the retailer and the manufacturer, respectively) denote the average cost of repair for party i. Thus, the cost of providing and managing the extended warranty of length w_e is $c_i w_e^2$, where $c_i = k\gamma_i$ for $i = r, m$.

Demand Functions

There are two demand functions in our model, the product demand and the extended warranty demand. First, consider the product demand. We assume that the demand is linearly decreasing in price:

$$q = 1 - bp, \tag{1}$$

where p is the retail price of the product, q is the product demand, and b is the price sensitivity of the consumers.

We next derive the demand function for the extended warranty. Note that only those consumers who buy the product are potential candidates for purchasing the extended warranty, i.e., the maximum demand for the extended warranty is limited by the product demand $1-bp$. How should the demand for the extended warranty, denoted by q_e, behave? Among the important characteristics, this demand should be decreasing in extended warranty price, and increasing in extended warranty length. In practice, extended warranties for the same product vary in price and duration of coverage; consumers consider both while evaluating an offering. Thus, the price per unit length of coverage, p_e / w_e, is an effective way to compare different extended warranties. Finally, if no extended warranty is offered, its demand should be zero. We use the simplest form of demand function that satisfies these properties and maintains tractability of the model. Let d denote the sensitivity of warranty demand to p_e / w_e. Then, the demand for the extended warranty is

$$q_e = \begin{cases} (1-bp) - d\dfrac{p_e}{w_e}, & \text{if } w_e > 0; \\ 0, & \text{if } w_e = 0. \end{cases} \tag{2}$$

Section 5.2 of the paper considers an extension where the demand of the extended warranty depends not only on its price but also on the product price.

Desai and Padmanabhan (2004) derive a demand function for the extended warranty by considering the product-failure probability and the utilities of risk-averse consumers. Their resulting demand function is linear and is of the following form (Equation 8 in their paper):

$$q_e = q - k_1 p_e + k_2,$$

where, k_1 and k_2 are constants. Similar to that of Desai and Padmanabhan (2004), our demand function for the extended warranty is linear in its price. However, unlike their work, the length of the extended warranty is also a decision variable in our model. Our non-linear demand function captures the effect of the length of extended warranty on its demand. Lambertini and Orsini (2001) use a similar non-linear demand function to model the effect of product quality on demand. We also note that we have experimented (analytically or numerically when analytical work was not possible) with several other forms of demand functions including non-linear demand functions. The results from these demand functions are qualitatively similar to those from our current demand function indicating that the recommendations of our model are quite robust. Section 5.2 of the paper further elaborates on these alternative demand functions.

We assume that there is no information asymmetry in the channel and that the manufacturer acts as a Stackelberg leader in the game. The manufacturer, therefore, can look ahead and anticipate the retailer's product pricing decision, as well as his extended warranty decisions, if applicable.

We will use subscripts and superscripts to facilitate the expression of the model variables. The superscripts will denote a model (C, R, or M); the subscripts r, m, 3, e, and $sys.$ will denote the retailer, the manufacturer, the third party, the extended warranty, and the system, respectively. We will also use the superscript "*" to denote optimal values. For example, π_m^{R*} is the optimal profit of the manufacturer in model R, while $\pi_{sys.}^C$ is the system profit in model C.

3.1 Model C: The Centralized System

A centralized system maximizes the total supply chain profit by simultaneously considering the product and extended warranty demands.

$$\underset{p, p_e, w_e}{Max}\ \pi^C = p(1-bp) + (p_e - c_m w_e^2)(1 - bp - d\frac{p_e}{w_e}) \tag{3}$$

Model C can be solved by using standard optimization techniques. Table 1 presents the optimal solutions to model C and the following two decentralized models, along with the optimal values of all decision variables.

3.2 Model R: Retailer Provides Extended Warranty

In model R, besides the retail price *p*, the retailer also decides on the extended warranty terms by specifying p_e and w_e. In the first stage of the game, the manufacturer chooses the wholesale price x that maximizes her profit. The optimization problem of the manufacturer is given by:

$$\underset{x}{Max}\,\Pi_m^R = x(1-bp)\,. \tag{4}$$

In the second stage, taking the wholesale price as given, the retailer maximizes his own profit. The retailer's problem is as follows.

$$\underset{p,p_e,w_e}{Max}\,\pi_r^R = (p-x)(1-bp)+(p_e-c_r w_e^2)(1-bp-d\frac{p_e}{w_e}) \tag{5}$$

We solve the model starting with the retailer's problem and working backwards. Noting that the second order conditions for the maximization problems are satisfied, the three first order necessary conditions of the optimization problem in (5) can be solved simultaneously to yield the retailer's best response retail price $p^*(x)$ in terms of the manufacturer's wholesale price *x*. The optimal wholesale price of the manufacturer can then be found by substituting this best response function into the manufacturer's profit maximization problem in (4) and optimizing with respect to the wholesale price. Once the optimal wholesale price is known, the optimal values of all other decision variables can be found from the first order conditions of the optimization problem in (5).

3.3 Model M: Manufacturer Provides Extended Warranty

In the first stage of the game, the manufacturer chooses the wholesale price x, and the extended warranty terms (p_e, w_e). In the second stage, the retailer takes the manufacturer's decisions as given and sets the retail price p. The manufacturer's optimization problem is as follows.

$$\underset{x,p_e,w_e}{Max}\,\pi_m^M = x(1-bp)+(p_e-c_m w_e^2)(1-bp-d\frac{p_e}{w_e}) \tag{6}$$

In the second stage, given the manufacturer's decisions, the retailer solves for p:

$$\underset{p}{Max}\,\Pi_r^M = (p-x)(1-bp)\,. \tag{7}$$

The solution procedure for model M is similar to that of model R.

4. RESULTS AND ANALYSIS

This section is divided into two sub-sections. Section 4.1 discusses the optimal design of the variables defining the extended warranty. We also elaborate on the interactions between the product market and the extended warranty market. Section 4.2 analyzes the profits of different parties offering the extended warranty. The results answer an important question: which model provides a higher supply chain profit? Our analyses are based on the optimal values of the decision variables summarized in Table 1.

4.1 Extended Warranty Design

Observation 1: *The extended warranty sensitivity parameter, d, must be higher than certain threshold values for the feasibility of each of the three models, C, M, and R.*

The conditions for the feasibility of the three models, C, M, and R, are $d \geq \sqrt{b/9c_m}$, $d \geq \sqrt{b/36c_m}$, and $d \geq \sqrt{b/12c_r}$, respectively. Similar requirements can be found in Gupta and Loulou (1998), and Savaskan et al. (2004). Observation 1 has interesting implications. The feasibility conditions are harder to satisfy when the product price sensitivity *b* is high and the repair cost is low. Such might be the case with products such as telephones and small kitchen appliances. As a result, an extended warranty is rarely offered on such products. On the other hand, the feasibility conditions are easily satisfied when the product price sensitivity *b* is low and the repair cost is high. The examples of such products are flat screen televisions, cars, and high-end camcorders, for which extended warranties are rather common.

Henceforth, we assume that the conditions described in Observation 1 are satisfied. We next characterize the properties of the optimal price and optimal length of the extended warranty.

Proposition 1: *For the three models, C, M, and R, the optimal price and the optimal length of the extended warranty are*

(a) decreasing in the repair cost c_i, *and the sensitivity parameter d.*

(b) increasing in the product price sensitivity parameter b.

The proofs of all results are included in the Appendix.

For a given price, as the repair cost increases, the provider reduces the coverage duration of the extended warranty. The price of the warranty also decreases in order to counter the reduction in demand due to the shorter duration. According to *The Wall Street Journal*, DaimlerChrysler reduced the extended warranty on its vehicles from 2006 because of higher repair costs resulting from higher labor costs and complicated technology (Saranow 2005). The findings of our model directly support this action.

The effect of a change in the parameter d is similar to that of the repair cost. As d increases, the demand for extended warranty goes down. A lower extended warranty price is necessary to counter this effect. Consequently, the duration of extended warranty decreases. It is easy to show that an increase in d (or the repair cost) also results in an increase of the product wholesale and retail prices. Under this scenario, a provider attempts to offset the lost revenue from extended warranty through additional revenue from the product sales.

What is the effect of changes in the product price sensitivity parameter? In absence of an extended warranty, both product price and demand would be decreasing in b. But with extended warranty, as b increases, the product retail price decreases and this decrease is large enough to produce an *increase* in the product demand. This, in turn, increases the size of the potential market for the extended warranty. Thus, a higher price can be charged for the extended warranty with a longer duration.

