

Differential Construct Definitions of Six Change Score Models within A Correlational Research Context

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Abstract

Six change score models were comparatively evaluated within the correlational research context. The models compared included raw change, corrections of raw change for unreliability in x, correction of raw change for unreliability in both x and y, a regression correction, the raw residual model, and the base-free measure of change. The data were simulated for nine different parameter conditions. The manipulated parameter values were reliability coefficient values for x, y and w where x and y were the components of change and w was an outside variable, relative variability of x and y, colinearity between x and y, and relative validity coefficients for x and y. A set of true and two sets of observed change scores (total of 18 models) were generated for 2000 cases under each condition. Correlations among scores between models within and across conditions were generated. A principal component analysis was used to investigate the commonality of the change score models regarding the construct definition of change when w was considered and when w was partialled from the change score models. The latter analysis investigated the possible differential impact of w on the construct definition of change.

The findings revealed that model differences do exist between the change scores under most of the parameter conditions, particularly for $\sigma_x = \sigma_y$ where $\rho_{xy} < .50$ and $\sigma_y > \sigma_x$ where $\rho_{xy} = 0.75$ when $\rho_{xx'} \neq \rho_{yy'}$. Selected parameter conditions had differential impact on discrepancy models versus residual models. Discrepancy models were more susceptible to manipulations of x and y variability, while the base-free measure of change was most affected by different reliability levels and colinearity coefficients. Removal of w had differential impact on the change score models.

The results of this study lead to a conclusion that change scores in the form of any of the models are not sufficiently stable across research conditions to provide confidence in their use. It is recommended that the researchers examine their data in light of the parameter conditions studied to decide if use of a particular change score model has any potential utility in correlations with a third variable. In any

event, those conditions most favorable to change scores are rare in practice and use of a single variable (y) will result in an equal amount of information. In less favorable conditions, an information increase can only be obtained by allowing both variables (x and y) to operate freely in a regression context to define the dominating linear composite in the data when relating to a third variable.

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Contents

Abstract	i
Acknowledgement	iii
Content	iv
List of Tables	viii
1 Introduction	1
1.1 Purpose	8
1.2 Rationale	14
2 Review of Literature	16
2.1 Change Scores as "Raw" Gain	16
2.1.1 Background Considerations	16
2.1.2 The Classical Test Theory Assumptions	17
2.1.3 The Measurement and Methodological Deficiencies of the Raw Gain	18
2.2 Modified Change Scores	33
2.2.1 Estimated "True" Gain Scores	34
2.2.2 Residual Gain Score	37
2.2.3 Combined True and Residualized Gain scores	38
2.3 The Merit of Alternative Approaches in Measurement of Change .	39
2.4 Reliability of Modified Changes Scores	44
2.5 Two - Part Indices as Psychological Constructs	46
2.5.1 Background	46
2.5.2 Methodological Adequacy and Psychometric Properties of Two-Part Indices	50
3 Method	55
3.1 Investigated Models	55
3.2 Simulation Parameters	56

3.3	Selection of the Parameter Values	57
3.4	Simulation of the True and Observed Change Scores	59
3.5	Data-Generation Procedures	59
3.6	Analysis of the Data	68
3.6.1	Verification of Simulated Data Accuracy	68
3.6.2	Parameter Effects on the Change Score Models	71
3.6.3	Differences in the Underlying Construct of Change Score Models	71
3.6.4	Differential Input of the Change Score Models within the Correlational Context	73
3.6.5	Stability of the Change Score Models for Estimation of Change .	73
4	Results	74
4.1	Simulation Accuracy Verification	77
4.1.1	Mean, Standard Deviations and Intercorrelations	77
4.1.2	Correlation of Change Score Models with X, Y and W	84
4.2	Relative Contribution of X and Y Components into the Definition of Change	86
4.3	Congruency of Change Score Models	88
4.3.1	Measurement Error and Colinearity Effects—Comparison of Change Score Models Within and Across ρ_{xy} levels of the Same Condition	89
4.3.2	Variability Effect—Comparison of Data Across Conditions I and III:	94
4.4	Consistency of the Change Score Models Across Six Different Pa- rameter Conditions	106
4.5	Correlation of Change Score Models with w	109
4.5.1	Commonality of the Models Considering w	109
4.5.2	Commonality of the Change Score Models Removing w:	112

5 Discussion	126
5.1 Input of x and y Components into the Definition of the Change Score Models	127
5.1.1 Conclusions	127
5.1.2 Relation to Other Studies	128
5.2 Congruency/Noncongruency of the Change Score Models	129
5.2.1 Conclusions	129
5.2.2 Relation to Other Studies	131
5.3 Effect of Various Parameter Conditions on the Change Score Models	132
5.3.1 Conclusions	132
5.3.2 Relation to Other Studies	133
5.4 Consistency of the Change Score Models Across Various Parameter Conditions	137
5.4.1 Conclusions	137
5.4.2 Relation to Other Studies	138
5.5 Commonality of the Change Score Models as they Relate to the Third Variable	138
5.5.1 Conclusions	138
5.5.2 Relation to Other Studies	139
5.6 Commonality of the Change Score Models Removing W from the Analyses	140
5.6.1 Conclusions	140
5.6.2 Relation to Other Studies	142
5.7 General Conclusion and Implications of the Results	142
5.7.1 Residual Gain Scores Versus Raw Gain Score	142
5.7.2 Estimated "true" gain scores versus Model 1	163
5.8 Summary	167
5.9 Implications and recommendations	168
5.10 Limitation of the Study	171

Appendix A	171
Appendix B	173
Appendix C	175
Appendix D	176
Appendix E	177
References	201

List of Tables

Chapter One

1.1 Redundant - Suppression Conditions Defining Underlying Discrepancy Change Score	11
1.2 Mathematical Models for the Estimation of Change	12
1.3 Simulation Population's Parameter Values	14

Chapter Three

3.1 Simulation Design	60
3.2 Variance - Covariance Matrix for Data in Condition II	62
3.3 Guidance to the Notation Used in This Document	65
3.4 Sample Condition Values Used to Compute Change Scores	69
3.5 Correlation of Change With the Third Variable	70

Chapter Four

4.1 Mean and Standard Deviation of the Change Scores Models and the Original Component of Change for Each Parameter Condition	78
4.2 Corellation Among True, Error and Observed Scores For Each Parameter Condition	81
4.3 Validity Coefficient of Change Scores Models and Their Correlation With X and Y Component For Nine Parameter Conditions	85
4.4 Percentage of Variance Explained by the First Factor of the Principal Component Across Three ρ_{xy} Levels of Condition I and II	92

4.5 Magnitude of the Loadings of the Change Score Models on the First Factor of the Principal Component for Different Parameter Conditions 93

4.6 Shared Variance of the Change Score Models With Factors 1 and 2 of the Principal Component combining change scores of Conditions I and III.....97

4.7 Percent Differences in the Shared Variance of the Change Score Models From Condition I and III 100

4.8 The Magnitude of the Increment/Shrinkage in the Shared Variance of the Change Score Models With Factor 1 When Moving Across Three Reliability Levels From: $\rho_{xx'} = \rho_{yy'} = 1.0$ to $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} = 0.70$ to $\rho_{yy'} = 0.90$ 101

4.9 Difference in the Shared Variance of the Change Score Models Due to Changes in ρ_{xy} Levels.....103

4.10 Magnitude of the loadings of Each Change Score Model on the First Factor of the Principal Component Across Nine Parameter Conditions and the Percent of the Variance Extracted by Each Model 107

4.11 Correlation of Change Score Models With W Accross Three Research Conditions 110

4.12 Percentage of the Total Variance Extracted by the First Factor of the Principle Component Across Three ρ_{xy} Levels of Condition I, II and III113

4.13 Magnitude of the Loadings of the Residual Change Score Models on the First Factor of the Principal Component for Different Parameter Conditions.... 114

4.14 Shared Variance of the Residual Change Score Models (when w was partialaed out) With Factor 1 and 2 of the Principle Component Analysis 115

4.15 Shared Variance of the Residual Change Score Models (when w was partialaed out) With Factor 1 and 2 of the Principle Component Analysis 116

4.16 Shared Variance of the Residual Change Score Models (when w was partialled out) With Factor 1 and 2 of the Principle Component Analysis	117
4.17 Shared Variance of the Combined Residual Change Score (when w was partialled out) and the Original Models With Factor 1 of the Principle Component Analysis	120
4.18 Shared Variance of the Combined Residual Change Score Models (when w was partialled out) and Original Change Score Models With the First Factor of the Principle Component Analysis	121
4.19 Shared Variance of the Combined Residual Change Score Models (when w was partialled out) and Original Change Score Modes With the First Factor of the Principle Component Analysis	122
4.20 Differences in the Shared Variance of the Original and Residualized Change Scores With Factor 1 of the Principal Component	123

Chapter Five

5.1 Reliability Coefficient of the Change Score Models Within Each ρ_{xy} Level Across Three Research Conditions (I, II, and III)	135
5.2 Range of the Reliability Coefficients for the Change Score Models Across Three Research Conditions	136
5.3 Shared Variance of the Change Score Models With W Across Three Research Conditions	144
5.4 Differences in the Shared Variance of the Change Score Models With W...146	
5.5 Shared Variance of Models 1 and 2 When W Was Included in and Removed From the Analyses.....	150

5.7 Shared Variance of Models 1 and 6 When W Was Included in and Removed From the Analyses..... 155

Chapter 1

Introduction

The use of “difference scores” for defining psychological constructs commonly called “change scores,” “gain scores” (change or gain over time) or theoretical constructs defined by two-part indices (the ones independent of time) have been popular among psychologists, educators, economists and other researchers for many years. The legitimacy of their widespread use as a single variable, however, has been the subject of much controversy among statisticians and psychometricians for more than four decades. “Change” or “raw” gain scores (G) are simply derived by subtracting two scores, each obtained at different times (usually pretest-posttest). By definition $G = y - x$, where x and y represent the same attribute measured at two times. These types of scores are commonly used in education as measures of learning or growth in achievement, in developmental studies for assessment of growth or changes in human attributes, in social psychology for estimation of attitudinal change, in clinical psychology for measuring personality changes, and in experimental studies for assessment of treatment effects. Another use of difference scores is as a “theoretical construct” operationalized as a discrepancy score between two variables (two-part indices). Two-part indices are derived by subtracting the scores on two variables. They are not explicitly dependent on time although their computational characteristics are identical to the “raw gain scores.” Some researchers (Cronbach & Furby, 1970; Glasnapp, 1984; Raciassi & Glasnapp, 1983) have treated them as they have treated the “change”

score construct. Examples of two-part indices as discrepancy scores are:

1. Efficiency Score (E) (Salkind & Wright, 1977).
2. Self-Concept Index (Wylie, 1970).
3. Job Satisfaction Index (Wanous & Lawley, 1972).
4. Attitude toward disability score (Cordaro & Shontly, 1969).
5. Learning disability index (Hanna & Holen, 1979).

The discrepancy composites have been used not only as the dependent variables in experimental conditions for studying “interindividual differences” but also as either the “predictor” or the “criterion” in correlational studies when studying intraindividual differences. The main intent of this project is to deal with the measurement and the methodological adequacy of the “discrepancy constructs” in the latter context only, that is, to examine and discuss the “intraindividual differences” and “two-part indices” in relation to a third variable.

