Shifting Preferences and Time-Varying Parameters in Demand Analysis: A Monte Carlo Study

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Abstract

Using Monte Carlo experiments, I address two issues in demand analysis. The first relates to the performance of local flexible functional forms in recovering the time-varying elasticities of a true model, and in correctly identifying goods as complements, substitutes, normal or inferior. The problem is illustrated with the nonlinear almost ideal demand system (NLAI) and the Rotterdam model (RM). For the AIDS, I also consider two versions of its linear approximation: one with simple formulas (LAISF) and the other with corrected formulas (LAICF). The second issue concerns the ability of the flexible functional structures to satisfy theoretical regularity in terms of the Slutsky matrix being negative semi-definite at each time period of time.

I tackle these issues in the framework of structural time series models, computing the relevant time-varying elasticities by means of Kalman filtered and smoothed coefficients. The estimated time-varying coefficients are obtained under the pure random walk and the local trend hypotheses. I find that both the NLAI and the RM qualitatively perform well in approximating the signs of the time-varying income and substitution elasticities. Quantitatively, the RM tends to produce values of the time-varying elasticity of substitution close to the true ones within separable utility branches while the NLAI tends to produce overestimating values. On the other hand, the RM produces time-varying income elasticities with values close to the true ones while the NLAI tends to produce constant values over time. The LAISF model qualitatively performs similarly to the NLAI, but the LAICF does not. Finally, the NLAI achieves higher levels of the regularity index under the local trend specification while the RM achieves higher regularity levels under the random walk specification. In contrast, the LAISF and the LAICF models achieve lower levels of regularity under both specifications of the time-varying coefficients. Globally, the LAICF which widely adopted in applied work performs poorly compared to the RM and the NLAI. These findings are robust to different values of the time-varying parameters in the utility function. Two implications emerge from this research. First, the LAICF model should be considered as a model on its own rather than as an approximation of the NLAI. Second, the choice between an AIDS-type model and the RM should be motivated by their performance with respect to the properties a hypothesized true model for the data at hand, especially when working with real data.

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Introduction and Motivation

This dissertation research is aimed at evaluating the performance of alternative demand structures in presence of shifting consumer preferences. Accounting for preference shifting factors in demand analysis is important for two major reasons. First, it allows to deepen our understanding of consumer behavior outside the neoclassical framework of fixed tastes. Further, the analysis helps break with the old tradition of considering the subject as pertaining to social disciplines other than economics. The analysis will feature shocks to the parameters in models of consumer preferences in the framework of local flexible functional forms.

Among the local flexible functional forms, the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980a,b), especially in its linear form, and the Rotterdam model (Barten, 1964, 1968, 1977; Theil, 1965, 1975a,b) have been widely adopted in applied research. Their attractiveness is explained by the fact that both demand specifications share desirable properties not possessed by other local flexible functional forms such as the Generalized Leontief (Diewert, 1971) and the Translog (Christensen et al., 1975): local flexibility, consistency with demand theory, linearity and parsimony with respect to the parameters. They also have identical data requirement so that no additional variable is required in order to estimate one specification whenever the estimation of the other is possible.

However, the two specifications lead to different results in some applications (Alston and Chalfant, 1991), prompting the question of the appropriateness of each specification for a given dataset. Nevertheless, the choice of either model for empirical applications has been purely arbitrary and possibly motivated by the personal acquaintance of the researcher with each of them. This is understandable since economic theory does not provide a basis to ex ante discriminate among the flexible functional forms in general, and between the AIDS and the Rotterdam model (RM) in particular.

The observed discrepancies between the outcomes from the two specifications raise the issue of adopting a research strategy that allows to discriminate between them not only based on the demand properties contained in the specific dataset, but also on their consistency with the particular maximization problem that has produced or that is believed to have produced the data. Thus, choosing the best approximating structure for the true underlying model should be the result of a well-defined methodology that straightforwardly establishes the true properties contained in the data as a benchmark. This applies whether consumer preferences are postulated to be fixed as in the neoclassical demand theory or otherwise subject to shifts of a specific nature.

Alston and Chalfant (1993) developed a statistical test of the linear-approximate AIDS against the RM. The test was then applied to the meat demand in the United States. The test concluded in favor of the acceptance of the RM, rejecting the AIDS. The same conclusion obtained with Barten (1993)'s test. However, the authors pointed out that their finding could not be interpreted as an evidence of the superiority of the RM on the AIDS in a general way. Moreover, their test may lead to a different conclusion if applied to a different data set.

On the other hand, Barnett and Seck (2008) conducted a Monte Carlo comparison of the nonlinear AIDS, the linear-approximate AIDS and the RM. They sought to determine which of the three specifications could perform better in terms of the ability to recover the elasticities of the true demand system. Their finding was that both the AIDS and the RM performed well when substitution among goods was low or moderately high. However, the nonlinear AIDS model performed better when the substitution among goods was very high. Finally, the RM performed better at recovering the true elasticities within separable branches of a utility function.

It is noteworthy that both papers postulated constant parameters in the demand functions and the underlying utility functions. However, when using real data, the consistency of the demand estimates can be compromised if one wrongly assumes the constancy of the parameters while they are actually random or varying over time. In this case the constant-coefficient model will not only fail to capture the possible long-run dynamics in the data but also will produce a poor approximation to the underlying data generation process (Leybourne, 1993). In addition, it is important that more investigation be conducted in order to determine whether or not the advantages of one demand specification on the other can be preserved when the constant-parameters assumption is abandoned in a Monte Carlo study.

The goal of this dissertation research is to contribute to the assessment of the performance of the nonlinear AIDS, the linear-approximate AIDS and the Rotterdam model when the coefficients of the demand system are permitted to vary over time. To the best of my knowledge, the assessment of the performance of these demand specifications under time-varying parameters has not been attempted yet. This research will contribute to the literature by filling the gap.

The motivation for undertaking this study can be put forth into a twofold argument. First, the real world economic system is constantly subject to shocks that translate into technological and institutional changes as well as shifts in consumer preferences. The interaction of these shocks leads to more or less permanent changes in economic behavioral relationships. Therefore, assuming time-varying parameters helps to capture dynamics of specific nature in these economic relationships. Second, both the RM and the AIDS are local first-order Taylor series approximations that are intended to approximate a true demand system derived from any utility maximization problem. When fitting the data to any of these flexible functional forms, an implicit assumption is that there exists an unknown true function of the variables of interest(e.g.: quantities and prices) that has generated the observed data given a set of parameters. Since the approximation provided by each functional form is only locally valid, assuming a single value for the parameter vector is unlikely to provide an adequate approximation of the true demand system that underlines the observed data. This idea has been expressed for the RM by Barnett (1979b) and Bryon (1984), and for the AIDS by Leybourne (1993).

I shall conduct the analysis in the framework of Harvey (1989)'s structural time series models. I first assume a pure random walk process for the parameters in the demand systems and compute the time-varying elasticities accordingly. Second, I assume a local trend model specification where the time-varying intercept is specified as a random walk with drift, with the drift itself being a random walk. The two approaches have been respectively used by Leybourne (1993) and Mazzocchi (2003) to estimate time-varying parameters in the linear-approximate AIDS. However, none of the papers attempted to compare the performance of the linear-approximate AIDS neither to that of the nonlinear AIDS nor to that of the RM.

The scope of the results in this dissertation will be limited to the approximating time-varying elasticities (elasticities of substitution, income and compensated price elasticities) with counterpart in the set of relevant elasticities derived from the true model. The approximating time-varying elasticities will be calculated using the estimated time-varying coefficients in each demand specification. I shall estimate the time-varying coefficients in each demand system by the Kalman filter and pass them through the Kalman smoother for their revision.

The dissertation research is divided in five chapters, including this introduction. Chapter 2 provides the literature review on accounting for shifting consumer preferences in demand analysis. A distinction is made between the effects on preferences of exogenous variables and the shocks to the parameters of the utility function. The analysis is then applied to the weak separable-branch utility tree by assuming that the parameters in the utility function vary over time according to a random walk process. The problem of time-varying parameters is considered in Chapter 3 for the two most used local flexible functional forms in empirical demand analysis, the AIDS and the RM. Chapter 4 presents the data generation procedure and the estimation method for the time-varying coefficients in AIDS and the RM as well as the estimation results. A discussion of the results and their robustness with respect to the performance of each demand system to recover the characteristics of the true demand system is presented in this chapter as well. Chapter 5 concludes the dissertation with a discussion of the implications of the findings, and ends with a brief outline of the future directions of the research.

2

Accounting for Varying Preferences in Demand Analysis

2.1 Introduction

It is a common knowledge today that the restrictions of the neoclassical consumer theory are unrealistic and that factors other than prices and income play an important role in the consumer's decision making process. Both economic and econometric theories recognize that technological and institutional shocks to an economy, together with shifts in consumer preferences may lead to permanent changes in economic behavioral relationships. Other sources of variations in economic behavioral relationships include model misspecification, nonlinearities, aggregation and the use of proxy variables in econometric models.

However, most of applications in empirical demand analysis continue to postulate fixed consumer preferences. Reluctance to account for preference shifting factors may be attributed to what Scitovsky (1945) qualified as the fear that such an account would wreak havoc on the whole theory of choice. The success obtained by economists in explaining human behavior under the neoclassical postulates may also be considered as a plausible reason. Why would we abandon a postulate with which we have been so successful? However, this does not preclude the requirement of consistency between real economic behaviors and the factors that are more likely to explain them.

Assuming varying or shifting preferences allows to analyze the effects of exogenous factors on consumers' behavior that cannot otherwise be captured under the neoclassical demand theory's fixed preferences hypothesis. Therefore, incorporating these variables in the analysis permits to explore and understand their interaction with consumers' tastes. For example, one can easily explain the impact on consumers' demand of advertisement, health and socio-demographic factors, interdependence of consumers' behavior and shocks to the economic system. Finally, accounting for exogenous factors and related shocks in demand analysis may also allows to explain the observed patterns in consumption and to capture the underlying dynamics.

There exists an extensive literature on accounting for taste changing factors and shocks to consumers' preferences both in static demand analysis and in dynamic demand analysis. Modeling the effects of exogenous shocks and shifting factors on consumers' preferences has consisted either in explicitly accounting for them in the consumer's optimization problem or in hypothesizing their impact on the parameters that determine consumers' tastes, or both. The next section reviews major contributions and trends in this literature.

2.2 Modeling shifting tastes in consumer demand literature

There are two approaches in the literature when it comes to accounting for taste changing factors in consumer demand analysis. The first is based on the premise that preference acquisitions arise as part of socialization and persistence of consumption habits. This approach hinges on the concept of interdependent preferences under which an individual's preferences depend on the consumption decisions of all the individuals in the society where his of her consumption decisions take place.

Gaertner (1974) and Pollak (1976, 1978) provided a theoretical framework for analyzing preference interdependence using the linear expenditure system (LES). In this framework, a consumer's preferences are assumed to depend on past consumption decisions of all the other individuals in the society. A special case of interdependent preferences is that of the habit formation hypothesis which postulates that an individual's preferences depend only on his or her past consumption decisions. This is referred as the *myopic* habit formation, in contrast to the *rational* habit formation which refers to a consumer who is not only backward looking but also forward looking.

In a rather game theoretic framework, Karni and Schmeidler (1990) considered interdependent preferences as based on a simultaneous determination of consumption by individuals in different social groups, allowing individuals to anticipate the actions of others. Their analysis of interdependent preferences hinges on the concept of extended commodities, defined as commodities that have social attributes in addition to standard attributes such as physical characteristics, delivery date, location or the state of nature if the good is a contingent commodity [see for example Malinvaud (1985)]. In this analysis, the social attributes of an extended commodity consist of information concerning its users and information concerning the users of other commodities. Sobel (2005) also considered interdependent preferences in a game theoretic framework, but as resulting from intrinsic reciprocity or consumer altruism. Under this assumption, an individual whose preferences reflect intrinsic reciprocity will sacrifice his or her own material consumption to increase the material consumption of others in response to kind behavior while, at the same time, he or she will be willing to sacrifice material consumption of others in response to unkind behavior.

Alessie and Kapteyn (1991) and Kapteyn et al. (1997) provided an empirical econometric treatment of interdependent preferences using the almost ideal demand system (AIDS) and the LES model and respectively. Considering three taste shifters, namely demographic factors, habit formation and preference interdependence, Alessie and Kapteyn (1991) found that all three shifters had significant effects. On the other hand, Kapteyn et al. (1997) estimated the LES that incorporated preference interdependence by using a cross-sectional data on consumer expenditure. They concluded that the interdependence of preferences was an important determinant of consumer behavior. Beside the difference in the functional forms used for the demand system, the two papers differed in the way of modeling the interdependence of preference. While Alessie and Kapteyn (1991) specified the current household budget share as a function of mean budget share in the household's reference group, Kapteyn et al. (1997) made the parameters in the LES to depend on the current quantities in the household's reference group.

The second approach to modeling variable preferences consists in considering the changes caused by exogenous factors in the parameters of the utility function. Ichimura (1950) and Tintner (1952) were the first to introduce this approach by explicitly expressing the shifts in demand functions in terms of the parameters of the utility functions. The idea is that the parameters of the utility function depend on the variations in the preference shifting variables. In a series of papers, Basmann (1954–1955, 1956, 1972) and Basmann et al. (2009) have considered prices and income as well as factors such as family size, schooling of the head of household, and other socioeconomic factors as preference shifting factors.

More specifically, Basmann et al. (2009) specifies a price-dependent direct utility function, the generalized Fechner-Thurstone (GFT) utility function, in which the parameters are functions of prices, income and other attributes that may impact consumer preferences. This strand of research shows that assuming the dependence of the parameters of the utility function on exogenous preference shifting variables does not contradict any of the deducted propositions of the neoclassic theory of consumer demand. Especially, the marginal conditions for maximum welfare[1] are satisfied independently of whether or not preferences are affected by exogenous factors[2]. However, demand functions derived from a utility function such as the GFT do not possess some of the nice properties of demand theory, such as the Slutsky symmetry that needs to be imposed.

Following Basmann and also Barten (1977), Brown and Lee (2002) considered a demographic variable as an argument in the consumer utility function to analyze its impact on marginal utilities in the Rotterdam model. The effect of the demographic variable is specified through a fundamental relationship between the price effect and the effect of the demographic variable on marginal utilities. More specifically, the change in the demographic variable was viewed as resulting in changes in adjusted prices, the latter being decomposed into actual price changes minus preference-variable-induced changes in marginal utilities. In an application to the demand of fruit, the demographic variable was found to have a significant effect on the demand of 3 out of 5 fruit included in the model. However the effect on marginal utilities was significant only for one of the fruit.

[1] The first condition is that of equal marginal rates of substitution between any pair of goods for any consumer who consumes both goods. The second condition is that of the equality between marginal rate of substitution between any two goods and the marginal technical rate of transformation between any two firms producing both goods.

[2] Somehow, this is the modern answer to Scitovsky (1945)'s argument.

Other exogenous factors that affect the parameters of consumers' preferences over goods are the strategies of the sellers to influence the willingness of consumers towards their products through advertising campaigns. As Basmann (1956) put it, "advertising costs are incurred by a seller in an effort to secure a favorable change in consumers' subjective evaluation of the commodity he offers for sale; in terms of economic theory, the seller seeks by advertising his goods and services to increase their marginal utilities of consumers with respect to the marginal utilities of commodities offered by other sellers". The parameters in the utility function may then be viewed as depending on sellers' advertising expenditures. Different ways of accounting for advertisement in demand analysis are found in Theil (1980b,a) and Duffy (1987, 1989, 1995) in the framework of both the Rotterdam model and the AIDS.

An important distinction needs to be pointed out between preference shifting factors that can more likely be observed such as prices, income and socio-demographic factors, and those that cannot be observed or that may be observed with considerable amount of measurement errors such as sellers' advertising expenditures and other shocks to the economy. For example, it's more reasonable to think that because each seller aims to obtain a favorable change in the consumers' subjective evaluation towards the commodity he or she sells, this can only be done to the detriment of goods that are sold by other sellers. Therefore, advertising strategies are antagonistic among the sellers of substitute goods and not always sympathetic among sellers of complementary goods. As a result, advertising expenditures that are aimed at influencing the marginal utilities of goods are less likely to be known with certainty and, to the best, more likely to be measured with errors.

Easily observable preference shifting factors such as demographic factors are more relevant for the analysis of shifts in static demand functions with constant properties; although the parameters in such demand functions may assume different values as a result of variations in the exogenous variables. However, exogenous factors that are subject to measurement errors or those that can be modeled as stochastic processes need an appropriate treatment. For example, Basmann (1985) explored the effect of stochastic taste shifters where the change in marginal rates of substitution is expressed as a multiplicative function with a systematic component and a stochastic component that can be modeled as a stationary process. In this specification the systematic factors satisfy the necessary and sufficient conditions to be the marginal utilities of a direct utility function that are free of stochastic parameters.

2.2.1 The time-varying parameter case

Preference shifters that affect marginal utilities may be considered as inducing changes over time through the parameters of the utility function on which depend marginal utilities. Barnett (1979b) considered a utility function that depended on a vector of stochastic preference shifting variables. He also showed that the parameters of the demand system derived from these preferences vary over time. Although this problem was theoretically studied in the framework of the Rotterdam model, the approach has never been attempted in applied work. In the contrary, the AIDS has received attention in this matter. Empirical studies with time-varying parameters in the AIDS model include Leybourne (1993) and Mazzocchi (2003). However, none of these papers explicitly postulated time-varying parameters in the utility function underlying their demand systems.

It is foremostly important to point out that assuming a time-varying structure in the parameters of demand functions would be reasonable only if the underlying preferences share the time-varying properties in their formulation. In addition, introducing time-varying parameters in demand analysis inescapably implies the recognition that the indifferent maps from which the demand functions are derived depend on time-varying parameters. This should not be surprising since the indifference maps and the corresponding demand functions share the same set of parameters, the characteristics of which jointly describe the state of consumer preferences.

2.3 The approach of this dissertation

I shall consider stochastic preference shifting factors that affect the parameters of the marginal utilities. My approach differs from Basmann (1985)'s in two aspects. First, I will not allow the multiplicative stochastic component in the expression of the marginal rates of substitution. Therefore, the focus will be only on systematic part. Second, the stochastic shocks to preferences will be modeled in a way to affect the parameters of the marginal rates of substitution over time. I shall assume that only the process that govern such stochastic shocks is relevant for the time-varying structure of the parameters in the utility function. This results in time-varying parameter indifference maps and demand systems. The approach used in this dissertation also differs from Barnett (1979b)'s in that I explicitly assume a time-varying stochastic process for the preference shifting variables and estimate the implied time-varying parameters in the demand functions. It is noteworthy that time-varying marginal utilities can obtain from Basmann's approach if one assumes a time-varying stochastic process for the stochastic component of the marginal utilities.

When preference shifting variables are perfectly observable, the properties of the demand functions derived from the corresponding preference ordering may be considered as producing demand elasticities that are constant over time. Of course, such a result assumes that the parameters of the utility function are fixed. The time-varying approach to the parameters in the utility function has the advantage of accounting for stochastic shocks to preferences that are more general in nature. In fact this approach permits to obtain a time path of demand elasticities, while allowing to explain how the marginal rate of substitution between any pair of goods evolves over time. As a result, the analyst will be able to observe how likely it is to substitute one good for another over time.

I shall explicitly assume time-varying parameters and apply the analysis to a known utility function. My purpose is to establish a framework for the comparison of approximating demand specifications with time-varying parameters. I shall use the properties of the demand system derived from such a known utility function with time-varying parameters as a benchmark and assess the performance of the AIDS and the Rotterdam model in recovering them.

It is important to mention that the kind of time-varying coefficients (TVC) structure that is specified for the parameter vector in this dissertation differs from other possible specifications in the literature. For example, in the case of random coefficients, a random shock produces the time variations around a common average value of the coefficient vector. The coefficient vector thus attains a value different from the average parameter vector at any time period. A selected number of papers in this literature include Swamy and Mehta (1975), Swamy and Tinsley (1980), Pratt and Schlaifer (1984, 1988), and Swamy and Tavlas (1995).

The coefficients in this specification have a *real-world* interpretation, as sums of three distinct components (Swamy and Tavlas, 2003): one corresponding to a direct effect of true value of a regressor on the true value of the dependent variable, a second part capturing omitted-variable biases, and a third part capturing the effect of mismeasuring the regressor. A simplest way to differentiate time-varying coefficients based on a Markov process-type specification and the random specification is to think of the coefficients in the random coefficient models as fluctuating around an average values that is common for all the time periods. Time-varying parameters are considered in the next section for the consumer maximization problem.

2.4 Introducing time-varying parameters in the utility function

From the onset of this section, I shall point out that the neoclassical theory of consumer demand hinges on four essential well-defined and regular functions (the direct utility function, the indirect utility function, the expenditure function and the distance function). These functions are alternatively used to characterize consumer preferences.

Useful duality relationships that relate these functions help specify four major categories of demand functions: ordinary demand functions, compensated demand functions, inverse demand functions and inverse compensated demand functions. Four important results of the duality theory, namely the Ville-Roy identity, the Hotelling-Wold identity, the Shephard lemma and the Shephard-Hanoch lemma, are used to derive the four major demand functions categories from the duality relationships defined among the four neoclassical functions listed above.

Ordinary demand functions are expressed as the partial derivatives of the indirect utility function using the Ville-Roy identity; inverse demand functions are derived by using the dual result to the Ville-Roy identity, the Hotelling-Wold identity, as the partial derivatives of the direct utility function; compensated demand functions are derived by using the Shephard lemma which states that the inverse demand functions are partial derivatives of the expenditure function.

Finally, the inverse compensated demand functions are derived by applying the Shephard-Hanoch lemma, which is dual to the Shephard lemma, to the distance function. One interesting feature of the demand functions derived from the duality relationships is that they have the same functional forms: the direct and inverse demand function on one hand, and the compensated and the inverse compensated demand functions on the other hand. For a recent review of consumer theory, see Barnett and Serlertis (2008) and Barnett and Serletis (2009a,b).

I shall illustrate the problem of maximizing consumer time-varying parameters preferences with the direct utility function. Consider a consumer whose objective is to maximize his or her preferences under the assumption of time-varying parameters. Let $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ be a vector in the consumer's commodity space. To account for variable preferences, I shall assume that the parameters of the utility function depend on a vector of time-varying stochastic taste determining factors that affect consumer's preferences over time. Therefore, these factors induce stochastic shocks into consumers' preference since they are assumed to be governed by a time-varying stochastic process.

Since the realized values of the preference shifting factors may not be known with certainty, and this is more likely to be the case in practice, I shall assume that only the nature of the stochastic process that govern the preference shifting shocks is relevant to the time-varying structure of the parameters in the utility function. More specifically, I shall assume that the parameters in the time-varying-parameter utility function follow the same stochastic process as the taste determining shocks, and that the realized value of the parameter vector at any time t uniquely determines the form of the utility function to be maximized by the consumer at that very specific time period. Therefore the consumer can be seen as having a family of utility functions and that he or she maximizes, at each time period $t=1,2, \ldots, T$, the one that is selected by the realization of the stochastic process governing the parameter vector.

I shall also assume that the parameters in the utility function follow a first order Markov process. However, a more general process such as an autoregressive or a moving average process of lower order can be postulated as well. The parameter variations are then seen as including a dominant component which is a realization of a stochastic process, in addition of whatever fixed components that may be related to observable factors. The stochastic part of the variation in the parameters is dominant in the sense that it determines the time path of the parameter vector. This consideration allows to include the constant-parameters case as a special case. Assuming that the parameter vector in the utility function is governed by a first order Markov process or by an autoregressive process of lower order naturally applies to models of time series.

