Distributed Tracking Control Design for Leader-Follower Multi-Agent Systems

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Abstract

Multi-agent systems (MASs) have been widely recognized as a key way to model, analyze, and engineer numerous kinds of complex systems composed of distributed agents. The aim of this dissertation is to study control design for leader-follower MASs such that a group of followers can track a specified leader *via* distributed decision making based on distributed information. We identify and consider several critical problems that have stood in the way of distributed tracking control synthesis and analysis. Specifically, they include: 1) limited information access by the followers to the leader, 2) effects of external disturbances, 3) complicated dynamics of agents, and 4) energy efficiency. To overcome the first three problems, we take a lead with the design of distributed-observer-based control, with the insight that distributed observers can enable agents to recover unknown quantities in a collective manner for the purpose of control. To deal with the fourth problem, we propose the first study of MAS tracking control conscious of nonlinear battery dynamics to increase operation time and range. The dissertation will present the following research contributions. First, we propose the notion of designing distributed observers to make all the followers aware of the leader's state and driving input, regardless of the network communication topology, and perform tracking controller design based on the observers. Second, we further develop distributed disturbance observers and observer-based robust tracking control to handle the scenario when all the leader and followers are affected by unknown disturbances only bounded in rates of change. The third contribution lies in treating a leader-follower MAS with high-order, nonlinear dynamics. Assuming the availability of very limited measurement data, we substantively expand the idea of observer-based control to develop a catalog of distributed observers such that the followers can reconstruct large amounts of information necessary for effective tracking control. Finally, we propose a distributed predictive optimization method to integrate onboard battery management with tracking control for long-endurance operation of an electric-powered MAS. The

proposed dissertation research offers new insights and a set of novel tools to enhance the control performance of leader-follower MASs. The results also have a promise to find potential applications in other types of MASs.

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Chapter 1

Introduction

1.1 Background

Multi-agent systems (MASs) have emerged as a crucial technology to advance cooperative autonomy for various applications across scientific, industrial, and civilian sectors. An MAS consists of a team of distributed agents, each equipped with information sensing, communication and computing capabilities. The agents exchange information and make decisions to coordinate with each other to collectively accomplish a goal [2]. Harnessing the power of cooperation among agents, an MAS can perform sophisticated tasks beyond the level of individual agents. For instance, a group of unmanned aerial vehicles can make fast exploration of large geographical areas or complex built environments, provide rapid surveillance and control of forest fires as well as other types of disasters, or enable efficient search and rescue missions [3, 4]. A key attribute of MASs is the distributed information interchange and decision making. This attribute allows the breakdown of a complex task into smaller parts amenable to execution by individual agents. It also yields much flexibility in the organization and coordination of agents—an MAS can adaptively remove agents from or add new ones to the team, thus robust to agent failures and scalable to handle large-scale tasks [5]. With these advantages, MAS-based autonomy has found its way into unmanned aerial, marine and ground vehicle teams [3, 6–11], mobile sensor networks [12–16], connected transportation systems [17–23], and even diverse power and energy systems [24–32].

The operation of MASs depends on distributed control. As the name suggests, distributed control enables an agent to make individual control decisions as an integral part of the team while interacting with its neighboring agents [33, 34]. Due to its importance, it has stimulated considerable interest from researchers in the past two decades. The study of this subject represents an extension of feedback control to distributed networked systems. But differing from centralized control, it focuses on leveraging local information to attain a global goal. Typical problems that have gained large volumes of research include but are not limited to the following: 1) leader-follower tracking, where a group of follower agents track the state of a leader agent, 2) dynamic consensus, where a group of agents mobilize to meet at an agreed location, or average consensus if the agents meet at the average of their initial locations, 3) formation control, where agents preserve a spatial pattern while tracking a certain trajectory, 4) coverage control, where a swarm of agents maneuver to optimally cover or sense a spatio-temporal field, and 5) flocking control, where agents form lattice-shaped meshes in space while maintaining aligned velocity and avoiding collision and obstacles.

Despite an ever-growing body of work, distributed control design still faces many open problems. A significant one among them is how to deal with the distributed information availability. Because the agents of an MAS can only exchange information with their neighbors due to network-based communication, information is distributed among the agents. The amount of information available to an agent is limited, especially if the communication links are sparse as a result of low communication capacity. The distribution of information necessitates distributed control while also making it hard for individual agents to make effective control decisions. To date, it remains a fundamental bottleneck in many task settings. There will arise even more challenges to further compound distributed control design if an MAS has complex dynamics (nonlinear, high-dimensional, etc.) or is subject to uncertainty. Another interesting open problem is how to instill an awareness of practical performance metrics in distributed control. Conventional MAS studies mostly focus on improving control properties, e.g., convergence guarantees or speed, but real-world autonomy tasks often require an MAS to meet more performance objectives. Long-endurance operation is an example in point—it is crucial for MASs to succeed in missions ranging from disaster response to delivery services. However, today's electric-powered MASs often struggle to operate for a long enough time, because currently available lithium-ion batteries, even considered as the best power source due to their high energy and power densities, can still sustain only a relatively short amount of runtime for one charge. This leads to the intriguing question of whether it is possible to achieve battery-aware distributed control.

1.2 Scope and Contributions of the Dissertation

This dissertation presents a study of distributed leader-follower tracking control. Leader-follower MASs represent an important class of MASs, in which a group of follower agents coordinate with each other to track a leader agent. Control design for them has recently attracted increasing research attention. Here, we aim to investigate several major issues that remain unresolved in the literature, outlined as below.

- For a leader-follower MAS, only a small subset of the followers can communicate directly with the leader to access its real-time state or maneuvering input. The fact that the majority of the followers cannot sense the leader makes it difficult to achieve effective tracking control. Existing studies often have to introduce some impractical assumptions to formulate tractable solutions.

- The current literature generally considers leader-follower MASs with linear, low-order dynamics and free of disturbances. However, uncertain disturbances and nonlinear, high-order dynamics are inevitable to appear in realistic settings. Control design in such cases will be much more challenging but has been poorly studied.

- An MAS in real world is often desired to operate for as long as possible or designed to perform long-endurance operations as aforementioned. Yet, prior distributed control methods have disregarded this need.

To address the above issues, the dissertation presents the following contributions.

- We propose a distributed-observer-based approach for leader-follower tracking control. To treat the issue that most of the followers have no direct exchange with the leader, we design distributed observers such that all the followers can use locally shared data to collectively infer about the leader in real time. The distributed observers can deal with inference of the leader's state, maneuvering input, and disturbance. They are then integrated with tracking control design to enable observer-based controller synthesis.

- We investigate leader-follower tracking under disturbances and high-order agent dynamics. In the case of disturbances, we consider the challenging situation that all the followers and leader suffer unknown disturbances. We develop distributed disturbance observers to make the followers gain an awareness of the disturbances for robust tracking. We then substantively expand the distributed observer approach to attain tracking for agents with high-order, nonlinear dynamics.

- We study energy-aware leader-follower tracking for electric-powered MASs. The key notion lies in leveraging the lithium-ion batteries' nonlinear dynamics to extract more energy to increase operation time and range. We develop a distributed model predictive control approach to handle tracking control and onboard battery use simultaneously. The approach by design can strike a balance between tracking performance and battery energy saving. This study also represents the first attempt to exploit distributed optimization for leader-follower tracking to our knowledge.

The dissertation is presented in a summary style. Chapters 2-5 are based on the research articles first-authored by the dissertation author. The first contribution spans and underlies Chapters 2-4; Chapter 3-4 present the second contribution; Chapter 5 addresses the third contribution. Finally, Chapter 6 concludes the entire dissertation.

Chapter 2

Observer-Based Distributed Leader-Follower Tracking Control: A New Perspective and Results[1]

2.1 Introduction

In a leader-follower MAS, a swarm of agents referred to as followers interchange information and apply local control to cooperatively track a leader agent's behavior. The past decade has witnessed a growing amount of research on control design to accomplish this objective, e.g., [36–44] and the references therein. Like other MAS control problems, this problem faces a fundamental challenge that a follower has limited access to information about the other agents (leader and other followers). A primary reason is that information exchange across an MAS is distributed and localized. That is, a follower can only exchange information with its neighbors, and only a subset of the followers can directly communicate with the leader. Adding to this situation, an agent may be unable to measure all of its state variables because sensing devices can be unavailable or too expensive. Consequently, significant research effort has been devoted to observer-based control design, in which followers run observers to estimate the leader's and/or their own state for the purpose of control. The literature includes two main types of approaches in this regard. The first type is about velocity or position observers designed for MASs based on a first- or second-order model, and the second type about state observers for MASs characterized by state-space models.

- *Velocity/position-observer-based control*. For a second-order MAS, the leader's velocity is useful for tracking control but inaccessible to followers when agents do not have velocity

[1]This chapter is based on the dissertation author's first-authored journal paper [35] and first-authored conference paper [36].

sensors. A lead is taken in [37] with the development of a distributed observer that allows a follower to estimate the leader's velocity. The notion is extended in [45] to achieve tracking control in a sampled-data setting and in [38] to enable finite-time leader-follower consensus. In [43], an observer is proposed for a follower to estimate its relative velocity with respect to the leader. Observer design can also be leveraged to estimate followers' velocities. In [46], a local velocity observer is proposed so that a follower can reconstruct its own velocity. A similar problem is investigated in [47]. The approach therein includes an observer, which, though not making explicit velocity estimation, is still meant to make up for the absent velocity information. Position-observer-based tracking control for a first-order MAS is studied in [48], in which a position observer is designed to allow followers to estimate the leader's position. However, it requires the leader's control law to take a specific linear form and be known by all the followers to ensure effective position estimation and tracking.

- *State-observer-based control.* When agents have dynamics modeled in the linear state-space form, a state observer is often needed to achieve output-feedback control. A Luenberger-like observer in [49] is proposed for a follower to estimate its local state, which adopts state correction using the follower's output estimation error relative to its neighbors'. Akin to this, state observers are designed and used in [50] for tracking control in the presence of switching topology and in [51, 52] for leader-follower synchronization with uncertainties.

The studies surveyed above not only provide a wealth of results regarding observer-based tracking control but also show the significance and potential of observers for this control problem. It is noted, however, that the observer design has been almost solely focused on estimating the state variables (e.g., velocity of a second-order agent or state vector of a state-space agent), either the leader's or a follower's. By comparison, estimation of the leader's input has received far less attention, even though it is evident that knowledge of a leader's maneuver input, if available in real time, can critically help a follower keep tracking the leader. Hence, we consider a new *perspective* to investigate leader-follower tracking control by developing distributed input observers that can enable every follower to estimate the leader's input. Since the input observers can bring a follower an awareness of the leader's maneuvers, the tracking control can be hopefully enhanced.

This perspective leads us to make a two-fold contribution through this work. First, we propose a novel input-observer-based tracking control framework. As a distinguishing feature, this framework includes distributed input observers run by followers to estimate the leader's control input. Compared to [48], such observers would neither require the leader's control law to take a special form nor demand it to be known by every follower. Second, following this framework, we systematically develop new tracking control approaches for both first- and second-order MASs. This involves the development of distributed input observers, together with some other observers for position or velocity estimation, and integrates them into tracking control laws. Theoretical analysis proves the effectiveness of the proposed approaches, which is further validated by simulation results. The proposed approaches will bring important benefits for tracking control, e.g., loosening some long-held assumptions and reducing the need for sensing devices, with a detailed discussion offered in the later sections.

The rest of this chapter is organized as follows. Section 2.2 summarizes graph theory used in this chapter. Section 2.3 formulates the problem of interest and presents the input-observer-based framework design for first-order leader-follower tracking. Section 2.4 studies the input-observer-based tracking for the second-order case. A simulation study is offered in Section 2.5 to illustrate the proposed approaches. Finally, Section 2.6 gathers our concluding remarks.

2.2 Graph Theory

We use a graph to describe the topological structure for information exchange among the leader and followers. First, consider a network composed of N independent followers. The interaction topology is modeled as an undirected graph. The follower graph is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the node set and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ contains unordered pairs of nodes. A path is a sequence of connected edges in a graph. The neighbor set of agent i is denoted as $\mathcal{N}_i$, which includes all the agents in communication with it. The adjacency matrix of $\mathcal{G}$ is $A = [a_{ij}] \in \mathbb{R}^{N \times N}$, which has non-negative elements. The element $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, and moreover, $a_{ii} = 0$

for all $i \in \mathcal{V}$. For the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$, $l_{ij} = -a_{ij}$ if $i \neq j$ and $l_{ii} = \sum_{k \in \mathcal{N}_i} a_{ik}$. The leader is numbered as vertex 0, and information can be exchanged between the leader and its neighbors. Then, we have a graph $\bar{\mathcal{G}}$, which consists of graph $\mathcal{G}$, vertex 0 and edges from the vertex 0 (i.e., the leader) to its neighbors. The leader is globally reachable in $\bar{\mathcal{G}}$ if there is a path from node 0 to every node i in $\mathcal{G}$. In order to express the graph $\bar{\mathcal{G}}$ more precisely, we denote the leader adjacency matrix associated with $\bar{\mathcal{G}}$ by $B = \text{diag}(b_1, \dots, b_N)$, where $b_i > 0$ if the leader is a neighbor of agent i and $b_i = 0$ otherwise. The following lemmas will be useful.

Lemma 1. [53] The Laplacian matrix $L(\mathcal{G})$ has at least one zero eigenvalue, and all the nonzero eigenvalues are positive. Furthermore, $L(\mathcal{G})$ has a simple zero eigenvalue and all the nonzero eigenvalues are positive if and only if $\mathcal{G}$ is connected.

Lemma 2. [54] The matrix $H = lB + L$ is positive stable (i.e., all the eigenvalues have a positive real part), where $l > 0$ is a positive coefficient, if and only if vertex 0 is globally reachable.

2.3 First-order Leader-follower Tracking

In this section, we first formulate the problem of first-order leader-follower tracking to be considered. Then, we develop an input-observer-based tracking control approach with convergence proof provided. In the end, the results are extended to a simplified yet meaningful case.

2.3.1 Problem Formulation and Proposed Algorithm

Consider a leader-follower MAS, where the followers are expected to track the leader's trajectory to accomplish an assigned mission. During the tracking process, the leader and followers maintain communication according to a pre-specified network topology to exchange their state information. Leveraging the information received, the followers can determine their control inputs and then steer themselves to track the leader. Suppose that the leader is numbered as 0 and that the N followers

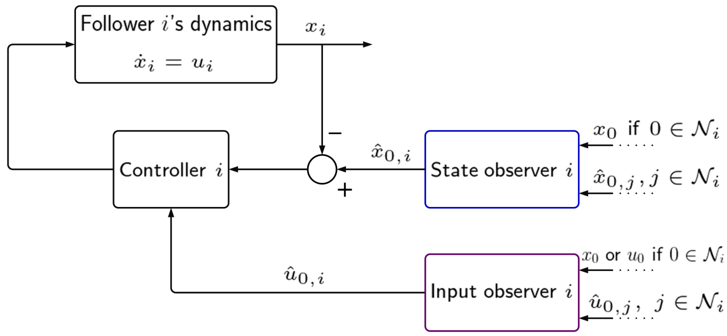

Figure 2.1: Input-observer-based framework for leader-follower tracking.

are numbered from 1 to N. Their dynamics is given by

$$\dot{x}_i = u_i, \quad x_i \in \mathbb{R}, \quad i = 0, 1, \ldots, N, \tag{2.1}$$

where x_i is the position and u_i the control input.

Given this problem setting, the aim is to design u_i for $i = 1, 2, \ldots, N$ such that follower i can asymptotically track the leader, i.e., $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$.

To achieve the above aim, we develop an input-observer-based tracking control design methodology. As a first step, we propose the conceptual design of a linear continuous controller. Because the leader's input u_0 can only be known by its neighbors, the proposed controller involves a local estimate of u_0. Similarly, it also entails a local estimate of the leader's position x_0. Hence, an input observer is designed, which can be used by a follower to infer the leader's input. Building on this input observer, another observer will be proposed for a follower to locally reconstruct the leader's position x_0. The design will be complete when the observers are integrated into the proposed controller. This methodology is illustrated in Figure 2.1.

To begin with, we consider the following control law for follower i:

$$u_i = -k_1(x_i - \hat{x}_{0,i}) + \hat{u}_{0,i}, \tag{2.2}$$

where $k_1 > 0$ is the control gain, and $\hat{x}_{0,i}$ and $\hat{u}_{0,i}$ are follower i's estimates of the leader's position and input, respectively. Here, the term $x_i - \hat{x}_{0,i}$ is meant to drive the follower approaching and tracking the leader, and the term $\hat{u}_{0,i}$ to ensure that the follower applies maneuvers consistent with the leader's driving input.

Proceeding further, we propose the following input observer for follower i to estimate the leader's input u_0:

$$\begin{aligned}
\dot{z}_i =& - b_i l z_i - b_i^2 l^2 x_0 - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) \\
& - d_i \cdot \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i(\hat{u}_{0,i} - u_0)\right], \qquad (2.3a)\\
\hat{u}_{0,i} =& z_i + b_i l x_0, \qquad (2.3b)\\
\dot{d}_i =& \tau_i \left|\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i(\hat{u}_{0,i} - u_0)\right|, \qquad (2.3c)
\end{aligned}$$

where z_i is the observer's internal state, l a scalar gain, d_i an adaptive gain and τ_i is a positive scalar. This design is inspired by an unknown disturbance observer developed in [55]. However, we introduce two significant modifications. First, the original design in [55] is a centralized observer for a single plant, whereas in this case it has been transformed to achieve distributed input estimation among a group of agents. Second, an adaptive mechanism is developed to enable a dynamic adjustment for the gain d_i, as shown in (2.3c), which helps avoid the cumbersome or inefficient gain selection procedure that would be necessary otherwise.

Building on the estimation of u_0 through (2.3), a position observer is designed as follows:

$$\dot{\hat{x}}_{0,i} = -c\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i} - \hat{x}_{0,j}) + b_i(\hat{x}_{0,i} - x_0)\right] + \hat{u}_{0,i}, \qquad (2.4)$$

where c is a scalar gain. Note that the term $-\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i} - \hat{x}_{0,j}) - b_i(\hat{x}_{0,i} - x_0)$ can help the observer overcome the error of the initial guess using neighborhood position estimation difference. The term $\hat{u}_{0,i}$ is to ensure that the observer's input is consistent with the leader's actual input u_0. With such a design, it is anticipated that $\hat{x}_{0,i}$ can converge to x_0.

Combining (2.2)-(2.4), we obtain a complete description of an input-observer-based controller. Next, we will prove its convergence.

2.3.2 Convergence Analysis

To analyze its convergence properties, the next assumption and lemmas are needed.

Assumption 1. The input $u_0 \in \mathcal{C}^1$, and its first-order derivative is bounded and satisfies $|\dot{u}_0| \leq w < \infty$, where w is unknown.

This assumption is mild and reasonable, since the leader's maneuver input u_0 should be smooth and bounded in rate-of-change due to practical control actuation limits. In addition, we assume that the bound for the rate-of-change does not have to be known. This reduces the amount of information about the leader that must be available to followers. It may also help avoid potential conservatism in control design caused by a bound set too large.

Define $e_{u,i} = \hat{u}_{0,i} - u_0$, which is the input estimation error. According to (2.3), the closed-loop dynamics of $e_{u,i}$ can be written as

$$\begin{aligned} \dot{e}_{u,i} =& \dot{\hat{u}}_{0,i} - \dot{u}_0 = \dot{z}_i + b_i l \dot{x}_0 - \dot{u}_0 \\ =& - b_i l e_{u,i} - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) - \dot{u}_0 \\ & - d_i \cdot \operatorname{sgn}\left[\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i (\hat{u}_{0,i} - u_0) \right]. \end{aligned} \tag{2.5}$$

Let us define $e_u = \begin{bmatrix} e_{u,1} & e_{u,2} & \cdots & e_{u,N} \end{bmatrix}^\top$. It then follows from (2.5) that

$$\dot{e}_u = -H_1 e_u - D \cdot \operatorname{sgn}(H_1 e_u) - \dot{u}_0 \mathbf{1}, \tag{2.6}$$

where $H_1 = lB + L$ and $D = \operatorname{diag}(d_1, \ldots, d_N)$. The convergence of e_u to zero is shown in the following lemma.

Lemma 3. If Assumption 1 holds, the input estimation $\hat{u}_{0,i}$ of (2.3) can track the input u_0 asymptotically with $\lim_{t\to\infty} e_u = 0$.

Proof: By Lemmas 1 and 2, H_1 is positive definite. Consider the Lyapunov function $V(e_u, d_i) = \frac{1}{2}e_u^\top H_1 e_u + \sum_{i=1}^{N} \frac{(d_i-\beta)^2}{2\tau_i}$ for the input estimation error dynamics in (2.6), where β is a positive constant. The derivative of $V(e_u, d_i)$ is given by

$$
\begin{aligned}
\dot{V} &= -e_u^\top H_1^2 e_u - e_u^\top H_1 D \cdot \mathrm{sgn}(H_1 e_u) - e_u^\top H_1 \dot{u}_0 \mathbf{1} + \sum_{i=1}^{N} \frac{(d_i-\beta)\dot{d}_i}{\tau_i} \\
&\leq -\sum_{i=1}^{N} d_i \left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i(\hat{u}_{0,i} - u_0) \right)^\top \\
&\quad \cdot \mathrm{sgn}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i(\hat{u}_{0,i} - u_0) \right) - e_u^\top H_1^2 e_u + \sum_{i=1}^{N} \frac{(d_i-\beta)\dot{d}_i}{\tau_i} + w\|H_1 e_u\|_1 \\
&= -\sum_{i=1}^{N} d_i \left| \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + l b_i(\hat{u}_{0,i} - u_0) \right| - e_u^\top H_1^2 e_u + w\|H_1 e_u\|_1 \\
&\quad + \sum_{i=1}^{N} (d_i - \beta) \left| \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right| \\
&= -e_u^\top H_1^2 e_u - (\beta - w)\|H_1 e_u\|_1.
\end{aligned} \tag{2.7}
$$

It is noted that $e_u^\top H_1^2 e_u \geq 0$. Then, given w, there always exists a β that guarantees $\beta \geq w$. So we can obtain $\dot{V} \leq 0$ from (2.7), which indicates that $V(e_u, d_i)$ is non-increasing. Therefore, one can see from the Lyapunov function that e_u and d_i are bounded. By noting that $\tau_i > 0$, it follows from (2.3c) that d_i is monotonically increasing. Thus, the boundedness of d_i indicates that each d_i converges to some finite value. In the meantime, $V(e_u, d_i)$ reaches a finite limit as it is decreasing and lower-bounded by zero. Let us define $s(t) = \int_0^t e_u^\top(\tau) H_1^2 e_u(\tau) d\tau$. It is obtained that $s(t) \leq V(0) - V(t)$ by integrating $\dot{V} \leq -e_u^\top H_1^2 e_u$. Thus, $\lim_{t\to\infty} s(t)$ exists and is finite. Due to the boundedness of e_u and $\dot{e}_u$, $\ddot{s}$ is also bounded. This shows that $\dot{s}$ is uniformly continuous. Hence, $\lim_{t\to\infty} \dot{s}(t) = 0$ by Barbalat's Lemma [56], indicating that $\lim_{t\to\infty} e_u = 0$. It is noted that (2.5) is globally asymptotically stable. ■

Lemma 3 indicates that each follower can successfully estimate the control input u_0 with the proposed input observer. We will next analyze the asymptotic stability of the position observer. The input-to-state stability lemma will be used.