Finally, it is important to clarify the distinction between Observation 1 and Proposition 1. Proposition 1 examines the sensitivity of the optimal solutions with respect to the model parameters in the region of feasibility, whereas Observation 1 simply imposes technical limits for the feasibility of our model.

The optimal extended warranty prices are not directly comparable across the three models as the optimal durations are different. We therefore compare the price per unit length of coverage, p_e^* / w_e^*, across the three models.

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Proposition 2: *Model M has the lowest extended warranty price per unit length of coverage, unless* $c_r > 3c_m$*, in which case model R has the lowest price per unit length of coverage. In particular, when the repair costs of the manufacturer and the retailer are equal, the extended warranty price per unit length of coverage is highest for model C and is lowest for model M.*

It is important to emphasize that price per unit length of coverage, p_e^* / w_e^*, is simply a construct that allows us to compare the three models. This ratio is *not* a decision variable for any provider of the extended warranty. This precludes the possibility of offering an infinitesimal warranty to maximize this ratio. It can be thought of as a measure of the value of the extended warranty to a consumer.

Per Proposition 2, both model M and model R are potential candidates for the lowest price per unit length of coverage. Consumers get the best value in terms of price per unit length of coverage when the extended warranty is provided by the manufacturer, unless the retailer's repair cost is substantially higher than that of the manufacturer. Why does this happen? The retailer in model R directly influences the demand of the product and the extended warranty by choosing the respective prices. The control over all three key decision variables (the product price, the extended warranty price, and the length of the extended warranty) allows the retailer to charge a higher warranty price per unit length of coverage. However, this advantage wears off as the repair cost of the retailer goes up. In fact, it is easy to establish that the ratio $\frac{p_e^*}{w_e^*}$ is decreasing in the repair cost of the provider (see proof of Proposition 2). Therefore, a substantial disadvantage in repair cost (i.e., $c_r > 3c_m$) eventually outweighs the benefit of setting product and warranty prices simultaneously. Under this scenario, model R has the lowest extended warranty price per unit length of coverage. Model C, being a centralized model, allows the provider to charge the highest price per unit length of coverage, and consequently provides the worst value to the customer.

Under most circumstances, the repair cost of the retailer is unlikely to be three times larger than that of the manufacturer, implying that model M is often likely to have the lowest extended warranty price per unit length of coverage. Direct support of this finding can also be found in the popular business press. According to *Business Week* (Armstrong 2004), electronics retailer

CompUSA charged \$369.99 for a three-year extended warranty plan on a Toshiba Satellite laptop computer, while the manufacturer Toshiba Corp. charged only \$199 for a better plan. Consumer experts often recommend buying an extended warranty directly from a manufacturer. Proposition 2 supports this recommendation.

While price per unit length of coverage (p_e^* / w_e^*) allows us to compare the warranty offerings in the three models, the aggregate measure of welfare from a purchase is formally measured by consumer surplus (CS). How does the consumer surplus from the product and the extended warranty purchase vary across the three models? It is easy to establish that when the repair costs of the manufacturer and the retailer are equal, model C offers the highest consumer surplus while model M offers the lowest ($CS^C \geq CS^R \geq CS^M$). Per proposition 2, when the repair costs are equal, the ratio $\frac{p_e^*}{w_e^*}$ is highest for model C and is lowest for model M. These two results together suggest an interesting interaction between the product and warranty sales. The product price is lowest in model C and is highest in model M ($p^{M*} > p^{R*} > p^{C*}$). As a result, the demand for the product, and hence the *potential* demand for the extended warranty, is highest for model C and is lowest for model M. Furthermore, the product demand in model C is so high that it results in the highest demand for the extended warranty (it is easy to establish from Table 1 that $q_e^{*C} \geq q_e^{*R} \geq q_e^{*M}$) in spite of having the most costly extended warranty in terms of unit length of coverage. This high demand results in highest CS for model C.

How do the product prices compare across model R and model M? Using the expressions from Table 1, it is easy to show that the centralized system offers the lowest product retail price. Furthermore, the product retail price in model R is higher than that in model M (i.e., $p^{R*} > p^{M*}$) if and only if $c_r > 3c_m$. This implies that when the repair costs of the manufacturer and the retailer are equal, the product retail price is highest when the manufacturer offers the extended warranty and lowest when the supply chain is centralized ($p^{M*} > p^{R*} > p^{C*}$). Thus, the product and the extended warranty prices per unit length of coverage exhibit opposite relationships across the three models.

An intuitive explanation is as follows. A lower product price results in a higher demand for the product and a higher potential market size for the extended warranty which allows a provider to charge a higher price for the extended warranty per unit coverage. When the manufacturer offers the extended warranty, the retailer, in setting the product price, is not concerned about the potential demand for the extended warranty and thus the price is higher.

4.2 Profit Analysis

In this sub-section, we compare the system profit for the three models, and examine the division of total profits between the manufacturer and the retailer. We define the system profit (or the supply chain profit) of any model as the sum of profits of the retailer and the manufacturer. Obviously, the centralized model C yields the highest system profit amongst the three models, as can be verified using the expressions from Table 1. As a result, we will restrict our attention to only model R and model M in this sub-section.

Theorem 1: *The optimal system profit of model M is higher than that of model R (* $\pi_{sys.}^{M*} > \pi_{sys.}^{R*}$ *) if and only if the repair cost of the retailer,* c_r*, is higher than a threshold value* Δ_s*, where* Δ_s *is larger than* c_m*. In particular, when* $c_m = c_r$*,* $\pi_{sys.}^{R*} > \pi_{sys.}^{M*}$*.*

The threshold Δ_s is a function of the parameters *b*, *d*, and c_m. The proof of Theorem 1 provides the analytical expression for it.

Theorem 1 reveals an interesting supply chain dynamics. When the repair costs of the manufacturer and the retailer are equal, the retailer offering the extended warranty yields a higher system profit compared to the scenario where the manufacturer offers the extended warranty. Why? In model R, the retailer decides p, p_e and w_e, while the manufacturer decides the wholesale price *x*. Thus, the retailer can simultaneously manipulate all three variables to maximize profits. These three variables allow him to directly influence the product demand as well as the extended warranty demand. On the other hand, when the manufacturer offers the extended warranty (model M), the retailer still determines the product retail price. The product price, in turn, determines its demand and maximum potential for the extended warranty. Under this scenario, the retailer has no incentive to control the double marginalization in the channel and the manufacturer has no mechanism to control it. This results in a lower supply chain profit. Theorem 1 further shows that a better system profit can be obtained if the extended warranty is offered by the retailer, instead of the manufacturer, even when the repair cost of the retailer is higher than that of the manufacturer as long as it is not too high ($c_r < \Delta_s$). Our extensive numerical experiment indicates that as the product price sensitivity parameter b increases, so does Δ_s. Thus, as the customers become more price sensitive, an extended warranty offered by the retailer will generate higher system profit even if the repair cost of the retailer goes up. When the retailer's repair cost increases beyond the threshold value of Δ_s, the cost advantage of model M outweighs the structural advantage (i.e., reduced double marginalization) of model R and hence, model M generates a better system profit.

We next look at the division of profit between the manufacturer and the retailer within a model, as well as between the two models.

Proposition 3:

(a) Comparison between model R and model M:

(i) Manufacturer's profit: $\pi_m^{R*} \leq \pi_m^{M*}$ *if and only if* $c_r \geq 3c_m$.

(ii) Retailer's profit: $\pi_r^{R*} \leq \pi_r^{M*}$ *if and only if* c_r *is greater than a threshold value* Δ_r, *where* Δ_r *is larger than* c_m.

(iii) In particular, when $c_m = c_r$, *we have* $\pi_m^{R*} > \pi_m^{M*}$ *and* $\pi_r^{R*} > \pi_r^{M*}$.

(b) Comparison within model R and model M:

(i) In model R, the profit of the manufacturer is higher than that of the retailer when $b < 11.8c_r d^2$, *and is lower than that of the retailer when* $11.8c_r d^2 < b < 12c_r d^2$.

(ii) In model M, profit of the manufacturer is higher than that of the retailer when $b < 33.75c_m d^2$*, and is lower than that of the retailer when* $33.75c_m d^2 < b < 36c_m d^2$.

The threshold Δ_r is a function of the parameters b, d, and c_m. An analytical expression for Δ_r can be found in the proof of Proposition 3(a).