Of the two types of discrepancy scores stated above, the “change score composites” have received the most attention by researchers in literature. This is indicated by the series of articles published in the 1960s (Bereiter, 1963; Harris, 1963; Lord, 1963, 1967, 1969) and in more recent writings (e.g., Cronbach & Furby, 1970; Glasnapp, 1984; Linn & Slinde, 1977; Raeissi & Glasnapp, 1983; Rogosa, Brandt, & Zimowski, 1982; Rogosa & Willett, 1983; Zimmerman & Williams, 1982a,b). “Raw” gain scores have been heavily criticized for their methodological and measurement problems such as unreliability, negative correlation with pretest scores, regression toward the mean, and their inadequacy as a definition of a construct. Several previously mentioned researchers have particularly questioned the measurement, theoretical and methodological adequacy of the “discrepancy composites” and have warned practitioners about the possible hazards of using “change” or “difference scores” in one way or another.

Bereiter (1963) referred to the problems of “raw gain” scores as a dilemma presenting his remarks under the subject of “Some Persisting Dilemmas in Measurement of Change.” Included in his list was the “unreliability-invalidity” prob-

lem of the “raw” change scores. Lord (1963) also questioned the merit of the “raw gain” scores based on their reliability coefficients. In discussing this problem, he commented, “...the difference between two fallible measures is frequently much more fallible than either” (p. 32). Other methodologists, i.e., Cronbach and Furby (1970), Glasnapp (1984), Linn and Slinde (1977), and Raeissi and Glasnapp (1983) questioned the appropriateness of a raw change score or discrepancy score as a definition of a construct in a correlational context. Cronbach and Furby (1970), for example, indicated,

- There is little reason to believe and much empirical reason to disbelieve the contention that some arbitrary weighted function of two variables will properly define a construct. More often, the profitable strategy is to use the two variables separately in the analysis so as to allow for complex relationships. (p. 79)

In examining the deficiency scores used in job-attitude research, Wall and Payne (1973) questioned the methodological adequacy of discrepancy composites as definitions of a construct and pointed out that the manner in which deficiency scores are derived may mask a psychologically meaningful relationship between deficiency scores and a dependent variable. Additional criticism of change score was levied by Glasnapp (1984) concerning the findings by Zimmerman and Williams (1982a,b) on the high predictive validity and reliability of the raw change score. Glasnapp (1984) empirically demonstrated that under the conditions that “change scores” or “difference scores” have high predictive validity ($\sigma_x \neq \sigma_y$ and $\rho_{xx'} \neq \rho_{yy'}$, Zimmerman & Williams, 1982a,b), one of two situations is evident: (1) the underlying model for the x and y composite is a weighted change score composite resulting from suppression conditions in the relationships among x, y, and w or (2) the effective weight of y in the change score y - x as it relates to w results in a high predictive correlation because of the domination of y.

By examining the conceptual and methodological adequacy of “discrepancy” and “additive” composites selected from the literature, Raeissi and Glasnapp (1983) demonstrated that three potential problems emerge when forming arbitrarily weighted constructs: (1) predictive information loss; (2) loss of the relative importance of individual variables forming the composite (i.e., miss of the model), and (3) presence of moderate or extreme suppression conditions in the data. Based on their findings, Glasnapp (1984) and Raeissi and Glasnapp (1983) are consistent with previous researchers in recommending that one should avoid the use of discrepancy scores as constructs and allow the individual variables to function separately in the analysis. Cronbach and Furby (1970), particularly, suggested, “Investigators who ask questions regarding gain scores would ordinarily be better advised to frame their questions in other ways” (p. 80).

In spite of the above recommendations and the large amount of criticism about “change scores” or “difference scores,” the appeal of the “concept of change” as a definition of constructs persists. In response to the continued use of the raw change scores, methodologists (Rogosa et al., 1982; Rogosa & Willett, 1983; Zimmerman & Williams, 1982a,b) have tried to identify those conditions under which raw change scores can be highly reliable and have high predictive validity. By mapping the pattern of intercorrelations between change and a third variable (w), Zimmerman and Williams (1982a,b) demonstrated that change scores can be highly reliable and valid if x and y have unequal validity coefficients ($\rho_{wx} \neq \rho_{wy}$), variability ($\sigma_x \neq \sigma_y$) or reliabilities.

Rogosa and Willet (1983) argue that some of the methodological deficiencies attributed to the “raw” gain scores are, in fact, due to the assumptions of the classical test theory on which most researchers have relied heavily (i.e., $\sigma_x = \sigma_y$ and $\rho_{xx'} = \rho_{yy'}$, and $\rho_{\epsilon_x \epsilon_y} = 0$) and not inherent problems of the change scores themselves. They commented,

- Psychometric properties, namely reliability, have been the predominant con-

cern in the behavioral science literature. Preoccupation with and misinterpretation of psychometric properties of measures of change have contributed to serious confusion in previous work. (p. 726)

Rogosa et al.,(1982), in defending the Zimmerman and Williams (1982a,b) statement that gain scores in research can be highly reliable, have shown why their statement should be taken seriously. They demonstrated empirically that, when there is high consistency (or stability) in individual changes, the dispersion (or variance) of change (i.e., σ_{β}^2) is small with little individual differences in true change to detect; consequently, the reliability of change is small. Based on the above logic and in defense of “raw gain scores” Rogosa et al. (1982) argue that low reliability does not necessarily mean lack of precision (p. 744), a point that is well demonstrated. In a context in which the current project is intended to analyze “change,” however, a low reliability coefficient undoubtedly would cause a serious problem. Recall that the magnitude of the validity coefficient cannot exceed the square root of the reliability coefficient (Brown, 1975; Lehman, 1978).

Glasnapp (1984) and Racissi and Glasnapp (1983) argue that, even under the conditions that change scores are highly reliable (i.e., fanspread pattern is present in the data, Rogosa et al., 1982, pp. 731-732) and have high-predictive-criterion-related validity, only one of its components is likely to dominate the relationship; and the conditions that both components have equal contribution into the correlation are, in fact, rare.

Besides the recent efforts to identify the conditions under which “raw gain scores” have high reliability or potential predictive validity, however, methodologists historically have proposed a variety of alternative approaches to the measurement of change to deal with the measurement and statistical deficiencies of the “raw change” scores (Cronbach & Furby, 1970; DuBois, 1957; Lord, 1956, 1958, 1968; Manning & DuBois, 1962; McNemar, 1958; Tucker et al., 1966). Some of these alternative approaches to the measurement of change are:

1. Estimated true gain scores, i.e.,
 - a. Correction by simple regression for error in x.

$$Y - \rho_{xx'}X$$

- b. Correction by simple regression for error in x and y.

$$\rho_{yy'}Y - \rho_{xx'}X$$

- c. Regression approach (the Lord Procedure).

$$\beta_1X + \beta_2Y$$

where

$$\beta_1 = \frac{(1 - \rho_{yy'})\rho_{xy}\left(\frac{\sigma_x}{\sigma_y}\right) - \rho_{xx'} + \rho_{xy}^2}{1 - \rho_{xy}^2}$$

$$\beta_2 = \frac{\rho_{yy} - \rho_{xy}^2 - (1 - \rho_{xx'})\rho_{xy}\left(\frac{\sigma_x}{\sigma_y}\right)}{1 - \rho_{xy}^2}$$

2. Residual gain scores, for example:

- a. Raw residual gain

$$Y - b_{xy}X$$

where

$$b_{xy} = \rho_{xy}\left(\frac{\sigma_y}{\sigma_x}\right)$$

3. Estimated true residual gain score, for example:

- a. Base free measure of change (Tucker, Damarin & Mesick, 1966, procedure)

$$Y - b_{xy}^*X$$

where

$$b_{xy}^* = \rho_{xy} \left(\frac{\sigma_y}{\rho_{xx'} \sigma_x} \right)$$

b. Estimated true residual gain – multiple regression method

$$B_{y\infty x\infty} (y - \rho_{xy} \frac{\sigma_y}{\sigma_x} x)$$

where,

$$B_{y\infty x\infty} = \frac{\rho_{xx'} \rho_{yy'} - \rho_{xy}^2}{\rho_{xx'} \rho_{xy} (1 - \rho_{xy}^2)}$$

Although the stated modified change score models all focus on correcting the measurement and statistical deficiencies of the raw change scores, in practice, unfortunately, they give minimal attention to the effects of the proposed modifications on the operational definition of the change score construct. The available comparative discussions of these models for dealing with change constructs have focused on their differential reliabilities and statistical properties rather than on the practical operational definition consequences when applying the different models' formulas. Corder-Bolz (1978) and Richards (1975), for example, compared different approaches through simulation studies but did not address questions regarding the resulting operational definition differences in their analysis. Glasnapp (1984), Raeissi and Glasnapp (1983) demonstrated that within the context of correlational studies when correlating raw change scores ($y - x$) with an outside variable (w), the variation in $y - x$ related to variation in w is primarily dominated by either x or y except for very restrictive situations. Even though x and y may remain constant in definition, the resulting change construct that is defined in the relationship with w changes depending on the intercorrelations among x , y , and w . Linn and Slinde (1977) indicated that the "alternative approaches to measuring change result in different correlations of change with other variables.

The different estimates have different theoretical and practical implications.” (p. 128).

While the latter statement directly supports the contention of the proposed study, Linn and Slinde did not identify or explore the magnitude of the differences to be expected among the alternative models under a variety of conditions, nor did they indicate which model might be the best estimate of the true change in correlational context. Cronbach and Furby (1970), however, indicated that estimated true residual gain (D_x) is proportional to the raw residual gain (y_x), which means both models will be perfectly correlated in the correlational context.

In a preliminary study examining a limited number of parameter values, Glasnapp and Raeissi (1985) did demonstrate that for selected models, differences do exist and these differences vary in magnitude as a function of parameter value levels. Their work was limited, however, and needs to be extended to clarify more systematically the magnitude and extent of the differences to be expected among the models.

1.1 Purpose

The principal objective of the current project was to investigate the extent to which six selected change score models differ in operation. Investigated differences focus on the operational definition of change as a construct, that is, the extent to which various change-score models measure different underlying constructs and have differential input into the correlational research context. To conduct the above analysis, parameter conditions were selected so that different levels of detectable change were simulated. This was initiated based on the comment of Rogosa et al., (1982) about the pattern of change from time 1 to time 2 and detectability of the individual changes by the raw change-score model.

Rogosa et al. (1982) concluded that when there is high consistency (or stability) in the individual changes (which means that ρ_{XY} is high), the dispersion

(or variance) of change (i.e., σ_{β}^2) is small with little individual differences in true change to detect. In this regard parameter conditions were selected producing 1) equal and unequal variability of the components of change (x and y) ($\sigma_x = \sigma_y = 1.0$ and $\sigma_x = 1.0$ and $\sigma_y = 2.0$); 2) equal and unequal validity coefficient of x and y ($\rho_{wx} = \rho_{wy} = .50$ and $\rho_{xy} = .30$ and $\rho_{wy} = .70$); and 3) low, moderate and high coefficient of colinearity between x and y (i.e., $\rho_{xy} = .25, .50$ and $.75$). As a result, a total of nine parameter conditions were generated for this project; ρ_{xy} was iterated from .25 to .75 for each of three research conditions (i.e., $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$; 2) $\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$; and 3) $\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$). Within each of these nine parameter conditions, the reliabilities for x,y and w were manipulated across three levels creating additional simulated scores conditions.

In summary, the conditions investigated were defined by selected values for the following parameters:

1. Reliability coefficients for the change score components, i.e., $\rho_{xx'}$ and $\rho_{yy'}$;
2. Reliability coefficient for the third variable ($\rho_{ww'}$);
3. Variability in the x component (σ_x);
4. Variability in the y component (σ_y);
5. Validity coefficient for y component (ρ_{wy});
6. Validity coefficient for x component (ρ_{wx});
7. Correlation between x and y components (ρ_{xy}).