Lemma 4. [56] Consider an input-to-state stable (ISS) nonlinear system $\dot{x} = F(x,w)$. If the input satisfies $\lim_{t\to\infty} w(t) = 0$, then the state $\lim_{t\to\infty} x(t) = 0$.

Define follower i's position estimation error as $e_{x,i} = \hat{x}_{0,i} - x_0$. The error vector for all followers is denoted as $e_x = \begin{bmatrix} e_{x,1} & e_{x,2} & \cdots & e_{x,N} \end{bmatrix}^\top$. It can be derived from (2.4) that

$$\dot{e}_x = -cH_2 e_x + e_u, \tag{2.8}$$

where $H_2 = B + L$.

Lemma 5. If Assumption 1 holds, the system in (2.8) is asymptotically stable with $\lim_{t\to\infty} e_x = 0$, if the observer gain c is chosen such that $c > 0$.

Proof: According to Lemmas 1 and 2, H_2 is positive definite. Then, the system in (2.8) is ISS, and as a result, $\lim_{t\to\infty} e_x = 0$ holds. ■

The above lemma shows the effectiveness of the proposed position observer for a follower to estimate the leader's x_0. Now, let us prove the convergence of the tracking control. Define follower i's tracking error as $e_i = x_i - x_0$, and put together e_i for $i = 1,2,\ldots,N$ to form the vector $e = \begin{bmatrix} e_1 & e_2 & \cdots & e_N \end{bmatrix}^\top$. Using (2.1) and (2.2), it can be derived that the dynamics of e is governed by

$$\dot{e} = -k_1 e + k_1 e_x + e_u. \tag{2.9}$$

The theorem below shows that e will approach 0 as $t \to \infty$.

Theorem 1. Suppose that Assumption 1 is satisfied. If the observer gain is chosen such that $k_1 > 0$ holds, the system in (2.9) is asymptotically stable, and $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ for $i = 1,2,\ldots,N$.

Proof: It can be obtained from Lemma 4 that the system in (2.9) is ISS if $k_1 > 0$. Therefore, $\lim_{t\to\infty} e = 0$ results from the analysis in Lemmas 3 and 5. This completes the proof. ■

Theorem 1 shows that the proposed tracking control approach would enable each follower to approach and track the leader as time goes by, with the position tracking error converging to zero. The following remark further summarizes its difference from some existing methods and advantages.

Remark 1. The input-observer-based tracking control approach proposed above presents a few advantages over many existing methods. First, for this approach, a follower only needs to interchange information with its neighbors. By comparison, some studies in the literature requires that the leader's input must be known by any follower even if it is not a neighbor of the leader, e.g., [1, 37, 42, 57]. Second, the followers do not have to be given information about the leader's controller. This contrasts with [48], which stipulates that every follower knows the leader's exact control law, and with [58], which requires the upper bound of the leader's control input to be known by all followers. Finally, the approach relaxes the assumption about the leader's control input. Here, a bound is only imposed on its rate-of-change rather than its magnitude as in [59]. This implies that this approach can apply to the case when the leader applies high-magnitude maneuvers. In particular, the bound of rate-of-change does not have to be known for the control design, further conducive to practical application of the proposed approach. •

2.3.3 Extension to a Simplified Case

A general case is considered above that the leader's input u_0 has a bounded rate-of-change. However, it is also practically meaningful in reality to consider a special case when the time derivative of u_0 becomes zero as time goes by. In other words, whatever the leader's movement is like at the beginning time, it gradually transitions to and maintains constant-speed movement. An example is a group of aerial vehicles tracking a leader that cruises at a stable speed to achieve high-quality photographing [60]. This setting is also of considerable interest in the literature, e.g., [61]. Along this line, let us consider that the rate-of-change of u_0 approaches zero, i.e., $\lim_{t\to\infty} \dot{u}_0(t) = 0$. To deal with this case, we can reduce the input observer in (2.3) to the following form, which is structurally more concise:

$$\dot{z}_i = -b_i l z_i - b_i^2 l^2 x_0 - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}), \tag{2.10a}$$

$$\hat{u}_{0,i} = z_i + b_i l x_0. \tag{2.10b}$$

When this observer is integrated into the controller in (2.2), effective tracking can be guaranteed under relaxed conditions. This argument is presented in the following corollary. The proof is straightforward and thus omitted here.

Corollary 1. Consider the systems in (2.1) and assume that $\lim_{t\to\infty} \dot{u}_0(t) = 0$. Suppose that the controller in (2.2) is applied together with the position observer in (2.4) and input observer in (2.10). Then, $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ for $i = 1, 2, \ldots, N$ if the control gain $k_1 > 0$ and the observer gain $c > 0$.

Remark 2. In addition to structural conciseness, it is noted that this input observer does not require the leader's input information if compared to the one in (2.3). This indicates that the leader does not even have to send its input to its neighbors in the considered setting, as a further advantage in practice. •

2.4 Second-order Leader-follower Tracking

This section considers leader-follower tracking for agents with second-order dynamics. Now, the leader and followers are described as

$$\begin{cases} \dot{x}_i = v_i, & x_i \in \mathbb{R}, \\ \dot{v}_i = u_i, & v_i \in \mathbb{R},\ i = 0, 1, \ldots, N, \end{cases} \tag{2.11}$$

where x_i is the position, v_i the velocity and u_i the input force. Still, agent 0 is the leader, and the other agents numbered from 1 to N are followers. It is considered here that no velocity sensor is used by the leader and followers, i.e., v_i for $i = 0, 1, \ldots, N$ is not measured. Akin to the first-order

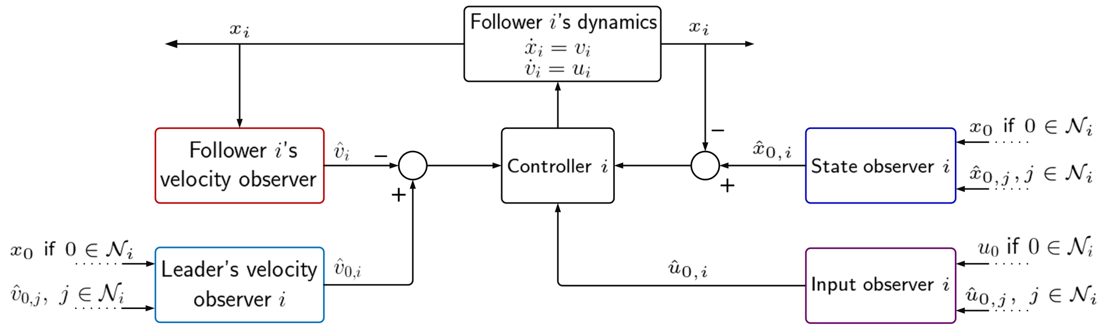

Figure 2.2: Input-observer-based framework for second-order leader-follower tracking.

case, our aim here is still to design a distributed control approach for each follower to track the leader, achieving $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ and $\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0$.

To address this second-order tracking problem, we continue to leverage the design thinking of input-observer-based control. The specific design can be laid out in two main steps. First, a linear continuous tracking controller is proposed for a follower, which uses the follower's position measurement and a few estimates, including its own velocity and the leader's position, velocity and input. Second, a series of observers are progressively developed to obtain the needed estimates. An input observer is designed such that the follower can reconstruct the leader's input. This is followed by the development of two observers that permit it to estimate the leader's velocity and position, respectively. Another observer is also proposed to help the follower determine its own velocity. Combining these observers with the controller then enables tracking control. This framework is schematically illustrated in Figure 2.2.

Along the above line, we start with proposing a control law for follower i, which is given by

$$u_i = -k_1(x_i - \hat{x}_{0,i}) - k_2(\hat{v}_i - \hat{v}_{0,i}) + \hat{u}_{0,i}, \tag{2.12}$$

where $k_1 > 0$ and $k_2 > 0$ are the controller gains. Here, $\hat{x}_{0,i}$, $\hat{v}_{0,i}$ and $\hat{u}_{0,i}$ are follower i's estimates of the leader's position x_0, velocity v_0 and input u_0, and $\hat{v}_i$ represents agent i's estimate of its own velocity v_i. Furthermore, the term $x_i - \hat{x}_{0,i}$ is used to propel follower i to move toward the leader, and the term $\hat{v}_i - \hat{v}_{0,i}$ to synchronize its velocity with the leader's. The term $\hat{u}_{0,i}$ is intended to maintain follower's maneuver at the same level with the leader. Next, we build observers to obtain $\hat{u}_{0,i}$, $\hat{v}_i$, $\hat{v}_{0,i}$ and $\hat{x}_{0,i}$.

We firstly propose an input observer to estimate u_0 as follows:

$$\begin{aligned}\dot{\hat{u}}_{0,i} =& -\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) - b_i(\hat{u}_{0,i}-u_0)\\ & - d_i\cdot \mathrm{sgn}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) + b_i(\hat{u}_{0,i}-u_0)\right], \end{aligned} \tag{2.13a}$$

$$\dot{d}_i = \tau_i\left|\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) + b_i(\hat{u}_{0,i}-u_0)\right|. \tag{2.13b}$$

Here, the term $-\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) - b_i(\hat{u}_{0,i}-u_0)$ is used to drive $\hat{u}_{0,i}$ toward approaching u_0; the $\mathrm{sgn}(\cdot)$ term is employed to maintain synchronization between $\hat{u}_{0,i}$ and u_0 in the presence of $\dot{u}_0$. It is seen that this observer does not require position x_0 measurement, differing from the one proposed earlier in (2.3). Note that this input observer is also applicable to the first-order case with provable asymptotic stability. In other words, if it replaces (2.3), the first-order tracking control can still be achieved under some mild conditions. This implies that one can design different kinds of observers to achieve estimation of the leader's input. Then, $\hat{u}_{0,i}$ can be used to estimate v_0 using the observer

$$\dot{z}_i = -b_i l z_i - b_i^2 l^2 x_0 - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{v}_{0,i}-\hat{v}_{0,j}) + \hat{u}_{0,i}, \tag{2.14a}$$

$$\hat{v}_{0,i} = z_i + b_i l x_0, \tag{2.14b}$$

where z_i, l, and $\hat{v}_{0,i}$ are the internal state of the observer, the observer gain, and the estimate of v_0, respectively. This velocity observer, as is seen, allows distributed estimation of the leader's velocity among all agents, even though it is not measured by a sensor. On such a basis, a position observer is designed for follower i to estimate x_0:

$$\dot{\hat{x}}_{0,i} = -c\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i}-\hat{x}_{0,j}) + b_i(\hat{x}_{0,i}-x_0)\right] + \hat{v}_{0,i}. \tag{2.15}$$

Finally, follower i uses the following observer to estimate its own velocity as it also has no velocity sensor:

$$\dot{\bar{z}}_i = -l\bar{z}_i - l^2 x_i + u_i$$
$$\hat{v}_i = \bar{z}_i + l x_i, \tag{2.16}$$

where $\bar{z}_i$ is the internal state of the observer. Putting together the above observers (2.13)-(2.16) with the controller (2.12), we can obtain a tracking control approach. Its convergence will be analyzed next. Yet before proceeding to the proof, we remark that Assumption 1 is also needed here and for simplicity do not restate it. In addition, the following lemmas will be used.

Lemma 6. [62] Let $Q = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where A, B, C, $D \in \mathbb{R}^{n\times n}$. Then $\det(Q) = \det(AD - BC)$, if matrix A, B, C and D commute pairwise.

Lemma 7. [63] Given a complex coefficient polynomial of order two as follows:

$$h(s) = s^2 + (a_1 + \mathbf{i}b_1)s + a_0 + \mathbf{i}b_0, \tag{2.17}$$

where $\mathbf{i} = \sqrt{-1}$; a_1, b_1, a_0 and b_0 are real constraints. Then, $h(s)$ is stable if and only if $a_1 > 0$ and $a_1 b_1 b_0 + a_1^2 a_0 - b_0^2 > 0$.

The following theorem is the main result regarding the convergence of the proposed tracking controller.

Theorem 2. Suppose that Assumption 1 holds and apply the proposed control approach (2.12)-(2.16) to the considered second-order systems in (2.11). If $k_1 > 0$, $k_2 > 0$, $l > 0$ and $c > 0$, then $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ and $\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0$.

Proof: It can be derived from (2.13) that the dynamics of the input estimation error e_u is given by

$$\dot{e}_u = -H_2 e_u - D \cdot \operatorname{sgn}(H_2 e_u) - \dot{u}_0 \mathbf{1}. \tag{2.18}$$

Along similar lines to the proof of Lemma 3, the above system is asymptotically stable, i.e., $\lim_{t\to\infty} e_u = 0$.

Define the velocity estimation error $e_{0v,i}$ as $e_{0v,i} = \hat{v}_{0,i} - v_0$. According to (2.14), the dynamics of $e_{0v,i}$ can be written as

$$\dot{e}_{0v,i} = \dot{\hat{v}}_{0,i} - \dot{v}_0 = -b_i l e_{0v,i} - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{v}_{0,i} - \hat{v}_{0,j}) + \hat{u}_{0,i} - u_0. \tag{2.19}$$

Further, let us define the vector $e_{0v} = \begin{bmatrix} e_{0v,1} & e_{0v,2} & \cdots & e_{0v,N} \end{bmatrix}^\top$. The dynamics of e_{0v} then can be obtained from (2.19), which is

$$\dot{e}_{0v} = -H_1 e_{0v} + e_u. \tag{2.20}$$

Because of $\lim_{t\to\infty} e_u = 0$ and the ISS result in Lemma 4, it can be concluded that $\lim_{t\to\infty} e_{0v} = 0$.

By (2.15), the position estimation error vector e_x, which shares the same definition as in the first-order case, is governed by the following dynamics equation:

$$\dot{e}_x = -cH_2 e_x + e_{0v}. \tag{2.21}$$

According to Lemma 4, the system in (2.21) is ISS. Since $\lim_{t\to\infty} e_{0v} = 0$, we have $\lim_{t\to\infty} e_x = 0$.

Now we consider a follower's estimation error for its own velocity. Denote $e_{v,i} = \hat{v}_i - v_i$ and $e_v = \begin{bmatrix} e_{v,1} & e_{v,2} & \cdots & e_{v,N} \end{bmatrix}^\top$. We can derive the dynamics of e_v from (2.16), which is

$$\dot{e}_v = -l e_v. \tag{2.22}$$

Obviously, $\lim_{t\to\infty} e_v = 0$ if $l > 0$.

Consider the leader and followers in (2.11) under the control law (2.12), one can obtain follower's closed-loop dynamics:

$$\dot{x}_i - \dot{x}_0 = v_i - v_0, \tag{2.23a}$$

$$\begin{aligned}\dot{v}_i - \dot{v}_0 = &-k_1(x_i - x_0) - k_2(v_i - v_0) - k_2(\hat{v}_i - v_i)\\ &+ k_2(\hat{v}_{0,i} - v_0) + k_1(\hat{x}_{0,i} - x_0) + \hat{u}_{0,i} - u_0,\end{aligned} \tag{2.23b}$$

for $i = 1, 2, \ldots, N$. Define $e = \begin{bmatrix} x_1 - x_0 & \cdots & x_N - x_0 & v_1 - v_0 & \cdots & v_N - v_0 \end{bmatrix}^\top$. Then, combining (2.18), (2.22) and (2.23), we have the closed-loop tracking error dynamics of the entire leader-follower system:

$$\dot{e} = F_1 e + F_2, \tag{2.24}$$

where

$$F_1 = \begin{bmatrix} 0 & I \\ -k_1 I & -k_2 I \end{bmatrix},\; F_2 = \begin{bmatrix} 0 \\ -k_2 e_v + k_2 e_{0v} + k_1 e_x + e_u \end{bmatrix}.$$

Furthermore, according to Lemma 6, the characteristic polynomial of F_1 is given by

$$\begin{aligned}\det(sI - F_1) &= \det\left(\begin{bmatrix} sI & -I \\ k_1 I & sI + k_2 I \end{bmatrix}\right)\\ &= \det(s^2 I + k_2 s I + k_1 I)\\ &= \prod_{i=1}^{N}(s^2 + k_2 s + k_1) = \prod_{i=1}^{N} h_i(s).\end{aligned} \tag{2.25}$$

Based on Lemma 7, $h_i(s)$ is stable when $k_1 > 0$ and $k_2 > 0$. With this result, the system (2.24) is ISS as $\lim_{t\to\infty} F_2 = 0$ from (2.18)-(2.22). Hence, $\lim_{t\to\infty} e = 0$, which implies $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ and $\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0$. This completes the proof. ■

Remark 3. This proposed tracking control approach offers some merits when compared with the literature. First, it does not require a follower to know the leader's input or velocity if they are not neighbors, differing from [37, 41, 42, 58]. This is similar to the approach in Section 2.3 and attributed to the input and velocity observers giving a follower a crucial "leader-awareness". Second, this approach can enable accurate tracking in the absence of velocity sensors. Recent years have seen a growing interest in tracking control without velocity measurements due to its practical benefits. Our proposed approach is different from the present methods in some interesting ways. Through the velocity observers, it makes an explicit estimation of the leader's and follower's velocities. This differs from [47, 64, 65], which make no velocity estimation and use only neighborhood position difference to achieve velocity-free tracking control. Velocity observer design is also considered in [42]. However, the design therein requires the leader's input force to be known by every follower. By contrast, our approach obviates this need because the input observer can infer the leader's input. •

2.5 Simulation Result

In this section, we provide an illustrative example to verify the effectiveness of the proposed distributed control algorithm. Consider an MAS consisting of one leader and five followers. The communication topology among them is shown in Figure 2.3a. Node 0 is the leader, and nodes 1 to 5 are followers. The leader will only send information updates to follower 1, and the followers maintain undirectional communication with their neighbors.

We consider the second-order tracking. The actual initial positions and velocities of the leader and followers are set to be $x(0) = \begin{bmatrix} 0 & 3 & 0 & -2 & 1 & -1 \end{bmatrix}^\top$ and $v(0) = \begin{bmatrix} 0 & 1 & -2 & 3 & 0 & -1 \end{bmatrix}^\top$. Figure 2.3 summarizes the simulation results when the tracking algorithm in Section 2.4 is applied. Looking at the position trajectories of all followers and the leader in Figure 2.3b, one can see that all followers catch up with the leader after around 25 seconds and then well continue the tracking. Associated with this position tracking, Figure 2.3c further illustrates the velocity tracking, which exhibits satisfactory convergence. The leader's velocity and the followers' estimation are shown in Figure 2.3d. It is seen that the velocity estimation by each follower converges to the truth at around the 12th second. Figure 2.3e demonstrates that each follower begins to get accurate estimate of its own velocity at around the tenth second and then keeps an accurate estimation. The time-based evolution of the leader's acceleration and its estimation by the followers is further shown in Figure 2.3f. From this figure, the input observers of all the followers can capture the truth quickly in about three seconds, showing the effectiveness of estimation. Figure 2.3g illustrates the leader's position and the locally estimated profiles, between which there is a good agreement. Finally, Figure 2.3h shows the leader and followers' control input profiles, which gradually become the same. Through the above results and many others simulation runs, we consistently observe that the proposed input-observer-based tracking control algorithms can provide effective performance.

2.6 Conclusion

Leader-follower tracking represents an important task in diverse MAS mission contexts, which has been seeing a rapid rise of interest from researchers. In this paper, we proposed a novel input-observer-based perspective into distributed tracking control design. Advancing the idea of observer-based tracking control in the literature, we highlighted that observers can be designed for a follower to directly estimate the leader's maneuver input and leverage the estimation to enhance tracking control. To this end, we developed distributed input observers along with some other observers and on such a basis, formulated a new tracking control framework. We conducted the study for second-order MASs, with a control approach developed for each case. We also pointed out that our approaches can help overcome a few limitations presented by some existing methods. We further validated its effectiveness by numerical simulation.

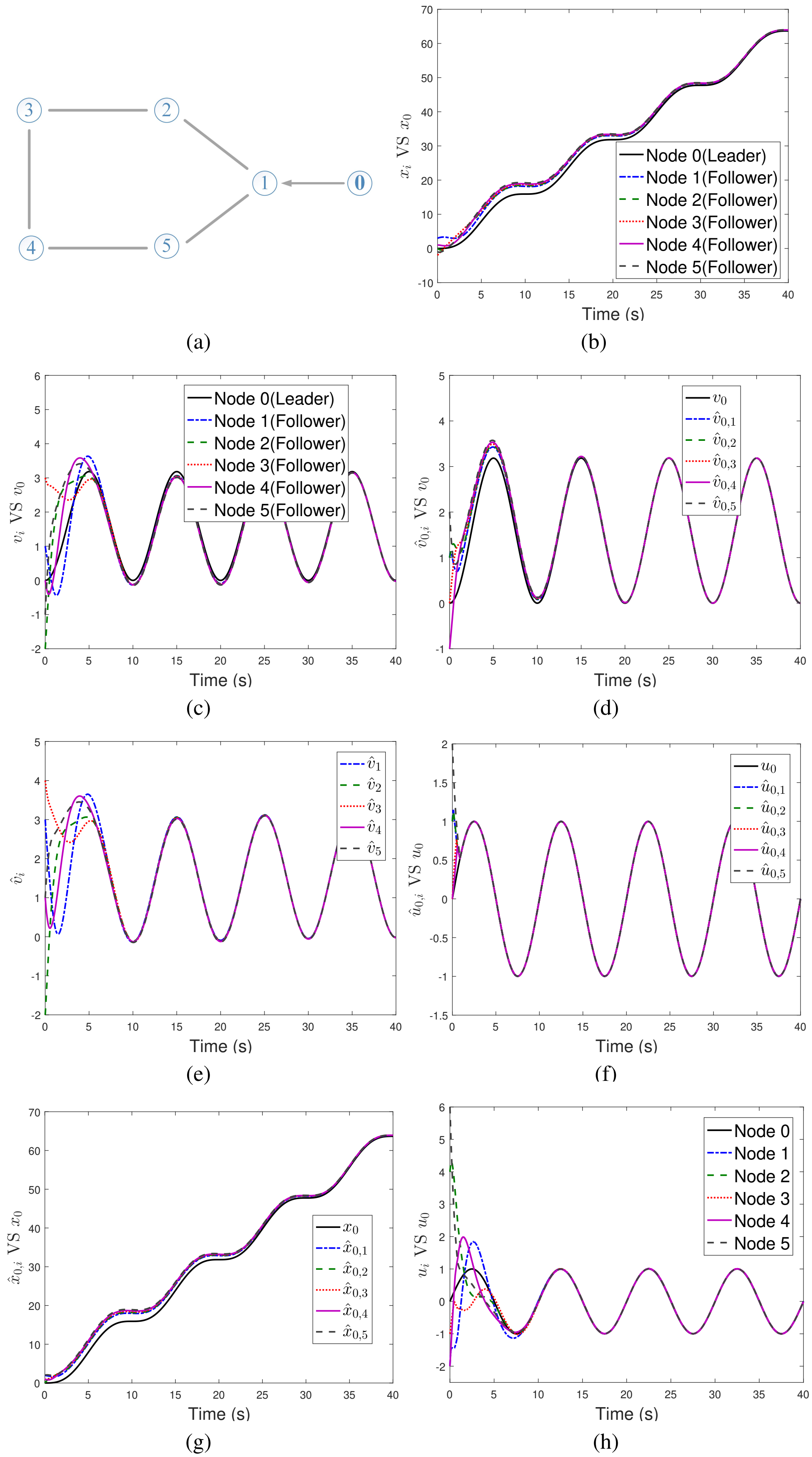

Figure 2.3: Tracking control for an MAS: (a) communication topology (b) position tracking; (c) velocity tracking; (d) followers' estimation of the leader's velocity; (e) follower's estimation of their own velocities; (f) followers' estimation of the leader's input; (g) followers' estimation of the leader's position; (h) followers' input in comparison with the leader's.