The proposition shows that, unless the manufacturer has a substantially lower repair cost, both the retailer and the manufacturer are better off with the retailer offering the extended warranty. Thus, under comparable repair costs, an extended warranty offered by a retailer not only generates a higher system profit (Theorem 1), but also makes both retailer and manufacturer better off than they would be if the manufacturer offered the warranty. Why does this happen? When the retailer offers the extended warranty, as discussed after Theorem 1, the double marginalization in the channel is lower than that when the manufacturer offers the extended warranty. This results in a higher demand for the product and the extended warranty for model R. Using the expressions from Table 1, it is easy to verify that for $c_m = c_r$, $q^{R*} \geq q^{M*}$ and $q_e^{R*} \geq q_e^{M*}$. It can further be shown that when the two repair costs are equal, the manufacturer's wholesale price is lower in model R compared to that in model M. Thus, the benefit of reducing the double marginalization in the channel is so significant that the manufacturer's profit from the product alone at a lower wholesale price (model R) is more than that of selling the product at a higher wholesale price and selling the extended warranty (model M). The retailer too enjoys a higher profit under model R because of higher demands for the product and the extended warranty compared to model M. These insights can serve as a useful qualitative guideline for the practitioners responsible for setting up or running extended warranty businesses. It also implies that an extended warranty can be used strategically to reduce the double marginalization in a supply chain.

Are the findings in Proposition 3(a) consistent with observed practices? *Warranty Week*, a major trade publication for the industry professionals, noted in its October 25, 2005 issue that on an aggregate basis, most extended warranties in the Unites States are sold by retailers (Source: http://www.warrantyweek.com/archive/ww20051025.html, retrieved on May 17, 2009). *Warranty Week* further noted in its January 19, 2005 issue that the manufacturers account for only 37.2% of the total market of the extended warranty administrators in the USA (Source: http://www.warrantyweek.com/archive/ww20050119.html, retrieved on May 17, 2009). These observations, once again, support the assertion in Proposition 3(a).

Proposition 3(b) states that, in both models M and R, the manufacturer's profit is higher than that of the retailer for lower values of the product price sensitivity parameter *b*. The manufacturer is the Stackelberg leader in our model. She optimizes her profit by anticipating the retailer's decisions, and is able to obtain a higher share of the system profit. However, when *b* becomes large, the customers become more price sensitive, and a large part of supply chain profit is derived from the product sales, giving the retailer more profit.

To provide or not to provide the extended warranty

During the 1980s extended warranties were offered only on large and expensive items. Today extended warranties are common on almost all durable goods. What are the profit implications of offering an extended warranty? We explore the answer to this question next. To accomplish our objective we compare our model with a *base case* where no extended warranties are sold (i.e., the manufacturer produces the product and sells it to the retailer at wholesale price and the retailer resells the product to the consumer at retail price). Under such a scenario models R and M are identical and that the system profit, the retailer profit, and the manufacturer profit are respectively, 3/(16b), 1/(16b), and 1/(8b). The system profit for model C is 1/(4b). We define the *rate of profit improvement* due to the extended warranty as the difference between the profit under the extended warranty and under the base case, expressed as a fraction of the base-case profit.

Proposition 4:

(a) For the system profit, model C has the highest rate of profit improvement due to the extended warranty. Model R has the smallest rate of profit improvement due to the extended warranty if and only if c_r is greater than a threshold value Δ_s, where Δ_s is greater than c_m. In particular, when $c_r = c_m$, model M has the smallest rate of profit improvement due to the extended warranty.

(b) In both models M and R, the retailer's rate of profit improvement due to the extended warranty is higher than that of the manufacturer.

The threshold Δ_s is a function of the parameters b, d, and c_m. An analytical expression is provided in the proof.

Proposition 4(a) shows that, providing an extended warranty improves the system profit more when it is sold by the retailer rather than the manufacturer, unless the retailer's repair cost is substantially larger than that of the manufacturer. When the retailer offers the extended warranty, he keeps the product price low to expand the potential market for the extended warranty, which in turn reduces the double marginalization in the channel resulting in a higher rate of profit improvement for model R. Proposition 4 shows that this reduction in double marginalization can even overcome some repair cost disadvantage for the retailer. However, when the retailer has a substantial repair cost disadvantage, model M offers better rate of profit improvement.

Under the base case, the manufacturer's profit is twice that of the retailer. However, with an extended warranty, the retailer's profit increases at a faster rate than that of the manufacturer. Indeed, as Proposition 3(b) indicates, when the price sensitivity parameter b is sufficiently large, the retailer's profit can even surpass the manufacturer's profit. This indicates that a retailer stands to gain more from an extended warranty than a manufacturer. This might explain why retailers are so actively involved in the extended warranty business across various industries.

5. EXTENSIONS

Our discussion thus far has focused on the case where either the manufacturer or the retailer underwrites the warranty and incurs costs associated with product failure. The objective of this section is two-fold: to explore an extension of our model to allow third-party extended warranties, and to study the robustness of our key findings under alternative demand functions. To accomplish our first objective, we explore an extension of our basic model, where the manufacturer or the retailer simply resells the extended warranty policies of a third party underwriter. We call these models the extended warranty *reseller* models (EWR) as opposed to the extended warranty *provider* models discussed thus far. As discussed in Section 1, Best Buy, CompUSA, and other retailers are often the resellers of the extended warranties from third party underwriters such as AIG, GE, and American Home Shield. Section 5.1 discusses these models. To accomplish our second objective, we show in Section 5.2 that the key findings of our paper continue to hold under various alternative demand functions and even when the demand of the extended warranty depends on the product price. We also discuss the limitations of our model.

5.1 Extended Warranty Reseller Models

In analyzing the EWR models, we keep the basic structure of our model unchanged, implying that the manufacturer still produces a single product and sells it exclusively through the retailer and that the manufacturer sets the product wholesale price while the retailer sets the retail price. However, the manufacturer or the retailer in the EWR models buys the extended warranty underwritten by a third party, and resells the policy to the end consumer at a retail price. The third party is responsible for managing the extended warranty, and decides on its length w_e and wholesale price x_e in its best interest. The manufacturer or the retailer sets the retail price p_e of the extended warranty.

We consider two EWR models, model 3R, where the retailer is the reseller, and model 3M, where the manufacturer is the reseller. Our objectives in this sub-section are two-fold: to compare the EWP and EWR models and to compare the two EWR models. The first comparison yields insights about whether it is more profitable for a manufacturer or a retailer to be a provider over a reseller of the extended warranty. The second comparison provides structural insights about the 3M and 3R models.

We model the EWR scenario as a three-stage game. In the first stage, as the Stackelberg leader, the manufacturer decides on the product wholesale price. In the second stage, the third party makes the extended warranty decisions (the length and the wholesale price of the extended warranty) after observing the product and evaluating the average repair cost c_3. Finally, the retail prices of the product and the extended warranty are determined simultaneously. The retailer decides on the product retail price, while the retail price of the extended warranty is set by either the manufacturer (Model 3M) or the retailer (Model 3R).

Model 3R

In the first stage of the game, the manufacturer optimizes her profit by deciding the product wholesale price x:

$$\underset{x}{Max}\,\Pi_m^{3R} = x(1-bp)\,. \tag{8}$$

In the second stage, the third party maximizes her profit by simultaneously deciding the wholesale price x_e and the length w_e of the extended warranty

$$\underset{x_e, w_e}{Max}\,\Pi_3^{3R} = (x_e - c_3 w_e^2)\cdot[(1-bp) - d(p_e / w_e)] \tag{9}$$

In the last stage, taking the length of the extended warranty, and the wholesale prices of the product and the extended warranty as given, the retailer simultaneously chooses the retail price of the product *p*, and the extended warranty retail price p_e to maximize his profit. The retailer's problem is as follows.

$$\underset{p, p_e}{Max}\,\Pi_r^{3R} = (p - x)(1-bp) + (p_e - x_e)\cdot[(1-bp) - d(p_e / w_e)] \tag{10}$$

We solve for the model using standard methodology by working backwards, starting from the retailer's problem in the last stage. Note that a special case of model 3R is the scenario where a retailer resells the extended warranty provided by the manufacturer. Desai and Padmanabhan (2004) report that Ford and Apple often use such arrangements.

Model 3M has a similar formulation and solution methodology and hence is omitted. The optimal solutions of the two models 3M and 3R are presented in Table 2.

Being a Provider vs. Being a Reseller

We compare the optimal profits of models 3M and 3R with those of models M and R respectively. This gives us insights about whether it is more profitable for a manufacturer or a retailer to be a reseller of the extended warranty instead of being a provider.

Proposition 5:

(a) The manufacturer gets a higher optimal profit by being a provider of the extended warranty if and only if $c_m < 4c_3$.