These simulated conditions created situations in which the concept of change was related to a weighted linear composite resulting in redundant or suppression conditions in the underlying regression model as well. Glasnapp (1984) contended that under the conditions that “change scores” or “difference scores” have high predictive validity ($\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$), one of the two situations is evident: (1) the underlying model for the x and y composite is a weighted change score composite resulting from suppression conditions in the relationships among x, y and w or (2) the effective weight of y in the change score y-x as it relates to w

results in high predictive correlations because of the domination of y . Table 1.1 represents conditions defining degrees of redundant or suppression relationships for which the raw change scores in this project were generated.

The correlation between change scores and w under given conditions were compared “across the models” for each set of parameter conditions. The higher the correlation between change-score models and w the more sensitive the models were for detection of the underlying change score for the given parameter condition. The obtained correlation for each change score model with w also was compared “across different conditions.” The inconsistency of the correlation of change with w across these conditions (within model comparison) would indicate the extent to which the change-score models differed for detection of the underlying change from one condition to another. The investigated differences indicate how and to what extent the change-score models differ in operation.

The change scores were generated via a simulation program by inputs of parameter values into derived or existing mathematical formulas presented in Table 1.2. More specifically, different sets of parameter values were systematically manipulated in a series of computer simulations resulting in data where true gain and its estimates were calculated. The obtained change scores were correlated with x and y components to demonstrate the differential input of x and y components into the definition of change across different models and conditions. The proportion of variance shared between the change score and x and y determined the degree of contribution of these components to the definition of change across the models.

Second, the simulated change scores were correlated to determine the degree of their congruency. The higher the shared variance between the stated change scores, the more congruent the underlying constructs measured by these models and the more similar the models as they rank the individuals.

Third, to demonstrate differential behavior (or input) of change-score models in the correlational research context, the obtained change scores were factor ana-

Table 1.1

Redundant - Suppression Conditions Defining
Underlying Discrepancy Change Score

Zimmerman and Willams' conditions for change score reliability	Raeissi Parameter Conditions	Condition	ρ_{xy}	reliability values			Glasnapp assumption about redundancy suppression cond	underlying regression model
				$\rho_{xx'}=1.0$ $\rho_{yy'}=1.0$	$\rho_{xx'}=0.90$ $\rho_{yy'}=0.90$	$\rho_{xx'}=0.70$ $\rho_{yy'}=0.90$		
(low) $\rho_{xx'} \neq \rho_{yy'}$	condition I: $\sigma_x = \sigma_y$ $\rho_{wx} = 0.50$ $\rho_{wy} = 0.50$	con11	0.25	GT*	GO1**	GO2***	0.25 γ $\frac{0.50}{0.50}$	redundant redundant redundant
		con12	0.50	GT	GO1	GO2	0.50 γ $\frac{0.50}{0.50}$	
		con13	0.75	GT	GO1	GO2	0.75 γ $\frac{0.50}{0.50}$	
(middle) $\rho_{xx'} \neq \rho_{yy'}$ $\rho_{wx} \neq \rho_{wy}$	condition II: $\sigma_x = \sigma_y$ $\rho_{wx}=0.30$ $\rho_{wy}=0.70$	con21	0.25	GT	GO1	GO2	0.25 γ $\frac{0.30}{0.70}$	redundant neg suppression neg suppression
		con22	0.50	GT	GO1	GO2	0.50 γ $\frac{0.30}{0.75}$	
		con23	0.75	GT	GO1	GO2	0.75 γ $\frac{0.30}{0.70}$	
(high) $\sigma_x = 2\sigma_y$ $\rho_{xx'} \neq \rho_{yy'}$ $\rho_{wx} \neq \rho_{wy}$	condition III: $\sigma_x \neq \sigma_y$ $\rho_{wx} = 0.30$ $\rho_{wy} = 0.70$	con31	0.25	GT	GO1	GO2	0.25 γ $\frac{0.30}{0.70}$	redundant neg suppression neg suppression
		con32	0.50	GT	GO1	GO2	0.50 γ $\frac{0.30}{0.75}$	
		con33	0.75	GT	GO1	GO2	0.75 γ $\frac{0.30}{0.70}$	

* GT represents true change scores (where $\rho_{xx'} = \rho_{yy'} = 1.0$) condition;

** GO1 represents observed 1 change scores (where $\rho_{xx'} = \rho_{yy'} = 0.90$) condition;

*** GO2 represents observed 2 change scores (where $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition;

Table 1.2

Mathematical Models for the Estimation of Change

Change Score	Notation	Formula	
Raw gain	D	$y - x$	
Raw residual gain	$D.x$	$y - b_{yx}x$	$b_{xy} = \rho_{xy}(\frac{\sigma_x}{\sigma_y})$
Estimated true gain corrected for errors in x	$D_{1\infty}$	$y = \rho_{xx'}x$	
Estimated true gain Corrected for error in x and y	$D_{2\infty}$	$\rho_{yy'}y - \rho_{xx'}x$	
Estimated true gain (Regression method)	$D_{3\infty}$	$B_1x + B_2x$	$B_1 = \frac{(1-\rho_{yy'})\rho_{xy}(\frac{\sigma_x}{\sigma_y}) - \rho_{xx'} + \rho_{xy}^2}{1-\rho_{xy}^2}$ $B_2 = \frac{\rho_{yy} - \rho_{xy}^2 - (1-\rho_{xx'})\rho_{xy}(\frac{\sigma_x}{\sigma_y})}{1-\rho_{xy}^2}$
Estimated true residual (based free measure of change)	$D_{\infty}.x_{\infty}$	$y - b_{xy}^*x$	$b_{xy}^* = \rho_{xy}(\frac{\sigma_y}{\rho_{xx'}\sigma_x})$
Estimated true residual multiple regression method	$D_{\infty}.x_{\infty}$	$B_{y\infty x\infty}(y - \rho_{xy}\frac{\sigma_x}{\sigma_y}x)$	$B_{y\infty x\infty} = \frac{\rho_{xx'}\rho_{yy'} - \rho_{xy}^2}{\rho_{xx'}\rho_{yy}(1-\rho_{xy}^2)}$

lyzed. This analysis was done for each subset of parameter condition as well as for combined levels of ρ_{xy} . The former analysis determined the degree of congruency of the change-score models within the set (for each ρ_{xy} value) and the latter determined the degree of congruency of the underlying change score across the sets (using three levels of ρ_{xy} values). Two types of correlation matrices were used for the factor analysis: 1) correlations among scores resulting from the change score models without consideration of their relationships to w and 2) the same type of correlation matrix, but with w partialled out from the correlation between the stated change scores.

Fourth, the relative sensitivity of each change-score model in detecting change was examined when change scores were correlated with w . (Recall that Rogosa et al. (1982) indicated that, when ρ_{xy} is high and $\sigma_x = \sigma_y$, change is not detectable when measured by $y - x$ variable.) The squared correlation coefficient of each change-score model with w was compared across different models and conditions. The higher the correlation of change score with w , the more sensitive the model in detecting the underlying change for the given condition. For this particular analysis the parameter values were selected such that dispersion (or variability) of change was high and low so that change was more likely detectable or not detectable along some continuum. The consistency of change-score models for detection of underlying change across different conditions also was examined in the above context. This analysis indicated how consistently each model estimated or detected the change as it correlates with w under different conditions.

In the above simulation analysis, change scores were calculated based on the different population parameter values, i.e., measurement errors ($\rho_{xx'}$, $\rho_{yy'}$ and $\rho_{ww'}$); variability effects (σ_x, σ_y , where a ratio of $\frac{\sigma_x}{\sigma_y} = \lambda$ indicates dispersion or variability of change; and validity coefficients (ρ_{wx}, ρ_{xy}) for x and y components as well as colinearity effect for the x and w variables. These parameter values (see Table 1.3 for a summary) were manipulated across identified values serving as

Table 1.3

Simulation Population's Parameter Values

parameter	notation	status
Correlation between x and y	ρ_{xy}	Low=0.25 Moderate = 0.50 High=0.75
Variability of x and y distribution	σ_x σ_y	$\sigma_x = \sigma_y, \frac{\sigma_x}{\sigma_y} = \lambda = 1$ $\sigma_x \neq \sigma_y, \lambda = \frac{1}{2}$
Validity coefficient for x (or colinearity)	ρ_{wx}	Low = 0.30 Moderate = 0.50
Validity coefficient for y	ρ_{wy}	Moderate = 0.50 High = 0.70
Reliability coefficient for x, y and w	$\rho_{xx'}$ $\rho_{yy'}$ $\rho_{ww'}$	perfect $\rho_{xx'} = \rho_{yy'} = \rho_{ww'} = 1.00$ nonperfect $\rho_{xx'} = \rho_{yy'} = 0.90$ nonperfect $\rho_{xx'} = 0.70, \rho_{yy'} = 0.90$

input into the simulation programs to generate nine simulated sets of data, i.e., 18 change scores were generated per set. From the 18 change scores six were true change scores and the rest (two sets of six each) were observed change scores for different error components. For each separate set of parameter values, 2000 x, y, and w scores were generated to form a single data set to which the change score models were applied.

1.2 Rationale

Despite the large number of criticisms about raw change scores, their methodological inadequacy as defining a construct, and their deficient psychometric properties, interest in the measurement of change has persisted and continues unabated. One of the main reasons, Knapp (1980) believes, is that most investigators are still unaware of the serious drawbacks of change scores. Beside using the

change in a variety of contexts (Knapp, 1980; Stake, 1971), researchers frequently use change scores as a predictor or a criterion in correlational studies, the context in which many of the methodological deficiencies of the “raw” change score would surface, i.e., unreliability-invalidity problem, loss of information, or miss of the construct due to the equality of weight assigned to its original components, etc. In this regard, the recommendation such as that which Cronbach and Furby (1970) stated, i.e., “...investigators who ask questions regarding gain scores would ordinarily be better advised to frame their questions in other ways” (p. 80) or avoid the use of change scores and allow the individual variables to function separately in the analysis (Cronbach & Furby, 1970; Glasnapp, 1984; Linn & Slinde, 1977; Raeissi & Glasnapp, 1983) is usually taken less seriously by practitioners. Thus, the main question that remains to be answered, as Knapp (1980, p. 149) has mentioned, is “Why this gap between expertise and practice?”

Perhaps lack of sufficient knowledge on the part of practitioners and the unavailability of enough empirical data at the present time can be contributing factors to this problem. The literature indicates that comprehensive studies, along with logically sound guidelines for practical purposes, are nonexistent; thus supporting the need for more work in this area. The findings of this project were intended to reveal what practitioners should expect to obtain in change analysis by application of alternative formulas and how the interpretation of their findings could vary as a result of such variations. Furthermore, the findings indicate the degree of sensitivity of the change-score models for detecting on underlying change score construct under a variety of conditions and the consistency of such estimation.

Chapter 2

Review of Literature

The concept of “difference” as a means of defining a “construct” has been very common in psychology, education, and the related fields for many decades. Two groups of constructs in the literature have mainly resulted from taking differences: (1) change scores or the so-called “raw gain scores,” and (2) “two-part indices.” Both have been referred to as “discrepancy scores” by researchers (Cronbach & Furby, 1970; Glasnapp, 1984; Raeissi & Glasnapp, 1983; Wylie, 1970). Examples of each type of construct were provided in Chapter 1. As previously explained, “change scores” represent a construct that is derived by subtracting the two measures of the same variable over time. Two-part indices as constructs, however, are independent of time and are defined by subtracting the scores on two variables. The main resemblance of these two groups of constructs is in their mathematical definition. To familiarize the reader with the properties of each group of constructs in the research context, each group of discrepancy scores is discussed separately.