Chapter 3

A New Encounter Between Leader-Follower Tracking and Observer-Based Control: Towards Enhancing Robustness against Disturbances[1]

3.1 Introduction

A large body of work has been developed recently to deal with leader-follower tracking control design under diverse challenging situations, e.g., complex dynamics, communication delays, noisy measurements, switching topologies, and limited energy budget, see [36, 37, 41, 44, 57, 68–78] and the references therein. However, a problem that has received inadequate attention to date is the case when the agents are subjected to disturbances. In a real world, disturbances can result from unmodeled dynamics, change in ambient conditions, inherent variability of the dynamic process, and sensor noises. They can cause degradation and even failure of tracking control if not well addressed.

A lead is taken in [37] with the study of disturbance-robust leader-follower tracking. It presents a distributed control design that achieves tracking with a bounded error when magnitude-bounded disturbances affect the followers. This notion is extended in [38] to make the followers affected by disturbances enter a bounded region centered around the leader in finite time. Another finite-time tracking control approach is offered in [39], where the sliding mode control technique is used to suppress the effects of disturbances. It is noted that, while the control designs in these works

[1]This chapter is based on the dissertation author's first-authored journal paper [66] and first-authored conference paper [67].

yield robustness, they are based on upper bounds of the disturbances. By contrast, a different way is to capture the disturbances by designing some observers and then adjust the control run based on the disturbance estimation. Obtaining an explicit knowledge of disturbances, this approach can advantageously reduce conservatism in control and thus enhance the tracking performance further. In [40, 79], disturbance observers are developed and integrated into tracking controllers such that a follower can estimate and offset the local disturbance interfering with its dynamics during tracking. The results in both studies point to the effectiveness of disturbance observers for improving tracking accuracy — for instance, the tracking errors can approach zero despite non-zero disturbances under certain conditions. However, other than these two, there are no more studies on this subject to the best of our knowledge. This leaves many problems still open. Meanwhile, the potential of the disturbance-observer-based approach is still far from being fully explored. It is noteworthy that observer-based tracking control has been investigated in a few works, e.g., [37, 38, 61, 80, 81], but observers in these studies are meant to infer various state or input variables rather than disturbances.

In this study, we uniquely focus on an open problem: can we enable distributed tracking control when not only the followers but also the leader are affected by unknown disturbances and when only the rates of change of the disturbances are bounded? The state of the art, e.g., [37–40, 79], generally considers that disturbances plague just the followers and that they are bounded in magnitude or approach fixed values as time goes by. The leader's dynamics, however, can also involve disturbances from a practical viewpoint. For example, consider an MAS composed of a few mobile ground robots, the changes in the slope of the road act as disturbances on every robot including the leader. The same can be said for the wind affecting a group of unmanned aerial vehicles. Such disturbances are more difficult to be rejected because the leader cannot measure them and share the information with any of the followers. Therefore, the tracking performance may be damaged when this occurs. Furthermore, it is usually desirable to relax the assumptions about disturbances to enhance the practical robustness of the control design. In [38–40, 79], the disturbances are assumed to be bounded in magnitude. However, we wish to require the disturbances to be bounded in rates of change. This relaxation will be realistically beneficial for dealing with large disturbances but also present more complexity to capture and suppress the disturbances. It must be pointed out that the observer designs in [37–40, 61, 79–81] cannot be extended to deal with the considered problem, due to the more challenging presence and nature of the disturbances. Hence, a solution is still absent from the literature.

To address the above problem, we develop a novel observer-based distributed tracking control framework, which is the main contribution of this paper. Different from the previous studies, this framework builds on the notion that a follower can gain a real-time awareness of not only its own but also the leader's dynamics through distributed estimation. We hence design a set of new observers and particularly, distributed disturbance observers that, for the first time, can enable the followers to collectively infer the disturbance affecting the leader. We perform the observer-based tracking control design for both first- and second-order MASs. We then conduct theoretical analysis of the proposed approaches. We show that, even though disturbances are imposed on all the agents, the tracking errors are still upper bounded (bounded-error tracking) as long as the rates of change of the disturbances are bounded. Further, the tracking errors will approach zero (zero-error tracking) if the disturbances converge to certain fixed points. We finally present simulations to validate the proposed approaches.

The rest of this chapter is organized as follows. Section 3.2 introduces some preliminaries. Section 3.3 considers a leader-follower MAS with first-order dynamics, develops an observer-based distributed tracking control approach, and analyzes its performance rigorously. Section 3.4 proceeds to study the second-order MASs and develops a more sophisticated tracking control approach. Numerical simulation is offered in Section 3.5 to illustrate the effectiveness of the proposed results. Finally, Section 3.6 gathers our concluding remarks.

3.2 Preliminaries

This section introduces notation and an assumption. The notation used throughout this chapter is standard. We let $\mathrm{diag}(\dots)$ and $\det(\cdot)$ represent a block-diagonal matrix and the determinant of a matrix, respectively. The eigenvalues of an $N \times N$ matrix are $\lambda_i(\cdot)$ for $i = 1, 2, \dots, N$. The minimum and maximum eigenvalues of a real, symmetric matrix are denoted as $\underline{\lambda}(\cdot)$ and $\bar{\lambda}(\cdot)$. A $\mathcal{C}^k$ function is a function with k continuous derivatives.

Throughout this chapter, we consider a leader-follower MAS with $N+1$ agents. The agents are numbered sequentially. The one numbered as 0 serves as the leader, and the other agents are followers. Each agent is driven by an input u_i and simultaneously affected by an external disturbance f_i for $i = 0, 1, \dots, N$. Here, we still apply Assumption 1 for leader's input. In addition, we make the following assumption.

Assumption 2. The external disturbance f_i for $i = 0, 1, \dots, N$ has a bounded first-order derivative, i.e., $\|\dot{f}_0 \mathbf{1}_{N\times 1}\| \leq q_0$ and $\left\| \begin{bmatrix} \dot{f}_1 & \dot{f}_2 & \cdots & \dot{f}_N \end{bmatrix}^\top \right\| \leq q_1$, where $q_0, q_1 \geq 0$.

3.3 First-Order Leader-Follower Tracking

This section studies first-order leader-follower tracking with disturbances. We develop an observer-based control approach, pivoting the design on a set of observers to make a follower aware of the leader's and its own disturbances. We further analyze the closed-loop stability of the proposed approach.

3.3.1 Problem Formulation

Consider an MAS with $N+1$ agents, in which agent 0 is the leader and the others are followers. An agent's dynamics is given by

$$\dot{x}_i = u_i + f_i, \quad x_i, u_i, f_i \in \mathbb{R}, \quad i = 0, 1, \dots, N, \tag{3.1}$$

where x_i is the position, u_i the control input equivalent to the velocity maneuver, and f_i the unknown disturbance. Suppose that Assumptions 1-2 hold. Here, the objective is to design a distributed control law for u_i such that each follower can control its dynamics to track the leader's trajectory *via* exchanging information with its neighbors.

Remark 4. Compared with previous studies, the problem setting here is more generic and applicable to a wide range of practical scenarios. Below, we outline a comparison with [37–40, 79], which are the main references about tracking control with disturbances and henceforth referred to as the existing literature. First, this work considers an input-driven leader, while the leader is usually assumed to be input-free in the literature. Assumption 1 only requires the leader's input to be bounded in rate of change (with the bound unknown), which can be easily satisfied since practical actuators only allow limited ramp-ups. Second, Assumption 2 imposes disturbances on all the leader and follower agents, while the literature assumes only followers to be affected by disturbances. Note that the case when a disturbance is inflicted on the leader is nontrivial. This is because the leader's disturbance is very difficult to be determined by the followers, especially in a distributed network where many followers cannot directly interchange information with the leader. Further, the disturbances are assumed to have only bounded rates of change rather than bounded magnitude as required in the literature. This can be greatly useful for dealing with very large disturbances. From the comparison, we conclude that the considered problem is less restrictive than the predecessors, which still remains an open challenge. •

3.3.2 Proposed Algorithm and Stability Analysis

Given the above problem setting, we propose an observer-based tracking control approach. The development begins with the design of a distributed linear continuous controller for a follower (say, follower i). It crucially incorporates the estimation of three unknown variables, u_0, f_0 and f_i, enabling follower i to maneuver through simultaneously emulating the input and disturbance driving the leader and offsetting the local disturbance. We subsequently construct three observers to achieve the estimation to be integrated with the controller.

Considering follower i, we propose to design its controller as follows:

$$u_i = -k\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0)\right] + \hat{u}_{0,i} + \hat{f}_{0,i} - \hat{f}_i, \tag{3.2}$$

where $k > 0$ is the control gain, $\hat{f}_{0,i}$ and $\hat{u}_{0,i}$ are follower i's respective estimates of the leader's disturbance f_0 and input u_0, and $\hat{f}_i$ is follower i's estimate of its own disturbance f_i. Note that $b_i > 0$ if the leader is agent i's neighbor and $b_i = 0$ if it is not. In (3.2), the term $-\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) - b_i(x_i - x_0)$ is employed to drive follower i approaching the leader; the term $\hat{u}_{0,i} + \hat{f}_{0,i}$ ensures that follower i applies maneuvers consistent with the leader's input and disturbance; the term $-\hat{f}_i$ is used to cancel the local disturbance. For this controller, we build a series of observers as shown below to estimate u_0, f_0 and f_i, respectively.

To begin with, we leverage observer design in (2.13) to obtain $\hat{u}_{0,i}$, and its stability property also holds at here. Then the following disturbance observer is proposed for follower i to estimate f_0:

$$\dot{z}_{f0,i} = -b_i z_i - b_i^2 x_0 - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{f}_{0,i} - \hat{f}_{0,j}) - b_i u_0, \tag{3.3a}$$

$$\hat{f}_{0,i} = z_{f0,i} + b_i x_0, \tag{3.3b}$$

where $z_{f0,i}$ is the internal state. The design of (3.3) is inspired by [55], in which a centralized disturbance observer is developed for a single plant. Here, transforming the original design, we build the distributed observer as above such that follower i can estimate f_0 in a distributed manner.

The last observer, designed as follows, enables follower i able to infer the disturbance f_i inherent in its own dynamics:

$$\dot{z}_{f,i} = -l z_{f,i} - l^2 x_i + u_i, \tag{3.4a}$$

$$\hat{f}_i = z_{f,i} + l x_i. \tag{3.4b}$$

Here, $l > 0$ is the observer gain, and $z_{f,i}$ is this observer's internal state.

Combining (2.13) and (3.2)-(3.4), we obtain a complete description of an observer-based distributed tracking controller. Next, we will analyze its closed-loop stability.

Now, consider the distributed observer for f_0. Define $e_{0f,i} = \hat{f}_{0,i} - f_0$, which is follower i's estimation error for f_0. Using (3.3), the dynamics of $e_{0f,i}$ is given by

$$\dot{e}_{0f,i} = -b_i e_{0f,i} - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{f}_{0,i} - \hat{f}_{0,j}) - \dot{f}_0.$$

Then, defining $e_{0f} = \begin{bmatrix} e_{0f,1} & e_{0f,2} & \cdots & e_{0f,N} \end{bmatrix}^\top$, we have

$$\dot{e}_{0f} = -He_{0f} - \dot{f}_0 \mathbf{1}. \tag{3.5}$$

The following lemma reveals the upper boundedness of e_{0f} under Assumption 2.

Lemma 8. If Assumption 2 holds, then

$$\|e_{0f}(t)\| \leq \|e_{0f}(0)\| + \frac{q_0}{\underline{\lambda}(H)}, \;\; t > 0, \tag{3.6a}$$

$$\lim_{t\to\infty} \|e_{0f}(t)\| \leq \frac{q_0}{\underline{\lambda}(H)}. \tag{3.6b}$$

Proof: Consider the Lyapunov function candidate $V_2(e_{0f}) = \frac{1}{2} e_{0f}^\top e_{0f}$ for (3.5). According to Assumption 2, we have

$$\dot{V}_2(e_{0f}) = -e_{0f}^\top H e_{0f} - e_{0f}^\top \dot{f}_0 \mathbf{1} \leq -\underline{\lambda}(H)\|e_{0f}\|^2 + \|e_{0f}\| \|\dot{f}_0 \mathbf{1}\| \leq -\underline{\lambda}(H)\|e_{0f}\|^2 + q_0\|e_{0f}\|.$$

The above inequality can be rewritten as

$$\dot{V}_2 \leq -2\underline{\lambda}(H)V_2 + \sqrt{2} q_0 \sqrt{V_2}.$$

It then follows that

$$\sqrt{V_2(t)} \leq \sqrt{V_2(0)} e^{-\underline{\lambda}(H)t} + \frac{\sqrt{2} q_0}{2\underline{\lambda}(H)} \left(1 - e^{-\underline{\lambda}(H)t}\right) \leq \sqrt{V_2(0)} + \frac{\sqrt{2} q_0}{2\underline{\lambda}(H)}. \tag{3.7}$$

Then, (3.6a) can result from (3.7) because $\sqrt{V_2} = \frac{\sqrt{2}}{2}\|e_{0f}\|$. Meanwhile, for the first inequality in (3.7), taking the limits of both sides as $t \to \infty$ would yield (3.6b). •

For $\hat{f}_i$, define the error as $e_{f,i} = \hat{f}_i - f_i$ and further the vector $e_f = \begin{bmatrix} e_{f,1} & e_{f,2} & \dots & e_{f,N} \end{bmatrix}^\top$. By (3.4), the dynamics of e_f is governed by

$$\dot{e}_f = -le_f - \dot{f}, \tag{3.8}$$

where $\dot{f} = \begin{bmatrix} \dot{f}_1 & \dot{f}_2 & \cdots & \dot{f}_N \end{bmatrix}^\top$. The next lemma shows that the error e_f is bounded under Assumption 2. Its proof is similar to that of Lemma 8 and thus omitted here.

Lemma 9. If Assumption 2 holds, then

$$\|e_f(t)\| \le \|e_f(0)\| + \frac{q_1}{l}, \ \ t > 0$$
$$\lim_{t\to\infty} \|e_f(t)\| \le \frac{q_1}{l}.$$

With the above results , we are now in a good position to characterize the properties of the tracking error. Define follower i's tracking error as $e_i = x_i - x_0$, and put together e_i for $i = 1, 2, \dots, N$ to form the vector $e = \begin{bmatrix} e_1 & e_2 & \cdots & e_N \end{bmatrix}^\top$. Using (3.1) and (3.2), it can be derived that the dynamics of e can be described as

$$\dot{e} = -kHe + e_{0f} + e_u - e_f. \tag{3.9}$$

The following theorem provides a key technical result.

Theorem 3. Suppose that Assumptions 1 and 2 hold. Then,

$$\|e(t)\| \le \|e(0)\| + \frac{\|e_{0f}(0)\| + \|e_u(0)\| + \|e_f(0)\| + \frac{q_0}{\underline{\lambda}(H)} + \frac{q_1}{l}}{k\underline{\lambda}(H)}, \tag{3.10a}$$

$$\lim_{t\to\infty} \|e\| \le \frac{\frac{q_0}{\underline{\lambda}(H)} + \frac{q_1}{l}}{k\underline{\lambda}(H)}. \tag{3.10b}$$

Proof: Take the Lyapunov function candidate $V_3(e) = \frac{1}{2}e^\top e$ for (3.9). Consider its derivative:

$$
\begin{aligned}
\dot{V}_3 &= -ke^\top He + e^\top e_{0f} + e^\top e_u - e^\top e_f \\
&\leq -k\underline{\lambda}(H)\|e\|^2 + \|e\| \cdot \|e_{0f}\| + \|e\| \cdot \|e_u\| + \|e\| \cdot \|e_f\|,
\end{aligned}
$$

where $\underline{\lambda}(H) > 0$. Equivalently, we have

$$
\dot{V}_3 \leq -2k\underline{\lambda}(H)V_3 + \sqrt{2}(\|e_{0f}\| + \|e_u\| + \|e_f\|)\sqrt{V_3}.
$$

Then,

$$
\begin{aligned}
\sqrt{V_3(t)} &\leq \sqrt{V_3(0)}e^{-k\underline{\lambda}(H)t} + \frac{\sqrt{2}(\|e_{0f}(t)\| + \|e_u(t)\| + \|e_f(t)\|)}{2k\underline{\lambda}(H)}(1 - e^{-k\underline{\lambda}(H)t}) \\
&\leq \sqrt{V_3(0)} + \frac{\sqrt{2}(\|e_{0f}(t)\| + \|e_u(t)\| + \|e_f(t)\|)}{2k\underline{\lambda}(H)},
\end{aligned}
$$

which, based on Lemmas 3 and 8-9, indicates (3.10a)-(3.10b). •

Theorem 3 shows that the proposed observer-based controller can make each follower track the leader with bounded position errors despite the disturbances. Therefore, we can say that the influence of the disturbances is effectively suppressed and that tracking is achieved in a practically meaningful manner.

Remark 5. For the proposed controller, the tracking performance will be further improved if the disturbances satisfy some stricter conditions. In particular, it is noteworthy that perfect or zero-error tracking can be attained if the disturbances see their rates of change gradually settle down to zero, i.e., $\dot{f}_i(t) \to 0$ as $t \to \infty$ for $i = 0, 1, \ldots, N$. The proof can be developed following similar lines as above and is omitted here. •

3.4 Second-Order Leader-Follower Tracking

This section considers leader-follower tracking control for a second-order MAS. An agent's dynamics now involves position, velocity, acceleration and disturbance:

$$\begin{cases} \dot{x}_i = v_i, \quad x_i \in \mathbb{R}, \\ \dot{v}_i = u_i + f_i, \quad v_i \in \mathbb{R}, \end{cases} \tag{3.11}$$

for $i = 0, 1, \ldots, N$. Here, x_i is the position, v_i the velocity, u_i the acceleration input, and f_i the disturbance. Still, agent 0 is the leader, and the others are followers numbered from 1 to N. We continue to apply Assumptions 1-2 here and set the objective of making the followers achieve bounded-error tracking of the leader in the presence of the disturbances.

For a general problem formulation, we further assume that no velocity sensor is deployed on the leader and followers. Hence, there are no velocity measurements throughout the tracking process. The absence of the velocity information, together with the unknown disturbances, makes the tracking control problem more complex than in the first-order case, thus requiring a substantial sophistication of the observer-based control approach in Section 3.3. Here, we will custom build an observer-based tracking controller and develop new velocity and disturbance observers.

Consider follower i. We propose the following distributed controller:

$$u_i = -k \left[\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \right] - (\hat{v}_i - \hat{v}_{0,i}) + \hat{u}_{0,i} + \hat{f}_{0,i} - \hat{f}_i, \tag{3.12}$$

where $k > 0$ is the control gain. In addition, $\hat{u}_{0,i}$, $\hat{v}_{0,i}$, $\hat{f}_{0,i}$, $\hat{v}_i$ and $\hat{f}_i$ are, respectively, follower i's estimates of u_0, v_0, f_0, v_i and f_i. The terms $-\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) - b_i(x_i - x_0)$ and $-(\hat{v}_i - \hat{v}_{0,i})$ are used to enable the follower to track the leader in both position and velocity; the term $\hat{u}_{0,i} + \hat{f}_{0,i}$ is used to make the follower steer itself with a maneuvering input close to the combined input and disturbance driving the leader; the term $-\hat{f}_i$ is meant to offset the local disturbance.

With the above controller structure, we need to construct observers that can obtain the needed estimates. First, it is noted that the input observer of u_0 in (2.13) can be applied here without any change, and its convergence property as shown in Lemma 3 also holds in this case. Then, we develop the following observer such that follower i can estimate the leader's unknown velocity:

$$\dot{z}_{v0,i} = -b_i z_{v0,i} - b_i^2 x_0 - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{v}_{0,i} - \hat{v}_{0,j}) + \hat{f}_{0,i} + \hat{u}_{0,i}, \tag{3.13a}$$

$$\hat{v}_{0,i} = z_{v0,i} + b_i x_0, \tag{3.13b}$$

where $z_{v0,i}$ is the internal state of this observer. Follower i's observer for the leader's disturbance is then proposed as

$$\dot{z}_{f0,i} = -z_{f0,i} - \hat{v}_{0,i} - \hat{u}_{0,i}, \tag{3.14a}$$

$$\hat{f}_{0,i} = z_{f0,i} + \hat{v}_{0,i}. \tag{3.14b}$$

The next observer enables follower i to estimate its own velocity:

$$\dot{z}_{v,i} = -l z_{v,i} - l^2 x_i + \hat{f}_i + u_i, \tag{3.15a}$$

$$\hat{v}_i = z_{v,i} + l x_i. \tag{3.15b}$$

Here, $l > 0$ is the gain for this observer, and $z_{v,i}$ the internal state. The final observer is aimed to allow follower i to infer its local disturbance. It is designed as

$$\dot{z}_{f,i} = -z_{f,i} - \hat{v}_i - u_i, \tag{3.16a}$$

$$\hat{f}_i = z_{f,i} + \hat{v}_i, \tag{3.16b}$$

where $z_{f,i}$ is the internal state.

From above, a complete observer-based distributed tracking controller can be built by putting together the control law (3.12) and the observers in (2.13) and (3.13)-(3.16). The following theorem is the main result about the closed-loop stability of the proposed controller.