(b) The retailer gets a higher optimal profit by being a provider of the extended warranty if and only if $c_r < \Delta_3$, *where* Δ_3 *is a function of the parameters b, d, and* c_3.

The analytical expression for the threshold Δ_3 can be found in the proof.

Proposition 5(a) shows that a manufacturer is better off being a provider unless her repair cost is substantially higher ($c_m \geq 4c_3$) than that of the third party. A manufacturer is likely to have better knowledge of her product, implying her repair cost is likely to be lower than, or comparable to, that of the third party. Thus, the relationship $c_m < 4c_3$ is likely to hold in practice. Therefore, being a provider is often the preferred way of offering extended warranty for a manufacturer. This is usually the case with the auto manufacturers, as discussed in Section 1. As afore mentioned and in consistent with Proposition 5(a), it is difficult to find an example where a manufacturer resells third-party extended warranties.

Proposition 5(b) describes the necessary and sufficient conditions for the retailer to be a provider of the extended warranty. Our numerical experiments indicate that the numerical value of Δ_3 falls in the range of $1.2c_3 \sim 2.5c_3$ for a wide range of values of the problem parameters. (We experimented with 100 scenarios by varying the price sensitivities *b* and *d* from 0.1 to 1 with a step of 0.1. The largest value of Δ_3 is $2.493c_3$, and the smallest is $1.202c_3$, with most of the values above $2c_3$.) Note that the retailer's condition for being a reseller in Proposition 5(b) is less restrictive than the manufacturer's in Proposition 5(a). In addition, a retailer might not have expertise in repairing the product from a manufacturer. Propositions 5(a) and 5(b) thus indicate that in practice, a retailer is more likely to be the reseller of an extended warranty than a manufacturer is. The examples in Section 1 support this notion.

Note that double marginalization is present in both channels in the EWR models: the product channel and the extended warranty channel. Comparing the two models M and 3M we see that when a manufacturer chooses to be a provider, she eliminates double marginalization in the warranty channel. This allows the manufacturer to use the resulting savings to lower the wholesale price of the product. It is easy to verify from Tables 1 and 2 that $x^{M*} \le x^{3M*}$. The lower wholesale price allows the retailer to charge a lower retail price for the product, resulting in higher demands for the product and the warranty. This benefits both the retailer and the manufacturer. Thus, a manufacturer prefers to be a provider of the warranty unless the cost disadvantage is overwhelming ($c_m < 4c_3$ in our model). Comparing the two models R and 3R we, once again, see that the retailer can eliminate the double marginalization in the warranty channel by choosing to be provider. However, unlike the models M and 3M, this does not benefit the manufacturer. The manufacturer maintains a higher wholesale price in this scenario, affecting the retailer's profit. As a result, a retailer chooses to be a reseller when the repair cost goes up and that critical value of the repair cost for switching from provider to reseller is lower for the retailer.

We conclude this section by noting that the scenario where a third party offers extended warranty directly to a consumer is beyond the scope of our paper. Such a scenario can easily be modeled as an insurance policy and is well studied in the literature of risk and insurance management (e.g., Lutz and Padmanabhan 1998, Manove 1983, Schlesinger 1983, and Taylor 1995).

Numerical Study: A Comparison of the two EWR Models

In this section, we present a numerical study to compare the two EWR models with respect to optimal price, demand, and profit. As the reader will see, the relationships between models 3M and 3R are qualitatively similar to those between models M and R, implying that the presence of a third party underwriter does not change the structural properties of our model.

The optimal solutions of the models 3R and 3M are summarized in Table 2. It is hard to compare the optimal solutions of these two models analytically. As a result, we numerically obtained the optimal solutions of the two models under various representative values of the model parameters. We choose the third party's repair cost $c_3 = 5$ (other feasible values of repair cost yield similar results) and let the price sensitivities *b* and *d* vary from 0.2 to 1 at a step of 0.4. Our numerical findings are summarized in Figures 3(a) – 3(g). In each of these figures we plot the difference between the optimal solutions of the models 3R and 3M for different values of the demand sensitivity parameters *b* and *d*. These figures are straightforward. As a result, we simply summarize our findings in the following two observations.

Observation 2:

(a) The optimal retail prices of the product in model 3R are always lower than those in model 3M (i.e., $p^{3R*} < p^{3M*}$).

(b) The optimal extended warranty prices per unit length of time in model 3R are always higher than those in model 3M (i.e., $(\frac{p_e^*}{w_e^*})^{3R} > (\frac{p_e^*}{w_e^*})^{3M}$.

Observation 3:

(a) *The optimal profits of the retailer, the manufacturer, and the third party warranty provider are higher in model 3R than those in model 3M, implying model 3R gives rise to a better system profit than model 3M (i.e.,* $\pi_r^{3R*} > \pi_r^{3M*}$, $\pi_m^{3R*} > \pi_m^{3M*}$, $\pi_3^{3R*} > \pi_3^{3M*}$, *and* $\pi_{sys.}^{3R*} > \pi_{sys.}^{3M*}$*).*

(b) *In the two models 3R and 3M, the optimal profit of the manufacturer is always higher than that of the retailer (i.e.,* $\pi_m^{3R*} > \pi_r^{3R*}$ *and* $\pi_m^{3M*} > \pi_r^{3M*}$*).*

The relationships between models 3R and 3M in Observations 2 and 3 are similar to those between the models R and M for $c_m = c_r$. (Note that the repair costs in models 3R and 3M are identical and equal to that of the third party). Thus, the presence of the third party does not significantly change the structure of our model. The intuitions behind these results have been discussed in detail in Section 4.

5.2 Sensitivity of Results with respect to Alternative Demand Functions

We discuss several alternative demand functions in this sub-section. First we consider the case where the demand of the extended warranty depends on product price. Then we present four other demand functions.

Demand of the Extended Warranty Depends on Product Price

In this section, we extend our basic model into a more complex scenario, where product price not only influences the maximum potential demand of the extended warranty ($1-bp$), but also has a direct impact on the extended warranty demand. Many consumers choose to buy extended warranties on expensive products such as cars, flat screen televisions etc., while few consumers buy extended warranties on products such as telephones or sewing machines. This observation suggests that the demand for the extended warranty relative to the product demand should be increasing with the product price. Our demand function in equation (2) readily accommodates such as extension by letting the parameter *d* to be a function of the product price *p*, i.e.,

$$q_e = (1-bp) - d(p)\frac{p_e}{w_e},\ \ d'(p) \le 0, w_e > 0\,.$$

We choose $d(p) = d/p$, where d is a constant, and write the demand function for the extended warranty as

$$q_e = (1-bp) - \frac{dp_e}{pw_e},\ \ for\ \ w_e > 0\,.$$

Note that this demand function satisfies the desired characteristics discussed in Section 3.

We solve the three models numerically and compare the insights with our original model. We choose the sensitivity parameter d equal to 0.4 and 0.7, and let the price sensitivity b vary from 1.05 to 1.5 in steps of 0.05, while the repair cost c_i varies from 0.05 to 4.0 in steps of 0.1. We seek the optimal solutions under the combinations of the parameters discussed. Our main findings are presented in Figures 4(a) – 4(d). Figure 4(a) shows how the extended warranty price per unit length of coverage, p_e / w_e, vary across the three models C, M, and R when the repair costs of the retailer and the manufacturer are equal. This finding is consistent with that in Proposition 2. Similarly, Figure 4(b) validates Theorem 1 for the more general modeling framework while Figures 4(c) and 4(d) validates Proposition 3(a). This suggests that the findings of our model are quite robust and that the key insights of our original model remain unchanged even when we let the demand of the extended warranty depend on the product price.

Alternative Demand Functions

While we have used the simplest form of demand function that satisfies the desirable properties *and* maintains analytical tractability, to check the robustness of our model, we have repeated our calculations and analyses (analytical or numerical when analytical calculations were not tractable) for each of the following alternative demand functions:

$$q_e = 1 - bp - dp_e + fw_e\,, \tag{11}$$

$$q_e = 1 - bp - dp_e - f / w_e\,, \tag{12}$$

$$q_e = 1 - bp - \frac{dp_e}{\sqrt{w_e}}\,,\ \text{and} \tag{13}$$

$$q_e = 1 - bp - dp_e + f\sqrt{w_e} \tag{14}$$

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Each of these demand functions satisfies the desirable properties of the extended warranty demand described in Section 3. Unlike our demand function in (2), each of the demand functions in (11), (12), and (14) allows different sensitivities for the price and the length of the warranty. We also test for both linear (equation 11) and non-linear (equations 12-14) relationships and allow complex relationship between the demand and length of the extended warranty by experimenting with four different exponents (-1.0, -0.5, 0.5, and 1.0) of w_e. We note that Desai and Padmanabhan (2004) use a linear additive demand function to study extended warranty. Our choices allow the generalization of their demand function to more complex and realistic scenarios. We were able to find analytically tractable optimal solutions for the three models corresponding to demand functions (11), (12), and (13). Models for demand function (14) was solved numerically (40 test problems for each model with varying range of the parameters b, d, and c_r or c_m). The analyses using each of the above four demand functions yield similar qualitative results as those of our original model discussed in Section 4 (i.e. same directional results with different values of the threshold parameters). These calculations, once again, suggest that the recommendations from our model are quite robust.