2.1 Change Scores as “Raw” Gain

2.1.1 Background Considerations

Theoretically, in the traditional definition of change (i.e., $G = D = y - x$), subtraction of a pretest score from the posttest score on the same variable would result in a gain score that is independent from the pretest measure. This independence will

seldom hold true at the practical level. Differences in the mathematical definition of a change score, as well as the “classical test theory” assumptions on which researchers have heavily relied in their analysis of change, have made the merit of the change score construct questionable (Cronbach & Furby, 1970; Glasnapp, 1984; Linn & Slinde, 1977; Raccisi & Glasnapp, 1983; Rogosa, Brandt, & Zimowski, 1982; Wylie, 1970).

To resolve the problem of the raw gain score, a number of modified change scores have been proposed for use. However, before the modified change scores can be introduced, classical test theory assumptions and the raw gain scores’ deficiencies need to be discussed.

2.1.2 The Classical Test Theory Assumptions

To simplify the analysis of changes, a number of assumptions traditionally have been made prior to the analysis. According to Kessler (1977, p. 47), these assumptions are:

1. Each variable is a combination of a true and an error component, i.e.,

$$V = V_t + e$$

where V_t = the true value of V and e = the error in measurement of V .

2. The true and error components of the observed scores are uncorrelated with each other. This means:

$$Cov(V_t, e) = 0$$

for all V where, Cov = the covariance of i and j .

3. The above assumption holds for all pairs of true and error components, that is,

$$Cov(x_t, e_x) = Cov(y_t, e_y) = Cov(y_t, e_t) = 0$$

4. Finally, error components of the observed scores are uncorrelated

$$Cov(e_v, e_w) = 0, v \neq w; Var(e_v), v = w$$

The above assumptions cause some methodological problems for the raw “gain” scores in the research situation. These are not inherent problems of the gain scores themselves.

An example of these problems is the negative correlation between the “gain” and the initial test score (Rogosa, Brandt, & Zimowski, 1982). Further details regarding this can be found under methodological problems of change in this chapter.

2.1.3 The Measurement and Methodological Deficiencies of the Raw Gain

Researchers’ criticisms about deficiencies of the “change” or the “raw” gain scores have been mainly related to measurement, statistical, and psychometric properties of the change score constructs. These areas include fairness of the measurement of individual change, precision of estimation, correlation of change and initial test scores, reliability-validity issues, and properties of the change scores as an indicator of a construct. The three major problems of change that have dominated discussions about the “change score” in literature center around (1) regression effects, (2) low reliability coefficients, and (3) inadequacies of the “raw” gain score as a predictor or criterion variable in correlation or regression context. A discussion of these major problems are presented as follows:

2.1.3.1 The “Raw” Gain Score and Regression Problem

One of the main disadvantages of “raw” gain score is that they are subject to the problem of regression toward the mean, which includes at least three problems, all of which result in a negative correlation between initial scores and raw gain (Kessler, 1977, p. 51). The first problem is “ceiling effects,” that is, the individuals at the extreme ends of the time 1 distribution of the time 1 variable cannot move any farther in the same direction and might move in the opposite direction, which consequently decreases the correlation between the initial status and the raw gain score. This problem arises from the measuring instrument. The second problem is

regression effects due to the correlation of the raw gain score with errors of both components. Since extreme scores are likely to have positive error components, they have a tendency to move toward the mean at the time of retesting. The third problem is regression caused by negative correlation between the raw gain and the initial status. This problem is caused by the arbitrary weighting in computing the change score and is independent of the two stated causes of regression (Kessler, 1977). This inherent problem will occur even if all component variables are perfectly measured.

The typical negative correlation of the change score with the pretest score has been considered as a major disadvantage of the “raw” gain score in literature. Thorndike (1924) first noticed that there is a spurious negative element in the correlation of an initial score with gain on the same test. The spuriousness is the result of the same errors of measurement occurring in the difference scores and in the variable with which it is correlated. In the correlation of $G = D = y - x$ with x , the same errors of measurement are positively weighted for x and negatively weighted for G , and the result is usually a spurious negative correlation (Bereiter, 1963; Linn & Slinde, 1977). The consequence of this negative correlation between gain and pretest score is regression effects; as has been mentioned by Kessler (1977), “It is this third aspect of the regression effect which is seen by most who have worked on this problem to be the most damaging to the use of the raw gain scores” (p. 52). In this regard Davidson (1972, pp. 13-14) also commented,

- Virtually all of the previous investigations argue that raw change or raw gain scores are of questionable utility and can easily lead to fallacious conclusions. One reason for this limited utility derives from the commonly observed negative correlation between the initial status score and the raw gain score (parenthetically, we might note that in a parallel fashion, one would also observe a positive correlation between the final status scores and the raw gain. In the usual case in which the variances of the initial and final status scores are approximately equal, this negative correlation will be observed regardless of the sign of the relationship between the initial and final scores.

The implication of a negative correlation between $G = D = y - x$ in correlational studies is that those variables positively related to the initial score more than the final score are also likely to show a negative relationship with the raw gain scores (Davidson, 1972; Lord, 1963). “However, it is by no means clear that these other variables affected a ‘real’ loss (negative change) in the criterion across the observed interval” (Davidson, 1972, p. 14).

Contrary to the previous researchers’ views, Zimmerman and Williams (1982b) demonstrated that correlation between change scores and pretest scores, $\rho(y - x, x)$, can be positive. For example, this correlation for data presented in their Table 2 was .23. Zimmerman and Williams (1982b) commented

- “Many psychometricians beginning with Thomson (1924), have supposed that change scores and pretest scores, or initial status, are negatively correlated, because errors of measurement appear with opposite signs in x and $y - x$. Actually, under realistic conditions, this correlation can be positive, as suggested by the results just obtained from Table 2” (p. 965).

From Zimmerman and Williams (1982b) point of view, these realistic conditions result when pretest and posttest measures have unequal standard deviations ($\sigma_x \neq \sigma_y$) and unequal reliability coefficients ($\rho_{xx'} \neq \rho_{yy'}$) and unequal correlations with another criterion ($\rho_{xz} \neq \rho_{yz}$). These researchers stated that, if $z = x$, the predictive-criterion validity of change scores,

$$\rho(y - x, z) = \frac{\lambda^{-\frac{1}{2}} \rho_{yz} - \lambda^{\frac{1}{2}} \rho_{xz}}{(\lambda + \lambda^{-1} - 2\rho_{xy})^{\frac{1}{2}}}$$

will reduce to

$$\rho(y - x, z) = \frac{\lambda^{-\frac{1}{2}} \rho_{yz} - \lambda^{\frac{1}{2}}}{(\lambda + \lambda^{-1} - 2\rho_{xy})^{\frac{1}{2}}}$$

which is equivalent to a well-known formula for the correlation of change and initial status (Lord, 1963; Stanley, 1971). "It is apparent that this correlation is positive if and only if $\rho_{xy} > \lambda$ " (Zimmerman & Williams, 1982b, p. 965). In the context of a causal relationship Kessler (1977) demonstrated algebraically that "when the component variables making up a gain score are either unrelated to each other or in a state of dynamic equilibrium" (i.e., $\sigma_x^2 = \sigma_y^2$ and $\rho_{xy} \neq 0$) "the gain score will be correlated negatively with the initial score. This negative correlation is normally thought of as a spurious correlation because the conditions of dynamic equilibrium and independence of x and y are taken by most analysts to represent conditions under which x has no causal impact on gain. If this is true, then by entering x into a multiple regression equation with raw gain as the dependent variable, the interpretation of results can be severely limited" (Kessler, 1977, p. 52-53).

2.1.3.2 Reliability of the Raw Change Score

The next major problem of the "raw" gain scores is their low reliability coefficient, one of the most seriously criticized issues in the literature. Due to this fact many psychometricians and researchers doubt their usefulness (see, for example, Bereiter, 1963; Linn & Slinde, 1977; Lord, 1963; Mehrens & Lehman, 1973; O'Connor, 1972). Lord (1963) commented, "The difference between two fallible measures is frequently much more fallible than either" (p. 32). Consistent with this statement are the findings of Mehrens and Lehmann (1973), who noted, "unfortunately, difference scores are considerably less reliable than single scores" (p. 117). Davis (1964) reported, "virtually all published tests display reliability coefficients lower than are desirable for measuring change..." (p. 237). Bereiter (1963) attributed the unreliability of the change scores to errors in the original component of the "raw" gain and the correlation between x and y. For example, he stated, "the best known 'fact' about change scores is that they are unreliable. It is also quite well known that this unreliability has two sources: unreliability in x and y and a positive correlation between x and y" (p. 8).

Based on the classical test theory assumptions, Kessler (1977, p. 47) also deduced a number of results, including the fact that raw change scores are correlated with measurement errors of both component score. As a consequence of this correlation, the raw gain score has a reliability no higher than that of the two component scores. Finally, this reliability can be increased only by decreasing the observed correlation between the two two components.

- ... The unreliability-invalidity dilemma stems from the fact that high reliability of change scores usually requires low test- retest correlations, with the implications that in such a case the test may not measure the same thing on the two occasions and the change scores will, therefore, be meaningless. It was concluded that the meaningfulness of change scores does not depend on a test's measuring 'the same thing'on two occasions, so that the dilemma is a false one (Bereiter, 1963, p. 20).

Bereiter (1963) and Kessler (1977) both stated that the reliability of the "raw" gain can be increased by either one of two methods: (1) the reliability of the component scores can be increased, (2) the observed relationship between the two component scores can be decreased. From a practical point of view manipulation of the above components for increasing the reliability coefficient has its own complexity, which Bereiter (1963) referred to as "the reliability-invalidity dilemma." He argues,

- If the correlation between the pretest and the posttest is reasonably high, we are inclined to ascribe change scores to changes in the individuals. But if the correlation is low or if the pattern of correlations with other variables is different on the two occasions, we may suspect that the test does not measure the same thing on the two occasions. Once it is allowed that the pretest and posttest measure different things, it becomes embarrassing to talk about change. There seems no longer any way to answer the question, 'change on what?' (p. 9)

Linn and Slinde (1977) stated,

- Of course one way to obtain a more reliable difference score is to have a low correlation between pre- and postscores. Under this circumstance, however, it is questionable that pre- and postmeasures are getting at the same construct, which would seem to be a prerequisite for difference scores to be interpreted as an index of growth. (pp. 123-124)

As Kessler (1977, p. 49) has indicated it is intuitively clear why reliability of gain can be affected by reliability of the component scores; but it might not be clear why it should be affected by changes in observed correlation between the two components. To understand this point, Kessler urges the readers to imagine a condition that the correlation between the component scores is $r = .90$. Then he argues,

- Given commonly accepted standards of reliability we would say that these two variables are almost identical to each other. In consequence, the difference between them would be considered little more than error. Since this difference is exactly what we mean by our gain score, it is clear that as the observed correlation increases between the components we have less and less faith in the substantive meaning of the difference between the two scores. (p. 49)

To show that there are realistic limits in manipulating the above parameters for attainment of greater reliability in the premeasures and postmeasures, Bereiter (1963, p. 10) provided the following examples:

- Consider a test where x and y have reliabilities of .80 and correlate .70 with each other. If their variances are the same, the reliability of their differences will be .33..., a fairly typical result in most psychological tests; the realistic upper limit of reliability is about .90; but, since increasing the reliability of x and y will also increase their correlation, raising the reliability of x and y to .90 will increase the reliability of their difference to only .53. On the other hand, leaving the reliability of x and y at .80 and reducing their correlation to zero would raise the reliability of their difference to .80. This tempting prospect is marred by two considerations: (a) Can it be done? and (b) Is it a reasonable thing to do?