Theorem 4. Suppose that Assumptions 1-2 hold and apply the proposed distributed tracking controller given in (2.13) and (3.12)-(3.16) to the MAS in (3.11). Then, there exist $\delta > 0$ and $\varepsilon > 0$ such that

$$\left\| \begin{bmatrix} x_i(t) - x_0(t) \\ v_i(t) - v_0(t) \end{bmatrix} \right\| \leq \delta, \ \ t > 0, \tag{3.17a}$$

$$\lim_{t\to\infty} \left\| \begin{bmatrix} x_i(t) - x_0(t) \\ v_i(t) - v_0(t) \end{bmatrix} \right\| \leq \varepsilon, \tag{3.17b}$$

for $i = 1, 2, \ldots, N$.

Proof: The proof is organized into three parts. Part a) proves that the coupled observers in (3.13)-(3.14) yield bounded-error estimation of v_0 and f_0; Part b) shows that the observers in (3.15)-(3.16) lead to bounded errors when estimating v_i and f_i; finally, based on Parts a) and b), Part c) demonstrates the upper boundedness of the position and velocity tracking errors when the proposed controller is applied.

Part a): Define the estimation errors of the observers in (3.13)-(3.14) as $e_{0v,i} = \hat{v}_{0,i} - v_0$ and $e_{0f,i} = \hat{f}_{0,i} - f_0$. According to (3.11) and (3.13)-(3.14), their dynamics can be written as

$$\dot{e}_{0v,i} = -b_i e_{0v,i} - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{v}_{0,i} - \hat{v}_{0,j}) + \hat{u}_{0,i} - u_0 + \hat{f}_{0,i} - f_0,$$
$$\dot{e}_{0f,i} = -b_i e_{0v,i} - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{v}_{0,i} - \hat{v}_{0,j}) - \dot{f}_0.$$

Defining $e_{0vf} = \begin{bmatrix} e_{0v,1} & \ldots & e_{0v,N} & e_{0f,1} & \ldots & e_{0f,N} \end{bmatrix}^\top$, we have

$$\dot{e}_{0vf} = Q_1 e_{0vf} + \ell_1, \tag{3.18}$$

where

$$Q_1 = \begin{bmatrix} -H & I \\ -H & 0 \end{bmatrix}, \ \ell_1 = \begin{bmatrix} e_{0u} \\ -\dot{f}_0 \mathbf{1} \end{bmatrix}.$$

The characteristic polynomial of Q_1 is given by

$$\det(sI-Q_1)=\det\left(\begin{bmatrix} sI+H & -I \\ H & sI \end{bmatrix}\right)=\det(s^2I+Hs+H)=\prod_{i=1}^{N}\left[s^2+\lambda_i(H)s+\lambda_i(H)\right].$$

As is seen from above, the poles of Q_1 is stable since H is positive definite. Then, there must exist a positive definite matrix P_1 such that

$$P_1Q_1+Q_1^\top P_1=-I.$$

For (3.18), take a Lyapunov function $V_4(e_{0vf})=\frac{1}{2}e_{0vf}^\top P_1 e_{0vf}$. Consider its derivative:

$$\begin{aligned}\dot{V}_4&=-\frac{1}{2}e_{0vf}^\top(P_1Q_1+Q_1^\top P_1)e_{0vf}+e_{0vf}^\top P_1\ell_1\\&\le-\frac{1}{2}\|e_{0vf}\|^2+\|e_{0vf}\|\|P_1\|\|\ell_1\|\\&\le-\frac{1}{2}\|e_{0vf}\|^2+\sqrt{e_u^2(0)+q_0^2}\|P_1\|\|e_{0vf}\|,\end{aligned}$$

where the fact suggested by Lemma 3 that e_u exponentially decreases to zero is used. The above inequality can be written equivalently as

$$\dot{V}_4\le-\alpha_1V_4+\beta_1\sqrt{V_4},$$

with $\alpha_1=1/\bar{\lambda}(P_1)$ and $\beta_1=\sqrt{2(e_u^2(0)+q_0^2)}\|P_1\|/\underline{\lambda}(P_1)$. Hence,

$$\sqrt{V_4(t)}\le\sqrt{V_4(0)}e^{-\frac{\alpha_1t}{2}}+\frac{\beta_1}{\alpha_1}(1-e^{-\frac{\alpha_1t}{2}})\le\sqrt{V_4(0)}+\frac{\beta_1}{\alpha_1}. \tag{3.19}$$

It then follows from (3.19) that

$$\|e_{0vf}(t)\| \leq \sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}}\|e_{0vf}(0)\| + \frac{\beta_1}{\alpha_1} = \sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}}\|e_{0vf}(0)\| + \frac{\bar{\lambda}(P_1)\sqrt{2(e_u^2(0)+q_0^2)}\|P_1\|}{\underline{\lambda}(P_1)},$$
$$\lim_{t\to\infty}\|e_{0vf}(t)\| \leq \frac{\sqrt{2}\bar{\lambda}(P_1)q_0\|P_1\|}{\underline{\lambda}(P_1)}.$$

Part b): Consider the observers for v_i and f_i. Define their respective estimation errors as $e_{v,i} = \hat{v}_i - v_i$ and $e_{f,i} = \hat{f}_i - f_i$. Their dynamics can be described as

$$\dot{e}_{v,i} = -le_{v,i} + e_{f,i},$$
$$\dot{e}_{f,i} = -le_{v,i} - \dot{f}_i.$$

Defining $e_{vf} = \begin{bmatrix} e_{v,1} & \dots & e_{v,N} & e_{f,1} & \dots & e_{f,N} \end{bmatrix}^\top$, we have

$$\dot{e}_{vf} = Q_2 e_{vf} + \ell_2,$$

where

$$Q_2 = \begin{bmatrix} -lI & I \\ -lI & 0 \end{bmatrix},\ \ell_2 = \begin{bmatrix} 0 & -\dot{f}_1 & -\dot{f}_2 & \cdots & -\dot{f}_N \end{bmatrix}^\top.$$

Following similar lines to Part a), we can obtain that e_{vf} is upper bounded:

$$\|e_{vf}(t)\| \leq \sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}}\|e_{vf}(0)\| + \frac{\beta_2}{\alpha_2} = \sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}}\|e_{vf}(0)\| + \frac{\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)},$$
$$\lim_{t\to\infty}\|e_{vf}(t)\| \leq \frac{\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)},$$

where P_2 is a positive definite matrix satisfying $P_2Q_2 + Q_2^\top P_2 = -I$.

Part c): Based on Parts a) and b), now let us move on to analyze the tracking performance under the controller in (3.12). Note that the position and velocity tracking errors are governed by

$$\begin{aligned}\dot{x}_i - \dot{x}_0 =& v_i - v_0,\\ \dot{v}_i - \dot{v}_0 =& -k_1\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0)\right] - (v_i - v_0) - (\hat{v}_i - v_i) + (\hat{v}_{0,i} - v_0)\\ &+ \hat{f}_{0,i} - f_0 + f_i - \hat{f}_i + \hat{u}_{0,i} - u_0,\end{aligned}$$

for $i = 1, 2, \ldots, N$. Define $e = \begin{bmatrix} x_1 - x_0 & \cdots & x_N - x_0 & v_1 - v_0 & \cdots & v_N - v_0 \end{bmatrix}^\top$. Then,

$$\dot{e} = Q_3 e + \ell_3, \tag{3.20}$$

where

$$Q_3 = \begin{bmatrix} 0 & I \\ -k_1 H & -I \end{bmatrix},\ \ell_3 = \begin{bmatrix} 0 \\ -e_v + e_{0v} + e_{0f} - e_f + e_u \end{bmatrix}.$$

The characteristic polynomial of Q_3 is

$$\det(sI - Q_3) = \det\left(\begin{bmatrix} sI & -I \\ kH & sI + I \end{bmatrix}\right) = \det(s^2 I + sI + kH) = \prod_{i=1}^{n}\left[s^2 + s + k\lambda_i(H)\right].$$

It is seen from above that Q_3 is stable because H is positive definite and $k > 0$. If Q_3 is stable, there exists a positive definite matrix P_3 such that

$$P_3 Q_3 + Q_3^\top P_3 = -I.$$

Define $V_5(e) = \frac{1}{2}e^\top P_3 e$ for (3.20). Then,

$$\begin{aligned}\dot{V}_5 &= -\frac{1}{2}e^\top(P_3 Q_3 + Q_3^\top P_3)e + e^\top P_3 \ell_3\\ &\leq -\frac{1}{2}\|e\|^2 + \|e\| \cdot \|P_3\| \cdot \|\ell_3\|\\ &\leq -\frac{1}{2}\|e\|^2 + (\sqrt{2}\|e_{vf}\| + \sqrt{2}\|e_{0vf}\| + \|e_u\|) \cdot \|P_3\| \cdot \|e\|.\end{aligned}$$

It can be rewritten as

$$\dot{V}_5 \leq -\alpha_3 V_5 + \beta_3 \sqrt{V_5},$$

where $\alpha_3 = 1/\bar{\lambda}(P_3)$ and $\beta_3 = \left(2\|e_{vf}\| + 2\|e_{0vf}\| + \sqrt{2}\|e_u\|\right) \|P_3\|/\underline{\lambda}(P_3)$. Therefore, we have

$$\sqrt{V_5} \leq \sqrt{V_5(0)} e^{-\frac{\alpha_3 t}{2}} + \frac{\beta_3}{\alpha_3}(1 - e^{-\frac{\alpha_3 t}{2}}) \leq \sqrt{V_5(0)} + \frac{\beta_3}{\alpha_3}.$$

It then follows that e satisfies

$$\|e(t)\| \leq \sqrt{\frac{\bar{\lambda}(P_3)}{\underline{\lambda}(P_3)}}\|e(0)\| + \frac{\beta_3}{\alpha_3} \leq \sqrt{\frac{\bar{\lambda}(P_3)}{\underline{\lambda}(P_3)}}\|e(0)\| + \frac{\bar{\lambda}(P_3)\|P_3\|}{\underline{\lambda}(P_3)} \left(2\sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}}\|e_{vf}(0)\| + \right.$$

$$\left. \frac{2\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)} + 2\sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}}\|e_{0vf}(0)\| + \frac{2\bar{\lambda}(P_1)\sqrt{2(e_u^2(0) + q_0^2)}\|P_1\|}{\underline{\lambda}(P_1)} + \sqrt{2}\|e_u(0)\| \right), \quad (3.21)$$

$$\lim_{t\to\infty} \|e(t)\| \leq \frac{\bar{\lambda}(P_3)\|P_3\|}{\underline{\lambda}(P_3)} \left(\frac{2\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)} + \frac{2\sqrt{2}\bar{\lambda}(P_1)q_0\|P_1\|}{\underline{\lambda}(P_1)} \right). \quad (3.22)$$

By (3.21)-(3.22), there exist δ and ε such that (3.17a)-(3.17b) hold. This completes the proof. •

Theorem 4 reveals that, for a second-order MAS, the proposed observer-based controller can enable a follower to track the leader with bounded position and velocity errors when the disturbances are bounded in rates of change. Such an effectiveness is mainly attributed to the proposed observers, through which a follower can estimate the disturbance and velocity variables for tracking control. Further, similar to Remark 5, the tracking error $e_i(t) \to 0$ as $t \to \infty$ if $\dot{f}_i(t) \to 0$.

3.5 Numerical Study

This section presents an illustrative simulation example to validate the proposed results. We consider a second-order MAS consisting of one leader and five followers, which share a communication topology shown in Figure 2.3a. For the topology graph, the edge-based weights are set to be unit for simplicity. The corresponding Laplacian matrix L is then given as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

Based on the communication topology, the leader adjacency matrix is $B = \text{diag}(1,0,0,0,0)$. We choose $l = 1$ and $k = 0.5$, respectively. The initial conditions include

$$x(0) = \begin{bmatrix} 0 & 3 & 0 & -2 & 1 & -1 \end{bmatrix}^\top, \; v(0) = \begin{bmatrix} 0 & 1 & -2 & 3 & 0 & -1 \end{bmatrix}^\top.$$

Further,

$$u_0(t) = -2\cos(0.1\pi t), \;\; f_0(t) = -\cos(0.1\pi t), \;\; f(t) = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 \end{bmatrix}^\top t.$$

Note that the disturbances enforced on the followers are bounded in rates of change but linearly diverge through time. This extreme setting is used to illustrate the effectiveness of disturbance rejection here. Apply the observer-based control approach in Section 3.4 to the MAS. The observer-based control approach in Section 3.4 is applied, with the simulation results outlined in Figure 3.1. Figure 3.1a and 3.1b demonstrate that the followers maintain bounded position and velocity tracking errors, which is in agreement with the results in Theorem 4. It is shown in Figure 3.1c that the observer for u_0 can gradually achieve accurate estimation through time. This is because the leader can send u_0 to its neighbor follower i, and with the implicit information propagation, the other followers can eventually estimate u_0 precisely. By comparison, the estimation of v_0, f_0, v_i and f_i is less accurate, as is seen in Figures 3.1d-3.1g, because there is no measurement of them available. However, the differences or estimation errors are still bounded, matching the expectation as suggested by the theoretical analysis.

3.6 Conclusion

MASs have attracted significant research interest in the past decade due to their increasing applications. In this chapter, we have studied leader-follower tracking for first- and second-order MASs with unknown disturbances. Departing from the literature, we have considered a much less restrictive setting about disturbances. Specifically, disturbances can be applied to all the leader and followers and assumed to be bounded just in rates of change. This considerably relaxes the usual setting that only followers are affected by magnitude-bounded disturbances. To solve this problem, we have developed observer-based tracking control approaches, which particularly included the design of novel distributed disturbance observers for followers to estimate the leader's unknown disturbance. A simulation result further demonstrated the effectiveness of the proposed approach.

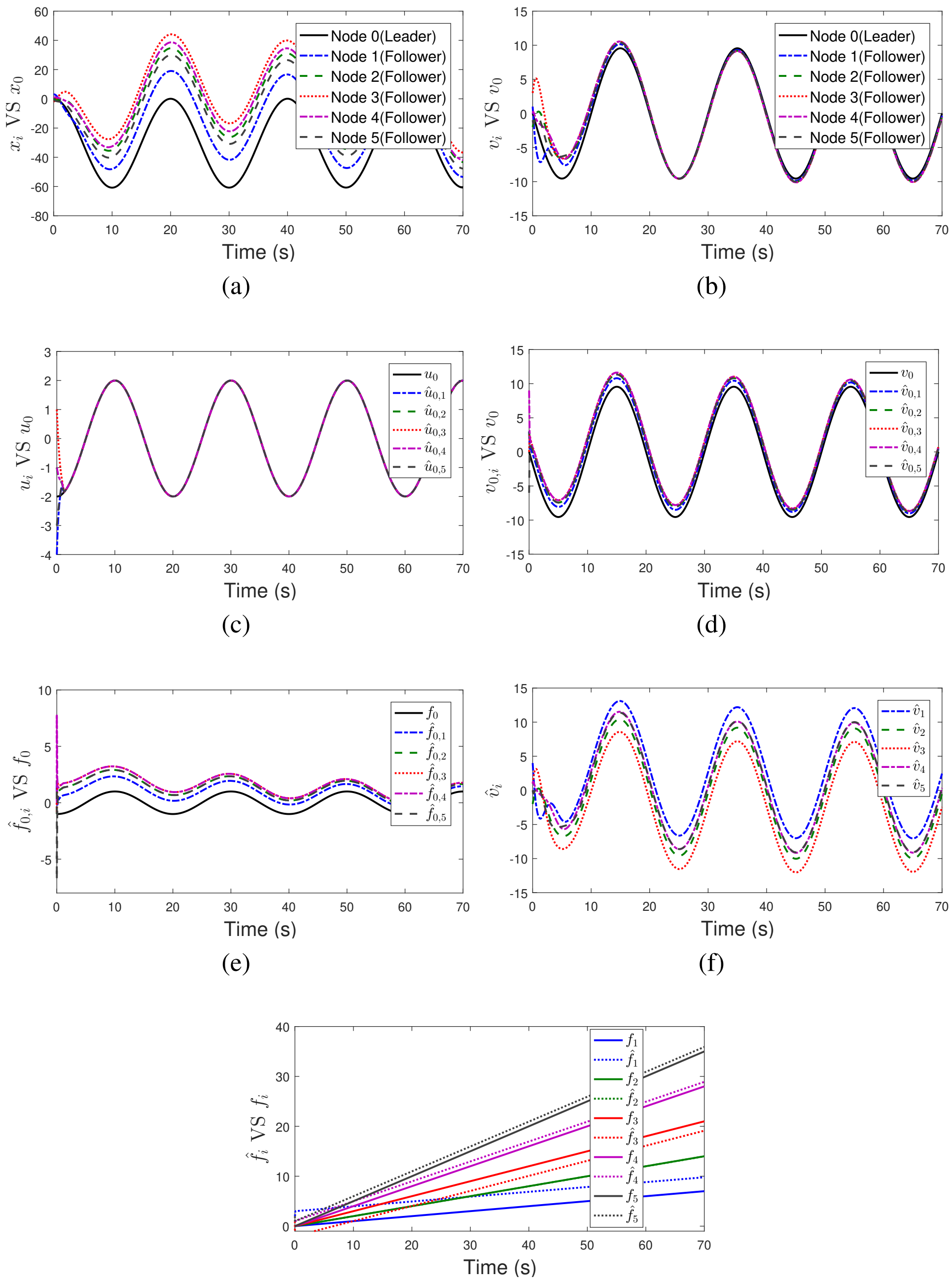

Figure 3.1: Second-order MAS tracking control: (a) leader's and followers' position trajectory profiles; (b) leader's and followers' velocity profiles; (c) leader's acceleration profile and the estimation by each follower; (d) leader's velocity profile and the estimation by each follower; (e) leader's disturbance profile and the estimation by each follower; (f) followers' estimation of their own velocities; (g) followers' disturbance profiles and the estimation on their own.

Chapter 4

High-Order Leader-Follower Tracking Control under Limited Information Availability[1]

4.1 Introduction

Cooperative autonomy based on MASs is finding ever-increasing application in a variety of sectors. This has driven a surge of research interest in distributed cooperative control for different tasks, including consensus, leader-follower tracking, synchronization, rendezvous, flocking, and coverage control [53, 84–88]. Most of the current literature considers agents governed by first- or second-order models. Although such low-order models can be useful as well as amenable to control design, they are often considered as inadequate for characterizing agents with more complex higher-order dynamics. It is also neither trivial nor easy to extend low-order cooperative control designs to high-order systems. Recent years hence have witnessed a growing amount of work focused on high-order MAS control synthesis [89].

A lead is taken in [90] with the study of high-order MAS consensus, which presents a distributed consensus control algorithm along with the corresponding sufficient and necessary conditions for convergence. This subject has since attracted considerable research efforts, with many studies proposed to deal with various challenges, e.g., directed communication topologies [91, 92], switching topologies [91, 93], output feedback design [94–97], bipartite consensus [98, 99], external disturbances [100, 101], and constrained energy budget [44]. In addition to consensus, high-order leader-follower tracking is emerging as another problem of great interest. It is in [90] that a

[1]This chapter is based on the dissertation author's first-authored journal paper [82] and first-authored conference paper [83].

basic form of this problem is introduced, which assumes that the leader agent continuously broadcasts its state information to all the followers. A consensus-based control algorithm is then developed therein to make the followers track the leader. The study [102] considers a more general setting where only a subset of the followers can receive information from the leader. It proposes a leader-follower tracking control method and proves that followers with small degrees must be informed by the leader to ensure tracking convergence. High-order nonlinear agents constitute a stronger challenge for leader-follower tracking control. This problem is investigated in [103], which proposes to adaptively estimate the nonlinearity inherent in an agent's dynamics using neural networks and then offset it in the control run. In [104], a finite-time tracking control approach is developed for a high-order nonlinear MAS subject to actuator saturation, and the work in [105] studies the problem of finite-time higher-order tracking with mismatched disturbances.

Despite the importance, the above studies generally assume that a follower can obtain a large amount of information to make control decisions. For example, it is required in [90, 102–104] that a follower must know all of its own states, all of the states of its neighbors, and if connected with the leader, all of the leader's states. This assumption can be hardly guaranteed in a real world where relevant sensors can be unavailable [102]. One can also find similar requirements in studies about high-order consensus control. This motivates us to explore a more realistic setting—when only the first state of every agent (leader or follower) is measured. It is unsurprising that the low information availability will complicate the tracking control design. To overcome the challenge, we propose to exploit the notion of observer-based control and make two major contributions. First, we design an observer-based tracking control approach for linear high-order MASs. We propose a set of distributed observers to compensate for the limited information, allowing a follower to comprehensively estimate the leader's maneuver input and states along with its own states. These observers are then combined with a nominal controller to form an observer-embedded tracking controller. We further characterize the convergence of tracking when the proposed controller is applied. As the second contribution, the study extends to the case when the agents' dynamics is not only high-order but also nonlinear. Extrapolating the design for the linear case, we develop an observer-based tracking control approach and rigorously analyze its convergence.

Our work is related with two lines of research. 1) Leader-follower tracking *via* output feedback, which generally considers a state-space model and uses local state or parameters observers to estimate certain unknown quantities [49, 51, 52, 106–108]. Differing from them, our study designs distributed observers to help the followers collectively infer the leader's input and states, removing the restriction that the leader's maneuver input must be either zero or partially known by all the followers. 2) Observer-based first- and second-order tracking control. The literature includes various kinds of observers designed to allow a follower to estimate its own velocity [46, 47], the disturbances acting on it [105], its velocity relative to the leader [43], or the leader's velocity [37, 45]. These results nonetheless cannot be readily generalized to the high-order MASs.

4.2 Leader-Follower Tracking with Linear High-Order Dynamics

In this section, we first formulate the leader-follower tracking problem for an MAS with linear high-order dynamics. We then propose an observer-based tracking control strategy and characterize its convergence properties.

Consider an MAS composed of $N+1$ agents, among which agent 0 is the leader and agents 1 to N are followers. Each agent has lth-order dynamics ($l \geq 3$) expressed as

$$\dot{x}_{i,m} = x_{i,m+1}, \quad m = 1, 2, \ldots, l-1, \tag{4.1a}$$

$$\dot{x}_{i,l} = u_i, \tag{4.1b}$$

for $i = 0, 1, \ldots, N$, where $x_{i,m} \in \mathbb{R}$ is the mth state of agent i, and u_i the maneuver input. The objective is to design a distributed control law u_i such that follower i for $i = 1, 2, \ldots, N$ can convergently track the leader with $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for $m = 1, 2, \ldots, l$.

Here, we assume that only $x_{i,1}$ for $i = 0, 1, \ldots, N$ is available. That is, only the first state of an agent is measured, regardless of whether it is the leader or a follower. This assumption considerably relaxes the usual requirement in the literature that substantial states of an agent must be measured.

However, it also implies that the accessible information about the agents is rather limited, which makes it more challenging to design an effective distributed tracking controller.