In the case of a purely random walk process the consumer's problem can be defined as that of maximizing the time-varying-parameter utility function:

$$
\begin{gathered}
u_t = u\left(\mathbf{x_t}; \boldsymbol{\Theta}_t\right) \\
\text{subject to} \\
\mathbf{p_t'x_t} = m_t \\
\boldsymbol{\Theta}_t = \boldsymbol{\Theta}_{t-1} + \varepsilon_{\boldsymbol{\Theta},\mathbf{t}}
\end{gathered}
\tag{2.4.1}
$$

where $\Theta_t = (\theta_{1t}, \theta_{2t}, \cdots, \theta_{nt})$ is the vector of parameters that describe the form of the ordinal utility function at each time period t = *1, 2, ..., T*; $\mathbf{p}_t = (p_{1t}, p_{2t}, \cdots, p_{nt})$ is the price vector and m_t is the consumer's expenditure. The specification in equation (2.4.1) implies that only the parameters of the utility function are time-varying and that the functional form of the utility function is time-invariant. This assumption allows to derive a unique functional form of the demand functions and consider their time-varying properties only as a result of the time-varying structure inherited by their parameters from the parameters of the utility function.

It is assumed that the specification of the time-varying structure of the parameter vector is such that the utility function u_t possesses nice properties at each time period t, that is u_t is assumed to be a well-behaved function that satisfies all the regularity conditions of consumer demand theory(increasingness, quasiconcavity, continuity, etc.). This implies that any theoretical constraint on the parameters of the utility function should hold at every single time period. In addition, the shocks to the parameter vector affect the marginal utilities and hence translate into demand functions with time-varying parameters.

2.4.1 Implications on the properties of demand functions

If one assumes that the parameters of the utility function just introduced are constant over time, then the analysis will reduce to the traditional framework of the neoclassical theory of consumer's demand. In such a framework consumer preferences are time-invariant and the utility function that is being maximized may be seen as preselected by fixing the value of its parameter vector once and for all. It is clear that the consumer's behavioral model that is based on this approach is a special case of the model that postulates time-varying parameters preferences. As a result, the implications of the consumer theory in the two approaches differ only in the fact that the properties (i.e. the elasticities) of the model are constants under fixed preferences whilst they are time series under the time-varying-parameter approach.

2.5 Illustration: The WS-Branch Utility Tree

The above considerations need to be illustrated with a known functional form that will serve as the true utility function. For this purpose, the weakly separable (WS-) branch utility function will be used. Given the implications of assuming time-varying parameters in the utility functions as outlined in the previous section, I shall only discuss the constant-parameters case in what follows.

The WS-branch utility function was first introduced by Barnett (1977) and subsequently used by Barnett and Choi (1989) as the underlying true utility function in testing weak separability in four demand specifications. This utility function, which is a macroutility function over quantity aggregator functions, is a flexible blockwise weakly separable utility function when defined over no more than two blocks with a total of two goods in each block. The constant-parameter homothetic form of the WS-branch utility function with two blocks q_1 and q_2 is defined as follows:

$$U = U(q_1(x_1, x_2), q_2(x_3)) = A\left[A_{11}q_1^{2\rho} + 2A_{12}q_1^{\rho}q_2^{\rho} + A_{22}q_2^{2\rho}\right]^{(1/2\rho)} \qquad (2.5.1)$$

where $\rho < 0.5$, the constants $A_{ij} > 0$ are elements of a symmetric matrix such that $A_{ij} = A_{ji}$ and $\sum_i \sum_j A_{ij} = 1$. The constant $A > 0$ produces a monotonic transformation of the utility function and thus can be normalized to 1 without loss of generality. Assume that there are only three goods and that the first block consists of the two first goods x_1 and x_2 while the second block consists only of the third good, x_3. Then the sub-utility functions q_1 and q_2 are defined as follows in terms of the vector of supernumerary quantities $\mathbf{y} = \mathbf{x} - \alpha$, where $\mathbf{x} = (x_1, x_2, x_3)$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is a vector of translation parameters:

$$q_1 = q_1(x_1, x_2) = B\left[B_{11}y_1^{2\delta} + 2B_{12}y_1^{\delta}y_2^{\delta} + B_{22}y_2^{2\delta}\right]^{(1/2\delta)} \qquad (2.5.2)$$

$$q_2 = q_2(x_3) = y_3 - \alpha_3 \qquad (2.5.3)$$

where $\delta < 0.5$, $B_{kl} > 0$ for $k,l = 1,2$, $B_{kl} = B_{lk}$ for $k \neq l$ and $\sum_k \sum_l B_{kl} = 1$. Notice that the specification of the aggregator function q_1 in equation (2.5.2) is the same as the specification of the macroutility function (2.5.1). Therefore, both functions share the same properties. For example, both functions are monotone and quasi-concave as a result of the restrictions on their parameters. These restrictions insure their theoretical regularity as well.

In the utility function specifications (2.5.1) - (2.5.3) the supernumerary goods y_1 and y_2 are weakly separable from the supernumerary good y_3. However, it is worthwhile noting that the WS-branch utility function contains a number of special cases. First, blockwise strong separability obtains when the interaction terms A_{12} and A_{21} equal to zero, which also implies additive separability in the two blockings q_1 and q_2. Second, when $B_{12} = B_{21} = 0$ the aggregator function q_1 is additively separable in y_1 and y_2, and therefore y_1 and y_2 are blockwise strongly separable. Third, the pure CES utility function obtains when $\rho = \delta$ and all the interaction terms in both the macrofunction U and the aggregator function q_1 are simultaneously zero. Finally, the Cobb-Douglas utility function obtains when all the interaction terms are zero and when $\rho = \delta = 0$ in the limit.

Barnett and Choi (1989) have shown that the elasticity of substitution between the two sub-utility aggregator functions is given by

$$\xi_{12} = \frac{1}{(1 - \rho + R)} \tag{2.5.4}$$

where

$$R = -\rho \frac{A_{11}A_{22} - A_{12}^2}{(A_{11}(\frac{q_2}{q_1})^{-\rho} + A_{12})(A_{12} + A_{22}(\frac{q_2}{q_1})^{\rho})} \tag{2.5.5}$$

Since the sub-utility function q_1 shares the same properties as the macrofunction U, it follows that the elasticity of substitution between the supernumerary goods y_1 and y_2 is given by the formula below (see also Hjertstrand (2009)):

$$\xi_{12}^* = \frac{1}{(1 - \delta + R^*)} \tag{2.5.6}$$

where

$$R^{*}=-\delta\frac{B_{11}B_{22}-B_{12}^{2}}{(B_{11}(\frac{y_2}{y_1})^{-\delta}+B_{12})(B_{12}+B_{22}(\frac{y_2}{y_1})^{\delta})} \tag{2.5.7}$$

Barnett and Choi (1989) have also shown that the elasticity of substitution between the elementary goods x_i and x_j obtains from the elasticity of substitution between the supernumerary quantities, say ξ_{ij} . Namely, the elasticity of substitution, σ_{ij}, between x_i and x_j and the income elasticity of demand, η_j, for the elementary quantity x_j *(j = 1,2,3)* are respectively

$$\sigma_{ij}=\xi_{ij}\left(\frac{1}{1-\overline{\mathbf{p}}'\alpha}\right)\frac{(x_i-\alpha_i)(x_j-\alpha_j)}{x_j x_i} \tag{2.5.8}$$

and

$$\eta_j=\left(\frac{1}{1-\overline{\mathbf{p}}'\alpha}\right)\frac{x_j-\alpha_j}{x_j} \tag{2.5.9}$$

where $\overline{\mathbf{p}}=(\overline{p}_1,\overline{p}_2,\overline{p}_3)$ is the income normalized price vector, $\mathbf{p}/m$, and $m=\mathbf{p}'\mathbf{x}$ is total expenditure. However this formula applies only when $\alpha_1=\alpha_2=0$ or when the aggregate function is defined in terms of the supernumerary quantities as in equations (2.5.2) and (2.5.3)[See Theorem 2.2 in Barnett and Choi (1989)].

Two demand systems, one for the aggregate quantities and one for the supernumerary quantities, can be derived from two utility maximization problems related to the WS-branch model. First, the aggregate quantities obtain from maximizing the macroutility function with respect to an appropriate income constraint. Second, the supernumerary quantities are chosen by maximizing the corresponding sub-utility function for each group of goods with respect to the group supernumerary income. Finally, the elementary quantities are derived from their relationship with the supernumerary quantities and the committed consumptions. Barnett (1977) and Barnett and Choi (1989) provided the details regarding these two optimization problems, which relate to the two-stage budgeting procedure.

Seck (2006) and Barnett and Seck (2008) used the WS-branch utility function as the true underlying utility function to test the ability of the AIDS model of Deaton and Muellbauer (1980a,b) and the Rotterdam model developed by Barten (1964, 1968, 1977) and Theil (1965, 1971, 1975a,b) to recover the true elasticities of substitution. Their analysis is based on the static specifications of both the AIDS model and the Rotterdam model with constant parameters. My aim in this dissertation research is to conduct a similar analysis by assuming time-varying parameters in both the AIDS and the Rotterdam model.

2.5.1 Time-Varying parameter specification

All the parameters in the model will be assumed to vary over time except ρ in the macroutility function (2.5.1) and δ in the microutility function (2.5.2). This time-varying specification is sufficient to obtain the time-varying properties of the derived marginal rates of substitution and demand functions. Note that Barnett (1977) does not make any assumption on the time variability of the parameters of the WS-branch utility function, especially that of the derived elasticity of substitution.

Assuming time variation in the model's parameters requires the specification of how the parameters change over time, and the resulting utility function shall have a time subscript to show its dependence on the time-varying parameter. I shall assume that the parameters in the WS-branch utility tree vary over time according to a pure random walk process.

It shall be noted that the time-varying assumption applies to the identified subset of the parameters of the utility function rather than the derived elasticities of substitution between aggregate quantities in the macrofunction or between the supernumerary quantities in the microutility function q_1. However, the change in the elasticities of substitution obtain over time through the time-varying characteristics of the parameters in the macroutility function and the microutility function. This is easily seen from the fact that the quantities R and R^* in equations (2.5.5) and (2.5.7) are now time varying through their dependence on the time-varying parameters of the utility functions. Finally, the elasticity of substitution between the elementary quantities x_i and x_j obtains from the derived time-varying elasticity of substitution between the supernumerary quantities y_j and y_j.

The time-varying cross-price elasticities obtain from the same formula as their constant counterparts which are derived from the Allen-Uzawa elasticities of substitution based on their relationship with cross-price elasticities and expenditure share (Barnett and Choi, 1989). The only difference is that the elasticity of substitution between the elementary quantities which is a component of this formula is now time-varying. It should be noted that the time variation in the demand elasticities is an implied property rather than the result of the direct and explicit realization of a stochastic process.

The time-varying compensated elasticity of the demand for the elementary good x_i with respect to price p_j is then given by

$$\eta^*_{ij,t} = \sigma_{ij,t} w_{j,t} \tag{2.5.10}$$

where $w_j = p_j x_j / \sum_k p_k x_k$ is the expenditure share for the elementary good x_j.

It is important to note that the formulas for the income elasticities of the elementary goods x_j, j =*1,2,3*, do not explicitly depend on time varying parameters. Their time-varying versions obtain by means of their indirect relationship with the time-varying parameters of the utility function through the optimal prices. In fact, the optimal prices obtain from the consumer maximization problem with the WS-branch utility function and thus depend on the time-varying parameters in the utility function. Moreover, the optimal prices also depend on the elasticities of substitution between the supernumerary quantities y_1 and y_2 on the one hand, and between the aggregator functions q_1 and q_2 on the other hand. In the framework that I'm considering in this dissertation, the two elasticities of substitution are assumed to be time varying. It then follows that their stochastic properties indirectly affect the income elasticities as well.

3

Time-Varying Parameters in the AIDS and the RM

3.1 Introduction

The analysis of time-varying coefficients demand systems is introduced in the general framework of Harvey (1989)'s structural time series models. The rationale behind such models is that observations are made up of an underlying level or permanent component that can be modeled as a random walk, and an irregular or transitory component that can be specified as a white noise process. These models are usually augmented to account for seasonal and cyclical effects. To allow for the discounting of past observations, the parameters in the structural time series models are allowed to vary over time. This makes the time series models to be regarded as regressions with time-varying parameters.

Chavas (1983) was the first to estimate a time-varying coefficients demand system using the Kalman filter. The parameter vector in the demand system was assumed to be generated by a stochastic difference equation and to depend on non-stochastic variables. The author claimed that this specification is more general in that it includes the constant-parameter model as a special case. However, the paper did not feature any of the flexible functional forms commonly used in empirical demand analysis. The next sections explore the specification of the structural time-varying coefficients demand systems featuring the AIDS and the Rotterdam model.

3.2 The AIDS and Rotterdam models

I shall introduce the AIDS and the Rotterdam model with constant parameters and explore their respective properties. The time-varying properties of each demand system obtain by adding a time subscript to their parameters.

3.2.1 The AIDS Model

The constant-coefficients specification of the AIDS was introduced by Deaton and Muellbauer (1980a) based on the price-independent generalized logarithmic (PIGLOG) class of preferences, allowing exact aggregation over consumers, and represented via the cost function. The logarithm of the PIGLOG cost function is defined as a convex combination of the logarithms of a subsistence function $a(p)$ and a bliss function $b(p)$, that is

$$\log c(u,p) = (1-u)\log\{a(p)\} + u\log\{b(p)\} \tag{3.2.1}$$

where the functional forms of the subsistence function and the bliss function are respectively

$$\log a(p) = a_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j \tag{3.2.2}$$

and

$$\log b(p) = \log a(p) + \beta_0 \prod_k p_k^{\beta k}. \tag{3.2.3}$$

The almost ideal demand system cost function, with parameters α_i, β_i and γ^*_{ij}, is then written as

$$\log c(u,p) = a_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod_k p_k^{\beta k} \qquad (3.2.4)$$

The cost function in equation (3.2.4) is linearly homogeneous provided that $\sum_i \alpha_i = 1$ and $\sum_j \gamma^*_{kj} = \sum_k \gamma^*_{kj} = \sum_j \beta_j = 0$, and possesses enough parameters to be a flexible functional form. The AIDS demand functions are derived from equation (3.2.4) by applying the Shephard lemma. For each good i in the demand system, the logarithmic differentiation gives the budget share as a function of prices and utility. By further exploiting the relationship between the cost function and the indirect utility function, Deaton and Muellbauer (1980a) produced the following AIDS demand functions in budget share form

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{x}{P}\right). \qquad (3.2.5)$$

where w_i is the budget share of good i, x is aggregate consumer expenditure on the n goods and P is the translog price index defined by

$$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \log p_k \log p_j \qquad (3.2.6)$$

For estimation purposes, especially when prices are closely collinear, Deaton and Muellbauer (1980a) suggested to replace the translog price index in equation (3.2.6) by the Stone's geometric price index

$$P^* = \prod_{i=1}^{n} p_i^{w_i}, \tag{3.2.7}$$

the logarithmic expression of which is

$$\log P^* = \sum_{i=1}^{n} w_i \log p_i. \tag{3.2.8}$$

The use of Stone's price produces what is known as the linear-approximate AIDS (LA-AIDS). The ith equation in this linear demand system is:

$$w_i = \alpha_i + \sum_{j} \gamma_{ij} \log p_j + \beta_i \log\left(\frac{x}{P^*}\right) \tag{3.2.9}$$

It is noteworthy that other linearizations of the AIDS in equation (3.2.5) can be produced by using other price indices such as the Törnqvist price index (Diewert, 1976) and the modified Stone price indexes (Moschini, 1995).

The following constraints are imposed on the parameters of both the nonlinear and the linear-approximate AIDS models to satisfy respectively linear homogeneity, adding-up and Slutsky symmetry:

$$\sum_{i=1}^{n} \gamma_{ij}^* = 0 = \sum_{i=1}^{n} \beta_i \tag{3.2.10}$$

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{3.2.11}$$

$$\gamma_{ij}^* = \gamma_{ji}^* \tag{3.2.12}$$

Deaton and Muellbauer (1980a) pointed out the striking similarity of the AIDS, when expressed in its first-differenced form, with the Rotterdam model that is discussed in the next subsection. They showed that the AIDS model in its first-differenced form has exactly the same right hand size as the Rotterdam model.

3.2.2 The Rotterdam model

The Rotterdam model, originated by Barten (1964) and Theil (1965), is one of the popularly used functional form in empirical demand analysis. Although it came under criticism almost a decade after its inception, a number of researchers did struggle to prove its merits and its consistency with demand theory. For example, it was shown to be flexible (Barnett, 1984; Mountain, 1988), to have negligible approximation errors and to be able to mimic any functional form (Barnett, 1979b, 1981; Bryon, 1984). Barnett (1979a,b) also showed that the Rotterdam model is derivable at the aggregate level under weaker assumptions than those needed to empirically acquire usable theoretical result at the aggregate level than other models. Mountain (1988) proved the same result for an individual consumer by considering the problem in the variable space.

I shall consider the absolute price version of the Rotterdam model, defined as

$$w_i \mathrm{dlog} x_i = \theta_i \mathrm{dlog} Q + \sum_{j=1}^{n} \pi_{ij} \mathrm{dlog} p_j \tag{3.2.13}$$

where dlog$Q$ represents the change in real income and defines the Divisia (1925)'s quantity index, that is

$$\mathrm{dlog} Q = \mathrm{dlog} y - \mathrm{dlog} P = \sum_{j=1}^{n} w_j \mathrm{dlog} x_j, \tag{3.2.14}$$

which is defined as the difference between the logarithmic change in income and the budget-share-weighted average of the n logged price changes that defines the Divisia price index,

$$\mathrm{dlog}P = \sum_{j=1}^{n} w_j \mathrm{dlog} p_j. \tag{3.2.15}$$

In terms of finite changes, equation (3.2.13) can be written as

$$\overline{w}_{it} Dq_t = \theta_i DQ_t + \sum_{j=1}^{n} \pi_{ij} Dp_{jt},\ \ i = 1, \ldots, n, \tag{3.2.16}$$

where $\overline{w}_{it} = (1/2)(w_{i,t-1} + w_{i,t})$ is an arithmetic average of the ith good income share over two successive time periods t and $t-1$; π_{ij} is the Slustky coefficient that gives the total substitution effect of the change in the price of good j on the demand for good i; DQ_t is the finite change version of the Divisia quantity index in equation (3.2.15). The income effect of the n price changes on the demand for good i is given by θ_i. When the parameters θ_i and π_{ij} are treated as constants, the model is referred to as the absolute price Rotterdam model. For estimation purpose, a stochastic version of equation (3.2.16) is specified by adding an uncorrelated zero mean disturbance term to each equation as follows:

$$\overline{w}_{it} Dq_t = \theta_i DQ_t + \sum_{j=1}^{n} \pi_{ij} Dp_{jt} + \nu_{it} \tag{3.2.17}$$

For more details on the derivation of the Rotterdam model, see Barten (1964), Theil (1965, 1971, 1975a,b, 1980a,b), Barnett (1979b), and Barnett and Serlertis (2008).

The following restrictions are imposed on the coefficients in order for the the Rotterdam model to satisfy Engel aggregation, linear homogeneity and symmetry are respectively:

$$\sum_{i=1}^{n} \theta_i = 1; \ \sum_{i} \pi_{ij} = 0 \tag{3.2.18}$$

$$\sum_{i=1}^{n} \pi_{ij} = 0 \tag{3.2.19}$$

$$\pi_{ij} = \pi_{ji} \tag{3.2.20}$$

When time-varying parameters are assumed in the Rotterdam model, a time subscript is added to each parameter in the above restrictions. In this case, the restrictions have to hold at every single period. The next section explores the specification of time-varying coefficients (TVC) in both the AIDS and the Rotterdam model (RM).

3.3 Structural time-varying coefficients AIDS and RM

In this section, I shall consider the AIDS and the Rotterdam model in the framework of Harvey (1989)'s structural time series models. This framework allows the time-varying specification of the parameters in each demand function and their estimation by means of the Kalman filter, after appropriately representing the demand systems in a state space form.

3.3.1 The structural TVC AIDS

The time-varying coefficient AIDS in its linearized version has been estimated by Leybourne (1993) and Mazzocchi (2003). Leybourne specified a random walk structure for all the time-varying parameters, including the intercepts. However, Mazzocchi incorporated Harvey's approach, by extending the static linear model accordingly and by estimating a local trend model. In contrast to Leybourne who estimated the model equation by equation, Mazzocchi jointly estimated the parameters of the demand system by considering the cross-equation restrictions on the time-varying coefficients. In the n-goods unrestricted model, the demand equation for the ith good is given by

$$w_{it} = \mu_{it} + \sum_{j=1}^{n} \gamma_{ijt} \log p_{jt} + \beta_{it} \log\left(\frac{x_t}{P_t^*}\right) + \phi_{it} + u_{it} \tag{3.3.1}$$

where w_{it}, x_t and P_t^* are defined as in the constant-coefficients model; μ_{it} and the ϕ_{it}'s are respectively the time-varying intercept and the seasonal components. Finally, u_{it} is a error term that is assumed to be a random noise process. Following Harvey (1989), the time-varying intercept is specified as a random walk with drift, with the drift itself following a pure random walk process. On the other hand, the seasonal dummies, ϕ_{it}'s, are constrained to sum to zero over a year. All the price and income coefficients in equation (3.3.1) are assumed to follow pure random walk processes.

From the similarity between the nonlinear AIDS in equation (3.2.5) and the linear-approximate AIDS in equation (3.2.9), the structural time-varying coefficient specification for the nonlinear AIDS obtains by using the appropriate price index in equation (3.3.1), that is the translog price index P_t defined in equation (3.2.6), to obtain:

$$w_{it} = \alpha_{it} + \sum_{j=1}^{n} \gamma_{ijt}\text{log}p_{jt} + \beta_{it}\text{log}\left(\frac{x_t}{P_t}\right) + \phi_{it} + u_{it} \tag{3.3.2}$$

The restrictions in equations (3.2.10), (3.2.11) and (3.2.12) apply to both models in equations (3.3.2) and (3.3.1) and are assumed to hold for every time period $t = 1, \ldots, T$.

3.3.2 The Structural TVC Rotterdam model

One important feature of the Rotterdam is that the constancy of its parameters obtains by assuming constant mean functions involved in the formulas of its maro-coefficients. However, Barnett (1979a,b) has shown that the macrocoefficients in the Rotterdam model are not necessarily constant. In contrast, they vary over time and are income proportional-weighted theoretical population averages of microcoefficients.

Regardless of this important theoretical finding, the Rotterdam model's coefficients have continued to be considered as constants during estimation. On the other hand, Barnett (1981) advocated testing for parameters constancy whenever the model is used. However, with modern computer capabilities and the advances in numerical methods, the computational difficulties involved in estimating evolving coefficients in the Rotterdam model can be easily overcome.

By admitting stochastic microparameters and macroparameters in the Rotterdam model, the implicit assumption is that the coefficients of the utility function that the Rotterdam is approximating are also stochastic. However, the neoclassical theory leaves open the question of how consumer preferences are affected by exogenous factors over time.