Kessler (1977, pp. 49-51) examined empirically the effect of varying component reliabilities and intercorrelations on the reliability of gain. In his calculation, Kessler assumed a “dynamic equilibrium” condition (Lord, 1963, p. 21), i.e., $\rho_{xx'} = \rho_{yy'}$ and $\text{var}(x) = \text{var}(y)$.

As a result of his finding, Kessler (1977) argues,

- The combined effects of unreliability in the components and high correlation between components can be devastating. It is reasonable to assume that the maximum reliability of any component score will be 0.9; higher reliabilities are uncommon. It is also reasonable to seek a reliability of at least 0.8 in the gain score. Thus, in order to obtain an acceptable reliability of the gain score under the most favorable conditions of component reliability, it is necessary that the correlation between the two components does not exceed .5. In many empirical longitudinal studies it is common, however, to find correlations far in excess of 0.5, implying that gain scores computed from these components will, of necessity, be unacceptably low.... (p. 51)

Linn and Slinde (1977, p. 124) commented, “An implication of the low reliability of difference score is that it is quite risky to make any important decision about individuals on the basis of gain from pre- to posttesting periods.” Stake (1971) reported, “Owing to unreliability, gain scores can appear to reflect learning that actually does not occur” (p. 587).

Furthermore, the low reliability coefficient of the “raw gain score” or “difference score” becomes a serious problem if change is used in correlational studies. The main reason is that a change score with low reliability does not correlate with other variables very much (Linn & Slinde, 1977). O’Connor (1972) and Woodrow (1946) concluded that in practice it is rather difficult to find variables that are highly correlated with changes.

From the statistical point of view, change scores with low reliability being used in correlational studies are judged to be a poor predictor or criterion in spite of the

fact that Rogosa, Brandt, and Zimowski (1982, p. 744) in defense of “change scores” stated, “Low reliability does not necessarily mean lack of precision.” Zimmerman and Williams (1982) also argue that change scores, like single scores, can be highly valid and reliable “...provided one makes other assumptions about the value of pretest and posttest reliability coefficients and standard deviations” (p. 149). Zimmerman and Williams (1982b) stated

- It is widely assumed that pretest and posttest measures are 'parallel' according to the usual test-theory definition. However,... changes are most valid and reliable when pretest and posttest measures have unequal standard deviations, unequal reliability coefficients and unequal correlations with other criteria. (p. 962)

Zimmerman and Williams (1982a) believe that the revised assumptions are more realistic than the usual ones in testing practice.

To demonstrate empirically that under “realistic experimental conditions change and growth measures determined from individual examinees’ test score can have excellent predictive value, Zimmerman and Williams (1982b) correlated the ”raw gain ($y - x$) scores with a third variable (z). In the following formula, e.g.,

$$\rho(y - x, z) = \frac{\lambda^{-\frac{1}{2}} \rho_{yz} - \lambda^{\frac{1}{2}} \rho_{xz}}{(\lambda + \lambda^{-1} - 2\rho_{xy})^{\frac{1}{2}}}$$

where ρ_{xz} , ρ_{yz} and ρ_{xy} are correlation coefficients and $\lambda = \frac{\sigma_x}{\sigma_y}$ is the ratio of the standard deviations of pretest and posttest scores. Given values were: $\rho_{xz}=0.45$; $\rho_{xy}=0.75$; $\lambda = 1, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{3}$. Zimmerman and Williams’ (1982b) findings revealed that as the magnitude of $\lambda = \frac{\sigma_x}{\sigma_y}$ approaches one the reliability coefficient of change scores decreases across the entire range of ρ_{xy} (i.e., .10, .30, .50, .70, .90), and the reliability of a difference score is the least when $\lambda = 1$, indicating that $\sigma_x = \sigma_y$.

Based on their analysis on the reliability of change scores, Zimmerman and Williams (1982a, p. 967) stated, “...if $\sigma_x = \sigma_y$ then $\rho_{cc'}$ ordinary is very low, often

lower than either $\rho_{xx'}$ or $\rho_{yy'}$, as many investigators have observed.” (Note: $\rho_{cc'}$ denotes the reliability of coefficient of $c = y - x$.) In general, by using a correlational context as above, these researchers concluded that change scores like single scores are valid and reliable, and “...the inequality of parameters which we have found to yield valid change scores also yield reliable change scores” (p. 968).

Furthermore, they stated, “Frequently one is more interested in the correlation between true criterion scores than in the observed score correlation $\rho(y - x, z)$. A high correlation between true change scores and a criterion is possible only if the reliability of the change score is high” (Zimmerman & Williams, 1982a, p. 967). Thus, in the reliability formula for change scores, i.e.,

$$\rho_{cc'} = \frac{\lambda\rho_{xx'} + \lambda^{-1}\rho_{yy'} - 2\rho_{xy}}{(\lambda + \lambda^{-1} - 2\rho_{xy})}$$

$\rho_{cc'}$ can be quite high if $\rho_{yy'} \succ \rho_{xx'}$ and $\sigma_y \succ \sigma_x$, or if $\rho_{yy'} \prec \rho_{xx'}$ and $\sigma_y \prec \sigma_x$ are met (Zimmerman & Williams, 1982a). “The fact that change scores can be both valid and reliable implies that the correlation between true change scores and true criterion scores can be high, provided the criterion scores also are reliable” (Zimmerman & Williams, 1982, p. 968). One interesting conclusion that can be inferred from Zimmerman and Williams (1982a,b) is that the low reliability coefficient of “raw” gain scores to a large extent are the function of the assumptions (i.e., classical test theory) which researchers have made in their analysis of change and not due to the inherent problem of the change scores themselves. Classical test theory assumptions were stated earlier in this document.

Rogosa, Brandt, and Zimowski (1982), by reanalyzing the statistical and psychometric properties of change along with the statistical assumptions used in analyzing the change concluded, “Many of the deficiencies that have been attributed to difference scores in the behavioral-science literature are a result of misunderstandings” (p. 730). Rogosa et al. (1982) especially focused on the reliability formulas and the type of assumptions that one can make and the effect of these assumptions on

the magnitude of the reliability of the difference score. These researchers start their arguments with the fact that reliability is a measure of interindividual differences and can only be defined for a group. Rogosa et al. (1982) argue, “Low reliability is cited as a major reason to eschew the difference score.” “...shunning the difference score because of these findings of low reliability is unwise” (p. 730). Their point is that past researchers (i.e., Linn & Slinde, 1977) used a series of assumptions (i.e., the so-called classical test theory assumptions) to demonstrate that $\rho(D)$ can be considerably smaller than $\rho(\xi)$, however, “...difference scores are not intrinsically unreliable, and, furthermore, the difference score can be changed even in situations where the reliability is low” (p. 730).

To clarify this issue Rogosa et al. (1982) pointed out that the key expression for $\rho(D)$ is the following formula:

$$\rho(D) = \frac{\sigma_{\beta}^2}{\sigma_{\beta}^2 + \frac{\sigma_{\epsilon^2 - \epsilon_1}}{(t_2 - t_1)^2}}$$

This formula shows the dependence of $\rho(D)$ on the variable of the errors of measurement for the true rate of change, that is, the magnitude of $\rho(D)$ decreases as the amount of measurement error increases. Also, true reliability increases as individual differences in true change increase. “If $\sigma_{\beta}^2 = 0$, the reliability is zero regardless of the precision with which the change of each individual is measured” (p. 731).

- Even though it is important to keep in mind that the reliability varies with the dispersion of scores, this does not alter the direct meaning of the reliability coefficient in any particular sample of people. The reliability coefficient is the ratio of true-score variance to obtained-score variance. If that ratio is small, measurement error will attenuate correlations with other variables. If the total group of subjects in a study has a standard deviation of scores which is not much larger than the standard error of measurement, it is hopeless to investigate the

variable in correlational studies (Nunnally, 1977, p. 242).

$$r_{xx} = 1 - \frac{\sigma_{\text{mea.error}}^2}{\sigma_x^2}$$

That is, in the configuration of individual time paths for which the rate of change varies little across persons, the difference score will have low reliability even if the estimates of each β_j are precise. (Note: β_j is rate of change for individual j from time 1 to time 2.) “Roughly speaking, the reliability indicates the accuracy with which individuals can be ranked on β on the basis of scores on D . If the β ’s are nearly identical, ... the reliability of D is small” (Rogosa et al., 1982, p. 731).

These findings imply that, although individual changes in growth are necessary for high reliability, the absence of such differences does not preclude meaningful assessment of individual change. The important message of Rogosa, Brandt, and Zimowski (1982) is that “low reliability does not necessarily imply lack of precision” (p. 731).

Furthermore, Rogosa et al. (1982, p. 731), by analyzing the relationship between the coefficient of stability and $\rho(D)$ shed some light on some of the misunderstandings of past researchers in this regard. They stated that “the stability of x , which is represented by the correlation $\rho_{x_1x_2}$, has a misplaced and misunderstood role in the difference score.” (Note: $\rho_{x_1x_2}$ is the same as ρ_{xy} .)

To show the clear dependence of the stability on σ_β^2 , Rogosa et al. (1982) used a special case such that $\rho_{\xi_1\beta} = 0$ (i.e., true initial status and rate of change uncorrelated). Under this condition the coefficient of stability is:

$$\rho_{\xi_1\xi_2} = \frac{1}{\sqrt{1 + \frac{\sigma_\beta^2(t_2 - t_1)^2}{\sigma_{\xi_1}^2}}}$$

As the above formula indicates, variability in the rate of change, i.e., σ_β^2 has an important role in determining the magnitude of $\rho_{\xi_1\xi_2}$. As σ_β^2 approaches zero, $\rho_{\xi_1\xi_2}$

approaches one, conversely, as σ_{β}^2 becomes large, $\rho_{\xi_1\xi_2}$ decreases toward zero.

According to Rogosa et al. (1982, p. 731), the stability of x is proportional to the stability of ξ when $\sigma_{\epsilon_1\epsilon_2} = 0$ with $\rho_{\xi_1\beta} = 0$; $\rho_{x_1x_2}$ will be high if and only if σ_{β}^2 is small and the reliability of x is high. Under either of the following conditions, i.e., large σ_{β}^2 or low reliability of x , the stability of x can be low.

Rogosa et al. (1982) reveals that when there are: (1) large individual differences in change, (2) a strong positive correlation between change and initial status, and (3) high stability in x , reliability of change can be high. These are the conditions under which Zimmerman and Williams (1982b) also have claimed “gain scores in research can be highly reliable.” Zimmerman and Williams (1982b) used only the extreme conditions.

Rogosa et al. in criticizing past researchers, for example, Bereiter (1963) Kessler (1977, p. 99) and O’Connor (1977), argue that “...stability has only an incidental role in understanding $\rho(D)$ ” (p. 733). They stated that the main source of confusion of the researchers is as $\rho_{x_1x_2}$ increases, $\rho(D)$ decreases. Bereiter (1963), for example, referred to such a misunderstood concept as a dilemma. But Rogosa et al. contended that both $\rho(D)$ and $\rho_{x_1x_2}$ depend on σ_{β}^2 .

- The major misconception that $\rho(D)$ is intrinsically small is a consequence of studying $\rho(D)$ only for very large $\rho_{\xi_1\xi_2}$. When x has high reliability and there exist individual differences to be detected, $\rho(D)$ will be respectable. (p. 733)

The data in Table 3 of Rogosa et al. (p. 733, 1982) indicate that, as $\rho_{\xi_1\xi_2}$ increases, the ratio of $\frac{\rho(D)}{\rho(X)}$ decreases. Rogosa et al. claimed that part of their goals of producing Table 3 was to “...document that it is only when $\rho_{\xi_1\xi_2}$ is very large that oft-quoted statement ‘the difference between two fallible measures is frequently much more fallible than either (Lord, 1963, p. 32) applies to $\rho(D)$. For example, with $\rho(X) = .90$ and $\rho_{\xi_1\xi_2} = .50$, the reliability of D is 91 percent of the reliability of x ” (p. 734).