4.2.1 Proposed Algorithm

We shall develop an observer-based control algorithm to enable convergent tracking in the above setting. To begin with, we consider the following controller for follower i:

$$\begin{aligned} u_i = & -k_1 \left[\sum_{j \in \mathcal{N}_i} a_{ij}(x_{i,1} - x_{j,1}) + b_i(x_{i,1} - x_{0,1}) \right] \\ & - \sum_{m=2}^{l} k_m(\hat{x}_{i,m} - \hat{x}_{0,i,m}) + \hat{u}_{0,i}, \end{aligned} \tag{4.2}$$

where k_m for $m = 1, 2, \ldots, l$ are gain parameters, $\hat{x}_{0,i,m}$ and $\hat{u}_{0,i}$ are follower i's estimates of the leader's state $x_{0,m}$ and input u_0, respectively, and $\hat{x}_{i,m}$ is follower's estimate of its own state $x_{i,m}$. The motivation behind (4.2) is to drive follower i toward its neighbors and the leader simultaneously. When all the followers do this, they can track the leader in a collective manner. Next, we design the observers so as to obtain the estimates as needed in (4.2).

It is noted that the input observer (2.13) still can be used here, and its stability property holds in this case. We further propose the next observer to estimate $x_{0,m}$ for $m = 2, 3, \ldots, l$:

$$\begin{aligned} \dot{z}_{0,i,2} = & -b_i c_{0,2} z_{0,i,2} - b_i^2 c_{0,2}^2 x_{0,1} \\ & - c_{0,2} \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) + \hat{x}_{0,i,3}, \end{aligned} \tag{4.3a}$$

$$\hat{x}_{0,i,2} = z_{0,i,2} + b_i c_{0,2} x_{0,1}, \tag{4.3b}$$

$$\dot{z}_{0,i,m} = -c_{0,m} z_{0,i,m} - c_{0,m}^2 \hat{x}_{0,i,m-1} + \hat{x}_{0,i,m+1}, \tag{4.3c}$$

$$\hat{x}_{0,i,m} = z_{0,i,m} + c_{0,m} \hat{x}_{0,i,m-1}, \quad m = 3, 4, \ldots, l-1, \tag{4.3d}$$

$$\dot{z}_{0,i,l} = -c_{0,l} z_{0,i,l} - c_{0,l}^2 \hat{x}_{0,i,l-1} + \hat{u}_{0,i}, \tag{4.3e}$$

$$\hat{x}_{0,i,l} = z_{0,i,l} + c_{0,l} \hat{x}_{0,i,l-1}, \tag{4.3f}$$

where $z_{0,i,m}$ and $c_{0,m}$ for $m = 2,3,\ldots,l$ are the observer's internal states and gain parameters, respectively. The development of (4.3) is inspired by [55], in which a centralized disturbance observer is designed for a single plant. Significantly transforming the original design, we develop the above observer, which has a distributed structure that is uniquely suitable for the considered MAS setting.

Finally, we design the following observer such that follower i can estimate its own states $x_{i,m}$ for $m = 2,3,\ldots,l$:

$$\dot{z}_{i,2} = -r_2 z_{i,2} - r_2^2 x_{i,1} + \hat{x}_{i,3}, \tag{4.4a}$$

$$\hat{x}_{i,2} = z_{i,2} + r_2 x_{i,1}, \tag{4.4b}$$

$$\dot{z}_{i,m} = -r_m z_{i,m} - r_m^2 \hat{x}_{i,m-1} + \hat{x}_{i,m+1}, \tag{4.4c}$$

$$\hat{x}_{i,m} = z_{i,m} + r_m \hat{x}_{i,m-1},\ m = 3,4,\ldots,l-1, \tag{4.4d}$$

$$\dot{z}_{i,l} = -r_l z_{i,l} - r_l^2 \hat{x}_{i,l-1} + u_i, \tag{4.4e}$$

$$\hat{x}_{i,l} = z_{i,l} + r_l \hat{x}_{i,l-1}, \tag{4.4f}$$

where $z_{i,m}$ and $r_{i,m}$ for $m = 2,3,\ldots,l$ are the internal states and gain parameters, respectively.

Putting together (2.13) and (4.2)-(4.4), we can obtain a distributed observer-based control algorithm for high-order leader-follower tracking. Its convergence is analyzed next.

Remark 6. We here highlight a comparison between the proposed approach and the study of output-feedback leader-following tracking control in [49, 51, 52, 106–108]. In these references, different observers are developed to enable a follower to estimate its own states or certain parameters. Therefore, they by design are local observers for estimation of local unknown quantities. Contrasting them, the proposed approach focuses more on distributed observer design—note that the observers in (2.13) and (4.3) have a distributed structure, where the followers exchange the estimates to collectively infer the leader's input and states. The new design hence allows the followers to keep tracking the leader driven by a maneuvering input, setting it apart from the references that restrictively require the leader to be input-free or the followers to have at least certain knowledge of the leader's input. •

4.2.2 Convergence Analysis

This section characterizes the convergence property for the algorithm proposed above. Before proceeding further, Assumption 1 is still applied here.

Now, we consider the observer in (4.3). Defining $e_{0x,i,m} = \hat{x}_{0,i,m} - x_{0,m}$, we have

$$\dot{e}_{0x,i,2} = -c_{0,2}b_i e_{0x,i,2} + e_{0x,i,3} - c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(e_{0x,i,2} - e_{0x,j,2}), \tag{4.5a}$$

$$\dot{e}_{0x,i,m} = -c_{0,m}c_{0,2}b_i e_{0x,i,2} + e_{0x,i,m+1} - c_{0,m}c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(e_{0x,i,2} - e_{0x,j,2}),$$

$$m = 3, 4, \ldots, l-1, \tag{4.5b}$$

$$\dot{e}_{0x,i,l} = -c_{0,l}c_{0,2}b_i e_{0x,i,2} + e_{u,i} - c_{0,l}c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(e_{0x,i,2} - e_{0x,j,2}). \tag{4.5c}$$

Define $e_{0x,m} = \begin{bmatrix} e_{0x,1,m} & e_{0x,2,m} & \cdots & e_{0x,N,m} \end{bmatrix}^\top$ and $e_{0x} = \begin{bmatrix} e_{0x,2}^\top & e_{0x,3}^\top & \cdots & e_{0x,l}^\top \end{bmatrix}^\top$. Then, (4.5) can be written into a compact form as below:

$$\dot{e}_{0x} = F_1 e_{0x} + \ell_1, \tag{4.6}$$

where

$$F_1 = \begin{bmatrix} -c_{0,2}H & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -c_{0,l-1}c_{0,2}H & 0 & \cdots & 0 & I \\ -c_{0,l}c_{0,2}H & 0 & \cdots & \cdots & 0 \end{bmatrix}, \; \ell_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ e_u \end{bmatrix}.$$

The next lemma shows the convergence of e_{0x}.

Lemma 10. If there exist $c_{0,2}, c_{0,3}, \ldots, c_{0,l} > 0$ such that the polynomials

$$h_i(s) = s^{l-1} + c_{0,2}s^{l-2}\lambda_i(H) + c_{0,2}\lambda_i(H)\sum_{z=0}^{l-3} c_{0,l-z}s^z \tag{4.7}$$

for $i = 1, 2, \ldots, N$ are Hurwitz stable, then the system in (4.6) is asymptotically stable, and $\lim_{t\to\infty} e_{0x} = 0$.

Proof: Using the Schur complement, we can find out that the characteristic polynomial of F_1 is $\prod_{i=1}^{N} h_i(s)$. Note that $\lim_{t\to\infty} \ell_1 = 0$ by Lemma 3. Hence, we have $\lim_{t\to\infty} e_{0x} = 0$ according to the input-to-state stability (ISS) theory [56]. •

Proceeding further, we define $e_{x,i,m} = \hat{x}_{i,m} - x_{i,m}$ for the observer in (4.4) and have

$$\begin{aligned}
\dot{e}_{x,i,2} &= -r_2 e_{x,i,2} + e_{x,i,3}, \\
\dot{e}_{x,i,m} &= -r_m r_2 e_{x,i,2} + e_{x,i,m+1}, \;\; m = 3, 4, \ldots, l-1, \\
\dot{e}_{x,i,l} &= -r_l r_2 e_{x,i,2}.
\end{aligned}$$

Define $e_{x,m} = \begin{bmatrix} e_{x,1,m} & e_{x,2,m} & \cdots & e_{x,N,m} \end{bmatrix}^\top$ for $m = 2, 3, \ldots, l$ and $e_x = \begin{bmatrix} e_{x,2}^\top & e_{x,3}^\top & \cdots & e_{x,l}^\top \end{bmatrix}^\top$. Then,

$$\dot{e}_x = F_2 e_x, \tag{4.8}$$

where

$$F_2 = \begin{bmatrix} -r_2 I & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -r_{l-1} r_2 I & 0 & \cdots & 0 & I \\ -r_l r_2 I & 0 & \cdots & \cdots & 0 \end{bmatrix}.$$

We can obtain the next lemma along lines similar to Lemma 10, with the proof skipped.

Lemma 11. If there exist $r_2, r_3, \ldots, r_l > 0$ such that the polynomials

$$s^{l-1} + r_2 s^{l-2} + r_2 \sum_{z=0}^{l-3} r_{l-z} s^z \tag{4.9}$$

are Hurwitz stable, then the system in (4.8) is asymptotically stable, and $\lim_{t\to\infty} e_x = 0$.

With the above results, we are now in a good position to investigate the state tracking errors, which are defined as $e_{i,m} = x_{i,m} - x_{0,m}$. The dynamics of $e_{i,m}$ is

$$
\begin{aligned}
\dot{e}_{i,m} &= e_{i,m+1}, \ \ m = 1,2,\ldots,l-1 \\
\dot{e}_{i,l} &= -k_1 \left[\sum_{j \in \mathcal{N}_i} a_{ij}(e_{i,1} - e_{j,1}) + b_i e_{i,1} \right] - \sum_{m=2}^{l} k_m (e_{x,i,m} - e_{0x,i,m}) \\
&\quad - \sum_{m=2}^{l} k_m e_{i,m} + e_{u,i}.
\end{aligned}
$$

Define $e_m = \begin{bmatrix} e_{1,m} & e_{2,m} & \cdots & e_{N,m} \end{bmatrix}^\top$ for $m = 1,2,\ldots,l$, and $e = \begin{bmatrix} e_1^\top & e_2^\top & \cdots & e_l^\top \end{bmatrix}^\top$. Here, e is the global tracking error with dynamics expressed as

$$
\dot{e} = F_3 e + \ell_3, \tag{4.10}
$$

where

$$
F_3 = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ -k_1 H & -k_2 I & \cdots & \cdots & -k_l I \end{bmatrix}, \quad \ell_3 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\sum_{m=2}^{l} k_m (e_{x,m} - e_{0x,m}) + e_u \end{bmatrix}.
$$

The following theorem outlines the convergence property of e. The proof is similar to that of Lemma 10 and thus omitted.

Theorem 5. If there exist k_m for $m = 1,2,\ldots,l$ such that the polynomials

$$
s^l + k_1 \lambda_i(H) + \sum_{z=2}^{l} s^{z-1} k_z
$$

for $i = 1,2,\ldots,N$ are Hurwitz stable, then the system in (4.10) is asymptotically stable, and $\lim_{t\to\infty} e = 0$.

Remark 7. The proposed controller only requires the neighboring followers to interchange $x_{i,1}$,

$\hat{u}_{0,i}$ and $\hat{x}_{0,i,2}$, because the observers can locally estimate other quantities necessary for control. This greatly reduces the amount of data to be exchanged between agents and makes the design more advantageous in terms of communication costs. •

4.3 High-Order Tracking for Nonlinear Dynamics

This section moves forward to study an MAS with nonlinear high-order dynamics. Extending the design in Section 4.2, we develop an observer-based tracking control algorithm and analyze its convergence properties.

4.3.1 Proposed Algorithm and Convergence Analysis

Suppose that agent i's dynamics is governed by

$$\dot{x}_{i,m} = x_{i,m+1} + f_m(\overline{x_{i,m}}), \quad m = 1, 2, \ldots, l-1, \tag{4.11a}$$

$$\dot{x}_{i,l} = u_i + f_l(\overline{x_{i,l}}), \tag{4.11b}$$

for $i = 0, 1, \ldots, N$, where $f_m(\overline{x_{i,m}}) : \mathbb{R}^m \to \mathbb{R}$ for $m = 1, 2, \ldots, l$ are nonlinear functions with $\overline{x_{i,m}} = (x_{i,1}, x_{i,2}, \ldots, x_{i,m})$. Following Section 4.1, we assume that only $x_{i,1}$ is measured and continue to hold Assumption 1. The control design objective here is still to enable convergent tracking, i.e., $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for $m = 1, 2, \ldots, l$ and $i = 1, 2, \ldots, N$.

To achieve the above objective, we propose the following distributed controller:

$$u_i = -k_1(x_{i,1} - \hat{x}_{0,i,1}) - \sum_{m=2}^{l} k_m(\hat{x}_{i,m} - \hat{x}_{0,i,m}) + \hat{u}_{0,i}. \tag{4.12}$$

This controller must be supplemented by corresponding observers. It is noted first that the observer in (2.13) can also be applied to obtain $\hat{u}_{0,i}$ here, so we continue to use it for the distributed estimation of u_0. We then construct the following state observer to allow follower i to estimate $x_{0,m}$ for

$m = 1, 2, \ldots, l$:

$$\dot{\hat{x}}_{0,i,1} = -c_{0,1}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,1} - \hat{x}_{0,j,1}) + b_i(\hat{x}_{0,i,1} - x_{0,1})\right] + \hat{x}_{0,i,2} + f_1(\overline{\hat{x}_{0,i,1}}), \tag{4.13a}$$

$$\begin{aligned}\dot{z}_{0,i,2} = &-b_i c_{0,2} z_{0,i,2} - b_i^2 c_{0,2}^2 x_{0,1} + \hat{x}_{0,i,3} + f_2(\overline{\hat{x}_{0,i,2}}) \\ &- c_{0,2}\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) - b_i c_{0,2} f_1(x_{0,1}),\end{aligned} \tag{4.13b}$$

$$\hat{x}_{0,i,2} = z_{0,i,2} + b_i c_{0,2} x_{0,1}, \tag{4.13c}$$

$$\dot{z}_{0,i,m} = -c_{0,m} z_{0,i,m} - c_{0,m}^2 \hat{x}_{0,i,m-1} + \hat{x}_{0,i,m+1} + f_m(\overline{\hat{x}_{0,i,m}}) - c_{0,m} f_{m-1}(\overline{\hat{x}_{0,i,m-1}}), \tag{4.13d}$$

$$\hat{x}_{0,i,m} = z_{0,i,m} + c_{0,m}\hat{x}_{0,i,m-1}, m = 3, 4, \ldots, l-1, \tag{4.13e}$$

$$\dot{z}_{0,i,l} = -c_{0,l} z_{0,i,l} - c_{0,l}^2 \hat{x}_{0,i,l-1} + \hat{u}_{0,i} + f_l(\overline{\hat{x}_{0,i,l}}) - c_{0,l} f_{l-1}(\overline{\hat{x}_{0,i,l-1}}), \tag{4.13f}$$

$$\hat{x}_{0,i,l} = z_{0,i,l} + c_{0,l}\hat{x}_{0,i,l-1}, \tag{4.13g}$$

where $\overline{\hat{x}_{0,i,m}} = (\hat{x}_{0,i,1}, \hat{x}_{0,i,2}, \ldots, \hat{x}_{0,i,m})$. To make a follower able to estimate its own states, we develop an observer as follows:

$$\dot{z}_{i,2} = -r_2 z_{i,2} - r_2^2 x_{i,1} + \hat{x}_{i,3} + f_2(x_{i,1}, \hat{x}_{i,2}) - r_2 f_1(x_{i,1}), \tag{4.14a}$$

$$\hat{x}_{i,2} = z_{i,2} + r_2 x_{i,1}, \tag{4.14b}$$

$$\dot{z}_{i,m} = -r_m z_{i,m} - r_m^2 \hat{x}_{i,m-1} + \hat{x}_{i,m+1} + f_m(x_{i,1}, \overline{\hat{x}_{i,m}}) - r_m f_{m-1}(x_{i,1}, \overline{\hat{x}_{i,m-1}}), \tag{4.14c}$$

$$\hat{x}_{i,m} = z_{i,m} + r_m \hat{x}_{i,m-1}, m = 3, 4, \ldots, l-1, \tag{4.14d}$$

$$\dot{z}_{i,l} = -r_l z_{i,l} - r_l^2 \hat{x}_{i,l-1} + u_i + f_l(x_{i,1}, \overline{\hat{x}_{i,l}}) - r_l f_{l-1}(x_{i,1}, \overline{\hat{x}_{i,l-1}}), \tag{4.14e}$$

$$\hat{x}_{i,l} = z_{i,l} + r_l \hat{x}_{i,l-1}, \tag{4.14f}$$

where $\overline{\hat{x}_{i,m}} = (\hat{x}_{i,2}, \hat{x}_{i,3}, \ldots, \hat{x}_{i,m})$.

Integrating observers in (2.13), (4.13)-(4.14) into the controller in (4.12) will yield a complete observer-based tracking controller. Before going further to analyze its effectiveness, we make the following assumption:

Assumption 3. There exist $\rho_m \geq 0$ such that

$$|f_m(\xi) - f_m(\varepsilon)| \leq \rho_m \|\xi - \varepsilon\|,\ m = 1, 2, \ldots, l,$$

where $\xi, \varepsilon \in \mathbb{R}^m$.

Assumption 3 implies that the nonlinear functions must be of Lipschitz class. It is commonly used in the literature on nonlinear MAS control and can be satisfied by many practical systems.

The following theorem shows the main result about convergence of the proposed controller.

Theorem 6. Assume that Assumptions 1 and 3 hold and that the controller proposed above is applied to (4.11). The state tracking error converges to zero, i.e., $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for $m = 1, 2, \ldots, l$ and $i = 1, 2, \ldots, N$, if there exist $c_{0,m}$, r_n and k_m for $m = 1, 2, \ldots, l$ and $n = 2, \ldots, N$ such that the polynomials (4.7), (4.9) and

$$\left(s^l + \sum_{z=1}^{l} s^{z-1} k_z \right)^N \tag{4.15}$$

are Hurwitz stable, and if there exist matrices $Q_i > 0$ and $\eta_i > 0$ for $i = 1, 2, 3$ such that

$$F_4^\top Q_1 + Q_1 F_4 = -\eta_1 I, \tag{4.16a}$$

$$F_2^\top Q_2 + Q_2 F_2 = -\eta_2 I, \tag{4.16b}$$

$$F_6^\top Q_3 + Q_3 F_6 = -\eta_3 I, \tag{4.16c}$$

$$\sum_{i=1}^{l} \|P_{0x,i}\| < \min \left\{ \frac{\eta_1}{2\|Q_1\|}, \frac{\eta_3}{2\|Q_3\|} \right\}, \tag{4.16d}$$

$$\sum_{i=2}^{l} \|P_{x,i}\| < \frac{\eta_2}{2\|Q_2\|}, \tag{4.16e}$$

where $P_{0x,i} = \mathrm{diag}\{\rho_1 I, \rho_2 I, \ldots, \rho_i I, 0I, 0I, \ldots, 0I\}$, $P_{x,i} = \mathrm{diag}\{\rho_2 I, \rho_3 I, \ldots, \rho_i I, 0I, 0I, \ldots, 0I\}$ for

$i = 1, 2, \ldots, l$, and

$$F_4 = \begin{bmatrix} -c_{0,1}H & I & 0 & \cdots & \cdots & 0 \\ 0 & -c_{0,2}H & I & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ \vdots & -c_{0,l-1}c_{0,2}H & 0 & \cdots & 0 & I \\ 0 & -c_{0,l}c_{0,2}H & 0 & \cdots & \cdots & 0 \end{bmatrix}, \quad F_6 = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ -k_1 I & -k_2 I & \cdots & \cdots & -k_l I \end{bmatrix}.$$

Proof: Let us define $e_{0x} = \begin{bmatrix} e_{0x,1}^\top & e_{0x,2}^\top & \cdots & e_{0x,l}^\top \end{bmatrix}^\top$, where $e_{0x,m}$ follows the same definition as in Section 4.2, and define $f_{0x,m} = \begin{bmatrix} \cdots & f_m(\overline{\hat{x}_{0,i,m}}) - f_m(\overline{x_{0,m}}) & \cdots & \end{bmatrix}^\top$ for $i = 1, 2, \cdots, N$. By (4.11) and (4.13), we have

$$\dot{e}_{0x} = F_4 e_{0x} + \ell_4, \tag{4.17}$$

where $\ell_4 = \begin{bmatrix} f_{0x,1}^\top & \cdots & f_{0x,l-1}^\top & f_{0x,l}^\top + e_u^\top \end{bmatrix}^\top$. We choose a Lyapunov candidate function

$$V_2(e_{0x}) = \frac{1}{2} e_{0x}^\top Q_1 e_{0x},$$

for which there exist $\alpha_1, \alpha_2 > 0$ such that

$$\alpha_1 \|e_{0x}\|^2 \leq V_2(e_{0x}) \leq \alpha_2 \|e_{0x}\|^2.$$

By (4.16a), we have

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2} e_{0x}^\top (Q_1 F_4 + F_4^\top Q_1) e_{0x} + e_{0x}^\top Q_1 \ell_4 \leq -\frac{1}{2}\eta_1 \|e_{0x}\|^2 + \|e_{0x}\| \|Q_1\| \|\ell_4\| \\ &\leq -\frac{1}{2}\eta_1 \|e_{0x}\|^2 + \|e_{0x}\| \|Q_1\| \left(\|P_{0x,1} e_{0x}\| + \|P_{0x,2} e_{0x}\| + \cdots + \|P_{0x,l} e_{0x}\| + \|e_u\| \right) \\ &= -\left(\frac{1}{2}\eta_1 - \|Q_1\| \sum_{i=1}^{l} \|P_{0x,i}\| \right) \|e_{0x}\|^2 + \|e_{0x}\| \|Q_1\| \|e_u\|. \end{aligned}$$

Define

$$\sigma_1 = \frac{1}{2}\eta_1 - \|Q_1\| \sum_{i=1}^{l} \|P_{0x,i}\|, \ \mathcal{X}(\|e_u\|) = \frac{\|Q_1\|\|e_u\|}{\sigma_1 \theta_1}$$

for any $0 < \theta_1 < 1$. By (4.16d), one can see that $\sigma_1 > 0$. It can be verified that

$$\|e_{0x}\| \geq \mathcal{X}(\|e_u\|) \Rightarrow \dot{V}_2 \leq -\sigma_1(1-\theta_1)\|e_{0x}\|^2.$$

Hence, V_2 is an ISS-Lyapunov function, implying that the system (4.17) is ISS [56]. Then, we have $\lim_{t\to\infty} e_{0x} = 0$ since $\lim_{t\to\infty} e_u = 0$ as indicated in Lemma 3.