I shall assume that shocks to preferences reflect into the utility function in the form of time-varying parameters. Hence the Rotterdam model is theoretically well suited to incorporate the analysis of change in preferences over time, the form of which I assume to be of time-varying parameters nature in the framework of Harvey (1989)'s structural time series models. The specification of the ith equation in the structural time-varying coefficients Rotterdam model is given in equation (3.3.3) as follows:

$$\overline{w}_{it} Dq_{it} = \varpi_{it} + \theta_{it} DQ_t + \sum_{j=1}^{n} \pi_{ijt} Dp_{jt} + \psi_{it} + \nu_{it} \tag{3.3.3}$$

where $\overline{w}_{it}$, DQ_t, Dp_t are defined as in the constant-coefficients case. The time-varying coefficients ϖ_{it} and ψ_{it}'s have the same meaning and follow the same stochastic processes as μ_{it} and the ϕ_{it}'s in equation (3.3.1). Each of the time-varying coefficients θ_{it} and π_{ijt}'s follows a pure random walk process.

The next section discusses the state space representation of the structural time series AIDS and Rotterdam models, a framework that allows to estimate their time-varying parameters using the Kalman filter. I shall consider two specifications of the time-varying parameters in the demand system: a model where all the parameters are assumed to follow a random walk process and a model where the intercept in each demand equation is assumed to follow a random walk with drift while all the other parameters follow a pure random walk process. The first model specification is referred to as the random walk model (RWM) while the second model is referred to as the local trend model (LTM). The estimation results in Chapter 4 will be presented for each of the two models.

3.4 State space Representation of the AIDS and the RM

3.4.1 Introduction

The state space representation of any model consists of two equations. The first equation, known as measurement equation, relates the observables $\mathbf{y}_t$ to the typically unobservable state vector, $\mathbf{x}_t$. The second equation, known as the state or transition equation, describes how the state vector evolves over time. The errors in both the measurement equation and the state equation are usually assumed to be independent, zero-mean and normally distributed. State space models are treated in more details by Harvey (1989), and Durbin and Koopman (2001).

Consider the following state space representation of the demand system:

$$\begin{aligned} y_t &= Z_t \alpha_t + w_t \\ \alpha_{t+1} &= S_t \alpha_t + v_t \end{aligned} \tag{3.4.1}$$

For an n-goods demand system, the $n \times 1$ vector y_t is the vector of the dependent variables in the demand system, the m vector α_t is the state vector of the m unknown parameters for t =*1*, . . . , *T*. The state space representation above has two matrices. The $n \times m$ matrix Z_t contains all the exogenous variables of the system while the $m \times m$ matrix S_t is the transition matrix that links the state vector at time period *t+1* to its current value, and the entries of which are supposed to be known.

It is worthy pointing out that although the state variables α_t are generally unobservable, they are known to be generated by a first-order Markov process as defined by the transition equation. Finally, the $n \times 1$ vector $\mathbf{w}_t$ and the $m \times 1$ vector $\mathbf{v}_t$ are the serially independent error vectors in the measurement equation and the transition equation respectively, with zero means and respective nonnegative definite covariance matrices H_t and Q_t, that is

$$E(\mathbf{w}_t) = 0 \text{ and } Var(\mathbf{w}_t) = H_t; \quad E(\mathbf{v}_t) = \mathbf{0} \text{ and } Var(\mathbf{v}_t) = Q_t; \quad t = 1, \ldots, T, \tag{3.4.2}$$

where H_t and Q_t are respectively of order $n \times n$ and $m \times m$. In addition, the error vectors in the state space model are assumed to be independent of each other at all time points, that is

$$E(\mathbf{w}_t \mathbf{v}_t^{'}) = \mathbf{0} \tag{3.4.3}$$

In what follows I shall provide the explicit formulation of different matrices in the state space model as they relate to the AIDS and the Rotterdam model. Although I shall only consider two specifications of the parameters' time varying structure, other stochastic processes can be specified for the time-varying coefficients as well, such as the autoregressive structure in Chavas (1983). While the random walk structure assumes that the coefficients in the model are varying violently over time, the autoregressive structure of second order, for example, assumes a slow drift in the coefficients over time (Saris, 1973).

Following Mazzocchi (2003), the homogeneity and symmetry restrictions are imposed by modifying the measurement equation and the transition equation accordingly. This procedure contrasts with the one suggested by Doran (1992) and Doran and Rambaldi (1997), consisting in augmenting the measurement equation prior to estimation of the state space model. The restrictions on the parameters of the demand systems shall be imposed to hold at each single time period. The random walk model and the local trend model are described for both the AIDS and the Rotterdam model in the next two subsections.

3.4.2 The Random Walk Model

I shall provide the matrices of the state space representation of the models with the restrictions imposed on the parameters. However, when linear homogeneity is imposed the disturbances become linearly dependent and their covariance matrix becomes singular. In order to circumvent this problem, one equation must be deleted from the demand system prior to estimation as suggested by Barten (1969). The parameters of the deleted equation will then be recovered by using the imposed restrictions or by estimating the system with a different equation deleted.

3.4.2.1 Representation of the structural TVC AIDS

In the 3-goods case, the measurement equation, with homogeneity and symmetry imposed on the coefficients and the third equation deleted, is as follows for every t:

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} = \begin{bmatrix} 1 & log\left(\frac{p_{1t}}{p_{3t}}\right) & log\left(\frac{p_{2t}}{p_{3t}}\right) & log\left(\frac{m_t}{P_t}\right) & 0 & 0 & 0 \\ 0 & 0 & log\left(\frac{p_{1t}}{p_{3t}}\right) & 0 & 1 & log\left(\frac{p_{2t}}{p_{3t}}\right) & log\left(\frac{m_t}{P_t}\right) \end{bmatrix} \times$$

$$\begin{bmatrix} \alpha_{1,t} \\ \gamma_{11,t} \\ \gamma_{12,t} \\ \beta_{1,t} \\ \alpha_{2,t} \\ \gamma_{22,t} \\ \beta_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

The transition equation is given by

$$\begin{bmatrix} \alpha_{1,t} \\ \gamma_{11,t} \\ \gamma_{12,t} \\ \beta_{1,t} \\ \alpha_{2,t} \\ \gamma_{22,t} \\ \beta_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{1,t-1} \\ \gamma_{11,t-1} \\ \gamma_{12,t-1} \\ \beta_{1,t-1} \\ \alpha_{2,t-1} \\ \gamma_{22,t-1} \\ \beta_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_t^{\alpha_1} \\ e_t^{\gamma_{11}} \\ e_t^{\gamma_{12}} \\ e_t^{\beta_1} \\ e_t^{\alpha_2} \\ e_t^{\gamma_{22}} \\ e_t^{\beta_2} \end{bmatrix}$$

3.4.2.2 Representation of the structural TVC RM

When linear homogeneity is imposed the ith equation of (3.3.3) in the n-goods model becomes

$$\overline{w}_{it} Dq_{it} = \varpi_{it} + \theta_{it} DQ_t + \sum_{j=1}^{n-1} \pi_{ijt}(Dp_{jt} - Dp_{n,t}) + \psi_{it} + \nu_{it} \tag{3.4.4}$$

With the constant and the seasonal dummies dropped from equation (3.4.4), the measurement equation of the state space representation of the Rotterdam model can be expressed explicitly as follows, in the 3-goods case when symmetry is imposed and the third equation deleted:

$$\begin{bmatrix} \overline{w}_{1,t} Dq_{1,t} \\ \overline{w}_{2,t} Dq_{2,t} \end{bmatrix} = \begin{bmatrix} DQ_t & (Dp_1 - Dp_3) & (Dp_2 - Dp_3) & 0 & 0 \\ 0 & 0 & (Dp_1 - Dp_3) & DQ_t & (Dp_3 - Dp_3) \end{bmatrix} \times$$

$$\begin{bmatrix} \theta_{1,t} \\ \pi_{11,t} \\ \pi_{12,t} \\ \theta_{2,t} \\ \pi_{22,t} \end{bmatrix} + \begin{bmatrix} \nu_{1,t} \\ \nu_{2,t} \end{bmatrix}$$

and the transition equation in matrix form is given $\forall t$ by

$$\begin{bmatrix} \theta_{1,t} \\ \pi_{11,t} \\ \pi_{12,t} \\ \theta_{2,t} \\ \pi_{22,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{1,t-1} \\ \pi_{11,t-1} \\ \pi_{12,t-1} \\ \theta_{2,t-1} \\ \pi_{22,t-1} \end{bmatrix} + \begin{bmatrix} e_t^{\theta_1} \\ e_t^{\pi_{11}} \\ e_t^{\pi_{12}} \\ e_t^{\theta_2} \\ e_t^{\pi_{22}} \end{bmatrix}$$

3.4.3 The Local Trend Model

The local trend model assumes that the intercept in each equation of both the AIDS and the Rotterdam model follows a random walk process with a drift, that is

$$\begin{aligned} \mu_{it} &= \mu_{i,t-1} + \lambda_{i,t-1} + e_{it}^{\mu} \\ \lambda_{it} &= \lambda_{i,t-1} + e_{it}^{\lambda} \end{aligned} \tag{3.4.5}$$

for the ith equation in the AIDS, and

$$\begin{aligned} \varpi_{it} &= \varpi_{i,t-1} + \omega_{i,t-1} + e_{it}^{\varpi} \\ \omega_{it} &= \omega_{i,t-1} + e_{it}^{\omega} \end{aligned} \tag{3.4.6}$$

for the ith equation in the Rotterdam model. All the other parameters of the demand systems follow the random walk process as in the random walk model. The measurement and transition equations are modified accordingly.

3.5 Elasticities in the AIDS and Rotterdam Model

The developments in this section are based on the constant-coefficients versions of both the AIDS and the Rotterdam model. The formulas naturally apply to the structural time series models of Section 3.3 by using the time-varying coefficients estimated for this purpose. The resulting elasticities are therefore time series rather than simple constants.

3.5.1 Demand elasticities in the AIDS

Both price and income elasticities in the PIGLOG AIDS model can be straightforwardly computed with formulas that use the estimated values of the parameters in the demand system. The income elasticity of good i is given by

$$\eta_i = 1 + \frac{\beta_i}{w_i}. \tag{3.5.1}$$

The Marshallian elasticity for good i with respect to the price of good j is given by

$$\eta_{ij} = -\delta_{ij} + \frac{1}{w_i}\left[\gamma_{ij} - \beta_i\left(\alpha_j + \sum_{k=1}^{n}\gamma_{jk} logp_k\right)\right] \tag{3.5.2}$$

where δ_{ij} is the delta Kronecker that is equal to 1 if $i = j$ and 0 otherwise. The corresponding compensated price elasticity is

$$\eta^*_{ij} = \eta_{ij} + w_j\eta_i = \eta_{ij} + w_j\left(1 + \frac{\beta_i}{w_i}\right). \tag{3.5.3}$$

However, when the Stone's price index is used to produce the linear-approximate AIDS, the above elasticities formulas have been shown to be inappropriate. More specifically, Moschini (1995) pointed out that the Stone's price index is not invariant to changes in the units of measurement. Therefore, its use as an approximation of the translog price aggregator results in a linear model that is not equivalent to the initial model when prices are scaled. This leads to a biased estimation of the behavioral parameters of the underlying model. On the other hand, Green and Alston (1990) pointed out the differences between the Marshallian elasticities in equation (3.5.2) and the Stone-index-corrected ones. They argued that these differences carry over directly into the calculations of the compensated cross-price elasticities. They derived a system of n^2 simultaneous equations of the uncompensated demand elasticities where each uncompensated cross-price elasticity is expressed in terms of itself and all the other elasticities. The typical equation in this system of equations is

$$\eta_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i}\left\{ w_j + \sum_k w_k \log p_k(\eta_{kj} + \delta_{kj}) \right\} \tag{3.5.4}$$

In matrix form, the system of uncompensated cross-price elasticities can be represented as follows:

$$\mathbf{E} = \mathbf{A} - (\mathbf{bc})[\mathbf{E} + \mathbf{I}], \tag{3.5.5}$$

where $\mathbf{A}$ is an $n \times n$ matrix with typical entry $a_{ij} = -\delta_{ij} + (\gamma_{ij}/w_i) - \beta_i(w_j/w_i)$; $\mathbf{b}$ is a $n \times 1$ vector with typical element $b_i = \beta_i/w_i$; $\mathbf{c}$ is a $1 \times n$ vector with elements $c_j = w_j \log p_j$, $j = 1,2, \ldots, n$ and $\mathbf{I}$ is an $n \times n$ identity matrix. Finally, $\mathbf{E}$ is an $n \times n$ matrix of uncompensated price elasticities with typical entry η_{ij}. Solving for $[\eta_{ij}]$ yields

$$\mathbf{E} = [\mathbf{bc}+\mathbf{I}]^{-1}[\mathbf{A} + \mathbf{I}] - \mathbf{I} \tag{3.5.6}$$

As described above for the uncompensated price elasticity, η_{ij}, the income elasticity of good i also depends on its own value and on the other income elasticities in the linear-approximate AIDS when the derivative of the Stone's price index with respect to income is accounted for in the calculations (Green and Alston, 1991). The following expression relates η_i to the other income elasticities:

$$\eta_i = 1 + \frac{\beta_i}{w_i}\left[1 - \sum_j w_j logp_j(\eta_j - 1)\right] \tag{3.5.7}$$

The corresponding n-equations system can be written in matrix form as:

$$\mathbf{n = b - bcn} \tag{3.5.8}$$

where $\mathbf{b}$ and $\mathbf{c}$ are defined as previously and $\mathbf{n}$ is an $n \times 1$ vector of income elasticities with components $n_i = \eta_i - 1$. Solving equation (3.5.8) for $\mathbf{n}$ yields

$$\mathbf{n} = [\mathbf{I}+\mathbf{bc}]^{-1}\mathbf{b} \tag{3.5.9}$$

It is important to mention that the above corrected formulas obtain, as mentioned by Green and Alston (1990), only if the budget shares in the Stone's price index are treated as exogenous. However, Alston et al. (1994) have shown in a Monte Carlo study that if the nonlinear AIDS is viewed as the underlying demand system and that the linear-approximate AIDS is indeed an approximation of it, the simple formulas of elasticities in equations (3.5.1) $-$ (3.5.3) provide a good approximation. In such a case, there is no need to use the corrected formulas.

Table 3.1. Time-varying demand elasticities in the AI and the Rotterdam models

Model	η_{it}	η_{ijt}	η^*_{ijt}
Rotterdam	$\frac{\mu_{it}}{wit}$	$\frac{\pi_{ijt}-\mu_{it}w_{jt}}{w_{it}}$	$\frac{\pi_{ijt}}{w_{it}}$
AI Model	$1+\frac{\beta_{it}}{w_{it}}$	$-\delta_{ijt}+\frac{\gamma_{ijt}}{w_{it}}-\frac{\beta_{it}\alpha_{jt}}{w_{it}}$ $-\frac{\beta_{it}}{w_{it}}\sum_k \gamma_{kjt}lnp_{kt}$	$\eta_{ijt}+w_{jt}\left(1+\frac{\beta_{it}}{w_{it}}\right)$
LA AI model	$1+$ $\frac{\beta_{it}}{w_{it}}\left[1-\sum_{jt} w_{jt}lnp_{jt}(\eta_{jt}-1)\right]$	$-\delta_{ijt}+\frac{\gamma_{ijt}}{w_{it}}-\beta_i\frac{w_{jt}}{w_{it}}$ $-\frac{\beta_{it}}{w_{it}}\left[\sum_k w_{kt}lnp_{kt}(\eta_{kjt}+\delta_{kjt})\right]$	$\eta_{ijt}+w_{jt}\eta_{it}$

The time-varying income and price elasticities obtain by straightforwardly applying the formulas derived for the constant-parameters AIDS, and replacing the constant parameters by the time-varying parameters. Table 3.1 summarizes the formulas for the time-varying income and price elasticities.

3.5.2 Demand elasticities in the Rotterdam model

Table 3.1 also shows the income and price elasticities for the time-varying parameter Rotterdam model. These are just the same formulas as those used for the constant-parameter Rotterdam model in which time-varying parameters are used instead.

3.5.3 Elasticities of substitution in the AIDS and Rotterdam Model

The concept of the elasticity of substitution was first introduced by Hicks (1932) to analyze changes in capital and labor income shares. This measure has a straightforward economic interpretation that, in the case of a two-variables production/utility function, the relative income share of factor/good i increases if the elasticity of substitution is greater than one and decreases when the elasticity of substitution is less than one.

Of the two generalizations of the Hickian concept to more than two variables by Hicks and Allen (1934), Allen (1938) and Uzawa (1962), namely the Hick's elasticity of substitution (HES) and the Allen-Uzawa elasticity of substitution (AUES), the second has been extensively used as a standard statistics in empirical studies in both production and consumption. The AUES between inputs or commodities i and j is expressed in terms of the cost function as follows

$$\sigma_{ij}^{AU}(\mathbf{p}, u) = \frac{C(\mathbf{p}, u)C_{ij}(\mathbf{p}, u)}{C_i(\mathbf{p}, u)C_j(\mathbf{p}, u)}) = \frac{\epsilon_{ij}(\mathbf{p}, u)}{w_j(\mathbf{p}, u)} \tag{3.5.10}$$

where $C(\mathbf{p}, u)$ is the cost function and the subscripts on $C(\mathbf{p}, u)$ are the partial derivatives with respect to the relevant prices; $\epsilon_{ij}(\mathbf{p}, u)$ is the Hicskian compensated elasticity of good i with respect to the price of good j; and $w_j(\mathbf{p}, u)$ is good j's expenditure share. The cost function in equation (3.5.10) depends on the price vector $\mathbf{p}$ and the utility level u and is assumed to satisfy all the regularity conditions. A regular cost function is continuous, nondecreasing, linearly homogeneous and concave in $\mathbf{p}$, increasing in u and twice continuously differentiable.

The AUES reduces to the original Hicksian concept in the two-dimensional case. However Blackorby and Russell (1989) recommend against the use of the AUES because it preserves none of the salient properties of the Hicksian concept, in general. More specifically, they argue that the AUES is not a measure of the ease of substitution; it provides no information about relative income shares; and cannot be interpreted as a logarithmic derivative of a quantity ratio with respect to the marginal rate of substitution. They conclude that the AUES has no meaning as a quantitative measure and adds no information to that already contained in the compensated price elasticity as a qualitative measure.

In contrast, the Morishima elasticity of substitution (MES), independently introduced by Morishima (1967) and Blackorby and Russell (1975) does preserve the salient characteristics of the original Hicksian concept. This measure of the elasticity of substitution is both quantitatively meaningful and qualitatively informative. Moreover, it is a measure of curvature or ease of substitution and a logarithmic derivative of a quantity ratio with respect to marginal rate of substitution (Blackorby and Russell (1981, 1989) and Blackorby et al. (2007)).

The formula to calculate the MES between goods i and j is given by

$$\sigma_{ij}^{M} = \frac{p_i C_{ij}(\mathbf{p}, u)}{C_j(\mathbf{p}, u}) - \frac{p_i C_{ii}(\mathbf{p}, u)}{C_i(\mathbf{p}, u}) = \epsilon_{ij}(\mathbf{p}, u) - \epsilon_{ii}(\mathbf{p}, u), \tag{3.5.11}$$

which is nothing but the difference between the compensated cross-price elasticity, $\epsilon_{ij}(\mathbf{p}, u)$, and the own-price elasticity, $\epsilon_{ii}(\mathbf{p}, u)$.

Both the MES and the AUES are used to classify inputs/goods as substitutes or complements; although they yield different stratification sets in general (Barnett and Serlertis, 2008). In fact, two Allen substitutes goods must be Morishima substitutes while two Allen complements may be Morishima substitutes.

I shall use the Morishima concept to characterize the time-varying elasticities of substitution. The compensated time-varying price elasticities will be obtained prior to the computation of all the relevant MES. However, this concept is used here under the reserve of the criticism formulated by de la Grandville (1997) on its interpretation by Blackorby and Russell (1989) as a measure of curvature. In fact, de la Grandville (1997) showed that there is no direct nor inverse relationship between the concept of elasticity of substitution and that of the curvature of the isoquant or the indifference curve. de la Grandville (1997) suggested that the concept of elasticity of substitution be interpreted as a measure of efficiency rather than a measure of curvature.

4

Estimation Method and Results

4.1 Introduction

Empirical demand system analysis with time-varying parameters usually proceeds by first choosing the functional form for demand curves and by specifying a time-varying structure for the parameters to be estimated. It is also customary to check how well the time-varying parameter specification fits the data compared to the constant parameter version or the dynamic specification of the selected functional form.

However, such a comparison needs to be conducted with caution for two reasons. First, both the constant parameter model and the time-varying parameter model are assumed to approximate the true model that is supposed to have generated the data being analyzed. But since the true model is usually unknown to the researcher so as to serve as the benchmark for the comparison, the conclusion of the analysis may be questioned. In addition, the conclusion may vary from one dataset to another. Second, the ignorance of the true model makes unrealistic the comparison of the performance of alternative functional forms for the demand system that may be applied to the same dataset. More specifically, it is unrealistic to compare the performance of one functional form to that of another when the parameters of the underlying true model are unknown.

In this chapter I shall conduct a Monte Carlo investigation of the performance of the AIDS and the Rotterdam model to recover the time-varying properties of the data generated from the true model, that I have specified to be the WS-branch utility tree. Such an exercise is more reasonable since the use of real data may lead to different conclusions when different dataset are used. Seck (2006) and Barnett and Seck (2008) used the same approach to compare the performance of the AIDS and the Rotterdam model with constant parameters. Barnett (1977)'s WS-branch utility function also served as the true underlying model for data generation.

This chapter is organized as follows. Section 4.2 presents the data generation procedure for the Monte Carlo experiments, and the estimation method is discussed in section 4.3. In section 4.4, the time-varying elasticities are presented. These elasticities are computed by using the values of the time-varying coefficients obtained for each demand system. The results are presented in the perspective of the performance of each individual demand system to recover the time-varying elasticities of the true model. The focus is on the ability to recover the signs of the true time-varying elasticities (qualitative performance) and to produce approximations of time-varying elasticities the values of which are close to the true ones (quantintative performamce). Section 4.5 reviews the performance of each model with respect to the theoretical regularity. A regularity index is thus calculated for each time period under the random walk specification and under the local trend model for each demand system. The comparison of the performance of each demand system to that of the other specifications is presented in section 4.6. Section 4.7 presents the results from a robustness-check Monte Carlo experiment. In this experiment, different paths of the time varying parameters in the utility function are used to generate the data.

4.2 Description of the data generation procedure

This section explains the steps used to generate the data for the Monte Carlo simulations. In this process, all the parameters in the utility functions in equations (2.5.1) and (2.5.2), except ρ and δ, are assumed to be time varying. The constancy of δ and ρ is assumed for convenience, since these parameters can be set time-varying as well. The reference dataset is generated by means of the following 12 steps:

Step 1: Set the value of the elasticity of substitution between the supernumerary quantities y_1 and y_2 in the microutility function in equation (2.5.2) for each time period, $t = 1, 2, ..., T.$, where $T = 60$.

Step 2: Generate the stochastic process for the time-varying parameters in the microutility function q_1. The parameters $B_{11,t}$,$B_{12,t}$,$B_{21,t}$ and $B_{22,t}$ are assumed to follow a random walk process and are constrained so that they satisfy the condition $\sum_k \sum_l B_{kl,t} = 1$ in each time period. In addition to this condition, $B_{12,t}$ and $B_{21,t}$ are constrained to be equal to each other at each time period by the symmetry condition. Under these constraints only the stochastic processes of $B_{11,t}$ and $B_{12,t}$ are needed to trace out the complete time-varying structure of the parameters in the utility function.

Step 3: Obtain the ratio between the two supernumerary quantities y_1 and y_2 at each time period from the formula of the elasticity of substitution between the two supernumerary quantities, using the values set in *Step 1*.