Although past researchers misunderstood the exact role of $\rho_{x_1x_2}$ in the analysis of $\rho(D)$, the above evidence identifies existing conditions where change scores can be highly reliable. These conditions are high variability for rate of change (i.e., large σ_β^2 Rogosa et al., 1982) or in other language indicates a fanspread condition (Campbell, 1969) in the data, and $\rho_{x_1x_2}$ or $\rho_{\xi_1\xi_2} < \sigma r = .50$ (Kessler, 1977; Rogosa et al. 1982).

- The problem of unreliability is manageable if the component score can be corrected for reliability and if these scores are not correlated above approximately .50. If the correlation exceeds this upper limit the estimation of stable correlates of change is made difficult. This problem can be understood as a special case of multicollinearity, where the collinear variables are one, a predictor and the other, a criterion score. (Kessler, 1977, p. 60)

2.1.3.3 Use of “ the Raw Change Score” as an Operational Definitions of a Construct in Correlational Studies

In addition to the other two major problems attributed to “raw” change scores in the literature, change scores have been criticized for their measurement and methodological adequacy, particularly as a definition of a construct in correlational studies; that is, not only does the low reliability coefficient of the “raw” gain scores restrict the correlation of the “change” with other variables but the equality of weights assigned to its component in correlational context also imposes its own restriction on the definition of the underlying construct of change, a matter about which many researchers in literature have been concerned. For example, Cronbach and Furby (1970) argue,

- There is little reason to believe and much empirical reason to disbelieve the contention that some arbitrarily weighted function of two variables will properly define a construct. More often, the profitable strategy is to use the two variables separately in the analysis so as to allow for complex relationships. (p. 79)

Linn and Slinde (1977), aware of the above problem, agreed with Cronbach and Furby (1970) that researchers should allow each variable to assume a weight in a

linear composite determined by the data rather than assign arbitrary weightings of “one” and “minus one.” Wall and Payne (1973), in examining the use of deficiency scores in job-attitude research, essentially reached the same conclusion. Wall and Payne’s (1973) reason was that there are inherent constraints in the relationships between deficiency scores and a dependent variable. Because of the manner in which deficiency scores are derived, a psychological meaningful relationship might be masked in change-score analysis. As a result, they stated, “We strongly agree with this advice offered by Cronbach and Furby (1970) that ‘deficiency,’ ‘change,’ or ‘gain’ scores should be avoided, and raw scores only should be used” (p. 326).

By analyzing the change score construct in a three variable regression model Glasnapp demonstrated that Zimmerman and Williams’ conclusions on high predictive validity and reliability of “change” score may be misleading .

Glasnapp (1984) argues,

- Under the unequal variability ($\sigma_x \neq \sigma_y$) condition, the raw change score weights may be equal giving the appearance of an ideal change score composite. However, when the weights are standardized, it is shown that a variable with greater variability will dominate the underlying weighted change score composite and its relationship with a criterion variable thus diminishing the importance of one of the variables in the change score composite. This is particularly true if the variable with the greater variability also has the higher validity coefficient (p.865).

Wylie (1970) made a similar argument about the self-concept discrepancy composite score (i.e., Ideal-Real self discrepancy).

Furthermore, Glasnapp (1984) introduced the problem of “suppression condition” present in the data when the “discrepancy” scores tend to have high predictive validity and reliability. Glasnapp (1984), by mapping the pattern of intercorrelations between “discrepancy score” and an outside criterion, demonstrated

- The high predictive potential of arbitrary weighted change score occurs for those conditions where the resulting least square regression model is one which also defines a weighted change score composite. These conditions are conditions where suppression will occur in the regression model. Even when suppressions are present, one of the change score variables will tend to dominate the composite as defined by their relative effective weights. (pp. 865-866)

Glasnapp's (1984) findings revealed that x and y had equal effective weights, and change scores ($y - x$) were perfectly defined only under the restrictive condition of reciprocal suppression (where validity coefficients for x and y were equal but of opposite sign). Consistent with Cronbach and Furby (1970), and Linn and Slinde (1977), Glasnapp (1984) recommends that "researchers should allow the individual variables to function separately in the analysis." His crucial point was;

- If a change score composite is a dominant variable in the data, suppression conditions will occur among the intercorrelations, the regression model will identify the effective weights in the change score composite and the relationship with the criterion will be maximized (p. 866).

Due to the fact that existence of composite constructs defined by suppression conditions is rare in practice, Glasnapp (1984) and Raeissi and Glasnapp (1983) commented, "The search for highly meaningful change score composites as predictor or criterion variable will be unrewarding." From these researchers' point of view, "...the condition under which change score composites will emerge as dominating variables limits the potential empirical verification of their importance" (p. 866). Raeissi and Glasnapp (1983), by using some examples from the literature, i.e., Impulsivity (I) and Efficiency (E) constructs (Salkind & Wright, 1977, cognitive style model), extended their analysis to "summed composite" scores. They used the two exemplary composites as predictors in a three-variable regression model to demonstrate the potential predictive validity losses for the "differential theoretical construct." As a result of

their findings, Raeissi and Glasnapp (1983) listed the following three specific problems associated with the composite constructs (both summated as well as discrepancy scores) in correlational studies:

1. Loss of information due to weakness in predictive validity power;
2. Misspecification of the model due to the preassigned weight to the component of the composite score;
3. Definition of the construct based on the suppression condition when the composite has high predictive validity and reliability.

The problems stated above are considered to indicate the inadequacy of “discrepancy scores” for defining a theoretical construct in the context of the regression model in literature.

Previously other researchers, i.e., Cronbach and Furby (1970) and Fuguitt and Lieberman (1973), also warned researchers against the use of discrepancy scores in the construction of conceptual variables. From the above researchers’ point of view, difference scores often are faulty indicators of the true theoretical concept that the analyst seeks to examine, and therefore should be used with caution (Cronbach & Furby, 1970; Fuguitt & Lieberman, 1973).

2.2 Modified Change Scores

Depending on the nature of the problem, one or more modified change scores have been proposed to resolve the stated problems of the “raw” gain scores. For example, modified change scores have been proposed for: (1) resolving the negative correlation between the gain score and the initial status (Garside, 1956); (2) resolving the problem of regression effects or the intercorrelation of x and y (Glass, 1968; Lord, 1963; Traub, 1967); (3) resolving the low reliability problem of the raw gain score (Cronbach & Furby, 1970; Lord, 1967, 1963, 1958; McNemar, 1958); (4) resolving

both unreliability and regression effects simultaneously (Tucker, Damarin, & Messick, 1966); and (5) purifying the change scores from the effects of other variables (i.e., obtaining an independent change score) for correlational studies (i.e., DuBois, 1957; Lord, 1958, 1963; Manning & DuBois, 1958, 1962). In general, the stated modified change scores can be categorized into three groups: (1) estimated true gain scores; (2) residualized gain scores; and (3) combined true and residualized gain score. these alternative approaches to the measurement of change are discussed under these three categories.

2.2.1 Estimated “True” Gain Scores

Numbers of researchers have created an estimated “true” gain score by means of various reliability adjustments to the component scores (Cronbach & Furby, 1970; Lord, 1956, 1958, 1963, 1967; McNemar, 1958).

a. Lord procedure

The Lord (1963) procedure is aimed at estimating a true difference or gain score, i.e., $G = Y - X$. For each individual a regression technique is employed, i.e.,

$$D_{\infty} = \beta_1 X + \beta_2 X + constant$$

$$\beta G_{x.y} = \frac{(1 - r_{yy})r_{xy} \frac{s_y}{s_x} - r_{xx} + r_{xy}^2}{1 - r_{xy}^2}$$

$$\beta G_{y.x} = \frac{r_{yy} - r_{xy}^2 - (1 - r_{xx}) \frac{s_x r_{xy}}{s_y}}{1 - r_{xy}^2}$$

The Lord procedure uses observed initial and final scores to estimate the true scores with the assumption of equality of error variance for initial and final

scores. Lord's concern in such estimation is to determine the regression coefficients of the above estimator. According to Lord, the "true" gain score can be obtained by:

$$D_{\infty} = \frac{1}{1 - \rho_{xy}^2} \left(\frac{y}{\sigma_y} (\sigma_y \rho_{yy'} - \sigma_y \rho_{xy}^2 + \sigma_x \rho_{xy} \rho_{xx'} - \sigma_x \rho_{xy}) - \frac{x}{\sigma_x} (\sigma_x \rho_{xx'} - \sigma_x \rho_{xy}^2 + \sigma_y \rho_{xy} \rho_{yy'} - \sigma_y \rho_{xy}) \right) + \text{constant}$$

b. McNemar procedure

The McNemar (1958) method aimed to provide a simple approximation method for estimating true gains, that is, the "true gain" is regressed on observed gain or fallible data where there is no restrictive assumption of equality of error variance for initial and final scores. The estimated true score obtained from this approach is called a "regressed score" (McNemar, 1958, p. 50). The corresponding formulas for this estimation are:

$$G_{\infty} = \beta_x X + \beta_y Y + \text{constant}$$

$$G_{\infty} = \beta_x \frac{\sigma_{gt}}{\sigma_x} X + \beta_y \frac{\sigma_{gt}}{\sigma_y} Y + \text{constant}$$

where,

$$\text{constant}(c) = G - \beta_x X - \beta_y Y$$

The key topic of the McNemar (1958) paper, as in Lord's paper, is the determination of the regression coefficient of the estimator of true change (i.e., G_{∞}).

c. Cronbach and Furby procedure

Cronbach and Furby (1970) extended and improved the Lord-McNemar procedure for the estimation of true scores. The Lord-McNemar procedure uses x

and y data while Cronbach and Furby brought in two further categories of w and z. The w and x are Time 1 measures but need not be simultaneous. The corresponding formulas for such estimation are:

$$D_{\infty} = \beta_1 X + \beta_2 W + \beta_3 Z + \text{constant}$$

By calculating X_{∞} , Y_{∞} , W_{∞} and Z_{∞} (if both w and z information are available) D_{∞} can be estimated.

Besides the above formula, Cronbach and Furby discussed three simpler approaches for the estimation of true change scores, i.e.:

1. Correction by simple regression for error in x:

$$D_{\infty} = Y - \rho_{xx'}X + \text{constant}$$

2. Correction by simple regression for error in x and in y:

$$D_{\infty} = \rho_{yy'}Y - \rho_{xx'}X + \text{constant}$$

3. The Lord/McNemar procedures: The estimated change score by (a) and (b) above, however, do not take the correlation between x and y into account. Thus, a more precise estimate of individual differences is possible by the regression approach as Lord (1963) and McNemar (1958) introduced it. These latter approaches are designed to take into account both the effects of regression and the correlation between x and y. As explained above, Cronbach and Furby's (1970) approach is an extended form of the Lord/McNemar procedure.

- d. Bereiter (1963) procedure.