6. SUMMARY AND CONCLUSION

In this paper, we have developed game-theoretic models to design extended warranties and compared different extended warranty schemes in a supply chain. We first analyzed two EWP models with the retailer and the manufacturer, respectively, being the provider of the extended warranty. We compared the two models in terms of system profits, retailer profits, manufacturer profits, product pricing decisions, and extended warranty decisions. We also compared the two models with a centralized system. The main results are as follows.

- Normally, the manufacturer provides the best extended warranty with respect to the lowest extended warranty price per unit length of coverage, unless the retailer's repair cost is substantially higher than the manufacturer's.
- The retailer, with a higher rate of profit improvement, benefits more from the extended warranty business than the manufacturer, unless he has substantial repair cost disadvantage.

- When the repair costs of the two models are equal, model R not only generates a higher system profit but also makes a Pareto improvement over model M. This fact arises due to the reduced double marginalization in the supply chain.

We then studied two EWR models where a third party offered the extended warranty, while the manufacturer or the retailer simply acted as the extended warranty reseller. Our results show that adding a third party provider does not change the dynamics of the system. By comparing the corresponding profits of the manufacturer and the retailer between EWP and EWR models, we outlined some general guidelines for practitioners as to when a firm should choose to be a provider versus a reseller of the extended warranty.

Our paper makes the following contributions to the operations management literature. First, it provides a unique demand function for the extended warranty, which explicitly takes into account product demand, product failure, and repair cost. In line with our observation in practice, we model the extended warranty as a "service product" with price and time duration for coverage. Second, while a majority of the literature in extended warranties focuses on consumer moral hazard and heterogeneity issues, our work addresses design issues and their role in channel performance. By adopting a game-theoretic approach, we study the strategic interaction between the manufacturer and the retailer as well as the interaction between the product sales and the extended warranty sales. The main results of our models are able to qualitatively explain some observed practices. For example, why do consumer experts always recommend the manufacturer's extended warranties? Why is Chrysler shortening the extended warranty on its new vehicles? Third, our model provides a new perspective in distinguishing the extended warranty sellers on the market: a provider or a reseller. We also provide guidelines for helping make the right choices. Fourth, our model implies that the extended warranty is not merely a source of revenue. Because of the unique character of its demand, which is dependent on product demand, the extended warranty could be used strategically in channel choices to improve system profits and product pricing decisions. These results provide useful insights as well as qualitative guidance to practicing managers involved in designing, implementing, and managing an extended warranty business.

Like any other model in operations management literature, our model is based on a set of assumptions. For simplicity, we studied the extended warranty business using a monopolistic setting. This allowed us to isolate the impact of the extended warranty on supply chain profits. An important extension of our model would be to include competition, either between manufacturers or between retailers. Incorporating demand uncertainties might be another useful extension. Our model assumes no information asymmetry. In practice, a retailer might have private information about consumer demand. In such a scenario, the retailer might use this information strategically to improve his profit. We have assumed a one-sided market where the demand for the extended warranty is influenced by the product demand but *not* vice-versa. This assumption seems to hold in practice for a wide range of consumer durables ranging from domestic appliances to consumer electronics. Retailers such as Wal-Mart and Sam's Club typically do not even pitch the sale of an extended warranty until the checkout. Admittedly, there might be scenarios where the availability of an attractive extended warranty will influence the product demand as well. Studying extended warranties under such two-sided markets remains one of our agendas for future research.

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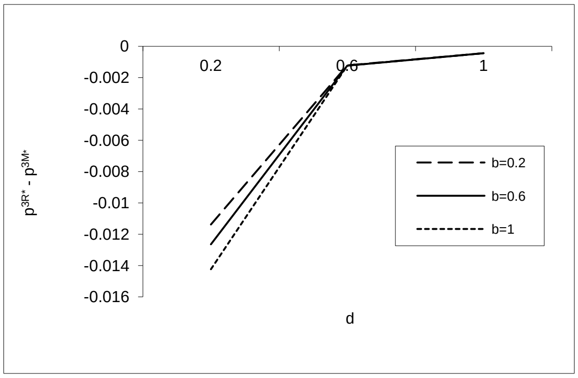

Figure 3(a): Comparison of Optimal Product Retail Price Between Models 3R and 3M

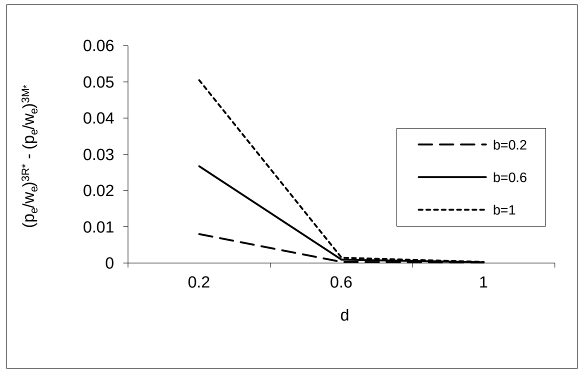

Figure 3(b): Comparison of Optimal Extended Warranty Price Per Unit Length of Time between Models 3R and 3M

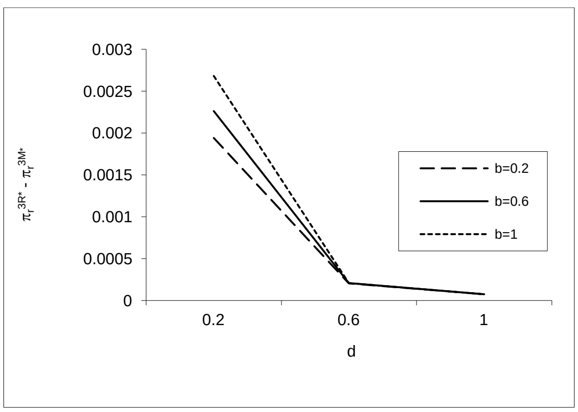

Figure 3(c): Comparison of Optimal Profit of Retailer Between Models 3R and 3M

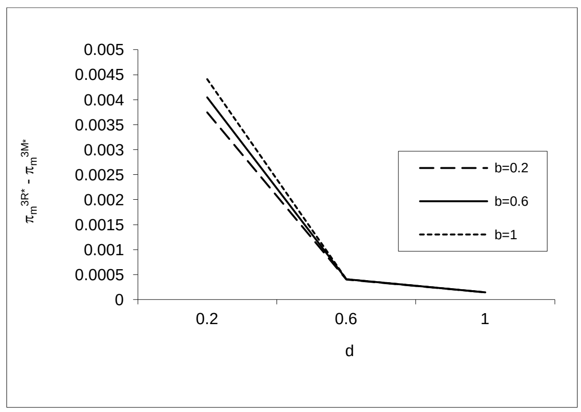

Figure 3(d): Comparison of Optimal Profit of Manufacturer Between Models 3R and 3M

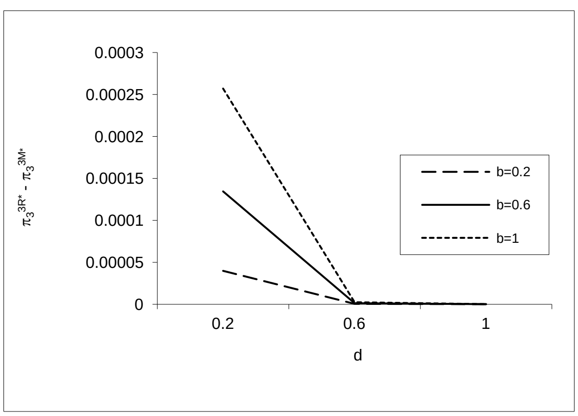

Figure 3(e): Comparison of Optimal Profit of Third Party Between Models 3R and 3M

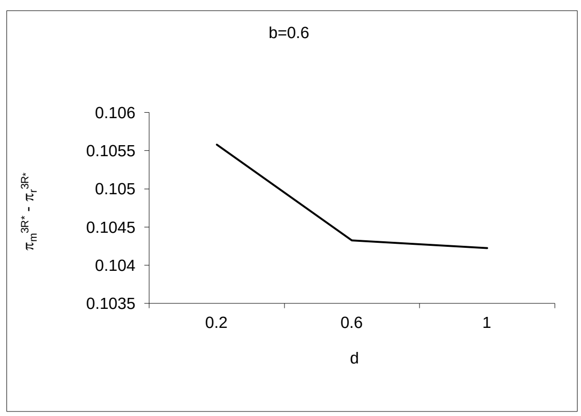

Figure 3(f): Comparison of Optimal Profit Between Manufacturer and Retailer in Model 3R

Li, Kumpeg, Suman Mallik, and Dilip Chhajed. "Design of Extended Warranties in Supply Chains under Additive Demand." *Production and Operations Management* 21.4 (2012): 730-46. Publisher's official version: http://dx.doi.org/10.1111/j.1937-5956.2011.01300.x. Open Access version: http://kuscholarworks.ku.edu/dspace/.