Step 4: Generate the first order autoregressive time series for the two supernumerary quantities y_1 and y_2 and the supernumerary income m_{1t}[1] and adjust the two supernumerary quantities time series so that the ratio y_2/y_1 corresponds to the one obtained in *Step 3* at each time period.

Step 5: Use the first order conditions for maximizing q_1[2] and the supernumerary budget constraint to solve for the price system (p_{1t}, p_{2t}), $\forall t$.

Step 6: Calculate the aggregate quantity q_1 at every time period and the corresponding price index using the Fisher factor reversal test.

Step 7: Set the value of the elasticity of substitution between the two aggregate quantities q_1 and q_2 in the WS-branch utility function in equation (2.5.1) and solve for the ratio q_2/q_1 from equation (2.5.4) for each time period $t = 1,2,\ldots,T$.

Step 8: Generate the time path for the time-varying parameters in the macroutility function. As for the time-varying parameters in the microutility function, the parameter vector in the macroutility function is assumed to follow a random walk process. The time-varying parameters are set in such a way that $\sum_i \sum_j A_{ij,t} = 1$ and $A_{12,t} = A_{21,t}$. $\forall t$. The only constant parameter in the macroutility function is ρ.

Step 9: Generate the supernumerary quantity $y_3 = q_2$ according a first order autoregressive process[3] and adjust the resulting time series so that the ratio q_{2t}/q_{1t} corresponds to the ratio obtained in *Step 7*.

[1] The autoregressive models for the supernumerary quantities and income are the following: $y_{1t} = 2 + 0.75y_{1,t-1} + e_{1t}$; $y_{2t} = 1 + 0.739y_{2,t-1} + e_{2t}$; $m_{1t} = 125 + 0.98m_{1,t-1} + e_{3t}$ where e_{1t}, e_{2t} and e_{3t} are zero mean and serially uncorrelated normal error terms with variance 1.

[2] See Barnett and Choi (1989)

[3] $y_{3t} = 3 + 0.69y_{3,t-1} + e_{4t}$

Step 10: Solve for p_{3t} from the first order conditions for the maximization of the macroutility function[4].

Step 11: Set the value of α_1,α_2 and α_3 and obtain the elementary quantities x_{1t}, x_{2t} and x_{3t} from their relationships with the supernumerary quantities, that is $x_i = y_i + \alpha_i$[5], *i=1,2,3* and calculate total expenditure on the elementary quantities.

Step 12: Add noises to the elementary quantities x_{1t}, x_{2t} and x_{3t} that constitute the reference dataset and estimate the time varying parameters of the resulting demand system, bootstrapping the model 2000 times while recalculating the total expenditure on x_{1t}, x_{2t} and x_{3t}. For the bootstrap procedure I have generated three vectors of 2000 seeds each to use in generating the random numbers that are added as shocks to the reference data. Relevant elasticities are calculated and stored at each replication from the estimated time-varying parameters. Finally, the income and compensated price elasticities as well as the elasticities of substitution at each time period are calculated as the averages of the values stored during the bootstrap procedure.

The true time-varying elasticities will be presented together with their approximations from the four demand systems in section 4.4. It is noteworthy that the true time-varying elasticities of substitution and income elasticities will be obtained in application of formulas in equations (2.5.4), (2.5.6), (2.5.8) and (2.5.9). The true time-varying cross-price elasticities will be obtained by using the relationship between the Allen-Uzawa elasticity of substitution and the Hicksian demand elasticities.

[4] See Barnett and Choi (1989) for the specification of this utility maximization problem.

[5] The values used to generate the data are: $\alpha_1 = 1$, $\alpha_2 = 10$ and $\alpha_3 = 4$. This specification is used for the random walk model. For the local trend model, each of the α_i's is specified as a random walk plus a shift, where the shift itself follows a random walk process.

4.3 The Estimation method

I estimate the time-varying parameters by the Kalman (1960)'s filter and then pass the filtered estimates through a Kalman smoothing algorithm in order to revise them. The Kalman filter is a sequential procedure for computing the optimal estimate of the state vector at time t using all the information available at time t, given some priors on the initial state and the covariance matrix. The Kalman smoother, on the other hand, is a backward procedure starting from the Kalman filtered state vector to produce smoothed estimates using all the available information in the data.

The following set of equations describes the Kalman recursion (Durbin and Koopman, 2001):

$$\begin{aligned} &\upsilon_t = y_t - Z_t a_t, && F_t = Z_t P_t Z_t' + H_t, && \\ &K_t = S_t P_t Z_t' F_t^{-1}, && L_t = S_t - K_t Z_t, && t = 1, \ldots, T, \\ &a_{t+1} = S_t a_t + K_t \upsilon_t, && P_{t+1} = S_t P_t L_t' + Q_t, && \end{aligned} \tag{4.3.1}$$

where a_1 and P_1 are respectively the mean and covariance matrix of the initial state vector, Z_t, S_t, P_t, α_t are as defined in equation (3.4.1); and H_t and Q_t as defined in equation (3.4.2).

The contemporaneous filtering equations are given by

$$\begin{aligned} &\upsilon_t = y_t - Z_t \alpha_t, && F_t = Z_t P_t Z_t' + H_t, && \\ &a_{t|t} = a_t + M_t F_t^{-1} \upsilon_t, && P_{t|t} = P_t - M_t F_t^{-1} M', && t = 1, \ldots, T, \\ &a_{t+1} = S_t a_{t|t}, && S_t P_{t|t} S_t' + Q_t, && \end{aligned} \tag{4.3.2}$$

where a_1 and P_1 are given and $M_t = P_t Z_t'$ is a matrix of order $m \times p$, and a_t and $a_{t|t}$ are $m \times 1$ vectors.

On the other hands, the smoothing recursion is defined by the following equations:

$$\begin{aligned} r_{t-1} &= Z_t' F_t^{-1} \upsilon_t + L_t' r_t, \quad & N_{t-1} &= Z_t' F_t^{-1} Z_t + L_t' N_t L_t \\ \hat{\alpha}_t &= a_t + P_t r_{t-1}, & V_t &= P_t - P_t N_{t-1} P_t \end{aligned} \tag{4.3.3}$$

with $r_T = 0$. The $m \times 1$ vector r_t in equation (4.3.3) is a weighted sum of innovations υ_h occurring after *t-1* for j = *1, . . . , T*. The $m \times m$ matrix V_t is the covariance matrix of the $m \times 1$ vector of smoothed states $\hat{\alpha}_t$. It is noteworthy that the smoothing recursion in (4.3.3) is referred to as the *fixed interval smoother* (de Jong, 1988, 1989; Kohn and Ansley, 1989; Durbin and Koopman, 2001) and that it differs from the *classical fixed interval smoother* suggested by Anderson and Moore (1979). However, this smoothing recursion can be derived from the one in Anderson and Moore (1979) as detailed by Durbin and Koopman (2001).

Under the normality assumption for the disturbance vectors $\mathbf{w}_t$ and $\mathbf{v}_t$ in equations (3.4.1), the distribution generated by the Kalman filter is given by

$$y_t | y_1, y_2, \ldots, y_{t-1} \sim N(Z_t' \alpha_t, \Lambda_t) \tag{4.3.4}$$

where $\Lambda_t = Z_t' P_{t|t-1} Z_t + Q_t$. The essential part of the likelihood function for the full sample, which is the objective function of the Kalman filter(smoother) is therefore

$$-\frac{1}{2} \sum_t log|\Lambda_t| - \frac{1}{2} \sum_t (y_t - Z_t' \alpha_{t|t-1})' \Lambda_t^{-1} (y_t - Z_t' \alpha_t). \tag{4.3.5}$$

I shall give the results from the above estimation method in the next section. The estimation was performed by using the exact Kalman filter (Koopman, 1997) for initial states and variances and implemented using the RATS software (Doan, 2010b,a, 2011; Estima, 2007a,b).

The AIDS models have been estimated in first-differenced form by assuming time-varying coefficients rather than constant coefficients like, for example, in Deaton and Muellbauer (1980a), Eales and Unnevehr (1988), Moschini and Meilke (1989), Brester and Wohlgenant (1991) and Alston and Chalfant (1993). An intercept is included in each demand equation.

4.4 Estimated values of the time varying elasticities

4.4.1 The RM approximation

The Rotterdam model correctly approximated the true time-varying elasticities of substitution with positive values at each time period. However, the approximating values were closer to the true ones only for the goods withing the same sub-aggregation group under the random walk specification than under the local trend model specification. Under the local trend model specification, the approximating values of the within-group time-varying elasticity of substitution tended to overestimate the true ones, especially when the values of the true time-varying elasticity were low (see Table 4.1). The model tended to underestimate the values of the elasticities of substitution for goods in different sub-aggregation groups under both the random walk specification and the local trend model specification. Nonetheless, the gap between the values of the true time-varying elasticities of substitution and the approximating values was smaller under the random walk specification when the true values were less than unity. Regardless of the magnitude of the approximating values, the Rotterdam model correctly identified each of the pair of goods (x_1, x_2), (x_1, x_3) and (x_2, x_3) as substitutes.

The Rotterdam model also approximated the true time-varying income elasticities with the correct positive sign. Under both the random walk specification and the local trend model specification, the Rotterdam model identified x_1, x_2 and x_3 as normal goods. In addition, each good was correctly identified as a necessity good or a luxury good. For most of the time periods, the approximating values of the time-varying income elasticities were close to the values of the true time-varying income elasticities.

Finally, the Rotterdam model correctly identified the pairs (x_1, x_2), (x_1, x_3) and (x_2, x_3) as substitutes by providing positive values of the time-varying compensated cross-price elasticities. The only exception occurred for x_2 and x_3 under the random walk specification for two consecutive time periods (see Table 4.9).

4.4.2 The AIDS model approximation

All the approximating values of the time-varying elasticities of substitution were positive both under the random walk specification and the local trend model specification. The model tended to overestimate the values of the time-varying elasticity of substitution (ESUB) within the same utility branch under both specifications of the structural time series demand system. For goods in different branches the results from the two specifications of the time-varying coefficients had different profiles. For example, the model tended to overestimate the values of the time-varying elasticity of substitution between x_2 and x_3 under the random walk specification while it tended to underestimate the true values of the same time-varying elasticity under the local trend model specification.

Table 4.1. True and approximating values of $\sigma_{12,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	RM	NLAI	LAISF	LAICF	True	RM	NLAI	LAISF	LAICF
1	0.399	0.303	1.638	-0.372	0.337	0.165	0.138	0.439	-0.910	-0.131
2	0.393	0.343	1.643	-0.420	0.307	0.184	0.114	0.419	-1.001	-0.182
3	0.386	0.342	1.636	-0.358	0.340	0.195	0.115	1.101	-0.975	-0.171
4	0.384	0.322	1.574	-0.227	0.392	0.203	0.142	1.100	-1.039	-0.213
5	0.413	0.321	1.617	-0.348	0.341	0.154	0.151	1.113	-1.077	-0.224
6	0.369	0.335	1.623	-0.359	0.337	0.169	0.152	1.109	-1.119	-0.236
7	0.329	0.314	1.538	-0.147	0.419	0.154	0.149	1.070	-1.098	-0.226
8	0.382	0.301	1.551	-0.183	0.383	0.107	0.156	1.078	-1.113	-0.231
9	0.329	0.298	1.523	-0.136	0.426	0.125	0.153	1.087	-1.289	-0.336
10	0.271	0.303	1.581	-0.266	0.376	0.115	0.146	1.083	-1.209	-0.288
11	0.273	0.311	1.555	-0.206	0.416	0.133	0.139	1.080	-1.097	-0.222
12	0.273	0.316	1.615	-0.320	0.357	0.078	0.136	1.068	-1.007	-0.167
13	0.405	0.334	1.640	-0.372	0.339	0.087	0.127	1.069	-1.007	-0.168
14	0.305	0.339	1.622	-0.372	0.336	0.067	0.122	1.064	-0.918	-0.113
15	0.266	0.346	1.687	-0.425	0.313	0.105	0.124	1.064	-0.999	-0.157
16	0.262	0.318	1.511	-0.139	0.429	0.069	0.120	1.065	-1.063	-0.191
17	0.387	0.312	1.639	-0.351	0.333	0.087	0.117	1.059	-1.004	-0.157
18	0.369	0.344	1.647	-0.415	0.288	0.101	0.117	1.062	-1.109	-0.218
19	0.370	0.351	1.665	-0.399	0.284	0.087	0.117	1.062	-1.093	-0.207
20	0.386	0.345	1.629	-0.344	0.319	0.106	0.114	1.053	-1.118	-0.223
21	0.380	0.342	1.639	-0.392	0.290	0.083	0.118	1.054	-1.043	-0.179
22	0.361	0.352	1.679	-0.408	0.263	0.075	0.118	1.056	-1.128	-0.226
23	0.373	0.363	1.687	-0.453	0.241	0.054	0.112	1.054	-1.035	-0.170
24	0.329	0.372	1.707	-0.478	0.212	0.125	0.110	1.054	-1.089	-0.204
25	0.404	0.374	1.696	-0.428	0.225	0.070	0.112	1.057	-1.152	-0.234
26	0.507	0.376	1.717	-0.506	0.209	0.088	0.110	1.058	-1.246	-0.292
27	0.465	0.368	1.668	-0.427	0.261	0.084	0.113	1.059	-1.268	-0.303
28	0.388	0.355	1.668	-0.417	0.301	0.085	0.115	1.060	-1.399	-0.378
29	0.373	0.345	1.641	-0.360	0.326	0.055	0.119	1.062	-1.330	-0.334
30	0.340	0.336	1.602	-0.347	0.321	0.062	0.124	1.052	-1.409	-0.379
36	0.429	0.306	1.573	-0.248	0.395	0.061	0.139	1.057	-1.666	-0.526
42	0.360	0.328	1.615	-0.316	0.389	0.061	0.158	1.064	-1.890	-0.651
48	0.341	0.346	1.649	-0.399	0.351	0.090	0.149	1.065	-1.925	-0.675
54	0.361	0.351	1.601	-0.324	0.381	0.060	0.154	1.068	-2.040	-0.734
60	0.406	0.327	1.631	-0.356	0.371	0.070	0.175	1.080	-2.505	-0.996

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Table 4.2. True and approximating values of $\sigma_{13,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	RM	NLAI	LAISF	LAICF	True	RM	NLAI	LAISF	LAICF
1	0.154	0.596	0.164	-0.303	0.467	3.050	0.301	1.060	1.250	1.289
2	0.595	0.771	0.195	-0.259	0.481	3.069	0.275	1.077	1.328	1.376
3	0.840	0.746	0.217	-0.223	0.494	2.964	0.405	1.082	1.461	1.408
4	1.122	0.673	0.348	-0.028	0.564	3.008	0.444	1.094	1.518	1.454
5	2.103	0.676	0.213	-0.237	0.487	3.002	0.455	1.254	1.464	1.422
6	2.002	0.746	0.199	-0.248	0.483	3.060	0.405	1.218	1.389	1.357
7	1.945	0.572	0.520	0.225	0.658	3.014	0.436	1.306	1.575	1.515
8	1.551	0.423	0.620	0.360	0.721	2.908	0.554	1.342	1.655	1.579
9	1.835	0.440	0.461	0.145	0.624	3.017	0.561	1.302	1.592	1.526
10	1.958	0.570	0.323	-0.067	0.552	3.008	0.531	1.294	1.573	1.510
11	1.134	0.655	0.296	-0.102	0.544	3.027	0.465	1.235	1.442	1.398
12	0.801	0.679	0.270	-0.143	0.526	2.946	0.433	1.253	1.481	1.432
13	1.306	0.726	0.187	-0.267	0.481	2.979	0.486	1.297	1.575	1.513
14	1.834	0.761	0.196	-0.255	0.486	2.848	0.583	1.365	1.729	1.644
15	2.387	0.747	0.217	-0.224	0.496	2.957	0.577	1.291	1.562	1.501
16	2.137	0.622	0.439	0.112	0.615	2.821	0.614	1.425	1.855	1.752
17	2.389	0.581	0.336	-0.051	0.556	2.744	0.760	1.426	1.863	1.757
18	2.256	0.614	0.365	-0.003	0.570	2.513	0.886	1.591	2.224	2.066
19	2.165	0.537	0.501	0.184	0.646	2.691	1.047	1.575	2.196	2.041
20	1.530	0.505	0.450	0.124	0.614	2.766	1.033	1.573	2.192	2.038
21	1.434	0.521	0.448	0.123	0.616	2.765	1.038	1.581	2.208	2.052
22	1.258	0.455	0.594	0.320	0.695	2.892	0.999	1.527	2.099	1.958
23	0.830	0.407	0.579	0.310	0.687	2.727	1.019	1.602	2.265	2.100
24	0.944	0.381	0.638	0.393	0.722	2.647	1.082	1.589	2.244	2.081
25	1.022	0.327	0.706	0.498	0.762	1.967	1.210	1.763	2.631	2.410
26	0.659	0.344	0.588	0.318	0.692	2.745	1.209	1.588	2.242	2.078
27	0.682	0.427	0.530	0.238	0.663	2.399	1.253	1.826	2.767	2.527
28	0.568	0.547	0.259	-0.166	0.505	2.833	1.233	1.571	2.204	2.046
29	0.430	0.676	0.303	-0.097	0.534	2.678	1.058	1.586	2.235	2.074
30	0.616	0.636	0.348	-0.027	0.561	2.712	1.066	1.580	2.222	2.063
36	1.391	0.684	0.245	-0.180	0.515	2.417	1.310	1.789	2.717	2.485
42	2.245	0.785	0.175	-0.283	0.496	2.948	1.132	1.491	2.039	1.906
48	2.897	0.746	0.201	-0.244	0.518	2.951	1.260	1.735	2.588	2.373
54	3.053	0.852	0.137	-0.340	0.475	2.657	1.534	1.905	2.966	2.695
60	2.762	0.905	0.008	-0.532	0.402	2.445	1.894	2.304	3.862	3.458

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

With regard to income elasticities, the AIDS model produced approximations that had positive values at each time period for the three goods. However, the model tended to produce time-varying income elasticities with unitary values(see Tables 4.4, 4.5 and 4.6). When the values of the true time-varying income elasticities were less the one, the approximating values overestimated the true values, and the gap was smaller only when the true values were very close to 1. On the other hand, the approximating values underestimated the true ones when the true values were greater than one. In this case the model failed to capture very high values of the time-varying income elasticities (see for example Table 4.6).

The nonlinear AIDS identified all the pairs of goods (x_1, x_2), (x_1, x_3) and (x_2, x_3) as substitutes under the random walk specification, with exception for few time periods at the end of sample for (x_1, x_3). In these cases, the two goods were identified as complements. Under the local trend model specification, the compensated TVC cross-price elasticities between x_1 and x_2 and between x_1 and x_3 had the correct positive sign at each time periods. However, the model produced an approximation of the time-varying cross-price elasticity between x_2 and x_3 the values of which were negative for most of the time periods. Thus, x_2 and x_3 were wrongly identified as complements for most of the time periods.

4.4.3 The LAISF model approximation

The LAISF model produced an approximation of the TVC ESUB between x_1 and x_2 the values of which were negative under both stochastic specifications of the time-varying coefficients in the demand system. The true TVC ESUB between x_1 and x_3 is approximated with positive values under the local trend model specification, but with both negative and positive values under the random walk specification.

Table 4.3. True and approximating values of $\sigma_{23,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	RM	NLAI	LAISF	LAICF	True	RM	NLAI	LAISF	LAICF
1	0.909	0.251	1.934	0.263	0.266	1.711	0.226	0.573	-0.410	-0.394
2	0.934	0.292	1.939	0.238	0.240	1.767	0.194	1.076	-0.333	-0.377
3	0.876	0.292	1.923	0.277	0.279	1.812	0.171	1.079	-0.351	-0.396
4	0.635	0.271	1.826	0.352	0.352	1.861	0.168	1.082	-0.429	-0.474
5	0.873	0.270	1.903	0.280	0.283	1.845	0.201	1.071	-0.472	-0.517
6	0.915	0.284	1.912	0.273	0.276	1.885	0.219	1.047	-0.438	-0.484
7	0.436	0.262	1.757	0.409	0.405	1.872	0.220	1.056	-0.436	-0.483
8	0.309	0.248	1.761	0.390	0.384	2.066	0.214	1.069	-0.500	-0.547
9	0.488	0.245	1.747	0.407	0.404	1.802	0.213	1.063	-0.463	-0.511
10	0.773	0.251	1.840	0.332	0.332	1.801	0.207	1.056	-0.426	-0.474
11	0.970	0.259	1.813	0.363	0.363	1.541	0.205	1.044	-0.423	-0.470
12	0.889	0.265	1.889	0.301	0.300	1.492	0.207	1.052	-0.365	-0.412
13	1.088	0.283	1.933	0.269	0.269	1.413	0.203	1.036	-0.410	-0.457
14	0.956	0.289	1.913	0.267	0.267	1.463	0.196	1.040	-0.459	-0.507
15	0.896	0.295	1.983	0.249	0.247	1.342	0.204	1.029	-0.619	-0.668
16	0.500	0.267	1.738	0.404	0.401	1.375	0.219	1.027	-0.572	-0.622
17	0.586	0.260	1.909	0.293	0.292	1.533	0.229	1.030	-0.634	-0.685
18	0.518	0.292	1.918	0.254	0.252	1.584	0.225	1.030	-0.630	-0.680
19	0.330	0.299	1.916	0.272	0.268	1.686	0.225	1.027	-0.608	-0.659
20	0.371	0.292	1.881	0.300	0.296	1.531	0.221	1.023	-0.619	-0.670
21	0.432	0.290	1.896	0.273	0.269	1.509	0.219	1.029	-0.662	-0.714
22	0.281	0.299	1.920	0.270	0.263	1.011	0.228	1.017	-0.737	-0.790
23	0.301	0.309	1.936	0.248	0.241	1.485	0.240	1.028	-0.637	-0.689
24	0.245	0.318	1.951	0.230	0.222	1.373	0.233	1.018	-0.849	-0.904
25	0.166	0.319	1.927	0.256	0.247	1.708	0.238	1.034	-0.715	-0.769
26	0.265	0.321	1.970	0.217	0.210	1.676	0.240	1.034	-0.737	-0.791
27	0.332	0.315	1.919	0.258	0.251	1.771	0.223	1.037	-0.812	-0.865
28	0.667	0.303	1.956	0.253	0.252	1.738	0.227	1.029	-0.903	-0.958
29	0.595	0.294	1.916	0.284	0.282	1.807	0.242	1.034	-0.947	-1.001
30	0.529	0.284	1.866	0.284	0.282	2.052	0.261	1.038	-0.891	-0.945
36	0.968	0.254	1.841	0.333	0.336	2.162	0.266	1.053	-1.037	-1.089
42	1.149	0.277	1.904	0.300	0.299	2.614	0.266	1.064	-1.016	-1.067
48	1.433	0.295	1.943	0.255	0.250	2.269	0.237	1.040	-1.430	-1.482
54	1.157	0.300	1.895	0.285	0.288	2.261	0.300	1.035	-1.731	-1.784
60	1.171	0.277	1.949	0.259	0.266	2.851	0.313	1.063	-1.737	-1.737