Define $f_{x,m} = \begin{bmatrix} \cdots & f_m(x_{i,1}, \overline{\hat{x}_{i,m}}) - f_m(x_{i,1}, \overline{x_{i,m}}) & \cdots \end{bmatrix}^\top$ for $i = 1, 2, \ldots, N$. and continue to adopt e_x as defined in (4.8). According to (4.14), its dynamics can be expressed as

$$\dot{e}_x = F_2 e_x + \ell_5, \tag{4.18}$$

where F_2 was defined in (4.8), and $\ell_5 = \begin{bmatrix} f_{x,2}^\top & \cdots & f_{x,l}^\top \end{bmatrix}^\top$. Following similar lines to the above, we can prove that $\lim_{t\to\infty} e_x = 0$ if (4.16b) and (4.16e) hold.

We proceed to consider the global tracking error when the controller in (4.12) is applied. We define $f_m = \begin{bmatrix} \cdots & f_m(\overline{x_{i,m}}) - f_m(\overline{x_{0,m}}) & \cdots \end{bmatrix}^\top$ for $i = 1, 2, \cdots, N$. The dynamics of the tracking error $e_{i,m} = x_{i,m} - x_{0,m}$ is

$$\dot{e}_{i,m} = e_{i,m+1} + f_m(\overline{x_{i,m}}) - f_m(\overline{x_{0,m}}), \tag{4.19a}$$

$$\dot{e}_{i,l} = -\sum_{m=1}^{l} k_m e_{i,m} - \sum_{m=2}^{l} k_m e_{x,m} + \sum_{m=1}^{l} k_m e_{0x,m} + e_{u,i} + f_l(\overline{x_{i,l}}) - f_l(\overline{x_{0,l}}), \tag{4.19b}$$

for $m = 1, 2, \ldots, l-1$ and $i = 1, 2, \ldots, N$. The notation of e in (4.10) is still adopted here. Now, combining (4.13), (4.14) and (4.19), the closed-loop system is indicated into a compact structure as below:

$$\dot{e} = F_6 e + \ell_6 + \ell_7, \tag{4.20}$$

where $\ell_6 = \begin{bmatrix} f_1^\top & \cdots & f_l^\top \end{bmatrix}^\top$ and

$$\ell_7 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\sum_{m=2}^{l} k_m e_{x,m} + \sum_{m=1}^{l} k_m e_{0x,m} + e_u \end{bmatrix}.$$

For (4.20), we can use the ISS theory to prove that it is asymptotically stable if (4.16c)-(4.16d) hold. Therefore, $\lim_{t\to\infty} e(t) = 0$ is established as $\lim_{t\to\infty} \ell_7(t) = 0$ due to (2.13), (4.17) and (4.18). Finally, we conclude that $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$, concluding the proof. •

Remark 8. The above design can be extended to the case when the nonlinear functions $f_m(\cdot)$ is unknown but can be approximated by a known function with bounded error. Specifically, suppose that there exist $g_m(\cdot)$ and $\varphi_m \geq 0$ such that

$$|g_m(\phi) - f_m(\phi)| \leq \varphi_m,\ m = 1, 2, \ldots, l,$$

for any $\phi \in \mathbb{R}^m$. We then can replace $f_m(\cdot)$ in (4.13)-(4.14) by $g_m(\cdot)$ and obtain a tracking controller based on the approximate nonlinearity. It can be proven that this controller will lead to bounded-error tracking under certain mild conditions. The analysis is omitted here for the sake of space.

4.4 Numerical Study

This section presents numerical simulation results to show the effectiveness of the proposed design. For the sake of space, we only illustrate the more sophisticated nonlinear leader-follower tracking. Consider a third-order MAS including one leader and five followers. The agents interchange information based on a communication topology shown in Figure 2.3a. The agents' dynamics is as described in (4.11), for which $f_m(\overline{x_{i,m}}) = \cos(\overline{x_{i,m}})^\top \mathbf{1} = \sum_{k=1}^{m} \cos(x_{i,k})$. The leader's maneuver input is set to be $u_0 = \sin(0.2\pi t)$. When implementing the proposed observer-based controller, we select $c_{0,1} = c_{0,2} = c_{0,3} = 5$, $r_2 = r_3 = 4$ and $k_1 = k_2 = k_3 = 3$. It is verifiable that such a selection can make the convergence conditions satisfied. The simulation results are summarized in Figure 4.1. Figures 4.1a-4.1c illustrate followers' and the leader's state trajectories, showing that the followers can manage to catch up with and then keep tracking the leader, despite they differ in initial states. Figure 4.1d show the estimation of the leader's input by the followers. For each follower, the estimation can quickly converge to the actual values. Meanwhile, the followers can also effectively estimate the leader's states using the designed observer, with the estimation errors approaching zero as shown in Figures 4.1e-4.1g. Figures 4.1h and 4.1i further present the followers' estimation of their own unmeasured states. These results validate that the proposed design can ensure convergent tracking even though there is nonlinearity and limited information available about the agents.

4.5 Conclusion

We studied leader-follower tracking control for high-order MASs in this paper. While this problem has recently attracted growing attention, we explored the challenging yet realistic case of high-order MASs where only the first state of an agent is measured, since the measurement information can be practically limited by the availability of sensors. We designed novel distributed observers, by which a follower can reconstruct unknown or unmeasured quantities about itself and the leader, and then performed distributed observer-based controller synthesis. We conducted the design for both linear and nonlinear MASs and characterized the convergence properties. A simulation result demonstrated the effectiveness of our design. Our future work will include extension of the results to directed graphs and completely unknown nonlinearity.

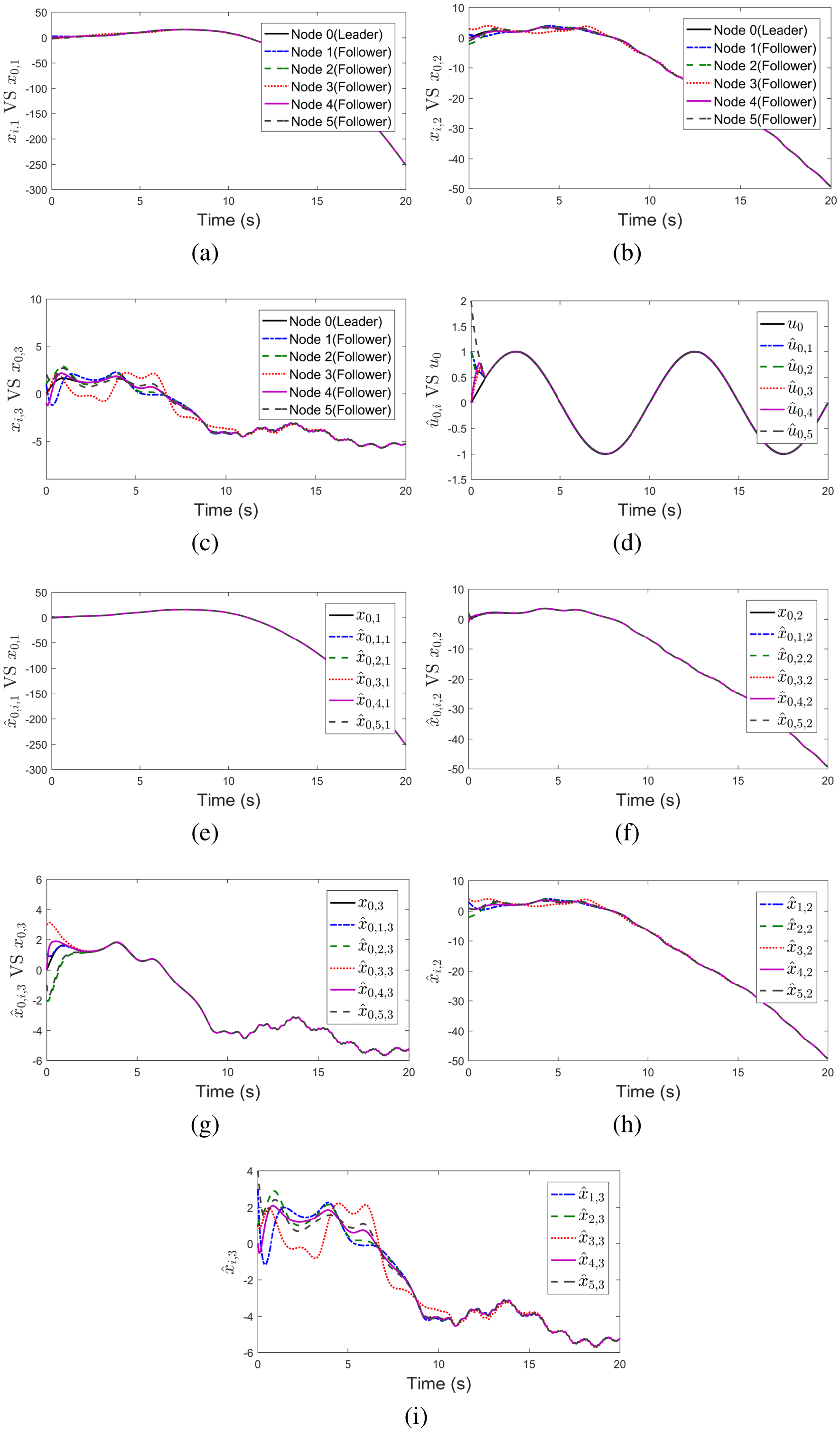

Figure 4.1: Third-order nonlinear MAS profiles: (a) leader's and followers' state trajectory profiles of $x_{i,1}$ for $i = 0, 1, \ldots, N$; (b) leader's and followers' state trajectory profiles of $x_{i,2}$ for $i = 0, 1, \ldots, N$; (c) leader's and followers' state trajectory profiles of $x_{i,3}$ for $i = 0, 1, \ldots, N$; (d) leader's input profile and the estimation by each follower; (e) leader's state trajectory profile of $x_{0,1}$ and the estimation by each follower; (f) leader's state trajectory profile of $x_{0,2}$ and the estimation by each follower; (g) leader's state trajectory profile of $x_{0,3}$ and the estimation by each follower; (h) followers' estimation of their own state trajectories of $x_{i,2}$ for $i = 1, 2, \ldots, N$; (i) followers' estimation of their own state trajectories of $x_{i,3}$ for $i = 1, 2, \ldots, N$.

Chapter 5

Energy-Aware Leader-Follower Tracking Control for Electric-Powered Multi-Agent Systems[1]

5.1 Introduction

Coordinated control design is central to the successful accomplishment of many MAS missions, having emerged as an active research field in the system and control community. In this vibrant field, problems of prime interest include group consensus [76, 110–114], swarming and flocking [115, 116], formation control [117–120], synchronization [121, 122], rendezvous [123], coverage control [124, 125], containment control [126, 127], and leader-follower tracking [42, 128]. However, despite these advances, the constrained operation time/range of an MAS often makes it unable to meet practical needs, which is a continual challenge in this area.

Most of electric-powered MASs depend on batteries for energy storage. The most favored choice is the lithium-ion batteries (LiBs) because of their high energy density, low self-discharge and long cycle life. Yet, though considered the best among all, LiBs still do not have the energy-weight ratio high enough to support long-duration tasks due to the electrochemistry-imposed constraints. For instance, many off-the-shelf unmanned aerial vehicles can only fly for 30 minutes at one charge, and ground robotic vehicles mounted with LiBs of much larger capacity will see power depletion in two to three hours, according to our survey. This issue is also pointed out in a few reports, e.g., [129, 130], raising concerns about the competence of MASs for long-endurance tasks. While the materials science and electrochemistry communities are making aggressive effort

[1]This chapter is based on the dissertation author's first-authored journal paper [44] and first-authored conference paper [109].

to develop batteries of higher energy and power densities, real-time control of battery use offers another promising way to improve the battery performance, as demonstrated by the rich literature in the area of battery management [131]. A question of interest then is: can an MAS have an extended operation time and range if its system-wide coordinated control is integrated with the battery control?

With its practical significance, energy awareness is a recurring subject in control design. It is conventionally handled by formulating a cost function weighing the relative importance of the considered control objective versus that of the input energy [132, 133] or through enforcing hard constraints on control inputs [134, 135]. These studies only emphasize reducing energy consumption in the control run, regardless of the dynamic features of the power sources. When it comes to MAS control design, a large body of work likewise rarely takes energy storage into consideration and almost unanimously assumes unconstrained power availability to drive an agent, e.g., [1, 136, 137], which though is not realistic. In reality, a battery not only has limited energy capacity and instantaneous power output but also is not an ideal linear power source as often assumed. A crucial factor contributing to a battery's nonlinear behavior is the well-known *rate capacity effect*, which states that the battery's total usable capacity goes down with an increase in discharging power [138]. That is, the higher the discharging power, the faster the battery will be drained, or equivalently, the available capacity will decrease at a slower rate given a lower discharging power. This phenomenon implies the promise of extracting more energy from the battery to support longer-time and wider-range operation if the discharging process is controlled to be appropriately conservative yet without much compromise to the mission control objective. Similar to this notion, the literature contains several studies on communication protocol design for wireless sensor networks aware of the rate capacity effect to increase operation time, e.g. [139]. However, the methodologies proposed therein are not suitable here due to the difference in problem contexts and structures. Meanwhile, the ever-widening use of batteries in electrified transportation, grid and buildings has motivated a growing interest on advanced battery management algorithms, which mainly focus on state-of-charge (SoC) and state-of-health (SoH) estimation [140–149], charging protocol optimization [150–152], thermal monitoring [153, 154], etc. Yet, they usually consider a standalone battery, without integrating the battery control with the system that it powers.

With this motivation, this chapter will investigate battery-aware time/range-extended leader-follower tracking. In a leader-follower MAS, the follower agents are distributedly controlled to track the trajectory of the leader agent with real-time information exchange among them according to a communication topology. Differing from the existing work, each follower in this study will be conscious of not only the tracking objective but also the rate capacity effect intrinsic to its battery. The challenge, however, lies in how to design an effective approach to control the joint MAS-battery dynamics in a distributed manner. To overcome it, this work will consider an MPC-based design for two reasons. First, the predictive nature of MPC will allow the battery use to be planned ahead, thus enabling consciousness of battery status. Second, MPC can accommodate state and input constraints [155, 156], which makes it a fit for handling battery use limits. Along this line, an MPC strategy based on distributed optimization is thus developed for battery-aware tracking.

The contributions of this work are as follows. 1) Formulation of battery-aware time/range-extended leader-follower tracking problem. It is presented in the form of receding-horizon optimization under constraints relevant to agent and nonlinear battery dynamics embodied by the battery's rate capacity effect. 2) Synthesis of a distributed MPC algorithm to address the problem. With this algorithm, each follower can use exchanged information with its neighbors to decide its control action in order to balance tracking performance and battery energy saving. The design builds on a distributed alternating direction method of multipliers (D-ADMM) method proposed in [157]. This study is the first one that we are aware of that exploits the battery's nonlinear dynamics to increase the operation time/range of an MAS.

The rest of this chapter is organized as follows. Section 5.2 describes the problem of leader-follower tracking with an awareness of the battery's rate capacity effect. Section 5.3 derives the distributed MPC strategy from the perspective of distributed optimization as a solution to the considered problem. A simulation study is offered in Section 5.4 to illustrate the effectiveness of the proposed strategy. Finally, Section 5.5 gathers our concluding remarks.

The notation throughout this chapter is standard. The set of real numbers is denoted by $\mathbb{R}$. The Euclidean norm of a vector is denoted as $\|\cdot\|$, and the Manhattan norm denoted as $\|\cdot\|_1$. Matrices, if their dimensions are not indicated explicitly, are assumed to be compatible in algebraic operations. Consider an MAS with one leader, which is labeled as 0, and N followers, which are labeled from 1 to N. The interaction topology among followers is modeled by an undirected graph. This graph is expressed as $G = (V, \mathcal{E})$, where $V = \{1, 2, \cdots, N\}$ is the node set and the edge set $\mathcal{E} \subseteq V \times V$ contains unordered pairs of nodes. A path is a sequence of connected edges in a graph. The neighbor set of agent i is denoted as $\mathcal{N}_i$, which includes all the agents in communication with it. Furthermore, combination of G with the leader gives a directed graph $\bar{G}$ since the information exchange is one-way from the leader to the followers directly connected with it. For $\bar{G}$, $\bar{\mathcal{N}}_i$ represents the neighbor set of agent i. Note that $\bar{\mathcal{N}}_i = \mathcal{N}_i \cup \{0\}$ if agent i can directly communicate with the leader and $\bar{\mathcal{N}}_i = \mathcal{N}_i$ otherwise.

5.2 Problem Formulation

This section formulates the problem of MPC-based battery-aware distributed leader-follower tracking control.

For a leader-follower MAS, the followers are expected to track the trajectory of the leader. During the tracking process, the leader and followers will maintain communication according to a pre-specified network topology to exchange their state information. Leveraging the information exchange, the followers will adjust control to themselves to achieve tracking. Suppose that a follower's dynamics is given by

$$z_i(t+1) = z_i(t) + \sigma_i u_i(t), \ i = 1, 2, \ldots, N, \tag{5.1}$$

where $z_i \in \mathbb{R}^n$ is follower i's current state, $u_i \in \mathbb{R}^n$ the control input with $\|u_i\|_1$ assumed to be the instantaneous discharging power drawn from the onboard battery, and $\sigma_i \in \mathbb{R}^{n\times n}$ a positive diagonal matrix of constant coefficient. While the model looks simple, it is capable of describing many types of MASs and thus often used in the literature [6]. In addition, it does not limit generalization of the following results to an MAS with higher-order dynamics. Without loss of generality, the leader's dynamics takes on a similar form:

$$z_0(t+1) = z_0(t) + \sigma_0 u_0(t), \tag{5.2}$$

where $z_0 \in \mathbb{R}^n$ and $u_0 \in \mathbb{R}^n$ denote the state and input of the leader, respectively, and $\sigma_0 \in \mathbb{R}^{n\times n}$ is a positive coefficient matrix. Since the leader's maneuver depends on the mission context, its control input u_0 is supposed to have been determined. Given the practical communication constraints, the leader is assumed to communicate with only part of the followers during the tracking progress.

A critical yet often neglected factor in MAS tracking control is the batteries mounted on the followers to provide power. Rather than a linear energy reservoir as widely assumed in the literature, a battery demonstrates nonlinear dynamics that can affect the amount of power and energy that it can offer. For a battery, its state is measured by the SoC, which is a percentage ratio between the available energy capacity and the maximum capacity. Denoting the SoC of follower i's battery as $x_i \in \mathbb{R}$, it is then given by

$$x_i(t) = \frac{Q_i(t)}{Q_{i,\max}} \times 100\%, \quad i = 1, 2, \dots, N,$$

where Q_i in Wh is the present capacity and $Q_{i,\max}$ is the nominal maximum capacity. Energy extraction from a battery and consequently, the change of SoC, is subjected to the rate capacity effect, which refers to the fact that the battery's usable capacity will decrease at a faster rate if the discharging power increases. This effect can be described by the well-known Peukert's law [158]:

$$C_p = I^{\beta} \bar{t}, \tag{5.3}$$

where C_p in Wh is the battery's nominal capacity calibrated at a one-ampere discharge rate, I is the actual discharging current, $\bar{t}$ is the actual time to discharge the battery, and β is the Peukert constant with $\beta > 1$. Along this line, the dynamics of SoC during discharging is governed by the following equation:

$$x_i(t+1) = x_i(t) - \alpha_i \|u_i(t)\|_1^{\beta}, \tag{5.4}$$

where $\|u_i\|_1$ is the discharging power as defined earlier, and $\alpha_i = t_s \delta_i / (3600 \cdot Q_{i,\max})$, with t_s being the sampling period in seconds and $\delta_i \geq 1$ the discharging efficiency coefficient. It should be noted that a battery's operation should be bounded for the consideration of battery safety and health [131]. Then to avoid overuse, a battery's SoC needs to be kept within a favorable range defined by the lower and upper limits $x_{i,\min}$ and $x_{i,\max}$, i.e.,

$$x_{i,\min} \leq x_i(t) \leq x_{i,\max}. \tag{5.5}$$

Meanwhile, the discharging power $\|u_i\|_1$ should also be bounded by an upper limit $u_{i,\max}$ in order to prevent life-damaging effects. Hence,

$$0 \leq \|u_i(t)\|_1 \leq u_{i,\max}. \tag{5.6}$$

As aforementioned, the objective of this paper is to develop a distributed control strategy for battery-aware leader-follower tracking in order to enhance the overall operation time/range. The battery awareness here mainly builds on an understanding of the rate capacity effect—if the discharging power is constrained, more energy can be extracted from the battery to support longer-duration tracking. However, a systematic control design must be performed in order to balance time/range extension and tracking performance, which will require us to answer a key question: how to jointly control the dynamics of a follower and its battery to enable energy-conscious tracking under the constraints relevant to battery operation? To address this challenge, an MPC approach is considered here. It frames the solution on the idea that the leader can inform its neighbor followers of its control decisions in an upcoming time window. Given preview of the leader's behavior, a follower can then make predictive control to extract power from its battery to track the leader.

Constrained optimization can then be formulated as a result of its battery's constraints, to which MPC can well lend itself. Through appropriate design, the MPC can be distributed among the followers such that each follower can apply individual control with cognizance of the global tracking objective.

To this end, we begin with a centralized MPC for energy-conscious tracking, which is laid out as follows:

$$\min \sum_{\tau=t+1}^{t+T} \sum_{i=1}^{N} \left[q_i \sum_{j \in \bar{\mathcal{N}}_i} \left\| z_i(\tau) - z_j(\tau) \right\|^2 + r_i \left(\| u_i(\tau-1) \|_1^{\beta} \right)^2 \right], \tag{5.7a}$$

$$\text{s.t.}\ z_i(\tau+1) = z_i(\tau) + \sigma_i u_i(\tau), \tag{5.7b}$$

$$z_0(\tau+1) = z_0(\tau) + \sigma_0 u_0(\tau), \tag{5.7c}$$

$$x_i(\tau+1) = x_i(\tau) - \alpha_i \| u_i(\tau) \|_1^{\beta}, \tag{5.7d}$$

$$x_{i,\min} \leq x_i(\tau) \leq x_{i,\max}, \tag{5.7e}$$

$$0 \leq \| u_i(\tau) \|_1 \leq u_{i,\max}, \tag{5.7f}$$

$$i = 1, 2, \ldots, N,\ \tau = t, t+1, \ldots, t+T-1.$$

Here, the MPC is considered for the time window from $t+1$ to $t+T$, and its cost function is quadratic and composed of two terms. The first term quantifies the squared sum of distances between follower i and its neighbors, including the leader or the other followers, and the second quadratic term expresses follower i's energy cost. Here, q_i and r_i are weighting factors to show the tradeoff between distance and energy consumption costs. The constraint (5.7d) serves as a predictive model able to indicate follower i's battery behavior within the window; similarly, follower i's system dynamics follows the constraint (5.7b). The battery's operational constraints are summarized in (5.7e)-(5.7f) as duplicates of (5.5)-(5.6). Next, we will distribute the MPC in (5.7) across the MAS, with the detailed development shown in Section 5.3.