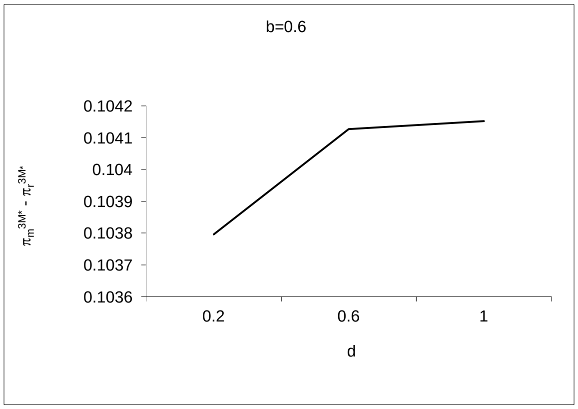

Figure 3(g): Comparison of Optimal Profit Between Manufacturer and Retailer in Model 3M

Note: We have omitted the curves for b = 0.2 and b = 1 from Figures 3(f) and 3(g) as the y-axis values are substantially higher for these curves. The qualitative nature of all curves, however, is similar.

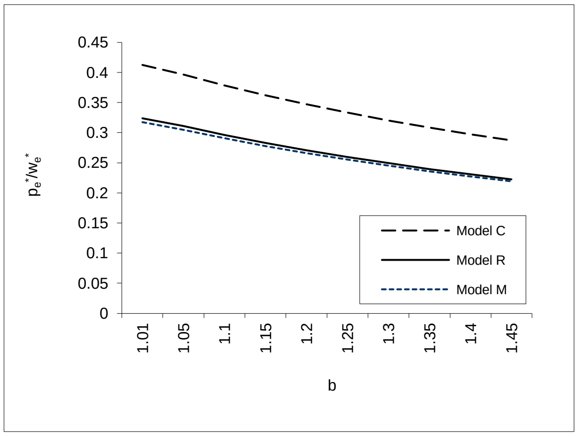

Figure 4(a): Comparison with Proposition 2, EW Price Per Unit Length of Coverage (c=0.5, d=0.4)

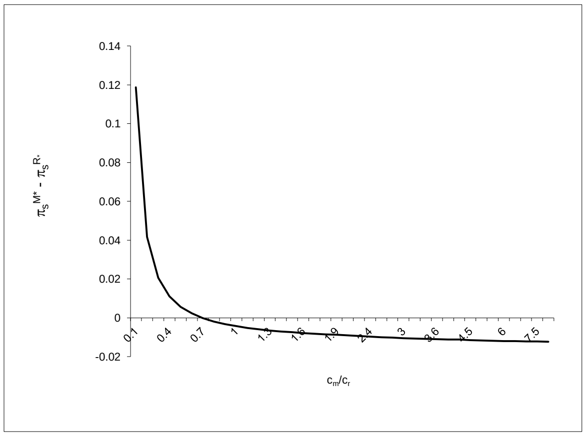

Figure 4(b): Comparison with Theorem 1, Optimal System Profit between Models R and M (d=0.4, b=1.01, c_r=0.5, c_m ranges from 0.05 to 4)

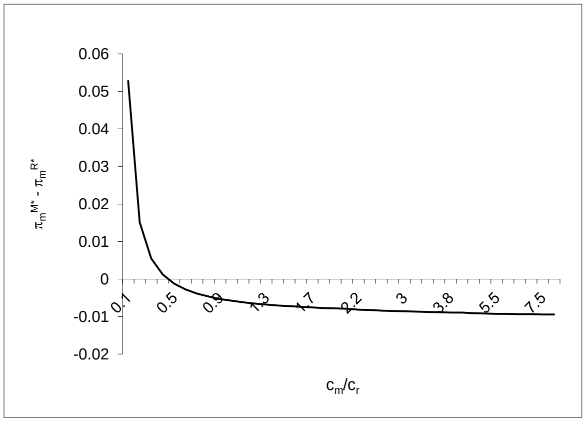

Figure 4(c): Comparison with Proposition 3(a), Optimal Manufacturer's Profit between Models R and M (d=0.4, b=1.01, c_r=0.5, c_m ranges from 0.05 to 4)

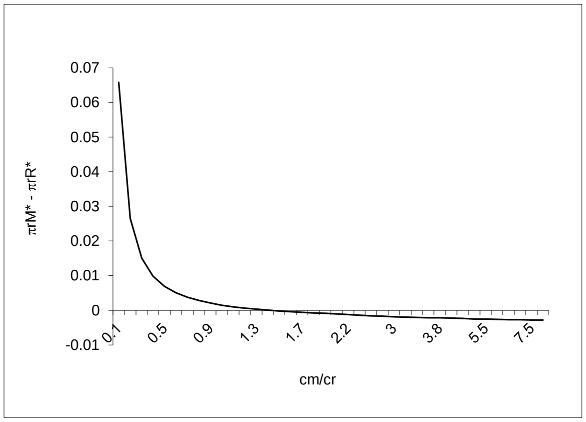

Figure 4(d): Comparison with Proposition 3(a), Optimal Retailer's Profit between Models R and M (d=0.4, b=1.01, c_r=0.5, c_m ranges from 0.05 to 4)

APPENDIX: PROOFS OF RESULTS

Proof of Proposition 1

We prove proposition 1 for Model R. The proofs for other two models are similar. Using the expressions from Table 1 we get:

(a) $\frac{\partial w_e}{\partial c_r} = \frac{-1}{2c_r\sqrt{3c_r(12c_r d^2 - b)}} < 0$; $\frac{\partial w_e}{\partial d} = \frac{-2[6c_r d - \sqrt{3c_r(12c_r d^2 - b)}]}{b\sqrt{3c_r(12c_r d^2 - b)}} < 0$;

$$\frac{\partial p_e}{\partial c_r} = \frac{-4d[-b + 24c_r d^2 - 4d\sqrt{3c_r(12c_r d^2 - b)}]}{b^2\sqrt{3c_r(12c_r d^2 - b)}} < 0;$$

$$\frac{\partial p_e}{\partial d} = \frac{-8c_r[-b + 24c_r d^2 - 4d\sqrt{3c_r(12c_r d^2 - b)}]}{b^2\sqrt{3c_r(12c_r d^2 - b)}} < 0.$$

(b) $\frac{\partial w_e}{\partial b} = \frac{-b + 24c_r d^2 - 4d\sqrt{3c_r(12c_r d^2 - b)}}{2b^2\sqrt{3c_r(12c_r d^2 - b)}} > 0$;

$$\frac{\partial p_e}{\partial b} = \frac{2[6c_r d(-3b + 48c_r d^2) - (48c_r d^2 - b)\sqrt{3c_r(12c_r d^2 - b)}]}{3b^3\sqrt{3c_r(12c_r d^2 - b)}} > 0.$$

Proof of Proposition 2

The extended warranty per length prices for model C, model R, and model M are:

$(\frac{p_e}{w_e})^{C*} = 2c_m[\frac{6d}{b} - \frac{1}{3c_m d - \sqrt{c_m(9c_m d^2 - b)}}]$, $(\frac{p_e}{w_e})^{R*} = 2c_r[\frac{4d}{b} - \frac{1}{6c_r d - \sqrt{3c_r(12c_r d^2 - b)}}]$,

and $(\frac{p_e}{w_e})^{M*} = 2c_m[\frac{12d}{b} - \frac{1}{6c_m d - \sqrt{c_m(36c_m d^2 - b)}}]$.