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Table 4.4. True and approximating values of η_{1t} at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	1.039	1.0003	1.072	0.074	1.054	1.000	1.0001	1.027	0.027	0.998
2	1.041	1.0003	1.072	0.075	1.028	1.001	1.0001	1.027	0.027	0.994
3	1.043	1.0003	1.072	0.075	1.030	0.997	1.0001	1.028	0.029	0.995
4	1.062	1.0003	1.073	0.078	1.040	0.995	1.0000	1.028	0.029	0.992
5	1.045	1.0003	1.072	0.076	1.040	0.996	1.0002	1.028	0.029	0.991
6	1.042	1.0003	1.072	0.076	1.031	0.996	1.0002	1.028	0.029	0.992
7	1.094	1.0003	1.075	0.082	1.055	0.991	1.0002	1.028	0.028	0.990
8	1.127	1.0004	1.077	0.086	1.094	0.992	1.0002	1.028	0.028	0.986
9	1.083	1.0003	1.075	0.080	1.089	0.990	1.0002	1.028	0.028	0.985
10	1.054	1.0003	1.073	0.077	1.057	0.987	1.0002	1.028	0.028	0.986
11	1.044	1.0003	1.073	0.075	1.044	0.978	1.0002	1.028	0.028	0.986
12	1.043	1.0003	1.072	0.076	1.040	0.984	1.0002	1.028	0.028	0.986
13	1.035	1.0003	1.072	0.074	1.033	0.984	1.0002	1.028	0.028	0.986
14	1.038	1.0003	1.072	0.074	1.029	0.985	1.0002	1.028	0.028	0.985
15	1.040	1.0003	1.072	0.075	1.029	0.990	1.0002	1.028	0.028	0.986
16	1.076	1.0003	1.074	0.079	1.047	0.987	1.0002	1.028	0.028	0.986
17	1.060	1.0003	1.073	0.078	1.054	0.992	1.0002	1.028	0.028	0.985
18	1.067	1.0003	1.073	0.079	1.044	0.993	1.0002	1.028	0.028	0.982
19	1.094	1.0003	1.074	0.082	1.056	0.998	1.0002	1.028	0.028	0.981
20	1.084	1.0003	1.074	0.081	1.063	0.995	1.0002	1.028	0.028	0.981
21	1.082	1.0003	1.074	0.081	1.059	0.994	1.0002	1.028	0.028	0.980
22	1.117	1.0004	1.076	0.086	1.074	0.996	1.0002	1.028	0.028	0.980
23	1.112	1.0004	1.076	0.086	1.088	0.996	1.0002	1.028	0.028	0.981
24	1.136	1.0004	1.077	0.088	1.097	0.996	1.0002	1.028	0.028	0.980
25	1.179	1.0004	1.080	0.092	1.127	0.998	1.0002	1.028	0.028	0.980
26	1.115	1.0003	1.076	0.086	1.116	0.998	1.0002	1.028	0.028	0.980
27	1.098	1.0003	1.075	0.084	1.080	0.993	1.0002	1.028	0.028	0.979
28	1.052	1.0003	1.072	0.077	1.053	0.988	1.0002	1.028	0.028	0.978
29	1.057	1.0003	1.072	0.078	1.036	0.987	1.0002	1.028	0.028	0.978
30	1.064	1.0003	1.073	0.079	1.042	0.986	1.0002	1.028	0.028	0.977
36	1.043	1.0003	1.072	0.075	1.042	0.970	1.0002	1.027	0.028	0.974
42	1.030	1.0003	1.072	0.071	1.029	0.966	1.0002	1.027	0.028	0.972
48	1.024	1.0003	1.072	0.069	1.029	0.958	1.0002	1.027	0.028	0.970
54	1.030	1.0003	1.072	0.070	1.021	0.963	1.0002	1.027	0.027	0.968
60	1.028	1.0003	1.071	0.070	1.021	0.967	1.0002	1.027	0.027	0.966

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Although the positive values under the local trend specification tended to underestimate the values of the true TVC ESUB between x_1 and x_3, the gap between the approximating values and the true values tended to be small when the true values ranged from 2.0 to 2.9. The approximation of the time-varying elasticity of substitution between x_2 and x_3 had values with opposite signs under the two specifications of the time-varying coefficients in the demand system. It has positive but underestimating values under the random walk specification, but negative values under the local trend specification.

The model produced positive values of the time-varying income elasticities for all the three goods under both specifications of the time-varying coefficients. While all the goods are correctly identifies as normal, the ability to classify them as luxuries or normal necessities differed under the two specifications. Under the random walk specification, the model produced a correct income classification of the three goods as luxury good and necessity goods respectively. However, the approximating values tended to be close to the true ones for the time-varying income elasticity of x_1 while those of the time varying income elasticities of x_2 and x_3 tended to respectively underestimate and overestimate the true ones. On the other hand, the model produced a correct income classification only for x_2 under the local trend specification.

As far as the compensated cross-price elasticities are concerned, the LAISF model produced time-varying values with opposite signs under the random walk specification and under the local trend model specification for x_1 and x_2. These goods were identified as substitutes under the random walk specification. However, they were identified as complements under the local trend specification. Finally, x_1 and x_3 on the one hand and x_2 and x_3 on the other hand, were wrongly classified as complements under both specifications of the time-varying coefficients in the demand system.

Table 4.5. True and approximating values of η_{2t} at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	0.449	0.996	0.279	-0.745	0.380	0.544	0.991	0.211	-0.794	0.571
2	0.485	0.996	0.255	-0.777	0.441	0.487	0.992	0.326	-0.677	0.580
3	0.470	0.996	0.283	-0.749	0.440	0.522	0.993	0.203	-0.803	0.604
4	0.435	0.997	0.356	-0.681	0.409	0.566	0.992	0.161	-0.846	0.643
5	0.497	0.996	0.291	-0.746	0.407	0.586	0.984	0.163	-0.845	0.659
6	0.487	0.996	0.286	-0.748	0.429	0.589	0.984	0.136	-0.872	0.669
7	0.420	0.997	0.392	-0.657	0.396	0.612	0.983	0.115	-0.893	0.688
8	0.435	0.997	0.366	-0.697	0.374	0.630	0.982	0.086	-0.923	0.707
9	0.405	0.997	0.407	-0.635	0.369	0.619	0.983	0.095	-0.914	0.715
10	0.444	0.997	0.333	-0.704	0.378	0.614	0.983	0.083	-0.926	0.716
11	0.424	0.997	0.366	-0.653	0.392	0.668	0.981	0.005	-1.006	0.751
12	0.434	0.996	0.304	-0.725	0.400	0.602	0.982	0.039	-0.971	0.769
13	0.471	0.996	0.276	-0.747	0.428	0.595	0.983	0.089	-0.921	0.736
14	0.440	0.996	0.279	-0.745	0.436	0.533	0.984	0.129	-0.880	0.701
15	0.430	0.996	0.241	-0.790	0.445	0.499	0.984	0.125	-0.884	0.687
16	0.351	0.997	0.405	-0.630	0.402	0.494	0.984	0.166	-0.842	0.672
17	0.433	0.996	0.281	-0.767	0.393	0.529	0.984	0.129	-0.880	0.671
18	0.439	0.996	0.253	-0.804	0.441	0.530	0.983	0.102	-0.906	0.697
19	0.436	0.996	0.251	-0.821	0.451	0.510	0.984	0.126	-0.882	0.697
20	0.426	0.996	0.285	-0.778	0.441	0.551	0.983	0.079	-0.930	0.706
21	0.458	0.996	0.261	-0.806	0.438	0.569	0.983	0.086	-0.923	0.722
22	0.476	0.996	0.239	-0.848	0.452	0.581	0.983	0.075	-0.935	0.724
23	0.491	0.996	0.211	-0.881	0.467	0.559	0.984	0.108	-0.902	0.715
24	0.489	0.996	0.194	-0.909	0.480	0.568	0.983	0.070	-0.940	0.717
25	0.455	0.996	0.208	-0.900	0.481	0.513	0.984	0.112	-0.897	0.715
26	0.479	0.996	0.182	-0.914	0.485	0.540	0.983	0.086	-0.924	0.710
27	0.456	0.996	0.235	-0.845	0.476	0.568	0.983	0.056	-0.954	0.732
28	0.449	0.996	0.248	-0.802	0.457	0.596	0.982	0.013	-0.999	0.761
29	0.442	0.996	0.280	-0.769	0.444	0.618	0.982	0.002	-1.009	0.784
30	0.426	0.996	0.297	-0.752	0.429	0.644	0.981	-0.056	-1.067	0.810
36	0.446	0.997	0.347	-0.675	0.384	0.781	0.980	-0.134	-1.144	0.882
42	0.411	0.996	0.302	-0.689	0.418	0.736	0.978	-0.240	-1.251	0.962
48	0.463	0.996	0.262	-0.715	0.445	0.838	0.976	-0.323	-1.332	1.051
54	0.438	0.996	0.305	-0.685	0.454	0.887	0.975	-0.396	-1.403	1.096
60	0.456	0.996	0.287	-0.705	0.418	0.968	0.973	-0.501	-1.508	1.167

Notes:
RM = Rotterdam model
NLAI = Nonlinear AI model
LAISF = Linear-Approximate AIDS with simple formulas
LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Table 4.6. True and approximating values of η_{3t} at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	0.695	0.997	0.213	-0.818	0.697	1.809	1.009	0.963	-0.032	1.932
2	0.645	0.997	0.241	-0.799	0.923	2.290	1.011	0.953	-0.043	2.270
3	0.633	0.997	0.264	-0.776	0.890	2.491	1.011	1.062	0.064	2.391
4	0.504	0.997	0.390	-0.652	0.798	2.652	1.018	1.073	0.075	2.597
5	0.594	0.997	0.253	-0.793	0.801	2.419	1.027	1.073	0.075	2.643
6	0.635	0.997	0.248	-0.795	0.887	2.187	1.024	1.063	0.065	2.367
7	0.358	0.998	0.555	-0.488	0.671	2.955	1.033	1.093	0.095	2.580
8	0.254	0.999	0.632	-0.414	0.464	3.023	1.039	1.109	0.111	3.300
9	0.412	0.998	0.507	-0.534	0.487	3.115	1.036	1.099	0.101	3.345
10	0.574	0.997	0.361	-0.682	0.670	3.314	1.035	1.095	0.097	3.160
11	0.756	0.997	0.341	-0.683	0.773	3.294	1.028	1.075	0.077	2.761
12	0.698	0.997	0.314	-0.719	0.806	3.175	1.031	1.081	0.083	2.571
13	0.766	0.997	0.234	-0.794	0.865	3.659	1.036	1.095	0.097	2.888
14	0.742	0.997	0.243	-0.786	0.910	4.457	1.043	1.118	0.121	3.481
15	0.706	0.997	0.263	-0.773	0.891	3.431	1.033	1.093	0.096	3.444
16	0.495	0.998	0.484	-0.553	0.736	5.020	1.048	1.137	0.140	3.669
17	0.480	0.998	0.374	-0.677	0.681	4.126	1.048	1.140	0.143	4.554
18	0.402	0.997	0.409	-0.646	0.719	5.160	1.067	1.195	0.199	5.319
19	0.267	0.998	0.524	-0.532	0.617	4.301	1.065	1.193	0.197	6.295
20	0.308	0.998	0.493	-0.560	0.578	4.450	1.065	1.192	0.196	6.217
21	0.317	0.998	0.491	-0.565	0.607	4.687	1.066	1.194	0.198	6.251
22	0.210	0.998	0.616	-0.437	0.513	3.828	1.061	1.177	0.182	6.009
23	0.218	0.999	0.613	-0.442	0.451	4.452	1.070	1.203	0.208	6.135
24	0.184	0.998	0.666	-0.386	0.417	4.370	1.068	1.200	0.205	6.523
25	0.143	0.999	0.737	-0.307	0.348	5.483	1.089	1.261	0.266	7.307
26	0.205	0.998	0.617	-0.439	0.370	4.182	1.068	1.202	0.207	7.301
27	0.259	0.998	0.562	-0.493	0.477	6.466	1.096	1.281	0.287	7.550
28	0.508	0.997	0.304	-0.751	0.637	5.407	1.066	1.195	0.200	7.428
29	0.485	0.997	0.346	-0.707	0.803	5.639	1.068	1.198	0.203	6.359
30	0.445	0.997	0.392	-0.658	0.749	5.511	1.067	1.196	0.201	6.412
36	0.716	0.997	0.289	-0.739	0.810	10.394	1.093	1.272	0.277	7.918
42	0.917	0.997	0.222	-0.765	0.941	7.461	1.057	1.168	0.172	6.849
48	0.965	0.997	0.247	-0.725	0.891	12.152	1.085	1.252	0.258	7.630
54	0.898	0.997	0.185	-0.801	1.028	12.881	1.105	1.310	0.317	9.300
60	0.878	0.996	0.062	-0.924	1.096	15.777	1.151	1.447	0.457	11.498

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Table 4.7. True and approximating values of $\eta^*_{12,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	0.015	0.066	0.016	-0.025	0.013	0.008	0.013	0.024	-0.008	0.044
2	0.015	0.065	0.015	-0.025	0.012	0.006	0.019	0.030	-0.008	0.048
3	0.015	0.067	0.016	-0.025	0.012	0.006	0.016	-0.004	-0.040	0.006
4	0.017	0.072	0.021	-0.026	0.012	0.006	0.015	-0.005	-0.040	0.005
5	0.016	0.067	0.017	-0.025	0.012	0.007	0.037	-0.005	-0.039	0.004
6	0.014	0.067	0.017	-0.025	0.012	0.007	0.036	-0.006	-0.040	0.004
7	0.015	0.075	0.023	-0.026	0.013	0.005	0.035	-0.006	-0.039	0.004
8	0.017	0.074	0.020	-0.027	0.013	0.005	0.034	-0.006	-0.038	0.005
9	0.016	0.076	0.024	-0.026	0.013	0.005	0.034	-0.006	-0.038	0.005
10	0.011	0.070	0.019	-0.026	0.013	0.003	0.033	-0.006	-0.038	0.004
11	0.012	0.072	0.022	-0.026	0.012	0.003	0.031	-0.008	-0.038	0.004
12	0.011	0.068	0.018	-0.026	0.012	0.003	0.032	-0.007	-0.038	0.004
13	0.016	0.066	0.016	-0.025	0.012	0.004	0.033	-0.006	-0.038	0.004
14	0.012	0.066	0.016	-0.025	0.012	0.002	0.034	-0.004	-0.037	0.004
15	0.010	0.064	0.014	-0.025	0.012	0.003	0.034	-0.004	-0.037	0.004
16	0.012	0.075	0.024	-0.026	0.012	0.002	0.036	-0.003	-0.037	0.004
17	0.015	0.067	0.016	-0.026	0.013	0.003	0.034	-0.004	-0.037	0.004
18	0.014	0.065	0.015	-0.026	0.012	0.002	0.033	-0.005	-0.037	0.004
19	0.014	0.066	0.014	-0.026	0.013	0.003	0.034	-0.004	-0.037	0.004
20	0.015	0.067	0.016	-0.026	0.013	0.003	0.032	-0.005	-0.037	0.003
21	0.014	0.066	0.015	-0.026	0.013	0.003	0.033	-0.005	-0.037	0.003
22	0.013	0.066	0.013	-0.027	0.013	0.003	0.032	-0.005	-0.037	0.003
23	0.013	0.065	0.012	-0.026	0.013	0.003	0.033	-0.004	-0.037	0.003
24	0.011	0.065	0.010	-0.027	0.013	0.002	0.032	-0.006	-0.037	0.003
25	0.014	0.066	0.010	-0.028	0.013	0.002	0.033	-0.004	-0.037	0.003
26	0.017	0.063	0.010	-0.027	0.013	0.004	0.032	-0.005	-0.037	0.003
27	0.017	0.065	0.013	-0.026	0.013	0.002	0.031	-0.006	-0.036	0.003
28	0.014	0.065	0.015	-0.025	0.012	0.002	0.030	-0.007	-0.036	0.003
29	0.014	0.067	0.016	-0.026	0.012	0.002	0.030	-0.007	-0.036	0.003
30	0.014	0.068	0.017	-0.026	0.012	0.002	0.028	-0.009	-0.036	0.003
36	0.018	0.071	0.020	-0.026	0.012	0.001	0.026	-0.010	-0.036	0.003
42	0.014	0.068	0.018	-0.026	0.012	0.002	0.024	-0.013	-0.036	0.003
48	0.013	0.065	0.015	-0.026	0.012	0.001	0.023	-0.014	-0.036	0.003
54	0.015	0.068	0.018	-0.025	0.012	0.001	0.022	-0.015	-0.036	0.003
60	0.016	0.067	0.017	-0.025	0.012	0.001	0.020	-0.016	-0.035	0.003

Notes:
RM = Rotterdam model
NLAI = Nonlinear AI model
LAISF = Linear-Approximate AIDS with simple formulas
LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

4.4.4 LAICF model approximation

The LAICF model produced approximations the values of which had opposite signs under the two specifications of the parameters in the demand system, for the time-varying elasticities of substitution between x_1 and x_2 on the one hand, and between x_2 and x_3 on the other hand. Only the elasticity between x_1 and x_3 was approximated with positive values at each time period. However, these values tend to underestimate the true ones.

Regarding the time-varying income elasticities, the model correctly identified x_1 as a normal goods under both specifications of the time-varying coefficients in the demand system. It also correctly identified x_3 as a normal good but under the local trend model specification only. The good x_2 was wrongly identified as an inferior good under both specification of the time-varying coefficients. In fact the approximations produced by the LAICF model for the time-varying income elasticity for x_2 had negative values at each single time period.

Based on the compensated time-varying cross-price elasticities, the LAICF correctly identified only x_1 and x_3 as substitutes under the local trend specification by approximating the relevant time-varying cross-price elasticity with positive values. In all the remaining cases, the goods were wrongly identified as complements. The approximating values of the corresponding time-varying cross-price elasticities were in fact negative at each single time period (Tables 4.7, 4.8 and 4.9).

It is important to keep in mind that all the true time-varying elasticities have positive values at every single period. Therefore, an approximation with negative values lead to wrong qualitative conclusion. This is a very important fact to note because the LAICF is the most used demand specification in empirical analysis.

Table 4.8. True and approximating values of $\eta^*_{13,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	0.101	0.005	0.027	-0.026	0.039	0.056	0.020	0.025	0.005	0.005
2	0.100	0.007	0.029	-0.026	0.038	0.043	0.015	0.020	0.005	0.004
3	0.100	0.009	0.031	-0.026	0.039	0.039	0.014	0.020	0.006	0.006
4	0.096	0.019	0.042	-0.027	0.039	0.036	0.013	0.018	0.006	0.006
5	0.094	0.008	0.030	-0.026	0.039	0.038	0.016	0.019	0.006	0.006
6	0.099	0.008	0.030	-0.026	0.039	0.045	0.018	0.021	0.006	0.006
7	0.097	0.041	0.066	-0.029	0.039	0.031	0.013	0.017	0.006	0.006
8	0.088	0.063	0.091	-0.031	0.041	0.027	0.012	0.015	0.006	0.006
9	0.100	0.032	0.057	-0.028	0.041	0.031	0.013	0.016	0.006	0.006
10	0.110	0.016	0.039	-0.027	0.040	0.031	0.013	0.017	0.006	0.006
11	0.136	0.014	0.036	-0.026	0.039	0.040	0.016	0.019	0.006	0.006
12	0.118	0.012	0.034	-0.026	0.039	0.036	0.015	0.018	0.006	0.006
13	0.118	0.007	0.028	-0.025	0.039	0.031	0.013	0.017	0.006	0.006
14	0.112	0.007	0.029	-0.026	0.038	0.024	0.011	0.014	0.006	0.006
15	0.110	0.009	0.031	-0.026	0.038	0.031	0.013	0.017	0.006	0.006
16	0.111	0.028	0.053	-0.028	0.039	0.020	0.010	0.013	0.006	0.006
17	0.087	0.018	0.041	-0.027	0.039	0.020	0.010	0.013	0.006	0.006
18	0.082	0.022	0.045	-0.028	0.039	0.013	0.008	0.011	0.006	0.006
19	0.070	0.038	0.063	-0.029	0.039	0.014	0.008	0.011	0.006	0.006
20	0.071	0.031	0.056	-0.029	0.040	0.015	0.008	0.011	0.006	0.006
21	0.078	0.032	0.056	-0.029	0.040	0.014	0.008	0.011	0.006	0.006
22	0.068	0.057	0.084	-0.031	0.040	0.016	0.008	0.012	0.006	0.006
23	0.069	0.056	0.083	-0.031	0.041	0.014	0.008	0.011	0.006	0.006
24	0.068	0.072	0.101	-0.032	0.041	0.013	0.008	0.011	0.006	0.006
25	0.064	0.099	0.131	-0.034	0.042	0.008	0.007	0.010	0.006	0.006
26	0.063	0.057	0.084	-0.031	0.042	0.014	0.008	0.011	0.006	0.006
27	0.071	0.044	0.070	-0.030	0.040	0.009	0.006	0.009	0.006	0.006
28	0.086	0.012	0.034	-0.027	0.039	0.015	0.008	0.011	0.006	0.006
29	0.083	0.015	0.038	-0.027	0.039	0.014	0.008	0.011	0.005	0.006
30	0.083	0.019	0.043	-0.028	0.039	0.014	0.008	0.011	0.005	0.006
36	0.120	0.010	0.032	-0.026	0.039	0.009	0.006	0.010	0.006	0.006
42	0.139	0.006	0.027	-0.024	0.038	0.018	0.009	0.012	0.005	0.006
48	0.158	0.007	0.029	-0.023	0.038	0.012	0.007	0.010	0.005	0.006
54	0.125	0.004	0.025	-0.024	0.038	0.009	0.006	0.009	0.005	0.006
60	0.106	-0.002	0.019	-0.023	0.038	0.005	0.005	0.008	0.005	0.005

Notes:

RM = Rotterdam model

NLAI = Nonlinear AI model

LAISF = Linear-Approximate AIDS with simple formulas

LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

Table 4.9. True and approximating values of $\eta^*_{23,t}$ at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
1	0.043	0.201	-0.062	-0.062	2.0E-05	0.031	0.058	-0.020	0.009	0.037
2	0.047	0.207	-0.066	-0.060	1.1E-05	0.021	0.043	-0.014	0.005	0.028
3	0.045	0.204	-0.061	-0.057	1.6E-05	0.020	0.050	-0.076	-0.053	0.017
4	0.039	0.198	-0.046	-0.046	1.5E-05	0.020	0.048	-0.084	-0.059	0.018
5	0.045	0.201	-0.060	-0.053	1.2E-05	0.022	0.003	-0.066	-0.060	0.018
6	0.046	0.203	-0.061	-0.056	1.9E-05	0.027	0.005	-0.069	-0.061	0.018
7	0.037	0.214	-0.031	-0.034	2.7E-05	0.019	0.000	-0.071	-0.064	0.019
8	0.034	0.243	-0.028	-0.029	1.3E-05	0.017	-0.003	-0.074	-0.067	0.019
9	0.037	0.202	-0.031	-0.037	2.3E-06	0.019	0.001	-0.073	-0.065	0.019
10	0.046	0.201	-0.051	-0.049	1.3E-05	0.020	0.001	-0.074	-0.066	0.019
11	0.055	0.191	-0.045	-0.056	7.0E-06	0.027	0.003	-0.081	-0.070	0.019
12	0.049	0.202	-0.057	-0.058	1.1E-05	0.022	0.002	-0.078	-0.068	0.020
13	0.054	0.202	-0.063	-0.065	2.6E-05	0.019	0.001	-0.073	-0.065	0.019
14	0.048	0.203	-0.062	-0.064	1.6E-05	0.013	0.001	-0.070	-0.062	0.018
15	0.046	0.212	-0.068	-0.063	1.5E-05	0.016	0.003	-0.070	-0.062	0.018
16	0.036	0.198	-0.033	-0.042	-5.9E-06	0.010	0.000	-0.066	-0.060	0.017
17	0.036	0.214	-0.059	-0.047	-1.3E-05	0.010	0.000	-0.070	-0.063	0.016
18	0.034	0.224	-0.062	-0.044	1.2E-05	0.007	-0.002	-0.072	-0.066	0.017
19	0.028	0.241	-0.058	-0.035	1.9E-05	0.007	-0.001	-0.070	-0.063	0.017
20	0.028	0.227	-0.054	-0.038	1.5E-05	0.008	-0.001	-0.074	-0.066	0.017
21	0.033	0.233	-0.058	-0.037	1.6E-05	0.008	-0.001	-0.074	-0.066	0.017
22	0.029	0.263	-0.056	-0.027	8.2E-06	0.010	-0.001	-0.075	-0.067	0.017
23	0.030	0.268	-0.062	-0.027	1.3E-05	0.008	-0.001	-0.072	-0.065	0.017
24	0.029	0.289	-0.063	-0.023	2.2E-05	0.008	-0.001	-0.075	-0.068	0.017
25	0.025	0.314	-0.054	-0.018	9.1E-06	0.004	-0.002	-0.072	-0.067	0.017
26	0.027	0.275	-0.068	-0.027	9.9E-06	0.008	-0.001	-0.074	-0.066	0.017
27	0.030	0.251	-0.060	-0.033	2.1E-05	0.005	-0.002	-0.076	-0.070	0.017
28	0.037	0.214	-0.065	-0.050	2.1E-05	0.009	-0.001	-0.080	-0.071	0.018
29	0.035	0.211	-0.059	-0.047	1.5E-05	0.008	-0.001	-0.081	-0.071	0.019
30	0.033	0.212	-0.056	-0.045	1.9E-05	0.009	-0.001	-0.086	-0.075	0.019
36	0.051	0.191	-0.051	-0.057	1.4E-05	0.007	-0.001	-0.092	-0.082	0.020
42	0.055	0.196	-0.059	-0.082	1.1E-05	0.013	0.000	-0.102	-0.084	0.021
48	0.071	0.206	-0.065	-0.095	1.1E-05	0.010	-0.002	-0.108	-0.091	0.023
54	0.053	0.193	-0.059	-0.080	1.6E-05	0.008	-0.003	-0.114	-0.097	0.023
60	0.047	0.191	-0.064	-0.082	2.5E-05	0.005	-0.005	-0.122	-0.105	0.025

Notes:
RM = Rotterdam model
NLAI = Nonlinear AI model
LAISF = Linear-Approximate AIDS with simple formulas
LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)

4.5 Regularity condition assessment

The regularity condition is defined as the non-violation of the negative semi-definiteness of the Slutsky matrix. Rather than being imposed as this the case for the other restrictions on the coefficients of the demand system (Engel aggregation, linear homogeneity and symmetry), negative semi-definiteness of the Slutsky matrix is usually just checked after estimation. In the case of a three-goods demand system where the third equation is dropped due to the imposition of linear homogeneity, the regularity condition is defined below for both the AI model and the Rotterdam model.