In correlational studies, instead of estimating the true gain and using it as a new dependent variable to be predicted from other variables, some researchers, i.e., Bereiter (1963) and Cronbach and Furby (1970), suggested the estimation

of the relationship between predictors and true scores directly. Bereiter (1963, p. 8) recommended the following formula,

$$r_{wy} \cdot X_t = \frac{r_{wy} - \frac{r_{wx}r_{xy}}{r_{xx'}}}{\sqrt{r_{xx'} - r_{wy}^2} \sqrt{r_{xx'} - r_{xy}^2}}$$

to estimate the correlation between an independent variable (w) and the final score (y) while the initial true individual's test score is partialled out from both w and y relationship. This process, of course, allows for the correction of the unreliability in the initial score. In the above formula, if the initial raw score rather than the true score is partialled out, the initial-score relationship can actually reverse the sign of the relationship based on the raw-score formula. The former approach provides the researchers with a set of change scores orthogonal to the estimated initial scores and the latter approach results in a set of scores orthogonal to the initial observed scores. The decision about either approach is not an empirical one but a personal one, depending on the type of data that analysts tend to seek. The Cronbach-Furby procedure is an extension of the Bereiter model since it introduces a new class of z variables—the variables that are measured at the time of (or after) the final measure. They will help further refine the estimate of true gain.

2.2.2 Residual Gain Score

To solve for the regression effect of the gain scores or the negative correlation between the gain and the initial score or other variables a series of residual gain scores have been proposed: a residualized gain score that is perfectly independent of the pretest measure (DuBois, 1957; Manning & DuBois, 1958, 1962). This means gain is residualized by expressing the posttest score as a deviation from the posttest-on-pretest regression line. In other words, removing that part of the posttest information that is linearly predictable from the pretest by partial correlation.

In the correlation of “change” or “gain” with an outside variable two choices are available to researchers, both yield similar results, i.e., part correlation ($\rho(X2.X1)Y1$) (DuBois, 1965, pp. 208-211) and partial correlation ($\rho(X2Y1.X1)$) (Pelz & Andrews, 1964). The corresponding formulas for calculation of the part and partial correlation given by Bohrnstedt (1969, p. 118) are as follows:

$$\rho(x2.x1)y1 = \frac{\rho_{x2y1} - \rho_{x1y2}\rho_{x1x2}}{\sqrt{1 - \rho_{x1x2}^2}}$$

$$\rho(x2y1.x1) = \frac{\rho_{x1y2} - \rho_{x1y1}\rho_{x1x2}}{\sqrt{1 - \rho_{x1y1}^2}\sqrt{1 - \rho_{x1x2}^2}}$$

where y is an outside variable and X_1 and X_2 have perfect reliability coefficient. Manning and DuBois (1958, 1962) actually used a part correlation technique and correlated the residual score with other residuals. According to Cronbach and Furby (1970, p. 74), the raw residual gain score is defined as follows:

$$D.X = Y - E(Y) | x = Y - Y - \beta_{Y.X}(X - X)$$

2.2.3 Combined True and Residualized Gain scores

For the simultaneous elimination of both unreliability and the regression effect of the raw gain score, researchers (Cronbach & Furby, 1970; Tucker, Damarin, & Messick, 1966) have proposed true residualized gain procedures. Since Manning and DuBois' (1958) residualized gain does not consider errors of measurement when x , y , and w are unreliable, the residual approach provides $r_w(y - x)$ when true correlation between the change and w $R_w(y - x)$ is required. These coefficients may even have opposite signs (O'Connor, 1972). Tucker, Damarin, and Messick (1966) proposed their “based free measure of change” approach (TDM). In this approach they partialled true x rather than raw x out of y and correlated this “adjusted” residual with another variable. The TDM (1966) is a part correlation technique that at least will always

have the same sign as the corresponding true-score part correlation but, nevertheless, would be a systematically biased estimate of $R_w(y.x)$. The estimate of $R_w(y.x)$ is:

$$RW(Y.X) = \frac{r_{wy} - \frac{r_{wx}r_{xy}}{r_{xx'}}}{\sqrt{1 - \frac{r_{xy}^2}{r_{xx'}}}}$$

The Base-free measure of change equal $Y - b_{yx}^*X$, and the correlation between the base-free measure of change and w would be:

$$r(W, Y - \beta_{yx}X) = \frac{r_{wy} - \frac{r_{wx}r_{xy}}{r_{xx'}}}{\sqrt{1 + \frac{r_{xy}^2}{r_{xx'}^2} - 2\frac{r_{xy}}{r_{xx'}}}}$$

$r(W, Y - b_{yx}X)$ has the same sign as $R_w(y.x)$ but a slightly different denominator.

Cronbach and Furby (1970) approach when w and z information are available, i.e.,

$$D_{\infty}.X_{\infty} = \beta_1X + \beta_2X + \beta_3W + \beta_4Z + constant$$

As Cronbach and Furby mentioned, correlation of the true residual score $D_{\infty}.X_{\infty}$ with any variable such as Q is possible by finding the following covariances.

$$\sigma(D_{\infty}.X_{\infty})Q = \sigma D_{\infty}Q - \frac{\sigma D_{\infty}X_{\infty}}{\sigma_{x_{\infty}}^2} \sigma X_{\infty}Q$$

2.3 The Merit of Alternative Approaches in Measurement of Change

Although estimated true scores were proposed to solve the unreliability problem of the raw gain score, their usefulness for practical purposes is questionable. This fact was reflected in the following message: "In most investigations there is nothing to be gained by estimating true scores..." (Nunnally, 1978, p. 217) or "...correcting for unreliability does not always result in an estimated true parameter which is larger

than the observed parameter. Such is the case for partial correlation” (Bohrstedt, 1969, p. 126).

Kessler (1977), in applying the Lord-McNemar/Cronbach-Furby procedure as a possible means of estimating a dependent variable in a correlational study, argues that using gain scores, which are computed from adjusted estimates of true x and true y can deal only with the unreliability problem attributed to gain scores and not with another equally important problem of gain scores such as intercorrelation of x and y .

- The use of residualized gain scores does not provide a mean of by-passing the problems of unreliability in the gain score . Problems of unreliability in the gain score. The more basic problem of regression toward the mean is not successfully resolved .Since each calculation ... makes use of the covariance between x and G , each suffers from the regression problem discussed above (Kessler, 1977 p. 60).

Kessler argues,

- ...when the components are quite reliable (0.9), it is still impossible to increase the reliability of the gain score to an acceptable level given component score correlations of 0.5 or higher. The adjusted score of true gain cannot deal with this problem and thus provides only a partial solution to the problem of unreliable gain score.... (p. 54)

Cronbach and Furby (1970) and O'Connor (1972) reviewed the major uses of “gain” or “change” scores, particularly estimated true or residualized true change, and concluded that for most research purposes the use of such change scores is not only unnecessary but in some cases may yield fallacious results. The measurement and methodological adequacy of residual gain scores also have been questioned in the literature. For example, Kessler (1977) questioned the merit of the residualized gain scores as a criterion variable. His point was that since the common part between gain

and x is partialled out before the gain score is computed, it is impossible to know the extent to which observed change is due to any given predictor. Kessler (1977) explained this problem more clearly by using an example. He argues,

- If $r = 0.9$, only 19% of the variance originally observed in raw gain remains to be explained by outside predictors, once the variable due to x has been partialled out. If a single predictor can explain 4% of this original variance, the observed correlation of this predictor with the residualized gain score will not be .20 (that is, 0.04), but rather 0.46 (that is $(4/19)$). In short, the base on which the influence of predictors is assessed is changed when residualized gain scores are used. (p. 55)

Another issue is that since residualization of gain score regression of y on x is determined before the gain score is regressed on other predictors, it is not possible to take into consideration the correlation between the initial score and these other predictors; consequently, the effect of other predictor(s) is underestimated if pretest (x) and other predictors (w) are correlated. The greater the intercorrelation between w and x , the greater is the underestimation of the effect of w on gain. The use of the residualized score assumes that w or any other predictor like w are uncorrelated with initial score (x). At the practical level this may not necessarily be true. In general Kessler (1977) concluded that,

- The use of residualized gain scores does not provide a means of by-passing the problems of unreliability in the gain score. The more basic problem of regression toward the mean is not successfully resolved. Since each calculation ... makes use of the covariance between x and G , each suffers from the regression problem discussed above. (Kessler, 1977, p. 60)

Contrary to Kessler's (1977) views, Bohrnstedt (1969) stated,

- The use of gain scores is less desirable than the use of residualized score since the former does not effectively remove the effect x_1 has on x_2 . As a result gain scores are likely to be negatively correlated with initial score, x_1 . (p. 121)

If x_2 and x_1 are linearly related, then residualization by x_1 leaves a score that has zero correlation or covariance with x_1 or initial test scores. Residualized x_2 is the deviation of the final score from its predicted value. This deviation score is then used to find the correlation of change even though this index is not a direct measure of the real change. One reason for using residualized gain is to guarantee "...that the variables found associated with gain are not found simply because they happen to be associated with initial status" (Lord, 1963, p. 24).

In spite of the methodological inadequacy of the residual gain, Manning and DuBois' (1962) findings revealed that residual gains in learning studies were more highly correlated with predictors, such as an aptitude test, than raw gain and were more intercorrelated. Inconsistent with these results, Bohrnstedt (1969) stated that, if the residual gain score rather than difference scores are used in correlational studies, the result is the same as that for part correlation. The pretest score, x is partialled out of the posttest score, y ; and the residual is correlated with a third variable, w . Cronbach and Furby (1970) and Linn and Slinde (1977) commented that residualizing actually removes that portion of the gain scores that linearly are predictable from pretest scores. They agreed that residual gain cannot be considered as a better measure of change. For example,

- One cannot argue that the residualized score is a 'corrected' measure of gain, since in most studies the portion discarded includes some genuine and important change in the person. The residualized score is primarily a way of singling out individuals who change more (or less) than expected. (Cronbach & Furby, 1970, p. 74)

Kessler (1977) also argues that "the use of residualized gain scores leads to no conclusions which could not also be obtained by the use of a raw gain score. ...even regression of the raw gain score gave no information which cannot also be obtained by regress of y on x " (p. 59). Bohrnstedt (1969) and Werts and Linn (1970) even believe that the use of difference scores in correlational studies is not necessary. For example, Bohrnstedt revealed that within the context of a linear model the relationship of a

variable, x , with change might be evaluated in terms of the pretest. Werts and Linn (1970) showed that the partial regression weights can be obtained from the partial regression coefficients in the regression of the posttest on w and x . Thus, there is no actual need for computing difference scores, and this is true with or without correction for unreliability of the measure.

As with estimated true gain and residualized gain scores researchers have questioned the merit of estimated residual gain scores. For example, in questioning the measurement and methodological adequacy of the “base-free measure of change,” Cronbach and Furby (1970) argue, “TDM offers an estimator that does not give the best least squares estimate of individual base-free scores because they seek instead estimates that correlate zero with x ” (p. 76). In this regard O’Connor (1972) commented,

- The point is not that the base-free approach is wrong, but that it produces a complex hybrid correlation that represents neither the raw score relationships nor the estimated true score relationships, but some combination of the two. It is far simpler mathematically to correct the appropriate partial correlation or multiple regression coefficient for attenuation without computing or conceptualizing in terms of change scores, residuals, or base-free measure of change. (O’Connor, 1972, p. 78)

Regardless of pro and con comments about each modified change score, Richards (1975) in a simulation study using the raw gain and eight other alternatives to the measurement of change (i.e., posttest score, gain adjusted for pretest error; gain adjusted for pretest and posttest error; Lord procedure, raw residual gain; estimated true residual gain; Tucker-Damarin-Messick procedure, and posttest score adjusted for initial academic potential) for measuring the individual’s growth concluded that, when one wants to order persons on growth, there is little reason for using complicated estimates of growth. According to Richards (1975, p. 15), the simple difference score (measured by subtraction of the pretest from the posttest) is much easier for

nonresearchers to compute and understand. He also argues, "Advocates of complex procedures should demonstrate practical, not just theoretical advantages for their techniques before researchers can be expected to take them seriously."

