Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/





# Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty

**By Tailan Chi and John Liu**
The University of Kansas

**Paper citation:**

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. *IIE Transactions*, 33 (9), 695-704.

**Abstract:**

A key characteristic of the product life cycle (PLC) is the depletion of the product's market potential due to technological obsolescence. Based on this concept, we develop a stochastic model for evaluating market entry and exit decisions during the PLC under uncertainty. The model explicates the conditions for the optimality of a two-threshold policy based on the estimated earnings potential of the product, and can be used by manufacturing firms to assess entry and exit decisions under such conditions. To aid the applications of the model in actual decision situations, we also provide the procedures for computing the exact and approximate values of the two thresholds.

# Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty

Tailan Chi and John Liu*

School of Business Administration
University of Wisconsin-Milwaukee
P. O. Box 742
Milwaukee, WI 53201-0742
USA

Accepted for publication in *IIE Transactions*, 33 (9), pp. 54-66, 2001.

* Address all correspondence to this author.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

# Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty

**ABSTRACT**

A key characteristic of the product life cycle (PLC) is the depletion of the product's market potential due to technological obsolescence. Based on this concept, we develop a stochastic model for evaluating market entry and exit decisions during the PLC under uncertainty. The model explicates the conditions for the optimality of a two-threshold policy based on the estimated earnings potential of the product, and can be used by manufacturing firms to assess entry and exit decisions under such conditions. To aid the applications of the model in actual decision situations, we also provide the procedures for computing the exact and approximate values of the two thresholds.

## 1. Introduction

Many technology-based products, such as consumer durables and office automation apparatus, have been found to exhibit a pattern of evolution that resembles the life of a living organism [1, 2, 3, 4]. The evolution process—commonly referred to as the product life cycle (PLC)—begins with the introduction of a new product (birth) and ends with the exhaustion of the product's market potential (death) due to technological obsolescence [5]. In general, a product's birth coincides with the commercialization of a technology innovator's R&D results, and its death coincides with the termination of the product's manufacture by the last remaining technology follower/laggard in the industry. Hence, firms that operate in the same industry but possess differing technological resources may enter and exit at different points in time during a product's life cycle. In addition, the innovating firm that has developed a new product sometimes does not have the production and marketing resources to commercialize the product by itself and may be compelled to sell or license its technology to a more established firm [6]. While the exit decisions of the innovator and follower are similar, their entry decisions differ significantly. Specifically, the innovator has the choice between commercializing the technology itself and selling or licensing it to another firm, and the follower faces only the decision of whether to acquire the technology for a price when it is available for purchase or license.

This study attempts to model the entry and exit decisions in the PLC from the perspective of a technology follower that can gain access to, at a price, the technology requisite for the entry into a given product market. The entry decision involves an evaluation of the potential earnings from the product against the price for acquiring the requisite assets. The exit decision arises in the later stages of the product's life cycle (i.e., sometime after the entry) and involves an evaluation of the remaining earnings potential against the salvage value of the existing assets. The key variable in both of these decisions is, obviously, the potential earnings from the product in the future. As has been demonstrated in numerous studies since the path-breaking work of Bass [1], a fundamental characteristic of the PLC is that the product's potential for future earnings depletes as its life cycle progresses. Existing models of the PLC, whose focus tends to be on marketing decisions such as advertising and pricing, have generally treated the depletion of the product's earnings potential during its life cycle as a deterministic process.[1] Given the extent of uncertainty about a product's future earnings, however, it is our position that analytical rigor in modeling the entry and exit decisions during the PLC calls for a more realistic representation of earnings depletion process through the use of a stochastic model.

Given one's necessarily incomplete knowledge of the natural world, the future earnings from a product can only be estimated imperfectly based on the current understanding of the

[1] For a review of various evolutionary functions that have been used in the literature to model a product's earnings or sales potential during the PLC, see Feichtinger, Hartl and Sethi [7].

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/  Open Access Version: http://kuscholarworks.ku.edu/dspace/

factors affecting its market potential. These factors may include (but are not limited to) the state and growth of the general economy, the availability of substitutes, and the pace of technological change in the industry. Since these factors tend to evolve over time with a significant degree of unpredictability, new information about them can be expected to arrive on a continuous basis during a product's life cycle. This possibility for continuously updating the estimation of the product's earnings potential can be accounted for through the use of a stochastic process (e.g., Brownian motion) to represent the evolvement of the estimation over time.

In this paper, we study the entry and exit decisions in the PLC by developing a stochastic model that treats the earnings potential of the product as a Brownian motion with negative drift to represent the earnings depletion process with random disturbances. This model is set up in section 2. As will be discussed in section 3, the model presents a challenging boundary-value problem represented in a system of second-order ordinary differential equations (ODEs), and a closed-form solution of the ODE system is not attainable. The section identifies the appropriate boundary conditions for obtaining a solution and also derives a power-series solution. Our result suggests that the optimal control of the PLC is characterized by a two-threshold policy for the entry and exit decisions. Specifically, the policy identifies both an entry threshold and an exit threshold such that the expected payoff from the product is maximized by entering the product market when its earnings potential is above the entry threshold and exit the product market when the potential falls to the exit threshold. Section 4 explains the methods for computing the two thresholds both under an exact solution and under an approximate solution and use numerical results to show how the thresholds vary with the key parameters of the system. The last section summarizes and concludes the paper.

## 2. Problem formulation

As explained in the introduction section, what we attempt to model in this paper is the market entry and exit decisions of a firm whose primary competencies lie in manufacturing and marketing. Suppose at time $t = 0$ the firm has an opportunity to acquire some technology that will enable it to introduce a new product into the market. Acquisition of the technology may involve the purchase or license of some patent rights from the current owner of the technology. Let $x \in (0,\infty)$ denote the current estimate of the future earnings from the product over its life time. The cost of acquiring the technology and any investment required to effectuate production and marketing of the product constitutes an initial entry cost $C(x)$. It is reasonable to assume this entry cost to have both a fixed component and a scale-dependent variable component that rises with $x$. A simple function that contains both of these components is

$$C(x) = I + bx,$$

with $b$ being a scale coefficient. The firm's entry decision essentially involves an assessment of whether the potential earnings from the product, $x$, justifies the initial entry cost, $C(x)$.