Remark 9. (Novelty of MPC-based leader-follower tracking). While the problem in (5.7) is motivated to enable time/range-extended MAS tracking with planned battery use, it also represents the first MPC-based formulation for leader-follower tracking control to our knowledge. It is observed

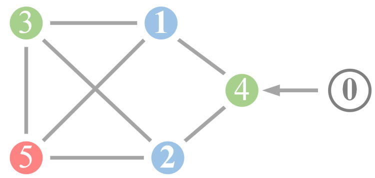

Figure 5.1: Illustration of a colored MAS communication topology, with the leader numbered as 0 and the followers numbered from 1 to 5. Note that the communication from the leader to a follower is unidirectional (directed) and that the communication between followers is bidirectional (undirected). Three colors, blue, green, and red, are used to mark the undirected follower graph such that no adjacent followers share the same color, and each follower is numbered in a color-based order. Thus, $\mathcal{C}_1 = \{1, 2\}$, $\mathcal{C}_2 = \{3, 4\}$, and $\mathcal{C}_3 = \{5\}$.

that this formulation brings two advantages. First, MPC would allow predictive management of the tracking mission under diverse practical operation constraints. Second, the problem formulated in (5.7) does not require the availability of u_0 at the end of the followers. This is a well-founded improvement to the literature, which often assumes that each follower has knowledge of the leader's instantaneous driving input u_0 even if it does not directly communicate with the leader [42, 128].

Remark 10. (Comparison with saturation-based formulation). Actuators are often subject to actuation limits, which are often modeled as control input saturation [159]. It is understood that the saturation can bring energy saving to a certain extent. However, this mechanism is passive and relevant to local control. By contrast, the MPC-based formulation not only includes actuation limits, i.e., (5.7f), but also introduces active optimization between energy use and tracking performance, more advantageous from a practical perspective.

5.3 Distributed MPC Design for Energy-Aware Tracking

This section distributes the centralized MPC in (5.7a)-(5.7f) across the followers such that each follower will perform local control yet achieve tracking as a group. The solution will build on the D-ADMM algorithm in [157], which decomposes a centralized optimization problem into a set of distributed separable ones. This algorithm is based on the ideas that the nodes can be marked according to a coloring scheme and that local actions follow a color-based order to enable distributed optimization. To see this, let us consider the follower graph. Suppose that P colors can be used to color the followers such that no adjacent ones share the same color. The first C_1 nodes have color 1, numbered as $\{1,2,\ldots,C_1\}$ and denoted as the set $\mathcal{C}_1$. Similarly, the C_p nodes have color p, collected in $\mathcal{C}_p = \{C_{p-1}+1, C_{p-1}+2, \ldots, C_{p-1}+C_p\}$. It then follows that $\sum_{p=1}^{P} C_p = N$. An example is shown in Figure 5.1. To proceed further, the following assumption is needed.

Assumption 4. The leader-follower graph is connected and time-invariant. Moreover, the follower graph is capable of implementing a coloring scheme.

In Assumption 4, a network is guaranteed to be connected if a path exists between every pair of nodes.

In what follows, we will develop the distributed MPC based on the coloring scheme.

5.3.1 Problem Manipulation

Here, we will reformulate the centralized MPC in (5.7a)-(5.7f) into a form suitable for distributed execution. Among distributed setting, a follower both measures its own variables composed of SoC, discharging power and position, and senses these variables from its neighbors. Along this line, let us first consider recasting the initial MPC problem into the consistency constraint form.

For follower i, its SoC variables from $t+1$ to $t+T$ can be stacked into a vector:

$$\hat{X}_i = \begin{bmatrix} \cdots & x_i(\tau) & \cdots \end{bmatrix}^\top, \tag{5.8}$$

for $\tau = t+1,\ldots,t+T$. Furthermore, to locally paint a picture of the global network, each follower has virtual copies of state and control variables of all the other nodes in addition to its own. Hence, follower i's state variable, $\bar{z}_i$, is defined by

$$\bar{z}_i(\tau) = \begin{bmatrix} \cdots & \left(z_j^i(\tau)\right)^\top & \cdots \end{bmatrix}^\top,$$

where $j = 1,2,\ldots,N$, and z_j^i represents the virtual replica of z_j in node i. Then, the predicted state vector within the window is obtained as

$$Z_i = \begin{bmatrix} \cdots & \bar{z}_i^\top(\tau) & \cdots \end{bmatrix}^\top, \tag{5.9}$$

for $\tau = t+1, \ldots, t+T$. Thus, Z_i represents node i's replica of all the states of all the followers in the network. In order to ensure consistency among all followers, the following constraint is enforced:

$$Z_i = Z_j, \tag{5.10}$$

for $j \in \mathcal{N}_i$. Similarly, we define

$$\bar{u}_i(\tau) = \begin{bmatrix} \cdots & \left(u_j^i(\tau)\right)^\top & \cdots & \|u_j^i(\tau)\|_1^\beta & \cdots \end{bmatrix}^\top,$$

for $j = 1, \ldots, N$, where u_j^i and $\|u_j^i\|_1^\beta$ represent the copy of u_j and $\|u_j\|_1^\beta$ stored at follower i, respectively. Note that we differentiate u_j and $\|u_j\|_1^\beta$ here, because the former is the power needed by the maneuver and the latter represents the actual power drawn due to the maneuver. The collection, $\bar{u}_j^i$ within the window, is then described as

$$U_i = \begin{bmatrix} \cdots & \bar{u}_i^\top(\tau) & \cdots \end{bmatrix}^\top, \tag{5.11}$$

for $\tau = t, \ldots, t+T-1$. To guarantee consistency among nodes, the following condition is needed to ensure all the control copies to be equal in an edge-based way:

$$U_i = U_j,\ \forall j \in \mathcal{N}_i. \tag{5.12}$$

We further define $X_i = \begin{bmatrix} Z_i^\top & \hat{X}_i^\top \end{bmatrix}^\top$ and rewrite the MPC problem using all the above definitions in

(5.7a)-(5.7f) into the following:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{N} f_i(X_i, U_i), && (5.13a)\\
\text{s.t.} \quad & A_i X_i + B_i U_i = c_i, && (5.13b)\\
& X_i \in \mathcal{X}_i, \quad U_i \in \mathcal{U}_i, && (5.13c)\\
& Z_i = Z_j, \text{ for } j \in \mathcal{N}_i, && (5.13d)\\
& U_i = U_j, \text{ for } j \in \mathcal{N}_i, && (5.13e)\\
& i = 1, 2, \ldots, N.
\end{aligned}
$$

Here, f_i is the cost function posed for follower i, which is deduced from (5.7a), (5.13b) is built on a combination of predictive model in (5.7b) and (5.7d), and (5.13c) summarizes the state and control constraints in (5.7e)-(5.7f).

With the followers marked in different colors according to the coloring scheme introduced at the beginning of this section, the next step is to convert the problem in (5.13a)-(5.13e) and make it ready for distributed optimization. To proceed, the state and control consistency constraints in (5.13d) and (5.13e) can be rewritten as

$$
\left(M^\top \otimes I_{nNT}\right)^\top \begin{bmatrix} Z_1 \\ \vdots \\ Z_N \end{bmatrix} = 0, \quad \left(M^\top \otimes I_{nNT}\right)^\top \begin{bmatrix} U_1 \\ \vdots \\ U_N \end{bmatrix} = 0, \tag{5.14}
$$

where M is the incidence matrix of the graph of followers, $I_{nNT} \in \mathbb{R}^{nNT \times nNT}$ is the identity matrix, and $\otimes$ denotes the Kronecker product. Then, we combine the states and control inputs of the followers marked in color p into new vectors:

$$
\bar{Z}_p = \begin{bmatrix} \cdots & Z_i^\top & \cdots \end{bmatrix}^\top, \quad \bar{U}_p = \begin{bmatrix} \cdots & U_i^\top & \cdots \end{bmatrix}^\top,
$$

for $i \in \mathcal{C}_p$. Since the followers are numbered in the order of colors, (5.14) is rewritten as

$$\bar{M}\begin{bmatrix} Z_1 \\ \vdots \\ Z_N \end{bmatrix} = 0, \quad \bar{M}\begin{bmatrix} U_1 \\ \vdots \\ U_N \end{bmatrix} = 0, \tag{5.15}$$

where $\bar{M} = \left(M^\top \otimes I_{nNT}\right)^\top$. It is noted that $\bar{M}$ can be decomposed in a color-based manner such that (5.15) can be expressed equivalently as

$$\sum_{p=1}^{P} \bar{K}_p \bar{Z}_p = 0, \quad \sum_{p=1}^{P} \bar{K}_p \bar{U}_p = 0,$$

where $\bar{K}_p$ is the p-th row block of $\bar{M}$. In the meantime, the states of the batteries mounted on the followers can be aggregated, one color after another:

$$\tilde{X}_p = \begin{bmatrix} \cdots & \hat{X}_i^\top & \cdots \end{bmatrix}^\top,$$

for $i \in \mathcal{C}_p$. We define $\bar{X}_p = \left[\bar{Z}_p^\top \ \tilde{X}_p^\top\right]^\top$. Hence, the MPC problem in (5.13) can be presented in a color-separable way:

$$\begin{aligned}
\min \quad & \sum_{p=1}^{P} g_p\left(\bar{X}_p, \bar{U}_p\right), && (5.16a)\\
\text{s.t.} \quad & \bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p = \bar{c}_p, && (5.16b)\\
& \bar{X}_p \in \bar{\chi}_p, \quad \bar{U}_p \in \bar{\mathcal{U}}_p, \quad p = 1, 2, \ldots, P, && (5.16c)\\
& \sum_{p=1}^{P} \bar{K}_p \bar{Z}_p = 0, && (5.16d)\\
& \sum_{p=1}^{P} \bar{K}_p \bar{U}_p = 0, && (5.16e)
\end{aligned}$$

where g_p, $\bar{\mathcal{X}}_p$, $\bar{\mathcal{U}}_p$, $\bar{A}_p$, $\bar{B}_p$ and $\bar{c}_p$ can be derived from the context. Now, the original MPC problem is changed into a color-based type, which can be broken down further to achieve full distribution among individual followers.

5.3.2 Distributed MPC

Here, we develop distributed MPC based on the problem in (5.16), leveraging the D-ADMM algorithm in [157]. To begin with, the augmented Lagrangian function for (5.16) is defined as

$$
\begin{aligned}
L = \sum_{p=1}^{P} \Big[& g_p\left(\bar{X}_p, \bar{U}_p\right) + \lambda_p^\top \left(\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p - \bar{c}_p\right) + \varphi^\top \bar{K}_p \bar{Z}_p + \eta^\top \bar{K}_p \bar{U}_p \\
& + \frac{\rho_1}{2} \left\|\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p - \bar{c}_p\right\|^2 \Big] + \frac{\rho_2}{2} \left\| \sum_{p=1}^{P} \bar{K}_p \bar{Z}_p \right\|^2 + \frac{\rho_3}{2} \left\| \sum_{p=1}^{P} \bar{K}_p \bar{U}_p \right\|^2, \qquad (5.17)
\end{aligned}
$$

where $\lambda_p > 0$, φ and η are dual variables, and ρ_1, ρ_2 and ρ_3 are positive penalty parameters. Using the alternating direction method of multipliers (ADMM), minimizing $\bar{X}_p$ and $\bar{U}_p$ can be achieved through the following iterative procedure:

$$
\begin{aligned}
\bar{X}_p^{k+1} &=_{\bar{U}_p \in \bar{\mathcal{U}}_p} L\left(\bar{X}_1^{k+1}, \bar{U}_1^{k+1}, \ldots, \bar{X}_p, \bar{U}_p^k, \ldots, \bar{X}_P^k, \bar{U}_P^k; \lambda_p^k, \varphi^k, \eta^k\right), && (5.18)\\
\bar{U}_p^{k+1} &=_{\bar{X}_p \in \bar{\mathcal{X}}_p} L\left(\bar{X}_1^{k+1}, \bar{U}_1^{k+1}, \ldots, \bar{X}_p^{k+1}, \bar{U}_p, \ldots, \bar{X}_P^k, \bar{U}_P^k; \lambda_p^k, \varphi^k, \eta^k\right), && (5.19)\\
\lambda_p^{k+1} &= \lambda_p^k + \rho_1 \left(\bar{A}_p \bar{X}_p^{k+1} + \bar{B}_p \bar{U}_p^{k+1} - \bar{c}_p\right), && (5.20)\\
\varphi^{k+1} &= \varphi^k + \rho_2 \sum_{p=1}^{P} \bar{K}_p \bar{Z}_p^{k+1}, && (5.21)\\
\eta^{k+1} &= \eta^k + \rho_3 \sum_{p=1}^{P} \bar{K}_p \bar{U}_p^{k+1}, && (5.22)
\end{aligned}
$$

where k is the iteration counter.

It is seen that (5.18) can be explicitly given as

$$
\begin{aligned}
\bar{X}_p^{k+1} =_{\bar{X}_p \in \bar{\mathcal{X}}_p} & g_p\left(\bar{X}_p, \bar{U}_p^k\right) + \left(\lambda_p^k\right)^\top \left(\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p^k - \bar{c}_p\right) + \left(\varphi^k\right)^\top \bar{K}_p \bar{Z}_p + \left(\eta^k\right)^\top \bar{K}_p \bar{U}_p^k \\
& + \frac{\rho_1}{2} \left\|\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p^k - \bar{c}_p\right\|^2 + \frac{\rho_2}{2} \left\|\bar{K}_p \bar{Z}_p + \sum_{i=1,\ldots,P,\, i\neq p} \bar{K}_i \bar{Z}_i^k\right\|^2 + \frac{\rho_3}{2} \left\| \sum_{i=1}^{P} \bar{K}_i \bar{U}_i \right\|^2 \\
=_{\bar{X}_p \in \bar{\mathcal{X}}_p} & g_p\left(\bar{X}_p, \bar{U}_p^k\right) + \left(\lambda_p^k\right)^\top \left(\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p^k - \bar{c}_p\right) + \left(\varphi^k\right)^\top \bar{K}_p \bar{Z}_p \\
& + \frac{\rho_1}{2} \left\|\bar{A}_p \bar{X}_p + \bar{B}_p \bar{U}_p^k - \bar{c}_p\right\|^2 + \frac{\rho_2}{2} \left\|\bar{K}_p \bar{Z}_p + \sum_{i=1,\ldots,P,\, i\neq p} \bar{K}_i \bar{Z}_i^k\right\|^2. \qquad (5.23)
\end{aligned}
$$

From above, the first three terms of the right-hand side of (5.23) are each a linear combination relative to the individual color-p followers. Further, the last term in (5.23) can be expressed as

$$\left\|\bar{K}_p\bar{Z}_p + \sum_{i=1,\dots,P,\, i\neq p} \bar{K}_i\bar{Z}_i^k\right\|^2 = \bar{Z}_p^\top \bar{K}_p^\top \bar{K}_p\bar{Z}_p + 2\bar{Z}_p^\top \bar{K}_p^\top \sum_{i=1,\dots,P,\, i\neq p} \bar{K}_i\bar{Z}_i^k + \left\|\sum_{i=1,\dots,P,\, i\neq p} \bar{K}_i\bar{Z}_i^k\right\|^2. \tag{5.24}$$

Since color-p nodes do not connect to each other, the term $\bar{K}_p^\top \bar{K}_p$ turns out to be a diagonal matrix with the diagonal elements being the degree of respective node, i.e.,

$$\bar{Z}_p^\top \bar{K}_p^\top \bar{K}_p\bar{Z}_p = \sum_{i\in\mathcal{C}_p} D_i \left\|\bar{Z}_i\right\|^2, \tag{5.25}$$

where D_i is the degree of node i. Meanwhile, $\bar{K}_p^\top \bar{K}_i = -I_{nNT}$ for $p \neq i$, and we have

$$\bar{Z}_p^\top \bar{K}_p^\top \sum_{i=1,\dots,P,\, i\neq p} \bar{K}_i\bar{Z}_i^k = -\sum_{i\in\mathcal{C}_p}\sum_{j\in\mathcal{N}_i} Z_i^\top Z_j^k. \tag{5.26}$$

With (5.24)-(5.26), (5.23) can be simplified as

$$\begin{aligned}\bar{X}_p^{k+1} =_{\bar{X}_p\in\bar{\mathcal{X}}_p} & g_p\left(\bar{X}_p,\bar{U}_p^k\right) + \left(\lambda_p^k\right)^\top\left(\bar{A}_p\bar{X}_p + \bar{B}_p\bar{U}_p^k - \bar{c}_p\right) + \frac{\rho_1}{2}\left\|\bar{A}_p\bar{X}_p + \bar{B}_p\bar{U}_p^k - \bar{c}_p\right\|^2 \\ & + \left(\mu_i^k - \rho_2\sum_{j\in\mathcal{N}_i} Z_j^k\right)^\top Z_i + \frac{\rho_2 D_i}{2}\|Z_i\|^2,\end{aligned} \tag{5.27}$$

where $\mu_i^k = \sum_{j\in\mathcal{N}_i} \mathrm{sign}(j-i)\varphi_{ij}^k$. We define an auxiliary dual variable $\phi_i^k = \mu_i^k - \rho_2 \sum_{j\in\mathcal{N}_i,\, j<i} Z_j^{k+1} - \rho_2 \sum_{j\in\mathcal{N}_i,\, j>i} Z_j^k$. The above equation, as a linear combination of Lagrangians of the color-p nodes, can then be split with respect to X_i for $i\in\mathcal{C}_p$, i.e.,

$$X_i^{k+1} =_{X_i\in\mathcal{X}_i} f_i\left(X_i,U_i^k\right) + \left(\gamma_i^k\right)^\top\left(A_iX_i + B_iU_i^k - c_i\right) + \frac{\rho_1}{2}\left\|A_iX_i + B_iU_i^k - c_i\right\|^2 + \left(\phi_i^k\right)^\top Z_i + \frac{\rho_2 D_i}{2}\|Z_i\|^2,$$

where $i \in \mathcal{C}_p$ and γ_i is the dual variable at the local node i, i.e.,

$$\lambda_p = \begin{bmatrix} \cdots & \gamma_i^\top & \cdots \end{bmatrix}^\top.$$

Now we consider the iterative update of $\bar{U}_p$ in (5.19), which can be expressed as

$$\begin{aligned}\bar{U}_p^{k+1} =_{\bar{U}_p \in \bar{\mathcal{U}}_p} & g_p\left(\bar{X}_p^{k+1}, \bar{U}_p\right) + \left(\lambda_p^k\right)^\top \left(\bar{A}_p \bar{X}_p^{k+1} + \bar{B}_p \bar{U}_p - \bar{c}_p\right) + \left(\eta^k\right)^\top \bar{K}_p \bar{U}_p \\ & + \frac{\rho_1}{2}\left\|\bar{A}_p \bar{X}_p^{k+1} + \bar{B}_p \bar{U}_p - \bar{c}_p\right\|^2 + \frac{\rho_3}{2}\left\|\bar{K}_p \bar{U}_p + \sum_{i=1,\ldots,P,\, i \neq p} \bar{K}_i \bar{U}_i^k\right\|^2. \end{aligned} \tag{5.28}$$

Following lines similar to the above, (5.28) is also separable with respect to U_i, which results in

$$\begin{aligned} U_i^{k+1} =_{U_i \in \mathcal{U}_i} & f_i\left(X_i^{k+1}, U_i\right) + \left(\gamma_i^k\right)^\top \left(A_i X_i^{k+1} + B_i U_i - c_i\right) + \frac{\rho_1}{2}\left\|A_i X_i^{k+1} + B_i U_i - c_i\right\|^2 \\ & + \left(\xi_i^k\right)^\top U_i + \frac{\rho_3 D_i}{2}\|U_i\|^2, \end{aligned} \tag{5.29}$$

where $\psi = \sum_{j \in \mathcal{N}_i} \operatorname{sign}(j-i) \cdot \eta_{ij}^k$ and $\xi_i^k = \psi_i^k - \rho_3 \sum_{j \in \mathcal{N}_i,\, j<i} U_j^{k+1} - \rho_3 \sum_{j \in \mathcal{N}_i,\, j>i} U_j^k$.

After the updates of X_i and U_i at follower i, the dual variable γ_i can be updated as

$$\gamma_i^{k+1} = \rho_1\left(A_i X_i^{k+1} + B_i U_i^{k+1} - c_i\right).$$

On the completion of updating X_i, U_i and γ_i for all the nodes, we can update μ_i and ψ_i

$$\begin{aligned} \mu_i^{k+1} &= \mu_i^k + \rho_2 \sum_{j \in \mathcal{N}_i}\left(Z_i^{k+1} - Z_j^{k+1}\right), \\ \psi_i^{k+1} &= \psi_i^k + \rho_3 \sum_{j \in \mathcal{N}_i}\left(U_i^{k+1} - U_j^{k+1}\right), \end{aligned}$$

where $i = 1, 2, \ldots, N$.

Up to this point, we have obtained a distributed MPC algorithm, which is summarized in Table 5.1. This algorithm fully distributes the original centralized MPC among the followers. As a result, a follower can learn about its neighbors' states and make individual control decision to min-

Table 5.1: Distributed MPC algorithm for battery-aware leader-follower tracking.

initialize: set $X_i^1 = 0$**,** $U_i^1 = 0$**,** γ_i^1**,** $\mu_i^1 = 0$**,** $\psi_i^1 = 0$
repeat
  $k \leftarrow k+1$
  for $p = 1, 2, \cdots, P$ **do**
    for $i \in \mathcal{C}_p$ or $i = C_{p-1}+1, \cdots, C_{p-1}+C_p$ **do**
      do

$$\phi_i^k = \mu_i^k - \rho_2 \sum_{j \in \mathcal{N}_i, j<i} Z_j^{k+1} - \rho_2 \sum_{j \in \mathcal{N}_i, j>i} Z_j^k$$

      find

$$\begin{aligned} X_i^{k+1} =_{X_i \in \mathcal{X}_i} & f_i\left(X_i, U_i^k\right) + \left(\gamma_i^k\right)^\top \left(A_i X_i + B_i U_i^k - c_i\right) \\ & + \frac{\rho_1}{2} \left\|A_i X_i + B_i U_i^k - c_i\right\|^2 + \left(\phi_i^k\right)^\top Z_i + \frac{\rho_2 D_i}{2} \|Z_i\|^2 \end{aligned}$$

      do

$$\xi_i^k = \psi_i^k - \rho_3 \sum_{j \in \mathcal{N}_i, j<i} U_j^{k+1} - \rho_3 \sum_{j \in \mathcal{N}_i, j>i} U_j^k$$

      find

$$\begin{aligned} U_i^{k+1} =_{U_i \in \mathcal{U}_i} & f_i\left(X_i^{k+1}, U_i\right) + \left(\gamma_i^k\right)^\top \left(A_i X_i^{k+1} + B_i U_i - c_i\right) \\ & + \frac{\rho_1}{2} \left\|A_i X_i^{k+1} + B_i U_i - c_i\right\|^2 + \left(\xi_i^k\right)^\top U_i + \frac{\rho_3 D_i}{2} \|U_i\|^2 \end{aligned}$$

      do

$$\gamma_i^{k+1} = \rho_1 \left(A_i X_i^{k+1} + B_i U_i^{k+1} - c_i\right)$$

    end for
  end for
  for $i = 1, 2 \cdots, N$ **do**
    do

$$\mu_i^{k+1} = \mu_i^k + \rho_2 \sum_{j \in \mathcal{N}_i} \left(Z_i^{k+1} - Z_j^{k+1}\right)$$

$$\psi_i^{k+1} = \psi_i^k + \rho_3 \sum_{j \in \mathcal{N}_i} \left(U_i^{k+1} - U_j^{k+1}\right)$$

  end for
until certain pre-set stopping criterion is met

imize a global cost function collectively. In this MPC setting, the follower takes into account its own local power consumption constraints and the global objective that weighs the overall energy cost against tracking performance. This will bring the advantage of drawing power at a slower rate from the battery, which implies that more energy can be drawn due to the rate capacity effect. Then, the battery runtime and MAS operation range are both extended.

