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EVALUATING NEGATIVE BENEFITS

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Evaluating investments by discounting anticipated future benefits at an exogenously determined risk-adjusted discount rate (hereafter referred to as the RADR approach) is well accepted in the canon of finance. If benefits (D_t) are to be received for T periods and if k, the discount rate, is constant over each of the t periods, then the discrete time net present value (NPV) is defined as:

$$\text{NPV} = \sum_{t=0}^{T} D_t/(1+k)^t. \tag{1}$$

A positive NPV characterizes a desirable investment.

A frequently offered criticism of the RADR approach centers on the fact that both risk and timing considerations are treated in the denominator of equation (1). The certainty equivalent (CE) method has been suggested as a way of distinguishing between the two effects. In computation of the CE-NPV, riskless benefits that are equal in utility to the risky projected benefits[1] are substituted in the numerator of equation (1). Since these benefits are by definition riskless equivalents, they are discounted at the pure rate of interest.

Another popular investment evaluation technique, the internal rate of return (IRR), compares that rate of discount which equates the NPV to zero to some hurdle value of k. Desirable investments are those with IRRs > k.

Nonsimple projects, those with more than one pair of sequential benefits different in sign, pose a well-known problem when the IRR technique is used. In particular, more than one real value can satisfy the IRR definition with nonsimple investments. The argument is frequently forwarded ([1, p. 93] [2, pp. 237-238] [5, p. 298]) that the RADR net present value should be utilized when faced with multiple IRRs since the NPV is unique with a specified k. The

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[1]A certainty equivalent *factor* is defined as the ratio of utility-equivalent riskless and risky benefits.

nongenerality of this "solution" is illustrated below.

Consider a nonsimple project with benefits of -$5,000, + $11,500, and -$6,600 in periods 0, 1, and 2 respectively. This project has IRRs of 10 percent and 20 percent. For the sake of simplicity of illustration, assume that the period 0 and period 1 benefits are nonstochastic and the period 2 benefit is the expected value of a 50-50 chance of -$6,200 and -$7,000.

According to conventional wisdom, the riskiness of this project is evaluated and the benefits discounted at an appropriate rate. If that rate is 9 percent, then the project would not be undertaken since its NPV is -$4.6293.

Assume a similar project exists (or a revision in expectations of the dispersion of the period 2 benefit occurs) such that the probability distribution of the period 2 benefit has greater variance and hence risk. (For example, the -$6,600 is from a 50-50 chance -$5,200 or -$8,000.) If this stream of benefits is discounted at a higher rate, say 11 percent, the NPV is +$3.6522. Such a result is paradoxical since the income stream's value has *increased* with increased risk.

The result is not surprising in a mathematical sense since the projects' NPVs plot as curves with maximums at k of 15 percent. Selection of rates greater than 15 percent would have resulted in decreasing NPVs with increasing capitalization rates. This observation notwithstanding, the fact remains that with many nonsimple projects, NPV increases with increasing capitalization rates over some range.

Moreover, the CE-NPV technique can be used to reconcile the paradox. For a return-seeking risk averter, a risky negative projected benefit has a certainty equivalent which is *more negative*. Hence, given a mean preserving increase in risk, the certainty equivalent will decrease. Since the discount rate is unchanged (the pure rate of interest), the NPV decreases, which is the appropriate result.

Moreover, the conclusion that the CE-NPV technique is superior to the RADR approach for evaluating investment projects is consistent with the conclusions of Robichek and Myers (hereafter, R and M) [3 and 4].[2] However, the example above illustrates the limited generality of the R and M works, limited in the sense that they considered only the special type of benefits with positive expected values *and* positive certainty equivalents. Indeed, R and M base their entire critique on the RADR method on the assumption that certainty equivalent factors are greater than or equal to zero and less than or equal to one [3, p. 80]. Using this, they claim that increasing risk causes the factors

[2] An anonymous referee pointed out the importance of emphasizing these observations.

to decrease in magnitude.

However, negative projected benefits have certainty equivalent factors which are *greater* than unity and which *increase* with increasing risk. Furthermore, a positive risky benefit could have a negative certainty equivalent,[3] yielding a factor which is negative and decreases (becomes more negative) with increasing risk. Finally, R and M did not consider the possibility of a risky projected benefit with an expected value precisely equal to zero, which yields an undefined certainty equivalent factor.[4]

Moreover, the use of negative benefits in no way "salvages" the RADR approach from the criticisms of R and M. Indeed, attempting to use the RADR rather than the certainty equivalent method provides nonintuitive results when negative benefits are considered.[5] However, the important conclusion is that the mathematical arguments of R and M ignore negative benefits entirely.

To summarize four points deserve reemphasis. (1) The risk-adjusted net present value is not a generally appropriate alternative to the internal rate of return when nonsimple projects are evaluated. (2) The risk-adjusted discount rate approach should not be applied to investment projects with negative benefits. (3) Robichek and Myers' conclusions regarding the risk-adjusted discount rate approach to valuation are only relevant for the special class of benefits they considered--risky benefits with positive expected values *and* positive certainty equivalents. (4) The conclusion reached here and by Robichek and Myers is essentially the same, namely, that the certainty equivalent net present value technique is a superior method for evaluating benefit streams than is the risk-adjusted discount rate technique.

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[3] For example, a decision unit could be just willing to pay \$50 to avoid participation in a lottery with equal probabilities of gaining \$1,200 or losing \$1,000.

[4] The certainty equivalent itself would of course be defined.

[5] As an illustration, if $X < 0$ is a projected risky benefit to be received one period hence with a certainty equivalent $Y < X$, then $X \div Y$ is less than one since both X and Y are negative. If i is the riskless rate, then the present value equals $Y \div (1 + i)$, which must also equal $X \div (1 + k)$. Hence $[X \div (1 + k)] = [Y \div (1 + i)]$ so $(1 + k) = (X \div Y)(1 + i)$ yielding $k < i$. This result is perverse, since the risk-adjusted rate supposedly equals the riskless rate *plus* some positive adjustment for risk.

REFERENCES

[1] Jean, William H. *Finance*. Hinsdale, Ill.: The Dryden Press (1973).

[2] Lorie, James H., and Leonard J. Savage. "Three Problems in Rationing Capital." *Journal of Business*, Vol. 28, No. 4 (October 1955), pp. 229-239.

[3] Robichek, Alexander A., and Stewart C. Myers. *Optimal Financing Decisio*: Englewood Cliffs, N.J.: Prentice-Hall, Inc. (1965).

[4] __________. "Conceptual Problems in the Use of Risk-Adjusted Discount Rates." *Journal of Finance*, Vol. 21, No. 4 (December 1966), pp. 727-730

[5] Weston, J. Fred, and Eugene F. Brigham. *Managerial Finance*, 5th ed. Hinsdale, Ill.: The Dryden Press (1975).