Once an entry decision is fully implemented, the firm will have acquired an additional bundle of assets in the form of technology, plant and equipment, and marketing expertise. If for some reason it decides to exit the market later, those assets may still have some salvage value. It is again reasonable to assume the salvage value of those assets to increase with $x$—the product's remaining earnings potential. Let the salvage value be an exponential function of $x$,

$$V(x) = V_0(\eta - \gamma^x),$$

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

with $V_0$ being a constant, $\gamma \in (0,1)$ and $\eta \in [1,2]$. The value of $V_0$ can be set in the vicinity of the value of the initial investment in the introduction of the product.[2] As can be seen, the value of $\gamma^x$ equals 1 at $x = 0$ and falls toward 0 as $x$ becomes very large. So, the lower bound of $V(x)$ is 0 if $\eta = 1$ and is $V_0$ if $\eta = 2$. Obviously, a reasonable function for $V(x)$ requires a coordinated choice of the values for $\gamma$ and $\eta$ based on how specialized the assets are. If the assets can not be used for any other purpose, they would lose all their value as $x$ falls to 0, implying a value of $\eta$ close to 1 and a relatively small $\gamma$. If the assets can be employed just as gainfully for another purpose, their value would depreciate little as the value of $x$ falls, implying a value of $\eta$ close to 2 and a value of $\gamma$ close to 1. The sensitivity of $V(x)$ to changes in $x$ is given by $V'(x) = -V_0\gamma^x \ln\gamma$. It should be noted that our problem is meaningful only if the new is worth more than the old, that is,

$$I + bx \geq V_0(\eta - \gamma^x).$$

Once the firm starts to manufacture and market the product, it will inevitably receive new information about the product's earnings potential on a continuous basis as a result of its direct involvement in the production and marketing activities. This type of learning can be modeled as a continuously updated forecast of the product's yield in the rest of its life. Let $X_t \in [0,\infty)$ denote the estimated remaining yield of the product as of time $t > 0$ after the firm's entry is carried out.

[2] One may wonder why we do not simply substitute $C(x)$ for $V_0$ since we consider the value of the initial investment to be an appropriate value for $V_0$. The reason is that there is likely a considerable lag between the time of entry and the time of exit. Entry necessarily occurs before exit, so the initial investment becomes a known constant after the entry decision is fully implemented. In addition, as a result of the time lag, the value of $x$ observed at entry time is likely to be very different from the value of $x$ observed at exit time.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

We treat $X_t$ as a stochastically evolving variable given the uncertainty about the evolution of the technological and market conditions during the product's life cycle. Specifically, we characterize the evolution of $X_t$ in the PLC using the following stochastic process:

$$dX_t = -f(X_t)dt + \sigma\rho(X_t)dW_t, \tag{1}$$

where $W_t$ is a Wiener process (i.e., a standard Brownian motion), $\sigma$ is a constant representing the maximum standard deviation of $dX_t$, and $\rho(X_t)$ is a scaling function defining the evolution of the standard deviation in the PLC. As the extent of uncertainty about the remaining yield is likely to diminish as $X_t$ approaches zero, we require $0 \le \rho(X_t) \le 1$ and $\lim_{X_t \to 0} \rho(X_t) \to 0$. As suggested by Pindyck [8], a simple function that embodies these properties is

$$\rho(X_t) = \sqrt{\lambda X_t},$$

where $\lambda$ is a constant that affects the sensitivity of the volatility to changes in $X_t$.

Since the evolution of the remaining yield is modeled as a stochastic depletion process, the drift term in (1) has a negative sign. Within the drift term, the function $f(X_t)$ represents the estimated yield in each time increment, which by definition is expected to reduce the remaining yield of the product by the same amount. We give $f(X_t)$ the following functional form:

$$f(X_t) = \frac{r(X_t)}{1+\xi(X_t)}, \tag{2}$$

where $r(X_t)$ and $\xi(X_t)$ are both increasing functions of $X_t$. As the total yield of the product in the rest of its life is given by $X_t$, a higher yield level in each time increment, $f(X_t)$, also means that the remaining yield will be depleted at a faster pace. The specification of $f(X_t)$ allows two opposing forces to operate as the remaining yield gets depleted with the accumulation of realized earnings. The component in the numerator, $r(X_t)$, represents the force of technology diffusion because a rise in this component accelerates the depletion process as a result of technological obsolescence.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

The rest of the function, $1/[1+\xi(X_t)]$, represents the force of market diffusion because the gradual fall in the value of $X_t$, which is expected to occur as sales accumulate with the product reaching more consumers, has a positive effect on the yield of each successive period $f(X_t)$ due to greater consumer awareness [1]. For simplicity, we assume $r(X_t) = \alpha X_t$ and $\xi(X_t) = \beta X_t$, with $\alpha$ and $\beta$ being constants. Based on these assumptions, we can rewrite (2) and (1) as, respectively,

$$f(X_t) = \frac{\alpha X_t}{1+\beta X_t}, \tag{3}$$

$$dX_t = -\frac{\alpha X_t}{1+\beta X_t}dt + \sigma\sqrt{\lambda X_t}\,dW_t. \tag{4}$$

It should be noted again that the function defined in (3) represents the instantaneous yield rate as well as the depletion rate of the remaining yield. As time $t$ does not enter explicitly in either the drift term or the volatility term, the stochastic process defined in (4) is stationary.

As the evolution of the remaining yield $X_t$ is defined as a stochastic depletion process, the value of $X_t$ will fall to zero at some $t < \infty$. Let $\tau$ denote the time at which $X_t$ reaches zero, i.e., $\tau \equiv \inf\{t \geq 0 \mid X_t \leq 0\}$. The life of the product obviously comes a natural end at $t = \tau$ as its yield potential is exhausted. But the optimal decision may entail the termination of the product before its life ends naturally. Let $\theta$ denote the exit time at which the firm decides to discontinue the product. Then, we have $\tau \leq \theta$ if the earnings are naturally depleted and $\theta < \tau$ if the product is discontinued before its earnings are depleted. The actual life of a product, therefore, is either $\tau$ or $\theta$, whichever comes first (i.e., $\theta \wedge \tau$).