In the AIDS model, the Slutsky matrix is negative semi-definite at each time period t if

$$\eta^*_{11t} < 0 \text{ and } \begin{vmatrix} \eta^*_{11t} & \eta^*_{12t} \\ \eta^*_{21t} & \eta^*_{22t} \end{vmatrix} = \eta^*_{11t}\eta^*_{22t} - \eta^*_{21t}\eta^*_{12t} > 0. \tag{4.5.1}$$

However, for the Rotterdam model one must have

$$\pi_{11t} < 0 \text{ and } \begin{vmatrix} \pi_{11t} & \pi_{12t} \\ \pi_{21t} & \pi_{22t} \end{vmatrix} = \pi_{11t}\pi_{22t} - \pi_{21t}\pi_{12t} > 0. \tag{4.5.2}$$

I shall report the percentage of replications producing the non-violation of the negative semi-definiteness as an index of regularity for each time period $t = 1, ..., T$. Table 4.10 reports the regularity index for the nonlinear AIDS, the linear-approximate AIDS with simple formulas, the linear-approximate AIDS with corrected formulas and the Rotterdam model. For each model, I calculated the regularity score under both specifications of the time-varying coefficients in the demand system.

Table 4.10. Regularity index by model and TVC specification

	NLAI		LAISF		LAICF		RM	
Period	RWM	LTM	RWM	LTM	RWM	LTM	RWM	LTM
1	84.3	72.5	47.8	53.3	66.9	51.7	100.0	98.0
2	86.1	71.9	50.0	62.4	64.8	60.3	100.0	98.1
3	87.6	71.9	49.1	40.8	66.6	34.8	100.0	96.1
4	95.3	71.3	56.1	39.5	71.7	32.8	100.0	95.9
5	87.7	96.2	48.8	40.6	67.0	32.7	100.0	95.4
6	85.0	95.9	48.7	35.5	66.7	28.6	100.0	95.3
7	96.5	95.9	73.2	37.2	75.2	28.4	100.0	94.2
8	96.1	95.4	71.3	38.2	73.1	26.8	100.0	95.3
9	96.5	95.4	67.4	38.9	75.7	27.9	100.0	95.3
10	93.4	95.5	54.7	37.8	70.8	27.3	100.0	95.3
11	94.3	94.3	52.1	29.4	72.6	21.8	100.0	95.4
12	91.7	94.8	52.3	33.3	68.0	24.1	100.0	95.5
13	87.0	96.0	48.8	38.2	65.8	27.9	100.0	95.3
14	86.5	96.4	49.1	44.1	65.9	31.9	100.0	95.3
15	88.2	96.2	52.1	42.1	64.7	31.7	100.0	95.2
16	96.3	96.4	67.7	50.2	75.4	36.3	100.0	95.2
17	93.4	96.1	57.8	46.8	67.0	32.9	100.0	94.5
18	91.8	95.8	60.7	47.8	64.6	30.7	100.0	94.3
19	94.4	96.2	67.2	50.7	65.0	32.8	100.0	93.6
20	94.7	95.4	65.6	46.5	66.5	28.9	100.0	93.7
21	93.7	95.7	63.0	46.8	65.2	29.5	100.0	93.3
22	94.8	95.3	65.7	44.5	64.5	29.0	100.0	93.1
23	93.8	96.1	64.7	49.3	62.5	31.1	100.0	92.8
24	94.2	95.6	61.4	45.9	61.1	28.2	100.0	93.2
25	94.3	96.2	61.9	52.5	62.2	31.3	100.0	93.1
26	93.9	95.8	63.2	48.1	61.2	29.8	100.0	91.9
27	94.3	95.3	65.0	49.5	63.6	28.1	100.0	91.1
28	90.0	94.2	53.4	42.2	64.8	25.0	100.0	91.0
29	91.7	93.9	54.9	42.2	66.8	24.3	100.0	91.1
30	92.6	93.3	55.5	37.6	67.6	21.4	100.0	90.9
36	90.9	91.5	48.6	35.8	71.2	18.3	100.0	91.2
42	86.6	90.4	48.1	25.1	68.3	14.1	100.0	91.9
48	88.5	88.2	53.2	27.0	65.3	12.3	100.0	91.8
54	83.0	87.2	46.3	26.6	68.1	11.0	100.0	92.0
60	64.7	85.8	42.8	28.3	67.2	9.8	100.0	92.1

Notes:
RM = Rotterdam model
NLAI = Nonlinear AI model
LAISF = Linear-Approximate AIDS with simple formulas
LAICF = Linear-approximate AI model with corrected formulas from Green and Alston (1990, 1991)
RWM = Random walk model
LTM = Local trend model

4.5.1 The Rotterdam model

The Rotterdam model satisfied the regularity condition under the random walk specification for every single replication and at every single time period. The regularity index is thus equal to 100, meaning that the Slutsky matrix was negative semi-definite for each replication of the bootstrap procedure and at every time period. Under the local trend model specification, the regularity index ranged from 91 to 98 by time period, showing that a minimum of 91% of the replications per time period satisfied the negative semi-definiteness condition of the Slutsky matrix.

4.5.2 The NLAI model

The NLAI achieved a minimum of the regularity index per time period of 64.7% and 71.3% respectively under the random walk specification and under the local trend model specification. The maximum regularity index was not very different under the two time-varying coefficients specification hypotheses (96.5% versus 96.4%). However, the regularity index was higher for most of the time periods under the local trend specification than it was under the random walk specification.

4.5.3 The LAISF model

The LAISF model achieved higher minimum and maximum values of the regularity index per time period under the random walk mode than under the local trend specification. A minimum of 20% of replications satisfied the regularity condition per time period under the local trend model, while the minimum was 41% of the replications under the random walk specification. The maximum regularity index per time period was also higher under the random walk specification(73.2%) than under the local trend model specification (62.4%).

4.5.4 The LAICF model

The LAICF model achieved a minimum regularity index as low as 9.8 under the local trend model, compared to 60.6 under the random walk specification of the time-varying parameters in the demand system. The maximum number of replications per time period that satisfied the regularity condition was also higher under the random walk model (76.0) than under the local trend model specification (60.3).

4.6 Discussion of the Results

This section shall focus on the discussion of the results from the previous section and assess the performance of each of the four models in recovering the characteristics of the true time-varying parameters elasticities. The following criteria are used to compare the performance of each model:

- consistency of the sign of the approximating values of the time-varying elasticities under different specifications of the time-varying parameters in the demand system, in agreement with the sign of the values of the true time-varying elasticities;

- closeness of the approximating values to the true values of the time-varying elasticities. This will be assessed by construction a 95% confidence intervals and checking whether or not the values of the true time-varying elasticity are inside the confidence interval;

- the ability of the approximating demand system to produce time-varying elasticities that reflect the variations in the true time-varying elasticities over time and that mimic the behavior of the paths of true time-varying elasticities;

- higher regularity index, which demonstrates the ability of the approximating demand models to conform to demand theory.

4.6.1 The Rotterdam model versus the Nonlinear AIDS

The approximation of the time-varying elasticity of substitution between x_1 and x_2 from the Rotterdam model had values that stayed close to the values of the true elasticity of substitution (Figure 4.1). In contrast, the values produced by the nonlinear AI model overestimated the true ones. Although the true values of the within-group time-varying elasticity of substitution were inside the 95% confidence intervals contructed for the approximations from the two models, the Rotterdam model tended to produce the approximation with more precision (see Figures A.1 and A.2). The nonlinear AI model tended to produce values that had more variability under the random walk specification than under the local trend model specification. On the other hand, the Rotterdam model tended to produce an approximation of the within-group time-varying elasticity that mimic the evolution of the true time-varying elasticity of substitution over time, but with a smoother path. However, none of the two models produced good approximations of the between-groups time-varying elasticities of substitution. Nevertheless, the approximations produced by both models correctly identified the signs of all the values of the time-varying elasticities of substitution.

Figures 4.2 and A.3 show that the time-varying income elasticities produced by the Rotterdam model track fairly well the evolution of the true time-varying income elasticities and that they are close to the true ones. In contrast, the nonlinear AIDS tended to produce time-varying income elasticities the values of which were close to unity. In addition, the model failed to capture very high values of income elasticities.

Figure 4.1. Path of the elasticities of substitution

Random Walk Model | *Local Trend Model*

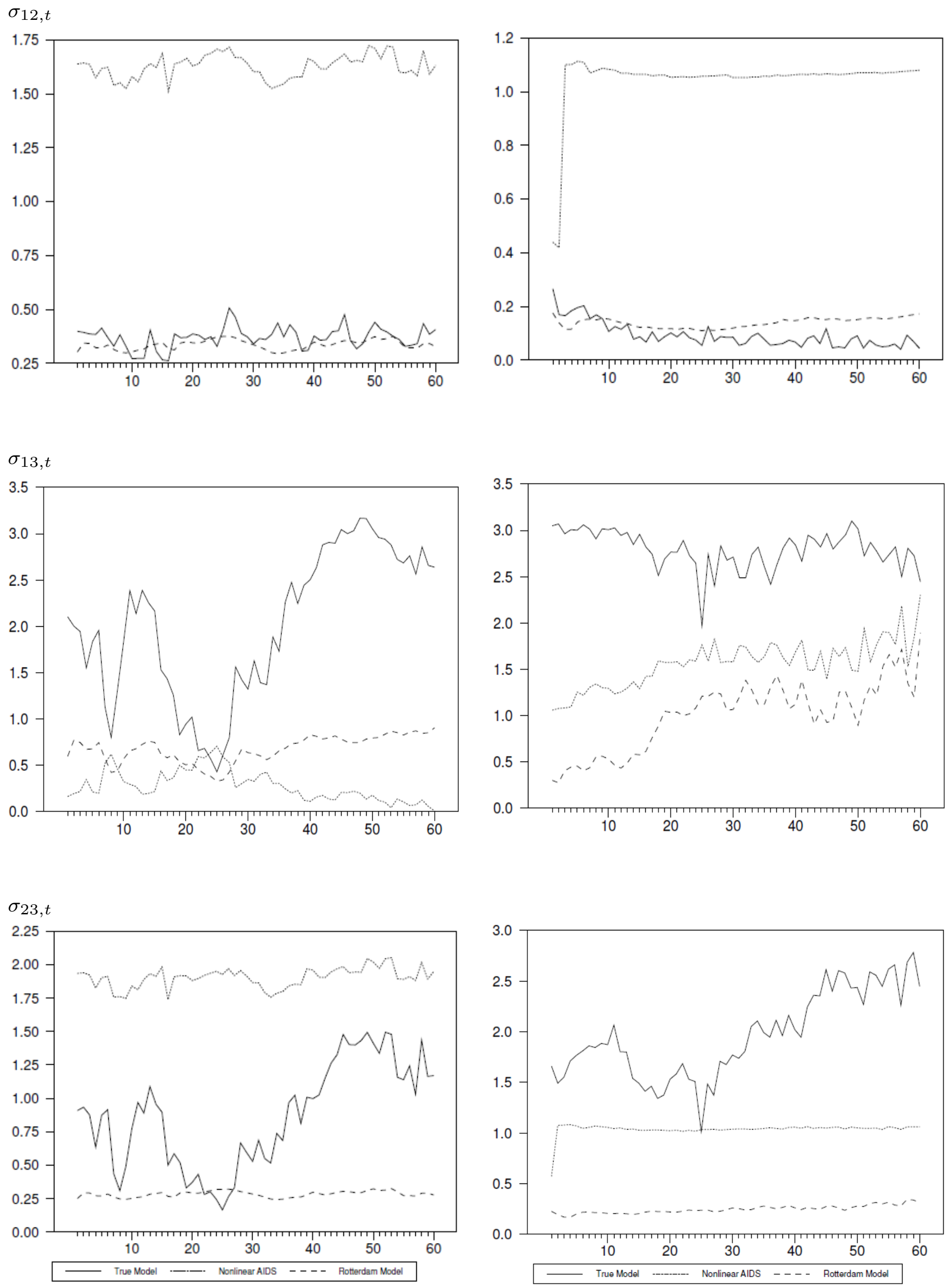
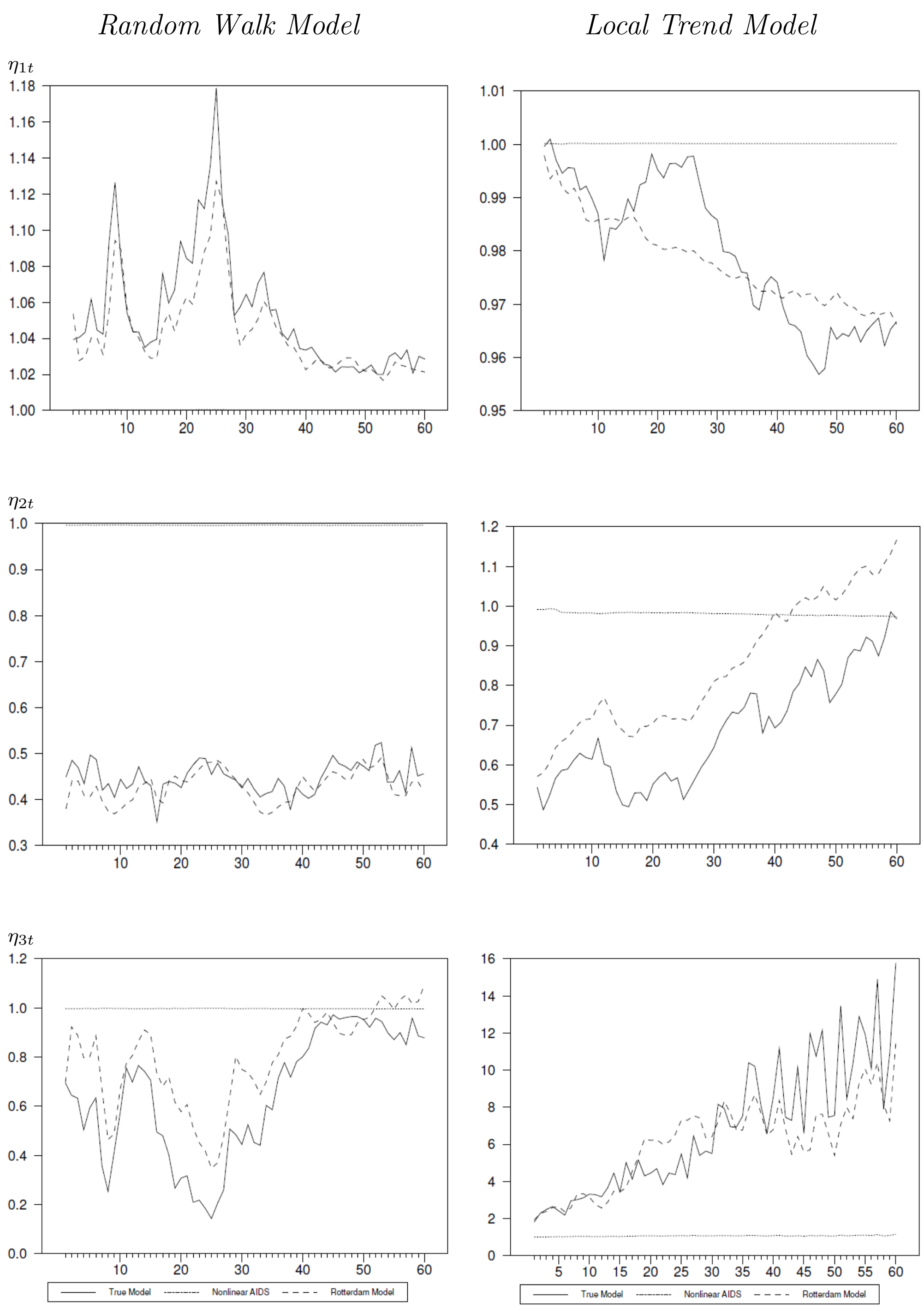


Figure 4.2. Path of the income elasticities



Finally, Figure 4.3 reveals that the values of the time-varying compensated cross-price elasticity between goods in the same sub-aggregation group were close to the true ones for the Rotterdam model than for the nonlinear AIDS. In addition, the nonlinear AIDS produced more negative values for the cross-price time-varying elasticities between good in different sub-aggregation groups. This behavior suggests that the model wrongly identified goods as complements where the values of the true time-varying elasticities indicated that the two goods were substitutes.

4.6.2 The LAISF and the LAICF versus the Nonlinear AIDS

It is not easy, in light the results from the previous chapter, to straightforwardly determine whether the LAISF model and the LAICF model provide better approximation of the time-varying elasticities produced by the nonlinear AIDS. In some cases, the two linear versions of the nonlinear AIDS produced elasticities that were close to the true time-varying elasticities (i.e. the time-varying elasticity of substitution between x_1 and x_3 under the local trend model specification). In some other cases, one or the other of the linear approximate AIDS, or both, produced elasticities with different sign from those produced by the nonlinear AI model.

4.6.3 The LAISF versus the LAICF

The LAISF model and the LAICF model tended to produce a same stratification of goods based on the time-varying elasticities under the local trend model specification and a divergent stratification under the random walk specification. The values of the time-varying elasticities in either case differed considerably in most of the cases. However, the two model estimated very close values of the time-varying elasticity of substitution between x_2 and x_3 under the random walk specification of the time-varying coefficients in the demand system.