First, we prove that the extended warranty per length price decreases in c_i , where $i = r$ *or* m.

$$\frac{\partial(p_e/w_e)^{C*}}{\partial c_m} = 2(\frac{6d}{b} - \frac{bc_m}{2\sqrt{c_m(9c_m d^2 - b)}(3c_m d - \sqrt{c_m(9c_m d^2 - b)})^2}) < 0$$

$$\frac{\partial(p_e/w_e)^{R*}}{\partial c_r} = 2(\frac{4d}{b} - \frac{3bc_r}{2\sqrt{3c_r(12c_r d^2 - b)}(6c_r d - \sqrt{3c_r(12c_r d^2 - b)})^2}) < 0$$

$$\frac{\partial(p_e/w_e)^{M*}}{\partial c_m} = 2(\frac{12d}{b} - \frac{bc_m}{2\sqrt{c_m(36c_m d^2 - b)}(6c_m d - \sqrt{c_m(36c_m d^2 - b)})^2}) < 0$$

Second, when $c_m = \frac{c_r}{3}$, $(\frac{p_e}{w_e})^{R*} = (\frac{p_e}{w_e})^{M*}$. Since $(\frac{p_e}{w_e})^{M*}$ is decreasing in c_m, so if c_m increases from $\frac{c_r}{3}$ to c_r, $(\frac{p_e}{w_e})^{M*}$ decreases and becomes smaller than $(\frac{p_e}{w_e})^{R*}$, i.e., when $c_r = c_m$, $(\frac{p_e}{w_e})^{R*} > (\frac{p_e}{w_e})^{M*}$. In addition, when $c_r = c_m$, $(\frac{p_e}{w_e})^{C*} > \frac{3}{2} \cdot (\frac{p_e}{w_e})^{R*} \Leftrightarrow 9b^2 > 0$, which shows $(\frac{p_e}{w_e})^{C*} > (\frac{p_e}{w_e})^{R*}$. In summary, we have $(\frac{p_e}{w_e})^{C*} > (\frac{p_e}{w_e})^{R*} > (\frac{p_e}{w_e})^{M*}$ when $c_r = c_m$. This also shows $(\frac{p_e}{w_e})^{C*}$ is always larger than $(\frac{p_e}{w_e})^{M*}$ as the two models have the same repair cost c_m.

When $c_m < \frac{c_r}{3}$, $(\frac{p_e}{w_e})^{R*} < (\frac{p_e}{w_e})^{M*}$, which shows the condition when model R has the lowest per length price.

<u>Proof of Theorem 1</u>

First, we prove that when $c_r = c_m$, $\pi_{sys.}^{R*} > \pi_{sys.}^{M*}$.

$\pi_{sys.}^{R*} > \pi_{sys.}^{M*} \Leftrightarrow$

$$33b^3 - 1377b^2 c_m d^2 + 69336b(c_m d^2)^2 - 414720(c_m d^2)^3 > (72b^2 + 8676bc_m d^2 - 69120(c_m d^2)^2)\sqrt{3c_m(12c_m d^2 - b)}$$

$\Leftrightarrow$

$121b^3 + 11826b^2 c_m d^2 + 97929b(c_m d^2)^2 - 1399680(c_m d^2)^3 < 0$, which holds when

$b \in (0, 7.326c_m d^2)$; or

$121b^3 + 11826b^2 c_m d^2 + 97929b(c_m d^2)^2 - 1399680(c_m d^2)^3 > 0$, which holds when

$b \in [7.326c_m d^2, 12c_m d^2]$.

For any value of c_r, we have $\pi_{sys.}^{R*} > \pi_{sys.}^{M*} \Leftrightarrow c_r < \Delta_s$, where Δ_s is equal to the value of

$$\frac{1}{135d^2(9b + 160c_m d^2)} \cdot [96b^2 + 2385bc_m d^2 + 11880c_m^2 d^4 + d(110b + 1980c_m d^2)\sqrt{c_m(36c_m d^2 - b)} + 4\sqrt{\delta},$$

where $\delta =$

$$576b^4 + 20b^3 d(163c_m d + 66\sqrt{c_m(36c_m d^2 - b)}) + 360b^2 c_m d^3(-1728c_m d + 271\sqrt{c_m(36c_m d^2 - b)}) +$$

$$4725bc_m^2 d^5(-2601c_m d + 470\sqrt{c_m(36c_m d \;\; - b)}) + 24300c_m^3 d^7(-2514c_m d + 661\sqrt{c_m(36c_m d^2 - b)})$$

Next, we prove $\Delta_s > c_m$:

When $c_r = c_m$, model R has higher system profit than model M. In addition,

$$\frac{\partial \pi_{sys.}^{R*}}{\partial c_r} = \frac{d[6(720c_r^2 d^4 - 93bc_r d^2 + 2b^2) - 9d(80c_r d^2 - 7b)\sqrt{3c_r(12c_r d^2 - b)}]}{9b^3\sqrt{3c_r(12c_r d^2 - b)}}$$

< 0. As c_r increases, model R's system profit decreases. When $c_r < \Delta_s$, model R's system profit is lower than that of Model M. Obviously Δ_s is required to be larger than c_m.

Proof of Proposition 3

(a) (i) $\pi_m^{R*} \le \pi_m^{M*} \Leftrightarrow c_r \ge 3c_m$.

(ii) $\pi_r^{R*} \le \pi_r^{M*} \Leftrightarrow c_r \ge \Delta_r$, where

$$\Delta_r = \frac{(6480c_m^2 d^4 - 1062bc_m d^2 - 4b^2) + 1080c_m d^3\sqrt{c_m(36c_m d^2 - b)} + \sqrt{\delta_r}}{54d^2(288c_m d^2 + b)},$$

where $\delta_r = 236196c_m^2 d^4(b + 288c_m d^2)[-b + 24d(6c_m d - \sqrt{c_m(36c_m d^2 - b)}] +$
$[(6480c_m^2 d^4 - 1062bc_m d^2 - 4b^2) + 1080c_m d^3\sqrt{c_m(36c_m d^2 - b)}]^2$

Similar proof as in Theorem 1 for Δ_r is larger than c_m.

(b) In model R, $\pi_m^{R*} > \pi_r^{R*} \Leftrightarrow b < \frac{189c_r d^2}{16} \approx 11.8c_r d^2$;

In model M, $\pi_m^{M*} > \pi_r^{M*} \Leftrightarrow b < \frac{135c_m d^2}{4} = 33.75c_m d^2$.

Proof of Proposition 4

Comparing the increasing rate of the system profit between model R and model M, we obtain

$$\frac{\pi_{sys.}^{R*} - \frac{3}{16b}}{\frac{3}{16b}} > \frac{\pi_{sys.}^{M*} - \frac{3}{16b}}{\frac{3}{16b}} \Leftrightarrow \pi_{sys.}^{R*} > \pi_{sys.}^{M*} \Leftrightarrow c_r < \Delta_s$$

. The value of Δ_s can be found in the proof of Theorem 1. The rest of the proofs are straightforward.

Proof of Proposition 5

(a) $\pi_m^{M*} > \pi_m^{3M*} \Leftrightarrow$

$c_m < Max[4c_3, \dfrac{4d(144c_3d^2 - b)\sqrt{c_3(144c_3d^2 - b)} - (6912c_3^2d^4 - 180bc_3d^2 + b^2)}{27(128c_3d^2 - b)}]$. For any value of $b \le 144c_3d^2$, $4c_3 \ge \dfrac{4d(144c_3d^2 - b)\sqrt{c_3(144c_3d^2 - b)} - (6912c_3^2d^4 - 180bc_3d^2 + b^2)}{27(128c_3d^2 - b)}$, so the final condition is $c_m < 4c_3$.

(b) $\pi_r^{R*} < \pi_r^{3R*} \Leftrightarrow \dfrac{d[b(9c_rd - 2\sqrt{3c_r(12c_rd^2 - b)}) + 12c_rd^2(6c_rd - \sqrt{3c_r(12c_rd^2 - b)})]}{9b^3} < \pi_r^{3R*}, \Leftrightarrow$

$c_r < \Delta_3$, where $\Delta_3 = \dfrac{b\cdot[(2 - 27b\pi_r^{3R*}) - (2 - 36b\pi_r^{3R*})\sqrt{(1 + 9b\pi_r^{3R*})}]}{27d^2(16b\pi_r^{3R*} - 1)}$.