Then, based on the stochastic process defined in (4) above, the expected payoff from the product over its life cycle, conditioned on an initial state of $x \in [0, \infty)$, can be expressed as

$$E_x\left[\int_0^{\theta\wedge\tau} \frac{\alpha X_t}{1+\beta X_t} e^{-\mu t} dt + V_0(\eta - \gamma^{X_{\theta\wedge\tau}})e^{-\mu(\theta\wedge\tau)}\right] - (I + bx), \tag{5}$$

where $\mu$ is an applicable discount rate. Within the expectation sign in (5), the first term gives the discounted value of the earnings over the product's life and the second term gives the discounted salvage value of the assets at the time of termination; the term outside the expectation sign is the initial entry cost at $t = 0$. After an entry decision is executed, the entry cost becomes a sunk cost, and the problem that the firm faces is reduced to finding the optimal stopping time such that the expected payoff in the product's remaining life is maximized. This optimal stopping problem can be stated as follows.

$$\pi(x) \equiv \max_{\theta} E_x\left[\int_0^{\theta\wedge\tau} \frac{\alpha X_t}{1+\beta X_t} e^{-\mu t} dt + V_0(\eta - \gamma^{X_{\theta\wedge\tau}}) e^{-\mu(\theta\wedge\tau)}\right]. \tag{6}$$

Then, the firm's optimal decision rule at the time of entry is just[3]

$$J(x) = \begin{cases} \text{Do not enter if } \pi(x) \le I + bx, \\ \text{Enter otherwise.} \end{cases}$$

## 3. Solution of the model

Based on the underlying stochastic process defined in (4), we can derive the following second-order differential equation from the optimal stopping problem stated in (6) using Ito's lemma [9, 10]:

$$\frac{1}{2}\sigma^2 \lambda x \frac{d^2\pi(x)}{dx^2} - \frac{\alpha x}{1+\beta x}\frac{d\pi(x)}{dx} - \mu\pi(x) + \frac{\alpha x}{1+\beta x} = 0\,. \tag{7}$$

Finding a solution to this differential equation entails the identification of appropriate boundary conditions. As the optimized payoff function defined in (6) is necessarily non-decreasing in $x$, there must exist two threshold values of $x$, $\hat{z} > \hat{x}$, such that entry is warranted for all $x \ge \hat{z}$ and

[3] Since the stochastic process defined in (4) is stationary, the value and shape of $\pi(x)$ remains the same whether $x$ is observed at $t = 0$ or any other time.

exit is warranted for all $x \leq \hat{x}$. These two thresholds provide two natural points of *x* that can be used in our derivation of the needed boundary conditions. First, optimality of the entry and exit decisions requires that the optimized payoff be equal to the value of the initial entry cost at the entry threshold $\hat{z}$ and be equal to the salvage value at the exit threshold $\hat{x}$. These requirements give us the following two boundary conditions:

$$\pi(\hat{z}) = I + b\hat{z}\,, \tag{8}$$

$$\pi(\hat{x}) = V_0(\eta + \gamma^{\hat{x}})\,. \tag{9}$$

Two additional boundary conditions come from the optimality requirement of "smooth pasting" that the marginal change in the optimized payoff be equal to the marginal entry cost at the entry threshold and be equal to the marginal change in the salvage value at the exit threshold, that is,

$$\pi'(\hat{z}) = b\,, \tag{10}$$

$$\pi'(\hat{x}) = -V_0\gamma^{\hat{x}} \ln\gamma\,. \tag{11}$$

The five equations specified in (7) to (11) in theory give a unique solution of $\pi(x)$ for the interval $x \in [\hat{x}, \hat{z}]$, as well as the two threshold values of *x*.

Although an analytical solution to the differential equation is not attainable, it is possible to derive a series solution that will enable us to examine the asymptotic properties of the solution and develop approximating algorithms. Because the derivation of the series solution is long and involves mostly technical details, we will present only the result in the text and leave the detailed mathematical operations in the appendix. As shown in the appendix, the following expression is a series solution to the differential equation specified in (7):

$$\tilde{\pi}(x; w_1, w_2) = w_2 + \sum_{n=0}^{\infty}[c_n + (w_1 + w_2 \ln x)\cdot a_n(1) + w_2 b_n'(0)]x^{n+1}\,, \tag{12}$$

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

where $a_n(1)$, $b_n'(0)$ and $c_n$ are given by (A16), (A20) and (A25), respectively. The two coefficients $w_1$ and $w_2$ need to be determined jointly with the two threshold values of $x$, $\hat{z}$ and $\hat{x}$, using the same boundary conditions given in (8) to (11). To be complete, we rewrite the four boundary conditions for the series solution $\tilde{\pi}(x; w_1, w_2)$ as

$$\tilde{\pi}(\hat{z}; w_1, w_2) = I + b\hat{z}\,, \tag{13}$$

$$\tilde{\pi}(\hat{x}; w_1, w_2) = V_0(\eta + \gamma^{\hat{x}})\,, \tag{14}$$

$$\tilde{\pi}'(\hat{z}; w_1, w_2) = b\,, \tag{15}$$

$$\tilde{\pi}'(\hat{x}; w_1, w_2) = -V_0\gamma^{\hat{x}} \ln\gamma\,. \tag{16}$$

As shown in the appendix, the power series given in (12) is convergent for $x \in (0, \frac{1}{\beta})$, that is, it constitutes a valid solution to the differential equation specified in (7) for $x \in (0, \frac{1}{\beta})$.

In practice, one can compute the exact values of the two control thresholds by solving numerically the ODE system specified in (7) to (11) or approximate their values using the series solution given in (12) to (16). The ensuing section will discuss how to compute the threshold values both under the exact solution and under the approximate solution and compare the results obtained with these two solution methods.

## 4. Numerical analysis: Procedures and results

The first part of this section will explain the procedure for obtaining the exact solution and then illustrate the solution with some numerical examples. The second part of the section will present the algorithm for obtaining the approximate solution and examine its accuracy.