Figure 4.3. Path of the compensated cross-price elasticities

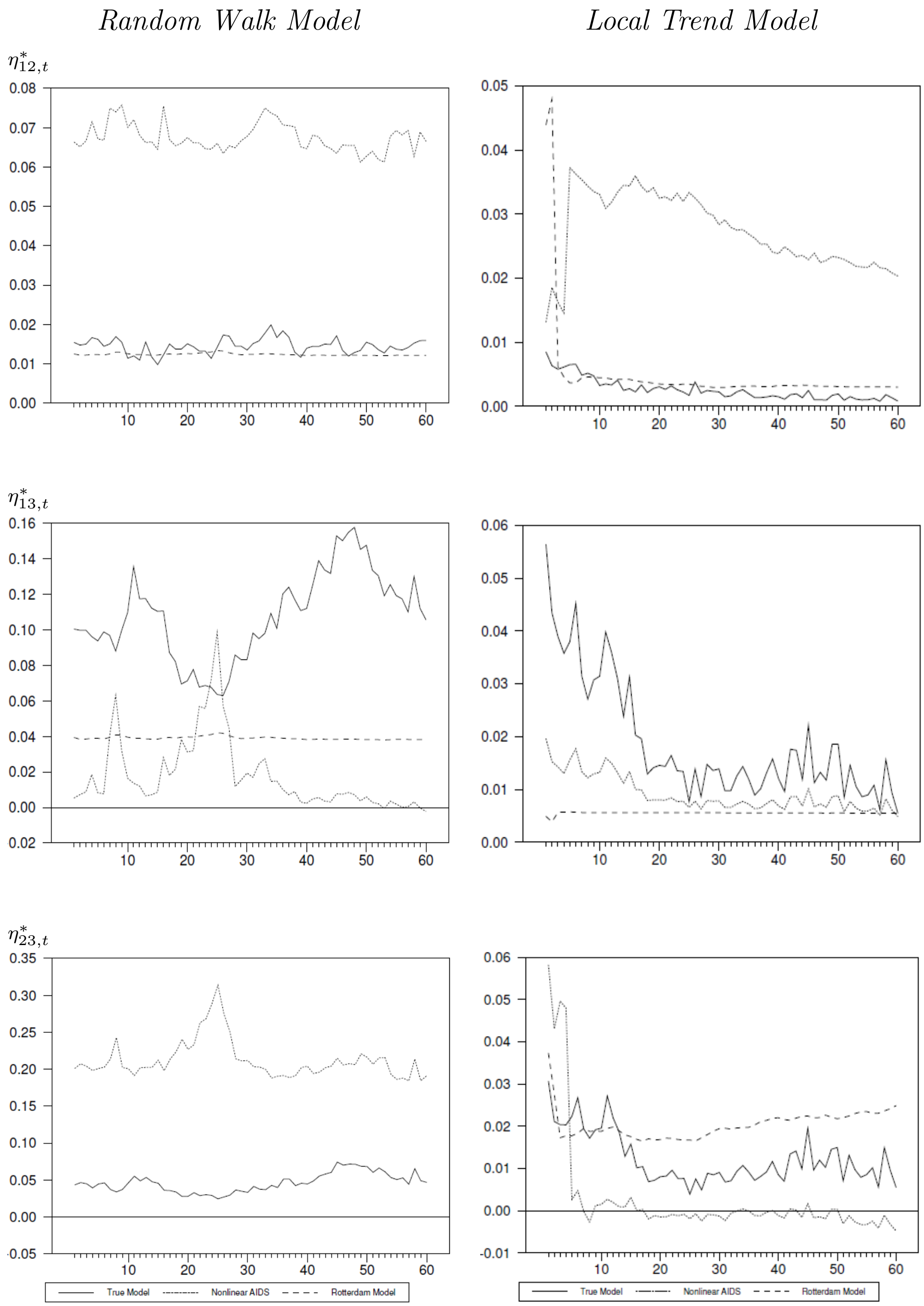
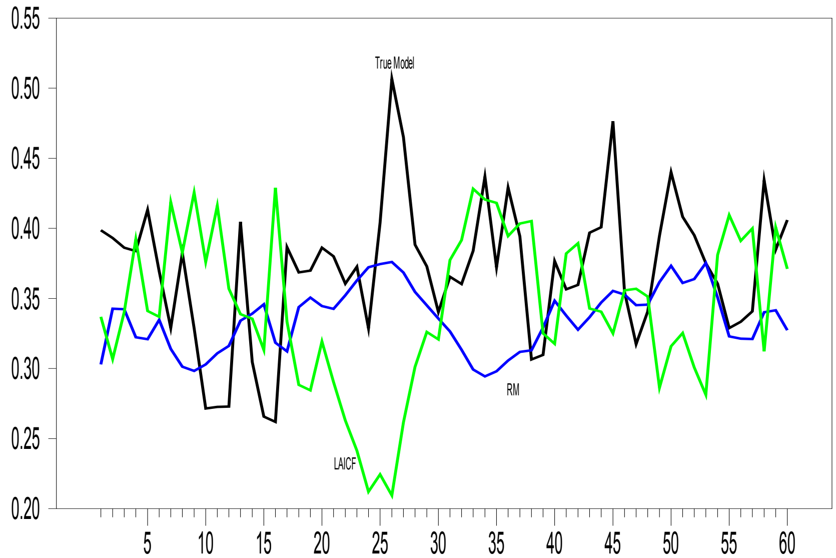


Figure 4.4. Path of $\sigma_{12,t}$: Rotterdam Model and LAICF Model



4.6.4 The LAISF and the LAICF vs. the Rotterdam model

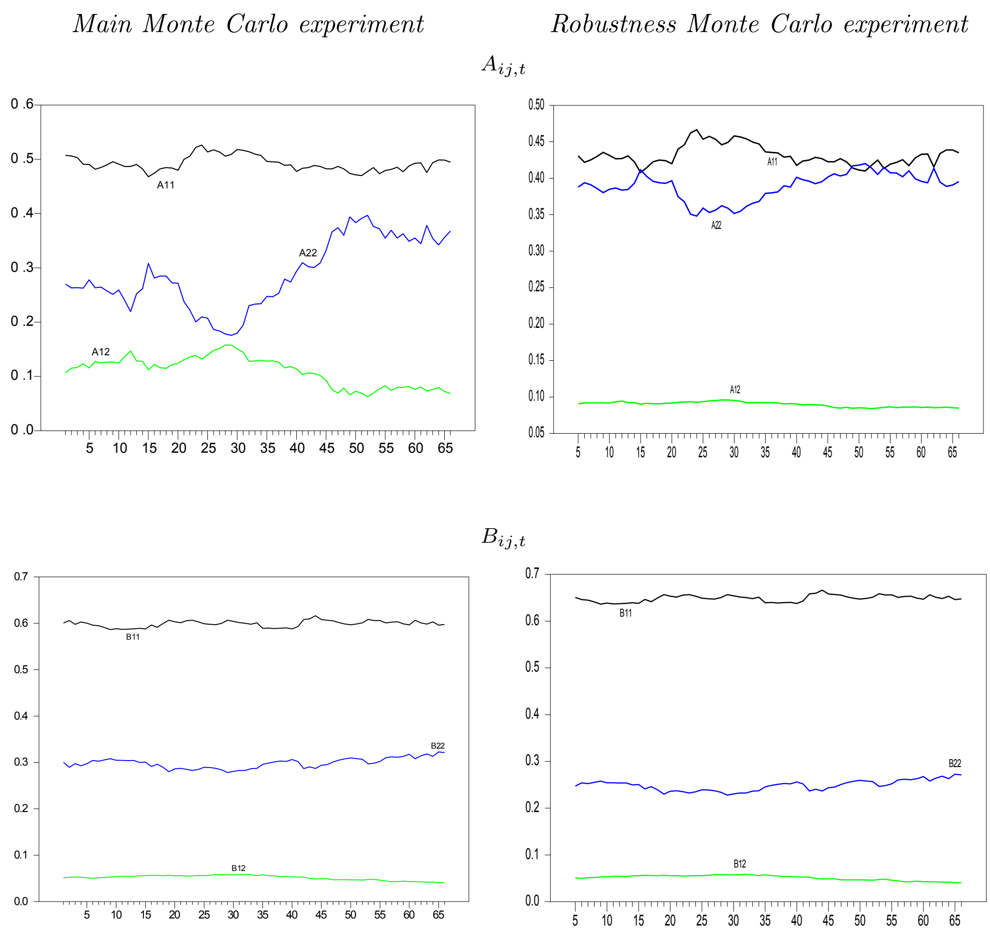
The LAISF model tended to produce time-varying income elasticities with same signs as those produced by the Rotterdam model. This is not the case for the LAICF model that produced negative values for most of the time-varying income elasticities. This observation also holds for the cross-price time-varying elasticities under the random walk specification. When the true elasticity of substitution were very high, the LAISF and the LAICF produce approximations that were closer to the true values than were those produced by the Rotterdam model. This was the case for the time-varying elasticity of substitution between x_1 and x_3 under the local trend model specification (Table 4.3). Under the random walk specification, the LAICF produced values of this time-varying elasticity that are comparable to both the true values and the approximation from the Rotterdam model as shown in Figure 4.4.

4.7 Robustness Checks

The previous sections presented and discussed the results from my main Monte Carlo experiment. I shall recall three important results from this experiment. First, the Rotterdam model performed better than the linear-approximate AIDS at recovering the time-varying elasticities. In particular, the RM was able to display similar paths of the income elasticities than did any of the alternative demand specifications. Second, the RM performed better in terms of the theoretical regularity that was defined as the negative semi-definiteness of the Slutsky matrix at each time period. Third, the LAICF poorly approximated the nonlinear AIDS model. These results are very important since the LAICF model is the most adopted demand specification in empirical studies.

In this section, I shall conduct an alternative Monte Carlo experiment to check the robustness of the previous results. This experiment uses the same autoregressive models for the supernumerary income and quantities. The key difference with the main experiment is that the time-varying parameters $A_{11,t}$, $A_{12,t}$, $A_{22,t}$, $B_{11,t}$, $B_{12,t}$, and $B_{22,t}$ are now generated by random walk processes that are different from those in the main experiment. Figure 4.5 display the time paths of the time-varying parameters for both experiments. Once again, the parameters δ and ρ were set to constant values, for convenience. Moreover, their values are the same as in the main Monte Carlo experiment. The committed quantities α_1, α_2 and α_3 were respectively set to 1, 25 and 15 to obtain the first data set on the elementary quantities x_1 x_1 and x_1. To obtain the second data set on elementary quantities, the committed quantities were specified as local trends, following exactly the same stochastic processes as in the main experiment.

Figure 4.5. Paths of the TVP in the utility functions



The specification of the time-varying parameters in the robustness-check Monte Carlo experiment is such that the optimal prices are different from those in the main experiment. I shall recall that the WS-branch utility function produces indirect demands functions, resulting from a two-stage budgeting procedure. The elementary quantities are then obtained from their relationships with the supernumerary quantities that enter as arguments in the utility function. In what follows, I shall assess the qualitative and quantitative performance of each model under this new Monte Carlo experiment.

4.7.1 Qualitative performance

The RM and the nonlinear AIDS qualitatively performed well at correctively recovering the positive sign of all the time-varying elasticities. This is true under both specifications of the time-varying coefficients in the demand system. However, the LAISF performed better on all the time-varying elasticities only under the RWM, while correctly recovering the positive sign of only four out of nine time-varying elasticities at every time period under the LTM. Table 4.11 shows that, under the LTM, the LAISF model approximated two elasticities with the wrong sign at each time period, and three elasticities with negative as well as positive values.

The LAICF model produced only two time-varying elasticities the values of which have the correct sign under the two specifications of the time-varying coefficients in the demand system. For the remaining time-varying elasticities, the signs of the approximating values are either opposite under the two specifications at each time period (4 elasticities) or with the wrong negative sign at every time period under both the RWM and the LTM (1 elasticity). Finally, Table 4.11 shows that the model produced two approximating elasticities with some values having the correct (positive) sign while some other values had the wrong (negative) sign.

It is important to point out again that the LAICF model is the most used demand specification in empirical analyses, compared to the Rotterdam model and the nonlinear AIDS model. The important message conveyed by Table 4.11 is that the Rotterdam model qualitatively performed better than the LAICF. This is consistent with the conclusion from the main Monte Carlo experiment. In addition, the LAICF model poorly approximated the qualitative properties of the nonlinear AIDS model as this was the case in the main experiment.

Table 4.11. Robustness checks: Signs of the time-varying elasticities

TVP Elasticity	NLAI		LAISF		LAICF		RM	
	RWM	LTM	RWM	LTM	RWM	LTM	RWM	LTM
σ_{12t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	+ and −	$+\ \forall t$	$+\ \forall t$
σ_{13t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$
σ_{23t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	+ and −	$+\ \forall t$	+ and −	$+\ \forall t$	$+\ \forall t$
η_{1t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$
η_{2t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	+ and −	$-\ \forall t$	$-\ \forall t$	$+\ \forall t$	$+\ \forall t$
η_{3t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$
η^*_{12t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	+ and −	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	$+\ \forall t$
η^*_{13t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$
η^*_{23t}	$+\ \forall t$	$+\ \forall t$	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	$-\ \forall t$	$+\ \forall t$	$+\ \forall t$

4.7.2 Quantitative performance

4.7.2.1 Time-varying elasticities of substitution

Under the random walk model specification, the RM, the NLAI model and the LAISF model produced approximations of the within-branch elasticity of substitution with values less than unity as the true ones. However, the values of the approximation produced by the Rotterdam model are closer to the true ones than are those from the NLAI and the LAISF. On the other hand, the LAICF produced an approximation with values greater than one at every time period. This is important to mention because the values of the time-varying elasticity of substitution higher than unity mean that, all else, the LAICF model will lead to a different conclusion in terms of the income shares of goods, compared to the other true models.

Table 4.12 shows that, under the random walk model, the Rotterdam model approximated the cross-branch elasticities of substitution with values less than unity when the true values were less than unity ($\sigma_{23,t}$), and with values greater than one when the true values were greater than one ($\sigma_{13,t}$). However, the approximating values underestimated the true ones in both cases. All the three versions of the AIDS model produced approximating time-varying cross-branch elasticities of substitution with values less than one when the true values were greater than one ($\sigma_{13,t}$), and with values greater than one when the true values were less than one ($\sigma_{23,t}$). This leads to a wrong conclusion in terms of the goods' income shares.

Under the local trend model, the RM and the nonlinear AIDS approximated the within-branch elasticity of substitution with values less than one like in the true model, while the LAICF produced both positive values (that were less than unity) as well as negative values. On the other hand, the LAISF produced a time-varying elasticity with negative values at each time period.

The AIDS models produced approximations of $\sigma_{13,t}$ with values greater than one, like the true values. However, the approximating values tended to underestimate the true ones; but the LAISF model produced some values very close to the true ones. The RM produced an approximation with 62% of values that were greater than one. On the other hand, the RM produced an approximation of $\sigma_{23,t}$ with 22% of values greater than one, although these values tended to underestimate the true ones. The NLAI produced an approximation of $\sigma_{23,t}$ the values of which were less than one. Finally, the LAISF and the LAICF produced approximations with negative values and positive values. The positive values of these approximations are all less than one.

Table 4.12. Robustness checks: TVC elasticities of substitution at selected periods

	Random Walk Model					Local Trend Model				
Period	True	RM	NLAI	LAISF	LAICF	True	RM	NLAI	LAISF	LAICF
					$\sigma_{12,t}$					
1	0.246	0.534	0.943	0.596	1.036	0.116	0.629	1.109	-0.283	0.481
2	0.241	0.546	0.947	0.613	1.036	0.141	0.685	0.927	-0.453	0.397
3	0.224	0.501	0.950	0.666	1.029	0.169	0.764	0.868	-0.624	0.320
4	0.253	0.494	0.945	0.622	1.034	0.170	0.781	0.862	-0.629	0.314
6	0.184	0.468	0.952	0.700	1.026	0.125	0.711	0.860	-0.536	0.363
9	0.227	0.559	0.945	0.603	1.038	0.038	0.875	0.702	-1.311	0.067
12	0.248	0.517	0.946	0.615	1.035	0.054	0.648	0.830	-0.394	0.421
18	0.197	0.505	0.949	0.641	1.032	0.081	0.643	0.787	-0.643	0.339
24	0.205	0.510	0.948	0.638	1.032	0.112	0.681	0.758	-0.843	0.254
36	0.230	0.485	0.946	0.662	1.030	0.044	0.742	0.755	-0.898	0.233
48	0.248	0.569	0.942	0.557	1.043	0.073	0.899	0.698	-1.345	0.047
60	0.263	0.536	0.946	0.604	1.035	0.055	1.041	0.650	-1.721	-0.098
					$\sigma_{13,t}$					
1	3.006	1.300	0.430	0.303	0.905	2.992	0.389	1.210	1.515	1.342
2	2.951	1.291	0.433	0.308	0.905	3.008	0.421	1.126	1.527	1.366
3	2.844	1.282	0.438	0.318	0.905	3.006	0.423	1.089	1.494	1.341
4	2.940	1.305	0.411	0.281	0.901	3.021	0.384	1.076	1.418	1.290
6	2.881	1.239	0.475	0.369	0.911	3.033	0.493	1.131	1.608	1.415
9	3.037	1.192	0.481	0.373	0.913	3.013	0.478	1.090	1.529	1.362
12	3.035	1.153	0.478	0.366	0.913	2.872	0.603	1.141	1.818	1.552
18	2.824	1.395	0.365	0.223	0.893	2.941	0.923	1.160	2.143	1.763
24	2.770	1.316	0.418	0.291	0.902	2.875	0.996	1.138	1.977	1.653
36	3.025	1.204	0.465	0.352	0.910	2.900	1.472	1.220	2.594	2.053
48	3.245	1.305	0.415	0.282	0.904	2.998	1.189	1.194	2.423	1.939
60	2.878	1.542	0.313	0.155	0.885	2.911	2.016	1.361	3.627	2.720
					$\sigma_{23,t}$					
1	0.883	0.476	2.175	1.941	1.925	1.113	0.674	1.133	0.380	0.336
2	0.836	0.489	2.147	1.916	1.901	1.334	0.734	0.974	0.290	0.248
3	0.696	0.440	2.032	1.812	1.806	1.553	0.816	0.978	0.216	0.169
4	0.865	0.433	2.137	1.904	1.892	1.523	0.829	0.966	0.225	0.178
6	0.638	0.404	1.937	1.734	1.733	1.460	0.759	0.914	0.240	0.195
9	0.747	0.379	2.003	1.797	1.788	1.358	0.719	0.926	0.290	0.243
12	0.853	0.455	2.105	1.887	1.870	1.110	0.699	0.975	0.276	0.225
18	0.631	0.448	2.136	1.891	1.886	1.335	0.701	0.948	0.143	0.087
24	0.619	0.451	2.103	1.873	1.864	1.442	0.730	0.925	0.080	0.022
36	0.736	0.422	2.018	1.808	1.799	1.472	0.817	0.971	-0.046	-0.106
48	0.956	0.514	2.272	2.027	2.003	1.979	0.962	0.912	-0.217	-0.277
60	0.870	0.484	2.249	1.987	1.974	2.521	1.149	0.981	-0.565	-0.634

4.7.2.2 Time-varying income elasticities

The Rotterdam model produced time-varying income elasticities the values of which tended to be close to the true ones under the two specifications of the time-varying coefficients in the demand system. In particular, Table 4.13 shows that the Rotterdam model has the ability to produce good approximation even for very high values of the time-varying income elasticities. This is illustrated by the time-varying elasticity of x_3 under the LTM. The NLAI model tended to produce the values of income elasticities with values close to one, as in the main Monte Carlo experiment.

The LAISF model produced an approximation of the time-varying income elasticity for x_1, η_{1t}, with values that underestimated the true ones. The models produced positive overestimating approximating values for η_{2t} under the RWM and both negative as well as positive but underestimating values under the LTM (Table 4.13). Finally, the approximating time-varying income elasticity for x_3 underestimated the true values under the RWM and the LTM.

Turning to the performance of the LAICF, the approximating time-varying income elasticity of x_1 obtained from this model tended to be around 0.06 under the RWM and around 0.02 under the LTM. In addition to the fact the values of this time-varying income elasticity tended to be constant over time, they all underestimated the true values. Furthermore, the model produced an approximating time-varying income elasticity of x_2 with negative values at each time period under the two specifications of the time-varying coefficients. Finally, the approximating time-varying income elasticity of x_3 had negative values at each time period under the RWM. Under the LTM, the values of this time-varying elasticity are positive but they underestimate the true ones (see Table 4.13).

Table 4.13. Robustness checks: TVC income elasticities at selected periods

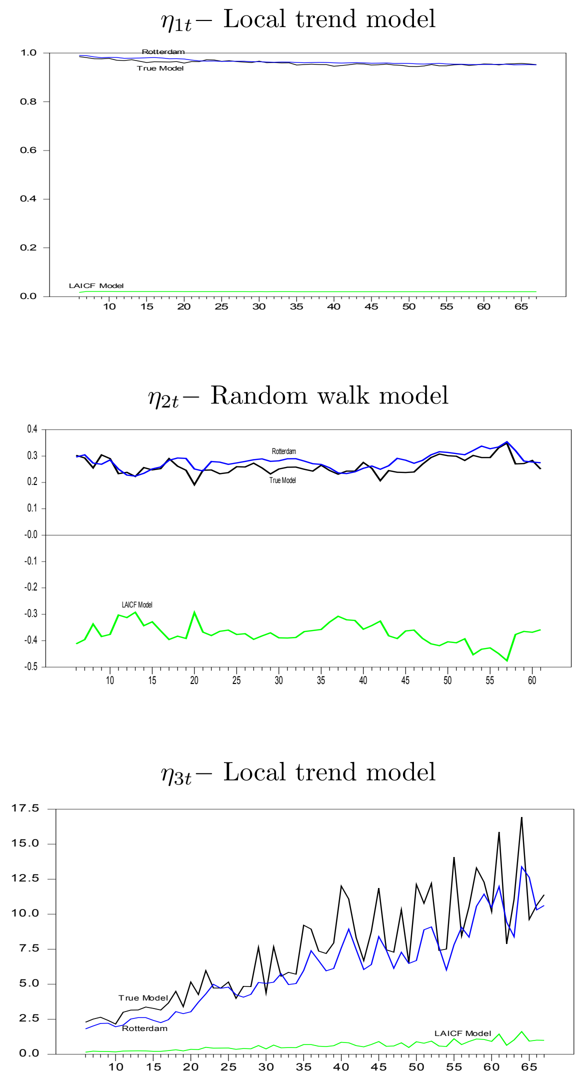
	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
					η_{1t}					
1	1.031	1.000	1.060	0.058	1.057	0.981	1.000	1.021	0.021	0.989
2	1.033	1.000	1.060	0.058	1.056	0.977	1.000	1.021	0.021	0.984
3	1.043	1.000	1.061	0.059	1.062	0.976	1.000	1.021	0.021	0.981
4	1.034	1.000	1.060	0.058	1.061	0.978	1.000	1.021	0.021	0.983
6	1.052	1.000	1.062	0.060	1.070	0.969	1.000	1.020	0.021	0.978
9	1.042	1.000	1.061	0.060	1.077	0.961	1.000	1.020	0.021	0.981
12	1.033	1.000	1.061	0.059	1.070	0.963	1.000	1.020	0.021	0.977
18	1.042	1.000	1.060	0.059	1.055	0.972	1.000	1.020	0.020	0.967
24	1.041	1.000	1.061	0.059	1.059	0.967	1.000	1.020	0.020	0.965
36	1.042	1.000	1.061	0.059	1.070	0.952	1.000	1.020	0.020	0.960
48	1.027	1.000	1.060	0.057	1.053	0.948	1.000	1.020	0.020	0.958
60	1.032	1.000	1.060	0.057	1.044	0.955	1.000	1.020	0.020	0.952
					η_{2t}					
1	0.303	0.998	0.571	-0.412	0.296	0.365	0.985	0.186	-0.834	0.478
2	0.293	0.998	0.589	-0.396	0.305	0.433	0.983	0.097	-0.923	0.522
3	0.255	0.998	0.653	-0.337	0.273	0.504	0.985	-0.009	-1.033	0.591
4	0.304	0.998	0.602	-0.384	0.269	0.493	0.985	-0.015	-1.040	0.628
6	0.233	0.998	0.691	-0.303	0.250	0.466	0.981	0.046	-0.976	0.583
9	0.256	0.998	0.647	-0.343	0.235	0.433	0.985	0.092	-0.930	0.557
12	0.290	0.998	0.590	-0.396	0.284	0.372	0.987	0.138	-0.883	0.548
18	0.233	0.998	0.627	-0.364	0.277	0.441	0.983	-0.035	-1.061	0.631
24	0.232	0.998	0.621	-0.370	0.280	0.485	0.982	-0.162	-1.194	0.689
36	0.253	0.998	0.646	-0.342	0.262	0.483	0.981	-0.185	-1.213	0.736
48	0.302	0.997	0.523	-0.453	0.321	0.625	0.976	-0.463	-1.497	0.894
60	0.312	0.998	0.585	-0.396	0.299	0.827	0.973	-0.694	-1.728	1.040
					η_{3t}					
1	0.965	0.998	0.393	-0.581	0.621	2.506	1.015	1.210	0.220	2.021
2	0.960	0.998	0.397	-0.579	0.616	2.643	1.030	1.196	0.202	2.201
3	0.926	0.998	0.403	-0.579	0.611	2.415	1.028	1.187	0.193	2.212
4	0.952	0.998	0.372	-0.605	0.624	2.154	1.024	1.161	0.166	1.965
6	0.906	0.998	0.446	-0.541	0.589	3.150	1.037	1.230	0.237	2.514
9	0.946	0.998	0.452	-0.531	0.564	3.277	1.032	1.203	0.209	2.421
12	0.973	0.998	0.449	-0.529	0.544	4.489	1.046	1.308	0.317	3.043
18	0.910	0.998	0.320	-0.663	0.670	4.730	1.073	1.428	0.441	4.711
24	0.930	0.998	0.381	-0.604	0.629	4.375	1.061	1.368	0.381	5.058
36	0.946	0.998	0.434	-0.547	0.571	8.314	1.100	1.596	0.613	7.474
48	0.980	0.998	0.376	-0.590	0.624	7.507	1.089	1.534	0.550	6.020
60	0.943	0.998	0.260	-0.704	0.747	10.64	1.163	1.976	1.001	10.30

One of the findings from the main Monte Carlo experiment was that the Rotterdam model had the ability to capture the variations in the true time-varying elasticities and to mimic their time paths. The results from the robustness-check experiment lead to the same evidence. It can be seen from Table 4.13 that the Rotterdam model approximated the high values of the time-varying income elasticity of x_3 quite well. To show the ability of this model to approximate the true time-varying income elasticities with closer values and to mimic their paths, I have displayed three of the six income elasticities that are considered in the analysis in Figure 4.6. The approximations from the Rotterdam model and the linear-approximate AIDS model with corrected formulas are shown together with the true time-varying income elasticities.