Note that $\pi_r^{3R*} = \dfrac{8d\sqrt{c_3}(b^2y - b + 9c_3d^2)\sqrt{c_3(b^2y + 9c_3d^2 - b)} + [3b^4y - 6b^3y + b^2(3 - 36c_3d^2y) + 36c_3d^2b - 216(c_3d^2)^2]}{16b^3}$,

and $y = \dfrac{4b^2 - 288bc_3d^2 + (171c_3d^2)^2 + \Phi(\Phi + 8b + 117c_3d^2)}{12b^2\Phi}$, where

$\Phi = \sqrt[3]{8b^3 - 378b^2c_3d^2 - 3888bc_3^2d^4 + 4330989c_3^3d^6 + 18d\sqrt{3c_3}(b - 90c_3d^2)\sqrt{8b^3 - 945b^2c_3d^2 + 71928bc_3^2d^4 - 793152c_3^3d^6}}$

.

Table 1: Optimal solutions for model C, model R, and model M

Models		**Model C**	**Model R**	**Model M**
Product decisions	p^*	$\frac{b-3d(3c_m d-\sqrt{c_m(9c_m d^2-b)})}{b^2}$	$\frac{b-d(6c_r d-\sqrt{3c_r(12c_r d^2-b)})}{b^2}$	$\frac{b-3d(6c_m d-\sqrt{c_m(36c_m d^2-b)})}{b^2}$
	x^*	N/A	$\frac{2b-2d(6c_r d-\sqrt{3c_r(12c_r d^2-b)})}{3b^2}$	$\frac{b-6d(6c_m d-\sqrt{c_m(36c_m d^2-b)})}{b^2}$
Extended warranty decisions	p_e^*	$\frac{12d(3c_m d-\sqrt{c_m(9c_m d^2-b)})-2b}{b^2}$	$\frac{8d(6c_r d-\sqrt{3c_r(12c_r d^2-b)})-2b}{3b^2}$	$\frac{24d(6c_m d-\sqrt{c_m(36c_m d^2-b)})-2b}{b^2}$
	w_e^*	$\frac{3c_m d-\sqrt{c_m(9c_m d^2-b)}}{bc_m}$	$\frac{6c_r d-\sqrt{3c_r(12c_r d^2-b)}}{3bc_r}$	$\frac{6c_m d-\sqrt{c_m(36c_m d^2-b)}}{bc_m}$
Demands	q^*	$\frac{3d(3c_m d-\sqrt{c_m(9c_m d^2-b)})}{b}$	$\frac{d(6c_r d-\sqrt{3c_r(12c_r d^2-b)})}{b}$	$\frac{3d(6c_m d-\sqrt{c_m(36c_m d^2-b)})}{b}$
	q_e^*	$\frac{d(3c_m d-\sqrt{c_m(9c_m d^2-b)})}{b}$	$\frac{d(6c_r d-\sqrt{3c_r(12c_r d^2-b)})}{3b}$	$\frac{d(6c_m d-\sqrt{c_m(36c_m d^2-b)})}{b}$

Optimal profits:

Model C Profit	$\pi_{sys.}^{C*}$	$\frac{d[b(9c_m d-2\sqrt{c_m(9c_m d^2-b)})-18c_m d^2(3c_m d-\sqrt{c_m(9c_m d^2-b)})]}{b^3}$
Model R Profits	$\pi_{sys.}^{R*}$	$\frac{d[b(63c_r d-8\sqrt{3c_r(12c_r d^2-b)})-60c_r d^2(6c_r d-\sqrt{3c_r(12c_r d^2-b)})]}{9b^3}$
	π_r^{R*}	$\frac{d[b(9c_r d-2\sqrt{3c_r(12c_r d^2-b)})+12c_r d^2(6c_r d-\sqrt{3c_r(12c_r d^2-b)})]}{9b^3}$
	π_m^{R*}	$\frac{d[b(18c_r d-2\sqrt{3c_r(12c_r d^2-b)})-24c_r d^2(6c_r d-\sqrt{3c_r(12c_r d^2-b)})]}{3b^3}$
Model M Profits	$\pi_{sys.}^{M*}$	$\frac{d[b(9c_m d-2\sqrt{c_m(36c_m d^2-b)})+36c_m d^2(6c_m d-\sqrt{c_m(36c_m d^2-b)})]}{b^3}$
	π_r^{M*}	$\frac{9c_m d^2[-b+12d(6c_m d-\sqrt{c_m(36c_m d^2-b)})]}{b^3}$
	π_m^{M*}	$\frac{d[2b(9c_m d-\sqrt{c_m(36c_m d^2-b)})-72c_m d^2(6c_m d-\sqrt{c_m(36c_m d^2-b)})]}{b^3}$

Table 2: Optimal solutions for model 3R and model 3M

Models		Model 3R	Model 3M
Product decisions	p^*	$\frac{6d\sqrt{c_3(b^2x+9c_3d^2-b)}+(b^2x+3b-18c_3d^2)}{4b^2}$	$\frac{b-72c_3d^2+6d\sqrt{c_3(144c_3d^2-b)}}{b^2}$
	x^*	$\frac{4b^2-288bc_3d^2+(171c_3d^2)^2+\Phi(\Phi+8b+117c_3d^2)}{12b^2\Phi}$	$\frac{b-144c_3d^2+12d\sqrt{c_3(144c_3d^2-b)}}{b^2}$
Extended warranty decisions	p_e^*	$\frac{(b^2x-b-24c_3d^2)\sqrt{b^2x+9c_3d^2-b}+(b^2x-b+72c_3d^2)d\sqrt{c_3}}{4b^2d\sqrt{c_3}}$	$\frac{10\cdot[-b+288c_3d^2-24d\sqrt{c_3(144c_3d^2-b)}]}{b^2}$
	x_e^*	$\frac{(b^2x-b-12c_3d^2)\sqrt{b^2x+9c_3d^2-b}-(b^2x-b-36c_3d^2)d\sqrt{c_3}}{4b^2d\sqrt{c_3}}$	$\frac{8\cdot[-b+288c_3d^2-24d\sqrt{c_3(144c_3d^2-b)}]}{b^2}$
	w_e^*	$\frac{3d\sqrt{c_3}-\sqrt{b^2x+9c_3d^2-b}}{b\sqrt{c_3}}$	$\frac{2\cdot[12c_3d-\sqrt{c_3(144c_3d^2-b)}]}{bc_3}$
Demands	q^*	$\frac{(-b^2x+b+12c_3d^2)\sqrt{b^2x+9c_3d^2-b}-(7b^2x-7b+36c_3d^2)d\sqrt{c_3}}{4b(\sqrt{c_3}d+\sqrt{b^2x+9c_3d^2-b})}$	$\frac{6\cdot[12c_3d^2-d\sqrt{c_3(144c_3d^2-b)}]}{b}$
	q_e^*	$\frac{(-b^2x+b-6c_3d^2+2d\sqrt{c_3}\sqrt{b^2x+9c_3d^2-b})\cdot d\sqrt{c_3}}{2b(\sqrt{c_3}d+\sqrt{b^2x+9c_3d^2-b})}$	$\frac{12c_3d^2-d\sqrt{c_3(144c_3d^2-b)}}{b}$
Profits	$\pi_{sys.}^*$	$p^*q^*+(p_e^*-c_3w_e^{*2})q_e^*$	$\frac{108d^2\cdot[-bc_3+288c_3^2d^2-24c_3d\sqrt{c_3(144c_3d^2-b)}]}{b^3}$
	π_r^*	$(p^*-x^*)q^*+(p_e^*-x_e^*)q_e^*$	$\frac{36d^2\cdot[-bc_3+288c_3^2d^2-24c_3d\sqrt{c_3(144c_3d^2-b)}]}{b^3}$
	π_m^*	x^*q^*	$\frac{4d\cdot[18bc_3d-1728c_3^2d^3+(144c_3d^2-b)\sqrt{c_3(144c_3d^2-b)}]}{b^3}$
	π_3^*	$(x_e^*-c_3w_e^{*2})q_e^*$	$\frac{4d\cdot[-36bc_3d+6912c_3^2d^3-(576c_3d^2-b)\sqrt{c_3(144c_3d^2-b)}]}{b^3}$

Note: The expressions for optimal solutions for model 3R are rather complicated. We have only provided the expression of the optimal wholesale product price x^{3R*} in Table 2, where

$$\Phi=\sqrt[3]{8b^3-378b^2c_3d^2-3888bc_3^2d^4+4330989c_3^3d^6+18d\sqrt{3c_3}(b-90c_3d^2)\sqrt{8b^3-945b^2c_3d^2+71928bc_3^2d^4-793152c_3^3d^6}}$$

The optimal values of other variables have been expressed in terms of x^{3R*}.