### 4.1 *Exact solution*

Although the system of ODEs specified in equations (7) to (11) are in principle solvable numerically, the computation of a numerical solution is complicated by the fact that the interval over which the solution must be evaluated, $x \in (\hat{x}, \hat{z})$, is unknown and thus does not have fixed endpoints. To overcome this difficulty, we need to convert the unknown interval to a known interval with two fixed endpoints. The conversion can be performed as follows. First, create a new independent variable $u \in [0,1]$ and define $q_0 = \pi(x)$, $q_1 = \dfrac{d\pi(x)}{dx}$, $q_2 = \hat{z}$ and $q_3 = \hat{x}$. Then, we can express the remaining yield in the relevant interval $(\hat{x}, \hat{z})$ as

$$x = q_2 + u(q_3 - q_2),$$

and perform a change of variable to obtain

$$\frac{dx}{du} = q_3 - q_2 = \hat{z} - \hat{x},$$

$$\frac{dq_0}{du} = \frac{d\pi(x)}{dx}\frac{dx}{du} = q_1(q_3 - q_2),$$

$$\frac{dq_1}{du} = \frac{d^2\pi(x)}{dx^2}\frac{dx}{du} = \frac{d^2\pi(x)}{dx^2}(q_3 - q_2).$$

Finally, using the redefined functions, we can set up the problem as a system of four first-order differential equations and solve it for the interval $u \in [0,1]$ with four boundary conditions:

$$\frac{dq_0}{du} = q_1(q_3 - q_2), \tag{17}$$

$$\frac{dq_1}{du} = \left\{ \frac{\alpha(q_1 - 1)}{1 + \beta[q_2 + u(q_3 - q_2)]} + \frac{\mu q_0}{q_2 + u(q_3 - q_2)} \right\}(q_3 - q_2)\frac{2}{\lambda\sigma^2}, \tag{18}$$

$$\frac{dq_2}{du} = 0, \tag{19}$$

$$\frac{dq_3}{du} = 0, \tag{20}$$

$$q_0(0) = I + bq_2, \tag{21}$$

$$q_0(1) = V_0(\eta + \gamma^{q_3}), \tag{22}$$

$$q_1(0) = b, \tag{23}$$

$$q_1(1) = -V_0\gamma^{q_3}\ln\gamma. \tag{24}$$

The problem defined in (17) to (24) can be easily implemented in such numerical solvers as MathCad and Maple V. We used the fourth-order Runge-Kutta method provided in MathCad Professional to solve the problem. The computation typically takes less than ten seconds on a PC with a 400MHz Pentium II processor, but requires 1-3 minutes with a slower 90MHz Pentium processor. Figure 1 shows an example of the solution to the differential equation (7).

--------------------------------
Insert Figure 1 about here
--------------------------------

In Figure 1, the dotted line on the top represents the initial capital $C(x)$ and the dashed line on the bottom represents the salvage function $V(x)$. The solid line in the middle represents the optimized payoff $\pi(x)$ for the interval $x \in (\hat{x}, \hat{z})$, and the intersections of this line with the other two lines indicate the values of the optimal entry and exit thresholds. It can be seen from the figure that the optimized payoff function $\pi(x)$ is strictly increasing in $x$ for the interval $(\hat{x}, \hat{z})$, validating the optimality of a two-threshold policy. As explained earlier, this policy calls for entry under $x \geq \hat{z}$ and exit under $x \leq \hat{x}$.

--------------------------------
Insert Figure 2 about here
--------------------------------

It is of particular interest to examine how the optimal entry and exit thresholds respond to the extent of volatility in the potential yield. Note that in our model the parameter $\sigma$ is the basic index of volatility. Figure 2 shows graphically the sensitivity of the entry and exit thresholds to the value of $\sigma$. As the figure indicates, a rise in volatility raises the entry threshold $\hat{z}$ and lowers the exit threshold $\hat{x}$, thus widening the distance between the two thresholds. The intuition behind this result is that greater uncertainty justifies more caution in the entry and exit decisions that are at least partially irreversible (e.g., due to sunk cost). It can also be seen that the entry threshold $\hat{z}$ is significantly more sensitive to a change in $\sigma$ than does the exit threshold $\hat{x}$. The reason lies in the fact that the volatility of the underlying stochastic process, as defined in (4), is determined by the function $\sigma\sqrt{\lambda x}$ and thus falls with the value of $x$. Since the value of $x$ near the exit threshold $\hat{x}$ is much smaller than its value near the entry threshold $\hat{z}$, the impact of a change in $\sigma$ on the volatility of the process is much weaker near the exit threshold than near the entry threshold.

### 4.2. *Approximate solution*

In the rest of this section, we sketch an algorithm for computing the approximate solution and assess its advantages and disadvantages as compared to the exact solution.

Given that the approximate solution is a convergent power series, the change in the value of the solution will diminish as the approximation order (i.e., the order of the power series) is increased. In order to achieve a proper balance between accuracy and computational demand, the algorithm being proposed here is designed to raise the approximation order successively until the fractional change in the results meets a prespecified convergence criterion. Let $\omega$ denote the convergence criterion, $L$ denote the starting order and $M$ denote the highest order the algorithm will go to. Then, the basic steps entailed in the algorithm can be outlined as follows.

1. Define the system.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

i. Input data on parameters such as $\alpha$, $\beta$ and $\gamma$.

ii. Define the polynomial coefficients $a_n(1)$, $b_n'(0)$ and $c_n$ in such a way that they are computed only when the relevant approximation order is reached.

iii. Define the objective function (12) and the four boundary conditions (13) to (16) in such a way that additional terms are added as the approximation order is increased.

2. Set the initial guess values for $\hat{x}$, $\hat{z}$, $w_1$ and $w_2$, and compute their values from the four boundary conditions (13) to (16) for order $n = L$.

3. Set the new guess values for $\hat{x}$, $\hat{z}$, $w_1$ and $w_2$ to their values in the previous step, and the recalculate their values for order $n = L + 1$.

4. Stop if the convergence criterion $\omega$ is met for each of the four variables; go back to step 3 otherwise until $n = M$ is reached.