It can be seen from Figure 4.6 that the RM produced an approximation of η_{1t} with values close to the true ones under the LTM specification. Moreover, the path of the approximating time-varying elasticity tracked that of the true one. However, while the true time-varying income elasticity displayed some variations over time, the approximation from the Rotterdam model tended to smooth them out. In contrast, the LAICF produced an approximation of η_{1t} that is off track and the values of which tended to be constant over time.

Figure 4.6 also shows that the RM performed better for the two other time-varying income elasticities. For example, while the approximating elasticity from the RM had positive values and tracked the path of η_{2t} fairly well under the random walk model, the LAICF approximation had negative values at every single time period. Finally, the RM approximated the high-valued income elasticity η_{3t} and tracked its path very well. Although the approximating elasticity produced by the LAICF showed some variations over time in this case, its paths is smoother with values too low compared to those of the true elasticity and those of the RM approximation.

Figure 4.6. Robustness checks: Selected time-varying income elasticities



The next subsection is devoted to the robustness checks for the compensated cross-price time-varying elasticities. As in the main Monte Carlo experiment, the most important concern is about the ability of each model to identify goods as complements of substitutes. I shall nevertheless briefly comment on the values of the time-varying cross-price elasticities.

4.7.2.3 Time-varying cross-price elasticities

Comparing the RM and the LAICF model, it follows from Table 4.14 that the RM approximated all the compensated cross-price elasticities with positive values. However, the approximating values tended to overestimated the true ones. The LAICF model produced three time-varying cross-price elasticities with negative values for all the time periods. It is important to keep in mind that values of the true time-varying cross-price elasticities are all positive at each single time period. The negative values of the approximating time-varying elasticities means that the LAICF wrongly identifies substitute goods as complement goods. For the cases where the LAICF produced positive values of the time-varying price elasticities, the values are either constant over time ($\eta^*_{12,t}$ under the RWM and $\eta^*_{13,t}$ under the LTM) or overly overestimating ($\eta^*_{23,t}$ under the RWM). It is noteworthy that the RM also produced a time-varying cross-price elasticity which tended to be equal to 0.01 for every time period ($\eta^*_{13,t}$ under the LTM).

The NLAI model produced approximations with positive values that tended to overestimate the true values for the goods in the same sub-aggregation group. For the goods in different groups, the NLAI model tended to underestimate the true values of $\eta^*_{13,t}$ and to overestimate the true values of $\eta^*_{23,t}$. The LAISF model also produced time-varying cross-price elasticities with positive values, except one elasticity under the local trend model. Table 4.14 shows that the LAISF model produced an approximation of the time-varying cross-price elasticity between x_1 an x_2 the values of which are comparable to those produced by the NLAI model. For the goods in different aggregation groups, the approximating values produced by the LAISF either overestimated or underestimated the approximating values from the NLAI model, depending on the specification of the time-varying parameters in the utility function.

Table 4.14. Robustness checks: TVC cross-price elasticities at selected time periods

	Random Walk Model					Local Trend Model				
Period	True	NLAI	LAISF	LAICF	RM	True	NLAI	LAISF	LAICF	RM
					$\eta^*_{12,t}$					
1	0.009	0.036	0.041	0.003	0.019	0.004	0.034	0.015	-0.016	0.020
2	0.009	0.038	0.042	0.003	0.019	0.004	0.025	0.011	-0.016	0.020
3	0.010	0.045	0.050	0.003	0.019	0.004	0.021	0.008	-0.016	0.020
4	0.010	0.039	0.044	0.003	0.019	0.004	0.021	0.008	-0.016	0.019
6	0.009	0.050	0.056	0.003	0.019	0.003	0.022	0.010	-0.016	0.018
9	0.007	0.044	0.049	0.003	0.019	0.002	0.022	0.011	-0.016	0.018
12	0.010	0.038	0.043	0.003	0.019	0.002	0.023	0.013	-0.016	0.018
18	0.008	0.042	0.047	0.003	0.018	0.002	0.018	0.009	-0.015	0.015
24	0.009	0.041	0.046	0.003	0.019	0.002	0.016	0.006	-0.015	0.015
36	0.010	0.044	0.049	0.003	0.019	0.001	0.015	0.006	-0.015	0.015
48	0.008	0.033	0.037	0.003	0.018	0.001	0.012	0.002	-0.015	0.015
60	0.010	0.038	0.042	0.003	0.018	0.001	0.009	0.000	-0.015	0.015
					$\eta^*_{13,t}$					
1	0.191	0.026	0.061	-0.008	0.084	0.074	0.030	0.034	0.009	0.010
2	0.189	0.026	0.061	-0.008	0.084	0.069	0.025	0.032	0.008	0.010
3	0.184	0.027	0.062	-0.008	0.085	0.073	0.026	0.034	0.008	0.010
4	0.181	0.024	0.058	-0.008	0.085	0.086	0.030	0.038	0.008	0.010
6	0.200	0.031	0.067	-0.008	0.085	0.061	0.022	0.029	0.008	0.010
9	0.214	0.032	0.068	-0.008	0.086	0.069	0.024	0.032	0.008	0.010
12	0.212	0.032	0.067	-0.008	0.085	0.043	0.017	0.024	0.008	0.010
18	0.160	0.019	0.053	-0.008	0.084	0.032	0.012	0.019	0.008	0.010
24	0.172	0.025	0.059	-0.008	0.084	0.036	0.014	0.021	0.008	0.010
36	0.206	0.030	0.065	-0.008	0.085	0.023	0.009	0.016	0.008	0.010
48	0.201	0.025	0.059	-0.008	0.084	0.026	0.010	0.017	0.008	0.010
60	0.150	0.015	0.049	-0.008	0.083	0.014	0.006	0.013	0.008	0.010
					$\eta^*_{23,t}$					
1	0.056	0.818	0.602	0.573	0.028	0.027	0.034	-0.069	-0.058	0.034
2	0.053	0.789	0.581	0.551	0.029	0.030	0.037	-0.078	-0.061	0.036
3	0.045	0.676	0.501	0.463	0.026	0.038	0.071	-0.090	-0.068	0.039
4	0.053	0.760	0.560	0.530	0.026	0.043	0.076	-0.091	-0.066	0.040
6	0.044	0.615	0.458	0.414	0.024	0.029	0.035	-0.084	-0.065	0.034
9	0.053	0.691	0.511	0.471	0.022	0.031	0.061	-0.078	-0.061	0.031
12	0.060	0.792	0.582	0.548	0.027	0.017	0.060	-0.074	-0.059	0.030
18	0.036	0.712	0.527	0.496	0.026	0.014	0.055	-0.088	-0.065	0.026
24	0.038	0.730	0.540	0.507	0.027	0.018	0.067	-0.101	-0.072	0.024
36	0.050	0.691	0.511	0.473	0.025	0.012	0.061	-0.104	-0.077	0.025
48	0.059	0.901	0.661	0.638	0.030	0.017	0.073	-0.130	-0.090	0.030
60	0.045	0.782	0.577	0.553	0.028	0.012	0.080	-0.150	-0.105	0.034

4.7.3 Regularity condition

Table 4.15 shows that the RM and the LAICF model satisfied the regularity condition – the negative semi-definiteness of the Slutsky matrix – at every replication and every time period under the RWM. Under this specification of the time-varying coefficients in the demand system, the NLAI and the LAICF models satisfied the theoretical regularity condition for more than 99% of the replications at each time period. Table 4.15 also shows that, under the LTM, the Rotterdam model achieved the highest levels of the regularity index – more than 99% – at every single time period, compared to the other models. In particular, the levels of the regularity index that were achieved by the LAICF model were the lowest at each time period.

Table 4.15. Robustness checks: Regularity index by model and TVC specification

	NLAI		LAISF		LAICF		RM	
Period	RWM	LTM	RWM	LTM	RWM	LTM	RWM	LTM
1	99.3	92.2	99.6	92.8	100.0	89.6	100.0	99.8
2	99.5	92.3	99.8	86.5	100.0	82.8	100.0	99.7
3	99.9	90.5	99.9	79.1	100.0	75.8	100.0	99.7
4	99.5	90.1	99.7	79.6	100.0	74.6	100.0	99.7
5	99.6	93.0	99.8	88.3	100.0	82.7	100.0	99.6
6	100.0	90.4	100.0	85.6	100.0	80.9	100.0	99.6
7	99.9	93.0	100.0	90.0	100.0	86.2	100.0	99.7
8	100.0	92.8	100.0	90.0	100.0	86.3	100.0	99.7
9	99.9	92.5	99.9	88.0	100.0	83.8	100.0	99.6
10	99.9	92.1	100.0	87.6	100.0	83.7	100.0	99.8
11	99.8	92.2	99.9	88.8	100.0	84.3	100.0	99.8
12	99.7	93.7	99.9	91.7	100.0	86.8	100.0	99.8
18	99.6	88.5	99.6	83.5	100.0	74.7	100.0	99.3
24	99.7	85.3	99.8	74.5	100.0	64.7	100.0	99.3
30	99.8	85.3	99.9	77.3	100.0	65.1	100.0	99.9
36	99.8	84.4	99.9	76.2	100.0	64.8	100.0	99.9
42	99.4	80.1	99.6	66.8	100.0	53.2	100.0	99.8
48	98.7	78.3	99.2	59.3	100.0	47.4	100.0	99.8
54	99.6	76.9	99.8	60.7	100.0	45.9	100.0	99.9
60	98.9	73.9	98.9	50.8	100.0	37.6	100.0	99.9

4.7.4 Final remarks on robustness checks

As a concluding remark to the robustness-check experiment, I shall say that its results support the findings from the main Monte Carlo experiment. In terms of the comparison between the Rotterdam model and the LAICF model, the new experiment shows that the RM performed better than the LAICF both in recovering the signs of the values of the time-varying elasticities and in producing the elasticity values that are close to the true ones. This experiment has also shown that the NLAI and the LAISF produced values of income and substitution time-varying elasticities that tended to be constant over time. But this is exactly one of the conclusions that emerged from the main Monte Carlo experiment. This experiment has also confirmed the ability of the RM to produce time-varying income elasticities that track the paths of the true ones, and the values of which are close to the true ones.

In terms of theoretical regularity, Table 4.15 showed that the four demand specifications achieved similar levels of theoretical regularity under the random walk model. However, the Rotterdam model and the LAICF model slightly performed better than the nonlinear AIDS model and the LAISF model. The performance of the four models under the local trend model, on the other hand, clearly shows the superiority of the Rotterdam model over the other three demand specifications. In fact, the Rotterdam model satisfied the negative semi-definiteness of the Slutsky matrix in more than 99% of the replications at every time period. However, each version of the AIDS demand specification achieved lower regularity levels compared to those of the Rotterdam model at every single time period. More importantly, the regularity index for the LAICF model is the lowest at each time period.

5

Conclusion

In this chapter, I shall review and summarize the results of my research, identify the main methods used and discuss the implications of the findings.

The main focus of this dissertation research was to assess the performance of the most used local flexible functional forms in demand analysis, namely the AIDS in its linearized form and the Rotterdam model, in recovering the elasticities of a true demand system when the parameters are varying over time. Monte Carlo simulations were used to generate data from a known utility function, the weak separable-branch utility tree. Two specifications of the time-varying parameters in the utility function were considered, resulting in two data sets on quantities demanded, prices and total expenditure. The first data set was generated by assuming that all the parameters in the true utility function follow a random walk process. The quantities in the second data set were generated as the sum of two components: a random-walk-with-drift component and a supernumerary-quantity component. Relevant time-varying elasticities for the true demand system were then obtained accordingly.

Four steps were used to assess the performance of the approximating demand systems: (i) estimate the time-varying coefficients using the Kalman filter and smoother by bootstrapping each demand specification; (ii) obtain the approximating time-varying elasticities by using the estimated time-varying coefficients; (iii) compare the performance of each model with respect to the values and paths of the the true time-varying elasticities; (iv) assess the ability of each approximating demand system to conform to demand theory by calculating a regularity index at each time period.

The results of the research were classified in terms of the sign and magnitude of the values of the approximating time-varying elasticities. The two classifications corresponded respectively to a qualitative assessment and a quantitative assessment of the performance of each approximating demand system. A particular attention was paid to the ability of each approximating demand system to perform in a consistent way under both specifications of the time-varying parameters. Based on the sign of the approximating values, goods were identified as substitutes, complements, normal or inferior. However, the magnitude of the approximating values was used to assess how close they are to the true ones.

Both the Rotterdam model and the NLAI model tended to provide a correct qualitative classification of goods based on the income elasticities and the elasticities of substitution at each time period. However, the Rotterdam model correctly identified goods as normal necessities or luxuries while the NLAI model failed to do so in most of the cases. For the cross-price-elasticities-based classification, the Rotterdam model tended to produce correct classification compared to the NLAI model. The LAICF model, which is widely used in applied research than the NLAI model, poorly approximated the NLAI model and performed poorly at recovering the true values of the time-varying elasticities in most of the cases.

On the other hand, the LAISF model qualitatively performed in the same way as the NLAI model only for the time-varying income elasticities. However, the approximation of the other time-varying elasticities was not consistent under the two specifications of the time-varying coefficients in the demand system. The consistency was evaluated in terms of the signs of the elasticity values under the two specifications of the time-varying parameters.

Concerning the magnitude of the estimated time-varying elasticities, the Rotterdam model tended to produce values of the time-varying elasticity of substitution that were close to the true ones within the branches of the utility function. In contrast, the NLAI tended to overestimate the values of the within-branch time-varying elasticity of substitution. Both models tended to produce poor approximations of the across-branch elasticities of substitution. On the other hand, the Rotterdam model tended to produce time-varying income elasticities that mimicked the paths of the true elasticities and the values of which were very close to the true ones. The NLAI tended to produce constant values over time and failed to capture very high values of time-varying income elasticities. Moreover, the NLAI overestimated the values of the time-varying cross-price elasticities within the branch of the utility function, while the Rotterdam model produced an approximation the values of which were close to the true ones.

To check the robustness of the above results, I conducted a new Monte Carlo experiment where I used different values of the time-varying parameters in the utility function. The results from this new experiment supported the conclusions from the main Monte Carlo experiment. More specifically, the RM performed better than all the other models, especially the LAICF. However, the LAICF was able to achieve similar levels of theoretical regularity as the RM under the RWM specification.

The findings in this dissertations lead to two important implications for the demand analysis with time-varying coefficients. First, with regard to the performance of the LAICF model, this model should not be considered as an approximation to the NLAI model. It should, in contrast, be considered as a model on its own. This is important since its outcomes may considerably differ from those of the NLAI with regard to the signs of the estimated time-varying parameters and elasticities on the one hand, and their magnitude on the other hand.

The second implication relates to the choice between an AIDS-type specification and the Rotterdam model in empirical studies. An important recommendation is that such a choice be made with respect to the performance of each model to better approximate the properties of an hypothesized true model. However, the results in this dissertation may be dependent on the structure of the true model and the particular Monte Carlo experiment that was implemented. Therefore, caution should be used in selecting the correct functional structure that is intended to approximate the properties that are contained in a given data set.

It is noteworthy that the comparison of the performance among different models included in this research mainly focused on how they can approximate the true model qualitatively and quantitatively. However, the comparison cannot be limited to the sole performance of this nature. A broad range of aspects can be considered as well. For example, future research efforts to assess the performance of the AIDS-type models and the Rotterdam model may focus on the forecasting abilities of each model. In the specific case of time-varying parameters, the two models can be judged in terms of their ability to produce time series of elasticities that recover the time series properties of the true time-varying elasticities.

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APPENDIX

A

Confidence Intervals for the Time-Varying Elasticities

Figure A.1. Time-varying elasticities of substitution in the RM

Random Walk Model *Local Trend Model*

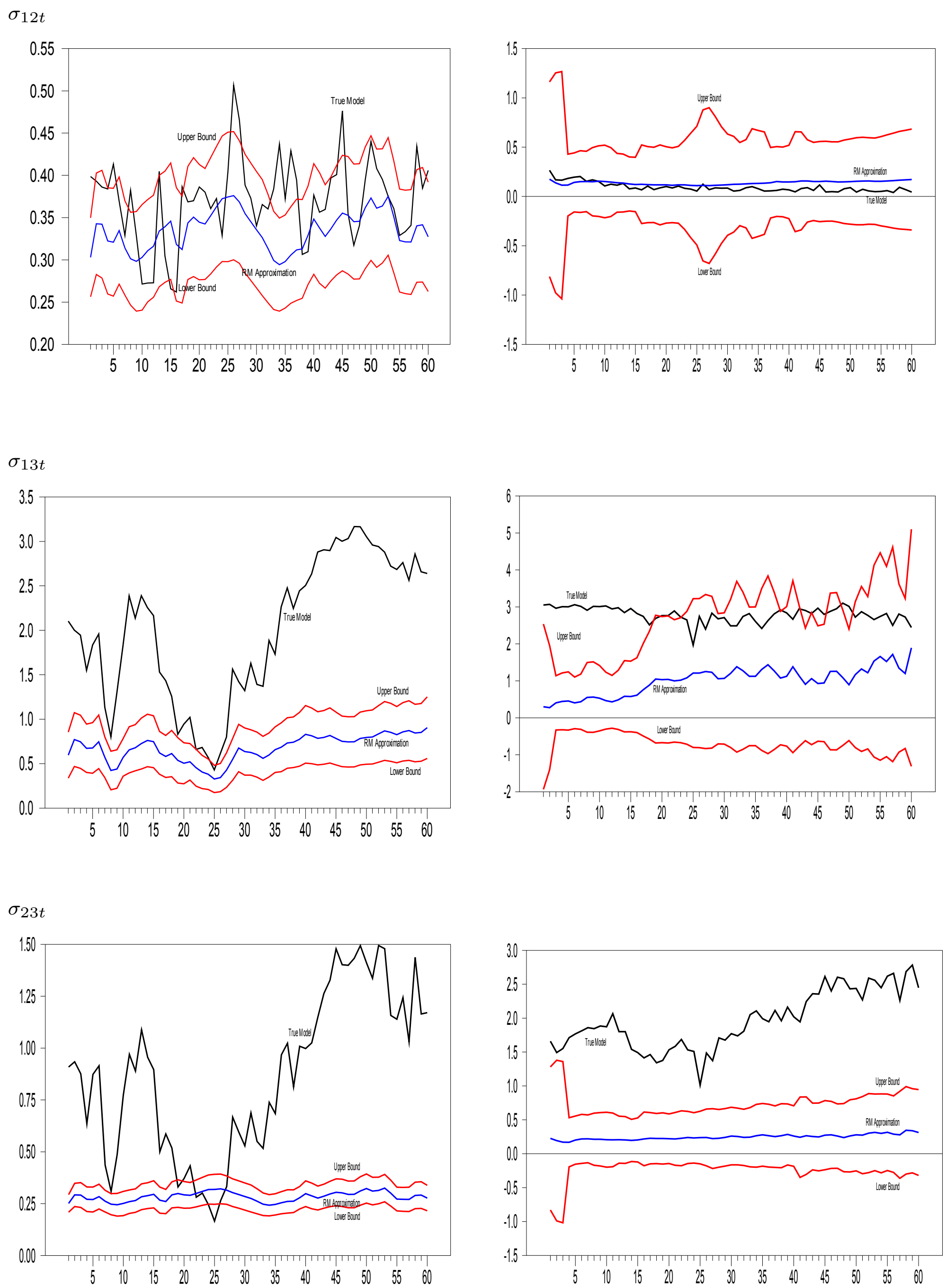

Figure A.2. Time-varying elasticities of substitution in the NLAI model

Random Walk Model *Local Trend Model*

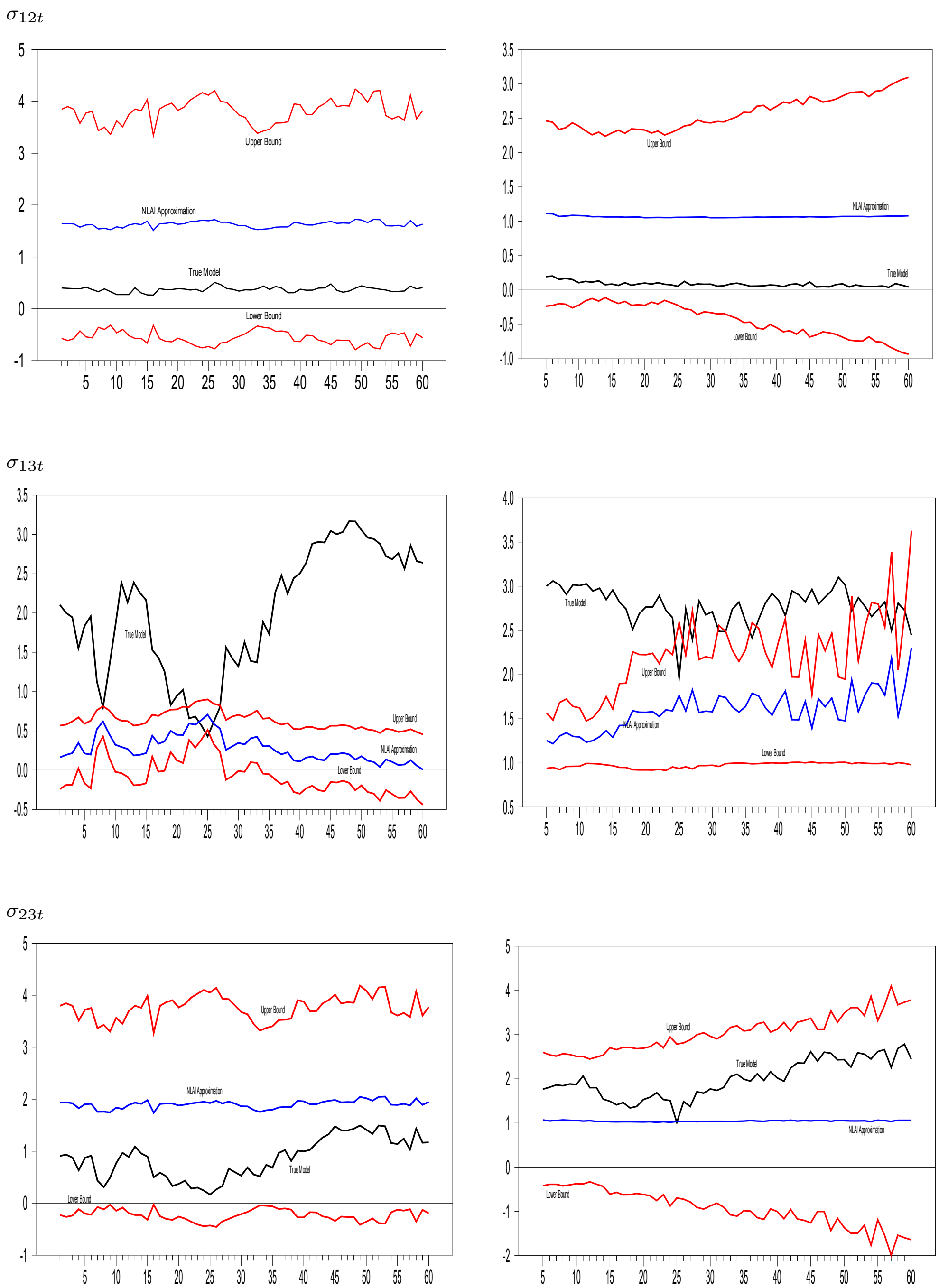

Figure A.3. Time-varying income elasticities in the RM