Theorem 7. Suppose that Assumption 4 holds, the proposed MPC approach enables an optimal discharging power generated in (5.29).

Proof: By [157], it is proven that D-ADMM algorithm will converge to optimal solution if the cost function is strongly convex and the network is connected. It is also noted that these conditions are satisfied according to (5.7) and when Assumption 4 holds. This completes the proof.

Remark 11. (Stability, feasibility and convergence). Stability and feasibility analysis have been studied in the literature for a few classes of MPC problems. The proofs are often based on prediction horizons of an infinite length or well-designed terminal costs [160]. Such conditions are not appropriate for leader-follower tracking because of the versatile situations in a tracking process.

Remark 12. (Computational complexity). To deal with nonlinear optimization problem in this paper, we use optimization toolbox of Matlab that admits an active-set algorithm. It divides the optimization problem into three main stages, which can then be solved step by step. One of these steps is quadratic programming that is used to optimize the Lagrangian function in this paper. Its computational complexity runs in time polynomial and is also largest among three steps. Therefore, it is difficult to calculate computational complexity for the whole nonlinear optimization problem. However, we may further apply large-scale algorithm in Matlab to reduce computational complexity. This is because it does not have to generate, store or operate full matrices, which thus results in saving execution time and reduces memory requirements.

Remark 13. (Extension to MASs with complex dynamics). The proposed results in this section can be readily generalized to an MAS with higher-order and even nonlinear dynamics, due to the wide applicability of the D-ADMM algorithm by design. The generalization can be performed along lines similar to the above development. It should also be noted that an increase in the complexity of dynamics will increase the computational costs of the resultant distributed optimization.

5.4 Numerical Study

In this section, we provide an illustrative example to verify the effectiveness of the proposed distributed MPC algorithm. Consider a battery-powered MAS consisting of one leader and five followers. The communication topology among them is shown in Figure 5.1. Node 0 is the leader, and nodes 1, 2, 3, 4 and 5 are followers. The leader will only send state updates to follower 1, and the followers maintain bidirectional communication with their neighbors.

Suppose that the i-th agent's dynamics can be described by a model used in [120]. Let (r_{xi}, r_{yi}), θ_i, and (v_i, ω_i) denote its Cartesian position, orientation, and linear and angular velocity, respectively. The kinematic equations are given by

$$\dot{r}_{xi} = v_i \cos(\theta_i), \tag{5.30a}$$

$$\dot{r}_{yi} = v_i \sin(\theta_i), \tag{5.30b}$$

$$\dot{\theta}_i = \omega_i. \tag{5.30c}$$

We apply feedback linearization to (5.30) around a fixed point denoted as (x_i, y_i), where $x_i = r_{xi} + d_i \cos(\theta_i)$ and $y_i = r_{yi} + d_i \sin(\theta_i)$ with $d_i = 0.15$m. Given

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) \\ -\frac{1}{d_i}\sin(\theta_i) & \frac{1}{d_i}\cos(\theta_i) \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix},$$

it follows that

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix}.$$

The initial states of the leader and followers are set to be $(4.02, 4.02)$, $(3.5, 3.45)$, $(2.8, 2.85)$, $(3, 3.05)$, $(2.7, 2.75)$ and $(3.2, 3.25)$. For each agent, it is assumed that $\sigma_i = 0.03 I_2$ for $i = 0, 1, \ldots, N$. The followers' batteries have different capacities, SoC and instantaneous power constraints, and initial states, which are summarized in Table 5.2. We assume that the maneuver input profile ap-

Table 5.2: Followers' battery parameters and operating bounds.

Follower	$Q_{i,\max}$ (Wh)	$x_{i,\min}$ (%)	$x_{i,\max}$ (%)	$x_i(0)$ (%)	$u_{i,\max}$ (W)
1	36	20	90	80	10
2	36	25	80	75	20
3	40	24	90	70	15
4	38	24	85	80	13
3	37	25	80	80	17

plied to the leader is given as

$$u_0(t) = \begin{cases} \begin{bmatrix} 5\sin(0.005\pi t)+5 \\ 5\cos(0.005\pi t)+5 \end{bmatrix} \text{(W)}, & 0 \le t \le 950 \text{ s}, \\ \begin{bmatrix} 5\sin(0.005\pi t)+10 \\ 5\cos(0.005\pi t)+10 \end{bmatrix} \text{(W)}, & 950 \text{ s} < t \le 1,100 \text{ s}, \\ \begin{bmatrix} 5\sin(0.005\pi t)+5 \\ 5\cos(0.005\pi t)+5 \end{bmatrix} \text{(W)}, & t > 1,100 \text{ s}. \end{cases}$$

The sampling period $t_s = 5$ s, and the window length $T = 3$, equivalent to fifteen-second or three-step-ahead prediction. For the batteries, the discharging efficiency coefficients are given as $\delta_i = 1$ for each follower i for simplicity. The Peukert constant is set as $\beta = \frac{5}{3}$. For the cost function in MPC, the weight coefficients are $q_i = 1$ and $r_i = 1$ for each follower i, and the penalty parameters involved in the distributed algorithm are set as $\rho_1 = 1$, $\rho_2 = 1$ and $\rho_3 = 1$.

The distributed MPC algorithm in Table 5.1 is applied to the above MAS for leader-follower tracking, with the simulation results shown in Figure 5.2. To make a comparison, a tracking control strategy proposed in [1] is also applied to the MAS. The results obtained are illustrated in Figure 5.3.

Figure 5.2a shows the trajectories of the leader and the followers using distributed MPC. It is seen that all the followers make an effort to track the leader. However, the followers are not rushing to approach the leader as soon as possible—there is an obvious tracking error maintained before the first about 5.8 Km in x-axis direction. This is because the followers are subject to power constraints and seek energy-aware tracking, refraining from fast maneuvers at fast energy consumption. After catching up with the leader, the followers then keep more accurate tracking. The evolutions of the position tracking errors are further shown in Figure 5.2b and 5.2c. From these two figures, the tracking errors of all the followers relative to the leader gradually decrease to zero at around 800 s in both x- and y-axis directions. Looking at the discharging profiles in Figure 5.2d, one can see that the followers adjust their individual power supply while obeying their own operation limits. In particular, the discharging power of followers 1 and 5 is zero at the first few seconds, which can be considered as "waiting" for followers 2, 3 and 4. They then jump to the upper power limit to start tracking the leader. The followers see their discharging power running at different levels due to their own battery constraints. At about the 800-th s, the discharging power of all the followers drops because the followers catch up with the leader. After sometime, the followers increase the discharging power again as the leader begins large maneuvers. In the final stage, the followers continue to adjust the discharging power to maintain tracking of the leader. The SoC profiles of the followers are shown in Figure 5.2e. It is observed that follower 3 is the first to see battery depletion, which ends the tracking process.

Let us now consider the case when the method in [1] is used. Figures 5.3a, 5.3b and 5.3c illustrate an exponential convergence of tracking, due to the assumed unlimited instantaneous power available to each follower. Associated with this, the discharging profiles in Figure 5.3d show that considerable power is applied in the initial stage, which leads to fast tracking. However, this comes at the expense of significantly short runtime. Figure 5.3e demonstrates the SoC decreasing more rapidly, and when follower 4 reaches its lower SoC limit, the operation time lasts for nearly only 1,800 s. By comparison, the total operation time of the MAS under the distributed MPC strategy is around 2,460 s, representing an improvement of 37%. With the time extension, the range of the MAS is also increased. According to Figures 5.2a and 5.3a, the leader's cruise distance reaches 5.68 Km from 3.70 Km, provided that all the followers can keep tracking. This counts as an increase of 53% in traveling distance. These results demonstrate the effectiveness of the proposed approach. It should also be pointed out that the time/range extension can be made more significant if one imposes stricter constraints on the power consumption limits. It is found that time/range extended operation may not be achieved through other simulation runs if we increase the value of q_i in (5.7). This is because the coefficient setting requires tracking objective to be completed at a fast speed. Hence, it result in large and fast power consumption by rate capacity effect. This scenario should be avoided during simulation runs.

5.5 Conclusion

This chapter considers energy-aware time/range-extended cooperative tracking. Electric-powered MASs nowadays often face limited energy budget and consequently, limited operation time/range, due to the limitations imposed by the onboard batteries. It thus becomes a pressing need to extract the maximum energy from the batteries to increase the time/range of an MAS, which, however, remains unexplored thus far. Motivated by this need, this chapter investigates the leader-follower tracking problem and proposes to integrate the MAS dynamics with battery dynamics. In this regard, the promising opportunity is identified that a battery's rate capacity effect can be exploited to help draw more energy from the battery. Then, an MPC problem is formulated to deal with MAS leader-follower tracking with a cognizance of the batteries' rate capacity dynamics and power/energy constraints. It is solved by a distributed optimization strategy, which leads to a distributed MPC algorithm. A simulation result shows its considerable effectiveness in extending the operation time/range of an MAS in comparison with an existing algorithm.

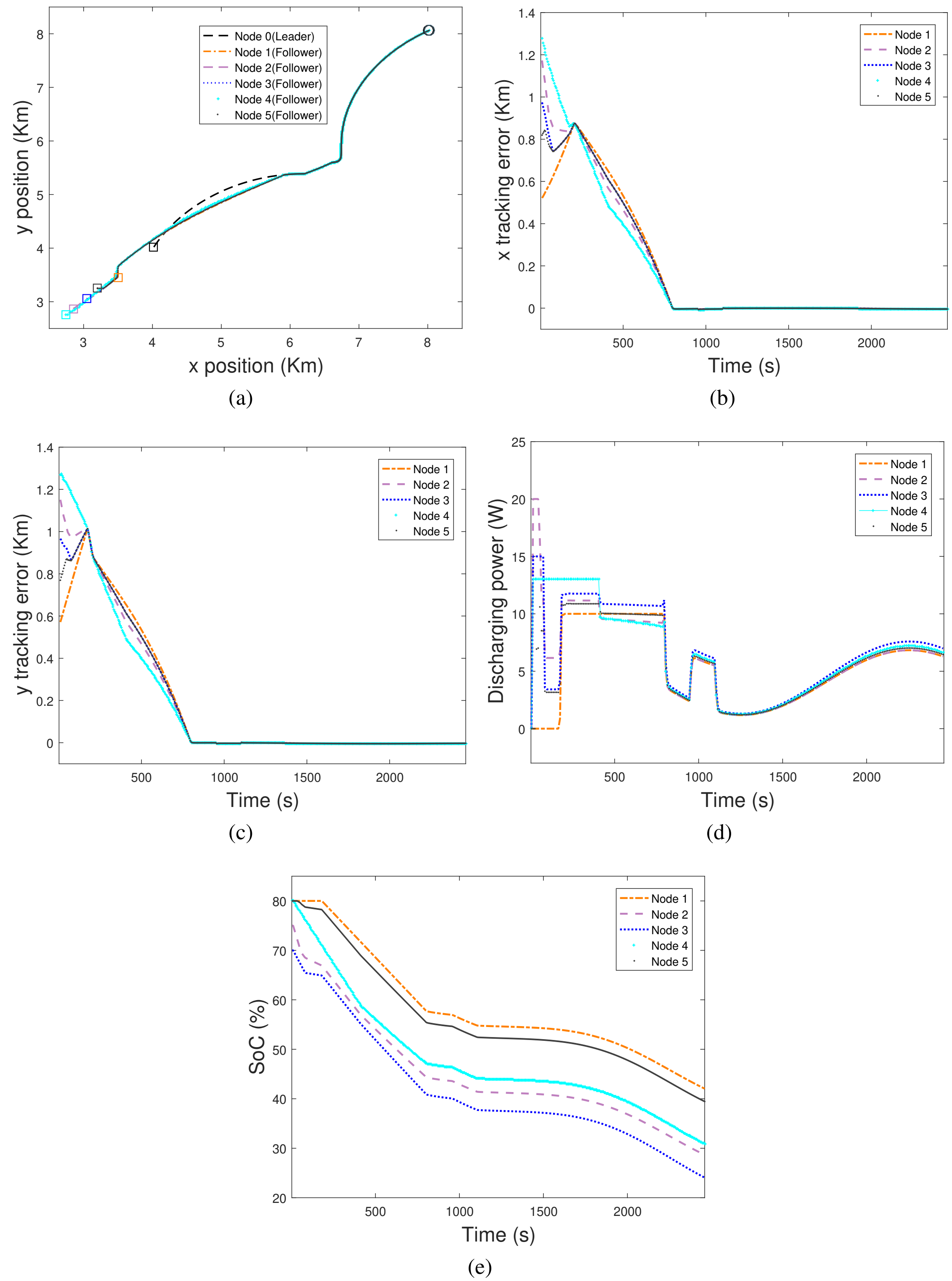

Figure 5.2: Leader-follower tracking using the proposed distributed MPC algorithm: (a) state trajectory in the MAS; (b) x tracking error in the MAS; (c) y tracking error in the MAS; (d) power discharging; (e) evolution of SoC.

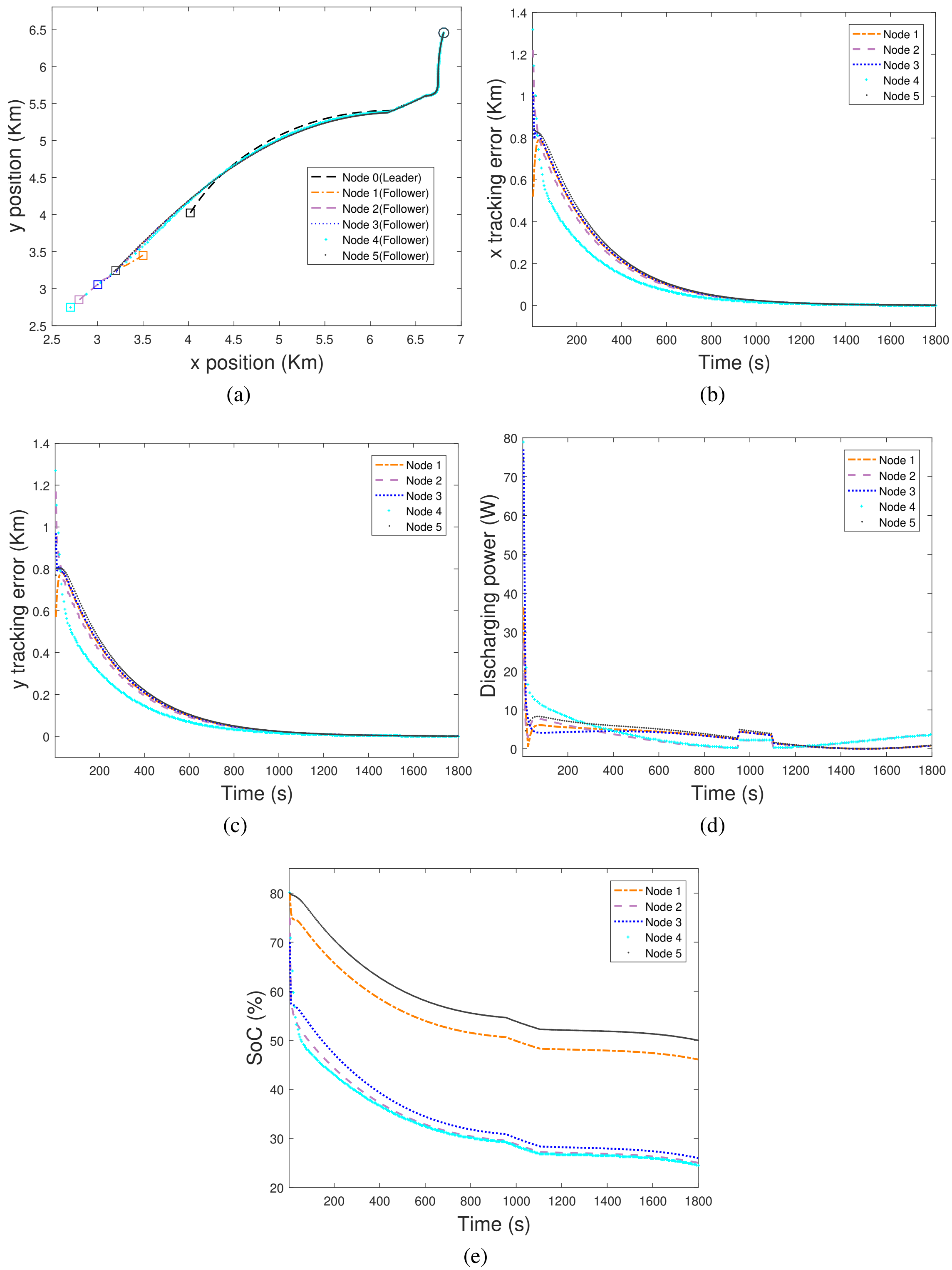

Figure 5.3: Leader-follower tracking using the distributed control algorithm in [1]: (a) state trajectory in the MAS; (b) x tracking error in the MAS; (c) y tracking error in the MAS; (d) power discharging; (e) evolution of SoC.

Chapter 6

Conclusion and Future Work

In this dissertation, we studied distributed control for leader-follower MASs. Our goal was to address several open problems that had persisted in the literature to challenge leader-follower tracking control design. They include: 1) lack of information access by most of the followers to the leader, due to network communication topology, 2) unknown disturbances affecting the agents' dynamics, 3) nonlinear, high-order dynamics governing the agents, and 4) energy awareness and efficiency for long-endurance tracking operation. To tackle the challenges, we investigated distributed observer-based control and distributed optimization to develop a series of studies. We summarize the contributions of the dissertation as follows.

In Chapter 2, given that the majority of the followers cannot interchange information with the leader, we proposed to enable all the followers to collectively estimate the leader's dynamic behavior. With this perspective, we developed distributed observers to allow a follower to infer the leader's state and driving input. Building upon them, we formulated an observer-based tracking control framework and approaches. We first considered the first-order tracking problem and then extended to the second-order tracking problem. We characterized the convergence properties of the proposed control approaches. The work highlights distributed observers as a promising way to overcome the distribution of information facing a leader-follower MAS.

In Chapter 3, we studied robust tracking control for a leader-follower MAS operating under external disturbances. We considered a challenging problem, which is characterized by all the leader and follower agents subjected to disturbances bounded only in rates of change. To deal with the problem, we took the observer-based tracking control approach. We designed a set of distributed observers to help the followers estimate the disturbance affecting the leader and the leader's state and maneuvering input. We then built tracking control approaches that exploit the estimation to enhance the tracking performance for both first- and second-order MASs . We proved that the approaches can lead to bounded-error tracking.

In Chapter 4, we investigated distributed tracking when the leader and followers are governed by high-order dynamics. Departing from the literature, we focused the scenario where only the first state of an agent is measured, considering restricted sensing capabilities. The scenario leads to very limited information availability for the agents. To address this challenge, we developed novel distributed observers to enable followers to reconstruct unmeasured or unknown quantities (e.g., state variables of higher than first order) about themselves and the leader. We further built observer-based tracking control approaches. We conducted the design for for high-order linear MASs first and then advanced to deal with high-order MASs with nonlinear dynamics in a generic form. We established some conditions to ensure convergence of the proposed approaches.

In Chapter 5, we studied integrating battery-based energy awareness with distributed tracking control synthesis to extend the operation time/range of an electric-powered leader-follower MAS. We proposed to leverage a battery's rate capacity effect to increase its runtime and formulated a model predictive control framework for battery-aware tracking control design. The framework is designed to strike a tradeoff between tracking performance and energy consumption rates, account for the battery's rate capacity dynamics, and incorporate the energy and power constraints. We applied a distributed optimization method to execute the framework. The obtained algorithm was shown to help an MAS gain longer endurance by mitigating the restriction of limited battery capacity.

Looking into the future, MASs will find ever-growing application across more sectors and domains. This will motivate many new MAS control problems worthwhile to explore. Among them, we highlight the following for future work:

- Existing MAS control research has concentrated largely on the case when agents are governed by deterministic linear or nonlinear dynamics. However, practical agents or systems are often subjected to the effects of random noise and thus better to be modeled as stochastic dynamic systems. Even control design of stochastic MASs has emerged recently as an important line of research, it is still far from reaching a level of maturity. Optimal estimation theory has proven as a useful tool to deal with stochastic systems. It is thus of our interest to extend the notion of observer-based control in this dissertation to develop novel distributed estimation methods and leverage them to enable distributed control design for stochastic MASs.

- Future MASs may generate large quantities of data due to increasing sensing and measurement capabilities. A stimulating question will be how to make effective use of data to enhance distributed control, especially when an MAS has sophisticated or even incomprehensible dynamics. Machine learning, with its widespread success in various data analysis and understanding tasks in the past years, can likely offer a path forward toward data-driven MAS control. To this end, machine learning must be profoundly blended with distributed control. Meanwhile, a growing number of machine learning problems, e.g., federated learning, are formulated in distributed frameworks for the sake of computation, scalability, and privacy. Distributed MAS control may provide inspirations or methods for distributed machine learning.

- Connected vehicle technologies have advanced rapidly, thanks to the sweeping progress in autonomy and vehicle-to-vehicle and vehicle-to-infrastructure communication. They hold a promise for pushing the efficiency, safety, and mobility of transportation to unprecedented heights. Connected vehicles running on roads can be modeled as large-scale MASs. It is hence promising to address related problems of automation and control for connected vehicles from the perspective MAS control. The studies in this dissertation and potential extensions may be useful in this regard. We will be interested to investigate the intersections of connected vehicles and distributed control.

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