--------------------------------
Insert Table 1 about here
--------------------------------

Table 1 provides a numerical comparison between the exact threshold values and their approximations to the 5th order (i.e., approximated by the first five terms in the series solution). The computation of the approximate solution is virtually instant using a nonlinear solver from MathCad Professional on a 400MHz Pentium II PC and only requires 1-2 seconds on a 90MHz Pentium PC. As can be seen in the bottom row of the table, the average approximation errors are 0.15% for the entry threshold $\hat{z}$ and 0.12% for the exit threshold $\hat{x}$, respectively. This suggests that the approximate solution can achieve a high level of accuracy with much less computation time than the exact solution, although the difference in computation time is not so significant on a more powerful PC. Hence, the main advantage of the approximate solution in our assessment lies in the fact that it can be implemented in a spreadsheet program such as Microsoft Excel and does not require the use of a specialized mathematics package or any skills in solving differential equations numerically. For those who have both experience in solving differential equations and access to specialize mathematics packages such as MathCad Professional, the exact solution may require a little less programming work.

## 5. Concluding remarks

The notion of product life cycle (PLC) was initially established and has received broad attention in the marketing literature. Existing research so far has focused on such questions as advertising and pricing and, in general, has been conducted under a deterministic framework. The model developed in this paper adopts a more realistic stochastic framework to examine the market entry and exit decisions during the PLC under uncertainty. Although the two-threshold policy derived from our model looks remarkably simple, our work suggests that determination of these thresholds in a given decision context poses many challenging tasks. Our model, and the solution procedure and computation algorithm derived in this paper, can aid manufacturing firms in making such decisions.

As pointed out in the introduction section, our model applies mainly to situations where the entry involves only the acquisition of some manufacturing assets (such as technology and equipment) via purchase or license, rather than the commercialization of one’s own R&D results. Extension of the model to evaluate such technology switching decisions represents clearly an interesting direction for future research.

## Acknowledgements

We thank three anonymous referees for their constructive comments. All remaining errors, however, are solely our own responsibility.

## References

[1] Bass, F. M. (1969) A new product growth model for consumer durables. *Management Science*, **15**(5), 215-227.

[2] Harrell, S.G., and Taylor, E.D. (1981) Modeling the product life cycle for consumer durables. *Journal of Marketing*, **45**, 68-75.

[3] Tigert, D. and Farivar, B. (1981) The Bass new product growth model: A sensitivity analysis for a high technology product. *Journal of Marketing*, **45**, 81-90.

[4] Mesak, H.I. and Berg, W.D. (1995) Incorporating price and replacement purchases in new product diffusion models for consumer durables. *Decision Sciences*, **26**(4), 425-449.

[5] Klepper, S. (1996) Entry, exit, growth, and innovation over the product life cycle. *The American Economic Review*, **86**, 562-583.

[6] Teece, D.J. (1986) Profiting from technological innovation: Implications for integration, collaboration, licensing and public policy. *Research Policy*, **15**, 285-305.

[7] Feichtinger, G., Hartl, R.F. and Sethi, S. (1994) Dynamic optimal control models in advertising: Recent developments. *Management Science*, **40**(2), 195-226.

[8] Pindyck, R.S. (1993) Investments of uncertain cost. *Journal of Financial Economics*, **34**, 53-76.

[9] Karlin, S. and Taylor, H.M. (1981) *A Second Course in Stochastic Processes*, Academic Press, Inc., San Diego, CA.

[10] Dixit, A.K. and Pindyck, R.S. (1994) *Investment under Uncertainty*, Princeton University Press, Princeton, N.J.

[11] Braun, M. (1983) *Differential Equations and Their Applications*, Third Edition, Springer-Verlag, New York.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

**Appendix**[4]

In this appendix, we derive a series solution to the differential equation specified in (7) and determine the solution's convergence region. The derivation follows standard methods that are explained in most textbooks on the theory of ordinary differential equations (see Chapter 2 of Braun [11], for instance).

Let $K = 2/\lambda\sigma^2$. Then, the differential equation derived in (7) can be represented as

$$\pi'' - \frac{\alpha K x}{x(1+\beta x)}\pi' - \frac{\mu K \pi}{x} + \frac{\alpha K x}{x(1+\beta x)} = 0\,. \tag{A1}$$

It can be easily verified that all the coefficient functions in (A1) are rational functions and that this ODE has regular singular points at $x = 0$ and $x = -1/\beta$. So long as the salvage value $V(x)$ is nontrivial, we only need attend to the case of $x = 0$. Given that (A1) has a regular singular point at $x = 0$ and contains only rational functions in its coefficients, the existence theorem of an ODE implies that this differential equation has at least one nontrivial power series solution around $x = 0$ that converges in an interval $|x| < R$, with $R > 0$ being the convergence radius.

Rearranging the terms in the form of polynomials, (A1) becomes

$$(1+\beta x)x\pi'' - \alpha K x \pi' - \mu K(1+\beta x)\pi + \alpha K x = 0\,. \tag{A2}$$

Write the homogeneous part of (A2) as

$$L(\pi) = (1+\beta x)x\pi'' - \alpha K x \pi' - \mu K(1+\beta x)\pi = 0\,. \tag{A3}$$

Let the solutions of (A3) be in the form of

$$\pi(x) = \sum_{n=0}^{\infty} a_n x^{n+s}\,,$$

[4] The appendix only presents the main steps of the derivation in order to save space. A more detailed derivation is available from the corresponding author upon request.

with $a_0 \neq 0$. Its first-order and second-order derivatives are

$$\pi'(x) = \sum_{n=0}^{\infty} (n+s) a_n x^{n+s-1}, \tag{A4}$$

$$\pi''(x) = \sum_{n=0}^{\infty} (n+s)(n+s-1) a_n x^{n+s-2}. \tag{A5}$$

Substitute (A4) and (A5) into (A3) and we get

$$\begin{aligned} L(\pi) &= (1+\beta x)x\sum_{n=0}^{\infty}(n+s)(n+s-1)a_n x^{n+s-2} - \alpha K x\sum_{n=0}^{\infty}(n+s)a_n x^{n+s-1} - \mu K(1+\beta x)\sum_{n=0}^{\infty} a_n x^{n+s} \\ &= \sum_{n=0}^{\infty}(n+s)(n+s-1)a_n x^{n+s-1} + \sum_{n=0}^{\infty}\beta(n+s)(n+s-1)a_n x^{n+s} - \sum_{n=0}^{\infty}\alpha K(n+s)a_n x^{n+s} \\ &\quad - \sum_{n=0}^{\infty}\mu K a_n x^{n+s} - \sum_{n=0}^{\infty}\mu\beta K a_n x^{n+s+1}. \end{aligned}$$

Combining terms with the same power of $x$, we obtain

$$\begin{aligned} L(\pi) &= a_0 s(s-1)x^{s-1} + \{s(s+1)a_1 + [\beta(s-1)s - \alpha K s - \mu K]a_0\}x^s \\ &\quad + \sum_{n=1}^{\infty}\{(n+s)(n+s+1)a_{n+1} + [\beta(n+s-1)(n+s) - \alpha K(n+s) - \mu K]a_n - \mu\beta K a_{n-1}\}x^{n+s}. \end{aligned}$$

Setting the coefficients of $x$ to zero gives

$$(s-1)s a_0 = 0, \tag{A6}$$

$$s(s+1)a_1 + [\beta(s-1)s - \alpha K s - \mu K]a_0 = 0, \tag{A7}$$

$$a_n = \frac{-[\beta(n+s-2)(n+s-1) - \alpha K(n+s-1) - \mu K]a_{n-1} + \mu\beta K a_{n-2}}{(n+s-1)(n+s)} \tag{A8}$$

for $n \geq 2$. The solution of equation (A6) is either $s = 0$ or $s = 1$, with the two solutions differ by an integer; hence, we can only use one of these two values in a solution to the differential equation. Note that any arbitrary value of $a_0$ satisfies (A6), whether $s = 0$ or $s = 1$. Suppose $s = 1$ and let $a_n(1)$ denote the value of $a_n$ for $s = 1$. Set $a_0(1) = 1$ and we obtain

$$a_1(1) = \frac{(\alpha+\mu)K}{2}, \tag{A9}$$

$$a_n(1) = -\frac{\beta(n-1)n - \alpha Kn - \mu K}{n(n+1)} a_{n-1}(1) + \frac{\mu\beta K}{n(n+1)} a_{n-2}(1) \quad \text{(A10)}$$

for $n \geq 2$. To reduce clutter in the derivation that follows, let

$$h_n = \frac{\beta(n-1)n - \alpha Kn - \mu K}{n} = \beta(n-1) - \alpha K - \frac{\mu K}{n}, \quad \text{(A11)}$$

$$d = \mu\beta K, \quad \text{(A12)}$$

$$A_0 = 1, \quad \text{(A13)}$$

$$A_1 = (\alpha + \mu)K, \quad \text{(A14)}$$

$$A_n = d \cdot A_{n-2} - h_n \cdot A_{n-1} \quad \text{(A15)}$$

for $n \geq 2$. Then, (A9) and (A10) can be expressed as

$$a_n(1) = \frac{A_n}{\Gamma(n+2)}, \quad \text{(A16)}$$

where $\Gamma(n+1) = n!$ . With $a_0(1) = 1$ and $a_n(1)$ given by (A16) for $n \geq 1$, the series

$$\pi_1(x) = \sum_{n=0}^{\infty} a_n(1) x^{n+1} \quad \text{(A17)}$$

is a general homogeneous solution of (A3).

Using $s = 0$, we can construct another linearly independent general solution of (A3) as follows. First, we derive the other homogeneous solution for $s = 0$ following the same procedure. Let $b_n(0)$ denote the value of $a_n$ for $s = 0$ and set $b_0(s) = s$. Substituting $b_0(0)$, $b_1(0)$ and $b_n(0)$ for $a_0(1)$, $a_1(1)$ and $a_n(1)$ in (A6), (A7) and (A8) gives us

$$b_0(0) = 0,$$

$$b_1(0) = \mu K,$$

$$b_n(0) = -\frac{\beta(n-2)(n-1) - \alpha K(n-1) - \mu K}{(n-1)n} b_{n-1}(0) + \frac{\mu\beta K}{(n-1)n} b_{n-2}(0)$$

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

for $n \geq 2$. Their first order derivatives with respect to $s$, $b_n'(0) = \left.\frac{db_n(s)}{ds}\right|_{s=0}$, are

$$b_0'(0) = 0\,, \tag{A18}$$

$$b_1'(0) = \beta + (\alpha - \mu)K\,, \tag{A19}$$

$$\begin{aligned} b_n'(0) = &-\frac{\beta(2n-3-\alpha K)(n-1)n - \left(\beta(n-2)(n-1) - \alpha K(n-1) - \mu K\right)(2n-1)}{(n-1)^2 n^2} b_{n-1}(0) \\ &- \frac{\beta(n-2)(n-1) - \alpha K(n-1) - \mu K}{(n-1)n} b_{n-1}'(0) - \frac{\mu\beta K(2n-1)}{(n-1)^2 n^2} b_{n-2}(0) + \frac{\mu\beta K}{(n-1)n} b_{n-2}'(0) \end{aligned} \tag{A20}$$

for $n \geq 2$. Using (A18), (A19) and (A20), we can write the other homogeneous solution with $s = 0$ as

$$y(x) = \sum_{n=0}^{\infty} b_n'(0) x^n\,. \tag{A21}$$

Then, making use of both (A17) and (A21), we can construct another linearly independent general solution of (A3) as

$$\begin{aligned} \pi_2(x) &= \ln x \cdot \pi_1(x) + y(x) \\ &= \ln x \cdot \sum_{n=0}^{\infty} a_n(1) x^{n+1} + \sum_{n=0}^{\infty} b_n'(0) x^n. \end{aligned} \tag{A22}$$

Finally, it can be verified that a particular non-homogeneous series solution of (A1) is

$$\pi^*(x) = \sum_{n=0}^{\infty} c_n x^{n+1}\,, \tag{A23}$$

where $c_0$ is a constant that needs to be selected,

$$c_1 = -\frac{\alpha K}{2} + \frac{(\alpha + \mu)K}{2} c_0\,, \tag{A24}$$

$$c_n = -\frac{\beta(n-1)n - \alpha K n - \mu K}{n(n+1)} c_{n-1} + \frac{\mu\beta K}{n(n+1)} c_{n-2} \tag{A25}$$

for $n \geq 2$. To select a proper $c_0$, we require the derivatives of the particular non-homogeneous solution $\pi^*(x)$ to possess properties that are similar to those of the salvage value function $V(x)$ near $x = 0$. This requirement can be expressed as $\lim_{x \to 0^+} \frac{d\pi^*(x)}{dx} \geq 0$ and $\lim_{x \to 0^+} \frac{d^2\pi^*(x)}{dx^2} < 0$, and translates to $0 < c_0 \leq \frac{\alpha}{\alpha + \mu}$. A simple selection can be, for example, $c_0 = \frac{\alpha}{\alpha + \mu}$. Using the particular non-homogeneous solution $\pi^*(x)$ and the two general homogeneous solutions $\pi_1(x)$ and $\pi_2(x)$, we can express a general non-homogeneous solution of (A1) as

$$\tilde{\pi}(x; w_1, w_2) = \pi^*(x) + w_1\pi_1(x) + w_2\pi_2(x), \qquad \text{(A26)}$$

where $w_1$ and $w_2$ are the integral constants to be determined by terminal conditions. Substituting (A17), (A22) and (A23) into (A26), we can rewrite the general solution as

$$\begin{aligned} \tilde{\pi}(x; w_1, w_2) &= \pi^*(x) + w_1\pi_1(x) + w_2[\ln x \cdot \pi_1(x) + y(x)] \\ &= \sum_{n=0}^{\infty} c_n x^{n+1} + (w_1 + w_2 \ln x)\sum_{n=0}^{\infty} a_n(1)x^{n+1} + w_2\sum_{n=0}^{\infty} b_n'(0)x^n \\ &= w_2 + \sum_{n=0}^{\infty}[c_n + (w_1 + w_2 \ln x)\cdot a_n(1) + w_2 b_n'(0)]x^{n+1}, \end{aligned} \qquad \text{(A27)}$$

where $a_n(1)$, $b_n'(0)$ and $c_n$ are given by (A16), (A25) and (A20), respectively.

As per the theory of differential equations, the series solution $\tilde{\pi}(x; w_1, w_2)$ is applicable only over its convergence region. In our case, the convergence region is an interval of $x$ over which $\tilde{\pi}(x; w_1, w_2)$ converges. In order to know whether the series solution gives a meaningful approximation, we need to determine the convergence region. We will establish the convergence region of the solution in two steps, using lemma 1 and lemma 2 derived below. Lemma 1 will show that there are two possible convergence regions depending on whether the approximation

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

oscillates, and lemma will show that the approximation does oscillate for a sufficiently large *n*.

The convergence region of the solution, as will be demonstrated below, is $x \in (0, \frac{1}{\beta})$.

**Lemma 1.** Suppose the asymptotic optimized payoff $\tilde{\pi}(x; w_1, w_2)$ is given by (A27). Then,

$$\tilde{\pi}(x; w_1, w_2) \text{ converges for } \begin{cases} x \in (0, \infty) \text{ if } \exists N > 0, \text{s.t. } A_n \cdot A_{n-1} \geq 0 \;\; \forall n \geq N, \\ x \in (0, \frac{1}{\beta}) \text{ if } \exists N > 0, \text{s.t. } A_n \cdot A_{n-1} < 0 \;\; \forall n \geq N, \end{cases}$$

where $A_n$ is given by (A15).

**Proof.** Obviously, the convergence radius *R* of $\tilde{\pi}(x; w_1, w_2)$ in (A27) is identical to that of $\pi_1(x)$ in (A17), which can be determined from

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{A_n\Gamma(n+3)}{A_{n+1}\Gamma(n+2)}\right| = \lim_{n\to\infty}\left|\frac{A_n(n+2)}{A_{n+1}}\right|. \tag{A28}$$

Let $\psi = \lim_{n\to\infty}\frac{A_n}{A_{n-1}}$. Dividing both sides of (A15) by $A_{n-1}$ yields an asymptotic equation

$$\psi = \frac{d}{\psi} - h_n.$$

Solve this equation for $\psi$ and we have

$$\psi_{1,2} = \frac{-h_n \pm \sqrt{h_n^2 + 4d}}{2}.$$

From (A11), we can see that $\lim_{n\to\infty} h_n = \infty$. If there exits an integer $N \geq 1$ such that $\frac{A_n}{A_{n-1}} \geq 0$ for all $n \geq N$, then we have $\psi \geq 0$, implying that $\psi_1 = (-h_n + \sqrt{h_n^2 + 4d})/2$ should be used in (A28) to determine the convergence region. If $\psi \geq 0$ is the case, the limit specified in (A28) is

$$R = \lim_{n\to\infty}\left|\frac{n+2}{\psi_1}\right| = \lim_{n\to\infty}\left|\frac{1+1/2}{(-h_n + \sqrt{h_n^2+4d})/2n}\right| = \left|\frac{2}{-\beta+\beta}\right| = \infty.$$

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

On the other hand, if $\frac{A_n}{A_{n-1}} < 0 \; \forall n \geq N$, then $\psi_2 = (-h_n - \sqrt{h_n^2 + 4d}\,)/2 < 0$ is the case, which renders the limit of (A28) as

$$R = \lim_{n\to\infty}\left|\frac{n+2}{\psi_2}\right| = \lim_{n\to\infty}\left|\frac{1+1/2}{(-h_n - \sqrt{h_n^2 + 4d}\,)/2n}\right| = \left|\frac{2}{-\beta-\beta}\right| = \frac{1}{\beta}.$$

This concludes the proof. □

**Lemma 2.** The optimized payoff $\tilde{\pi}(x; w_1, w_2)$ converges for $x \in (0, \frac{1}{\beta})$.

**Proof.** Based on Lemma 1, it is sufficient to show that $\lim_{n\to\infty} A_n \cdot A_{n+1} < 0$. By (A13) and (A14), we have $A_0 > 0$ and $A_1 > 0$. By (A15), we know

$$A_{n+1} = dA_{n-1} - h_{n+1}A_n$$

for $n \geq 2$. From (A11), we see that $h_n$ increases with $n$, leading to $\lim_{n\to\infty} h_n = \infty$. Then by induction, there must exists an $n^* < \infty$ such that $h_{n^*} > 0$, $A_{n^*-1} \geq 0$, $A_{n^*} \geq 0$ and $A_{n^*+1} < 0$. Furthermore, it can be verified that $\frac{A_n}{A_{n+1}} < 0$ for all $n \geq n^*$, that is, $\lim_{n\to\infty}\frac{A_n}{A_{n+1}} < 0$. The proof of the Lemma 2 is thus concluded. □

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

**Biographical Sketches**

Tailan Chi is an Associate Professor at the School of Business Administration, University of Wisconsin-Milwaukee. He received his B.E. degree from the University of International Business and Economics, Beijing, China, his M.B.A. degree from University of San Francisco, and his M.A. degree in economics and Ph.D. degree in business administration from the University of Washington. His research interests are in the area of organization and decision economics. He has published in journals such as *Management Science, Strategic Management Journal*, *Decision Sciences*, and *IEEE Transactions on Engineering management*. He is a member of the Academy of International Business, Academy of Management, American Economic Association, INFORMS, and Strategic Management Society.

John Liu is a Professor at the School of Business Administration, University of Wisconsin-Milwaukee. He received his M.S. degree in Engineering-Economic Systems from Stanford University and Ph.D. degree in Industrial Engineering from the Pennsylvania State University. His primary research interests are stochastic optimization models, and manufacturing and distribution systems. His current research focuses on technology acquisition and diffusion, and quality regulation in manufacturing and service. He has published in journals such as *Operations Research*, *Management Science, the European Journal of Operational Research*, and *the Journal of Production and Operations Management*. He is a member of INFORMS and IIE.

**Complete Addresses of Authors**

Tailan Chi
School of Business Administration
University of Wisconsin-Milwaukee
P. O. Box 742
Milwaukee, WI 53201-0742
USA
Tel: (414) 229-5429
Fax: (414) 229-6957
Email: chi@uwm.edu

John Liu
School of Business Administration
University of Wisconsin-Milwaukee
P. O. Box 742
Milwaukee, WI 53201-0742
USA
Tel: (414) 229-3833
Fax: (414) 229-6957
Email: jjl@uwm.edu

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/ Open Access Version: http://kuscholarworks.ku.edu/dspace/

**Table 1**. Comparison of Approximations with Exact Solutions: Entry and Exit Thresholds

| | Exact Solution | | Approximation | | Approximation Error | |
|---|---|---|---|---|---|---|
| $\sigma$ | Entry $\hat{z}_e$ | Exit | Entry | Exit | $\frac{\lvert \hat{z}_e - \hat{z}_a \rvert}{\hat{z}_e}$ | $\frac{\lvert \hat{x}_e - \hat{x}_a \rvert}{\hat{x}_e}$ |
| 0.1 | 3.000 | 1.705 | 3.000 | 1.704 | 0.0000 | 0.0006 |
| 0.2 | 3.092 | 1.682 | 3.090 | 1.679 | 0.0006 | 0.0018 |
| 0.3 | 3.212 | 1.653 | 3.200 | 1.656 | 0.0037 | 0.0018 |
| 0.4 | 3.372 | 1.633 | 3.372 | 1.630 | 0.0000 | 0.0018 |
| 0.5 | 3.560 | 1.629 | 3.572 | 1.629 | 0.0034 | 0.0000 |
| Average | | | | | 0.0015 | 0.0012 |

Values of Other Parameters: $\lambda = 1$, $\alpha = .2$, $\beta = .1$, $I = 1.4$, $b = .3$, $V_0 = 2$, $\eta = 1.15$, $\gamma = .5$, $\mu = .1$.

Chi, Tailan. (2001) Product Life Cycle, and Market Entry and Exit Decisions Under Uncertainty. IIE Transactions, 33 (9), 695-704.
Publisher's Official Version: http://dx.doi.org/10.1023/A:1010901813523/  Open Access Version: http://kuscholarworks.ku.edu/dspace/

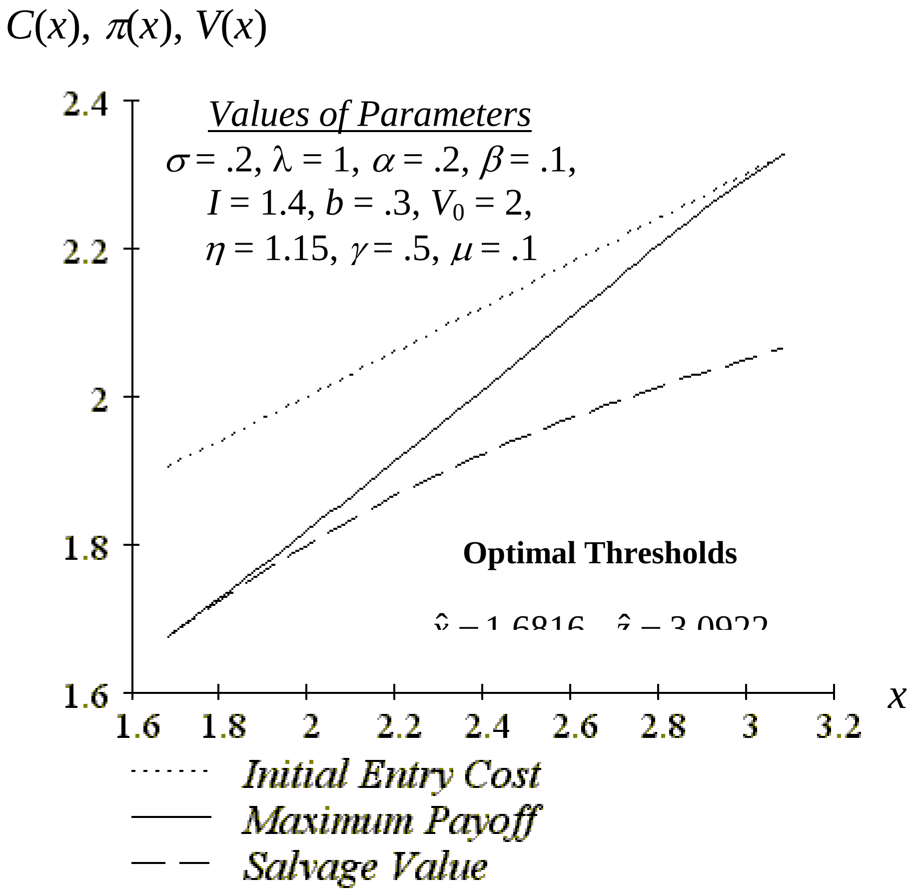


**Fig. 1**. Sample trajectory of optimal payoff.