Rogosa et al. (1982) in criticizing previous researchers for overlooking and misunderstanding the crucial aspects of the raw gain scores argues that many current recommendations about the measurement of change are unsound (p. 726). Their recommendation is that the measurement of change must be based on a model for change. In this regard they reported that "the often cited deficiencies of the difference score - low reliability and negative correlation with initial status - are more illusory than real....The important deficiency (overlooked in the literature) lies in the data, not the measure of individual change" (p. 735).

Regarding the use of modified change scores, such as residual change, they believe that residual change measures are not a replacement or substitute for the estimation of the true change for each individual. Rogosa et al. contended that

- Residual change does not attempt to answer the simple question, How much did a person change on the attribute (ξ) Instead residual change addresses a far more difficult and arguably intractable question. (p. 740)

In correcting the change score with other variables, however, Linn and Slinde (1977) reported, "The alternative approaches to measuring change result in different correlations of the measure with other variables. The different estimates have different theoretical and practical implications"(p.128).

2.4 Reliability of Modified Changes Scores

As mentioned before, one form of residual gain is obtained by subtracting the predicted posttest scores ($\hat{x}_2 = a + bx_1$) from the corresponding observed posttest (x_2). The major advantage of the residual gain is that it is independent of the pretest score, one of the severely criticized topics about change scores. Researchers' findings

(Linn & Slinde, 1977) indicate that residualized gain scores like “difference scores” are unreliable.

- Although the residual score reliabilities ... are somewhat better than corresponding difference score reliabilities..., they are still disappointingly small whenever the correlation of pre- and postscores is large. Furthermore, residuals are usually of most interest in situations where the pre-post correlation is large relative to the reliability of the part. Thus, the same cautions due to unreliability of difference scores also apply to residual scores. (p. 125)

The above findings support Cronbach and Furby’s (1970) views that the residualized gain scores are not any better than the “raw” gain scores, especially in the context of the regression model. Such claims would hold true due to the fact that a residual-gain score with a low reliability coefficient still would be a poor predictor or criterion variable.

2.5 Two - Part Indices as Psychological Constructs

2.5.1 Background

Subtraction of the scores on two separate variables for defining a third construct has been common practice among psychologists and educators. For example, Salkind and Wright (1977) defined an individuals' cognitive style by two composite scores called Impulsivity and Efficiency

$$I_i = Ze_i - Zl_i$$

$$E_i = Ze_i + Zl_i$$

where Ze_i is a standard score for the i th individuals' total errors and Zl_i is a standard score for the i th individuals' mean latency on the Matching Familiar Figures Test. In this model large positive I scores are indicative of impulsivity, and large negative I scores indicate reflectivity. High positive E scores indicate inefficiency, and high negative E scores indicate efficiency. In this model the two dimensions of efficiency and impulsivity are defined to be conceptually orthogonal to each other.

In the area of self-concept analysis "Insight" and "Self - regard" have been operationalized as follows (Wylie, 1970, p. 88):

$$\text{Insight} = \text{Self} - \text{others}$$

$$\text{Self-regard} = \text{real} - \text{ideal self}$$

"Real self" (the concept of the ordinary person) is the individual's recognized perception of the attributes, feelings, and behavior of people in general. The "ideal self" represents the attributes, feelings, and behavior the individual admittedly would like to possess (Rogers & Dymond, 1957, p. 85). The smaller the discrepancy between Real-Ideal self, the higher self - regard. Furthermore, the subject's "global" self-regard is operationalized as a summated self-ideal discrepancy score. This means the single discrepancy across trait scales is summed to obtain a total self-ideal discrepancy score for S (Bill's 1954 Index of Adjustment and Values (IAV)); Leary's (1957) The

Interpersonal Checklist (ICL); and Worchel's (1957) Self-Activity Inventory (SAI)).

The attitude-toward-disability index (Cordaro & Shontz, 1969) is another construct defined by a discrepancy score, i.e.,

$$\text{attitude toward disability} = \text{spread score} - \text{Isolation score}$$

The test classifies the subjects as isolators or spreaders. The negative items on the test are indicative of higher agreement with spread items, and positive scores reflect the subjects' frequent tendency for selecting the isolation items.

The job satisfaction index is another composite construct, operationalized both as a change construct (difference between real and ideal ratings of job facets) and as a summated composite of ratings of job facets (Wanous and Lawler, 1972, p. 96), i.e.,

$$JS = \Sigma \text{ facets (should be - is now)}$$

Porter (1961) defines job satisfaction as the difference between responses to a "how much is there now" item and responses to a "how much should there be" item when these two items are asked for a number of job facets or needs. The difference between these two types of items is computed, and the differences are summed across the job facets to yield a measure of overall satisfaction (Wanous & Lawler, 1972, p. 96).

$$JS = \Sigma \text{ facets (would like - is now)}$$

In the Minnesota Satisfaction Questionnaire (MSQ) used in analysis of work adjustment, job satisfaction also was operationalized as the difference between what the individual would like to receive and what he/she receives. Another discrepancy score used to define job satisfaction as presented in Wanous and Lawler (1972, p. 96) is:

$$JS = \Sigma \text{ facets (Importance - is now)}$$

The above formula shows the discrepancy between the importance of a job facet and the perception of fulfillment from a facet.

A measure of learning disability also has been defined as a discrepancy score (Hanna, Dyck, & Holen, 1979); that is, the individual's achievement test score is subtracted from his aptitude-test score to indicate how well he/she is performing

with respect to the expected performance level .

$$\text{LDI} = \text{Aptitude test score} - \text{obtained achievement test score}$$

Depending on what cut score the researcher uses, the subject will be categorized either as learning disabled or a normal individual based on the above index.

Two-part indices like change score composites (Dubois, 1957; Lord, 1956; Manning & Dubois, 1962; Roff & Payne, 1956; Roff, Payne, & Moore, 1954; Simral, 1947; Woodrow, 1938a, 1939a,b,c, 1945) have been used in correlational studies as well as in other research contexts. For example, Loper and Hallahan (1980) in comparing different statistical procedures for determining the relationship between cognitive tempo and reading achievement, correlated the students' achievement scores with their performance scores on the Matching Familiar Figures Test (i.e., with latency, error, impulsivity score (I) and efficiency score (E)). Rollins and Genser (1977) also correlated I and E scores with students' performance on concept attainment (i.e., cognitive task with few dimensions and cognitive task with many dimensions). In their study, consistent with Loper and Hallahan's (1980) findings, Rollins and Genser (1977) concluded that the impulsivity dimension predicted the performance of students very well while the efficiency dimension did not.

Wanous and Lawler (1972), in analyzing the degree of congruence between the operational measure of job satisfaction, three of which were defined as a discrepancy score (i.e., Importance-Is Now, Should Be-Is Now, and Would Like - Is Now) used different measures of satisfaction to predict absenteeism and turnover. Their findings indicate that only three discrepancy measures (as stated above) significantly correlated with absenteeism. Based on these findings, Wanous and Lawler (1972) argue that it does not appear to be safe to assume that two measures of satisfaction necessarily have high correlation with a third variable. "Had not a number of operational measures been used here, conclusions about the job satisfaction- absenteeism relationship would have been determined by the choice of which job satisfaction measure to use" (p. 103). These researchers reported that

- It appears quite likely that some of the conflicting results reported in studies of satisfaction are due to the different measures of job satisfaction that have been used. This is illustrated by the research on the relationship of satisfaction to performance where different results have been reported and different measures have been used (p. 103).

The real-ideal discrepancy score also has been widely used in research as an index of personal adjustment (Block & Thomas, 1955; Borislow, 1962; Butler & Haigh, 1954; Chodorkoff, 1954; Hanlon, Hofstaetter & O'Connor, 1954; Rogers & Dymond, 1954; Scott, 1958; Smith, 1958; & Turner and Vanderlippe, 1957); self-concept (Morrison, 1962; Purkey, 1970; Rogers & Dymond, 1954; Yarworth & Guthier, Jr., 1978); self-acceptance (Crowne & Stephens, 1968); self-regards (Rogers & Dymond, 1954; & Wylie, 1974); self-esteem (Kwal & Fleshler, 1973); self-image (Katz & Zigler, 1967; & Phillips and Zigler, 1980); and cognitive development (Achenbach & Zigler, 1963; Katz & Zigler, 1967; Katz, Zigler, & Zalk, 1975; Phillips & Zigler, 1980; Zigler, Balla, & Watson, 1972). The major use of real-ideal discrepancy scores, however, has been to index self-regard (Hoge & McCarthy, 1983; Lazzair, Fioravati, & Gough, 1978; Lombardo, Fantasia, & Poulos, 1975, Mahoney & Hartnett, 1973; & Yanagide & Marsell, 1978) - the smaller the discrepancy, the higher the self-regard. The real-ideal discrepancy scores, like other discrepancy scores also have been used in the correlational research context. For example, Yanagida and Marsella (1978) investigated the relationship between real-ideal self-concept discrepancy and depression among different generations and age groups of Japanese-American women in Hawaii. Researchers also have analyzed the self-concept discrepancy score in the context of causal relationships (Blalock, 1964; Crane, Kenny, & Campbell, 1972; Yee & Gage, 1968). Bixler (1965) used the cross-lagged panel technique to investigate the causal effect of teachers' and peers' influence on changes in students' self-concepts.

2.5.2 Methodological Adequacy and Psychometric Properties of Two-Part Indices

Like raw gain scores, two-part indices have come under increasing methodological evaluation and criticism, particularly in correlational studies. Cronbach and Furby (1970), for example, included in their discussion of gain scores the use of two-part scores as definitions of constructs, and as stated before, their argument was,

- There is little reason to believe and much empirical reason to disbelieve the contention that some arbitrarily weighted function of two variables will properly define a construct. More often, the profitable strategy is to use the two variables separately in the analysis so as to allow for complex relationships.

Raeissi and Glasnapp (1983), in examining the potential predictive power of impulsivity (I) and efficiency (E) (Salkind & Wright, 1977) in a three-variable regression model demonstrated that the arbitrarily weighted (I) and (E) scores are methodologically deficient. The following two reasons were behind such a conclusion: misspecification of the model; and loss of information with respect to free regression of a third variable on the components of the raw gain. Raeissi and Glasnapp's (1983) recommendation was,

- To allow the individual error and latency variables to function separately in the analysis. If an E score composite is a dominant variable in the data, suppression conditions will occur among the intercorrelations, the regression model will identify the effective weights in the change score composite, and the relationship with the criterion will be maximized. If an I-score composite is a dominant variable in the data, redundant conditions will occur among the intercorrelations, the regression model will identify the effective weights for errors and latency in the summated score composite and the relationship with the criterion will be maximized. (p. 116)

Raeissi and Glasnapp's (1983) recommendation partly stems from the fact that the findings on the relative importance of response latency and accuracy in defining

MFF efficiency/ impulsivity are inconsistent. Block, Block and Harrington (1974), for example, concluded that MFF accuracy, not latency, carries the important psychological variance while Kagan and Messer (1975) suggested that both accuracy and latency carry significant variance, and reflection-impulsivity construct demand simultaneous measurement of both variables.

Consistent with Kagan and Messer (1975), Loper and Hallahan (1980), who investigated the relationship between reading achievement and performance on the Matching Familiar Figure Test (MFF) concluded that latency and errors have the same importance for predicting achievement (p. 95). Rollins, and Genser (1977) also concluded that the latency measure on the cognitive style test is an important predictor of performance on a cognitive task (i.e, concept attainment).

Using I and E composite scores as a predictor of reading achievement (Lopper & Hallahan, 1980) and concept attainment on a cognitive task (Rollins & Genser, 1977), both groups of researchers concluded that the impulsivity dimension predicted performance of the students very well while the efficiency dimension did not. These findings support Raeissi and Glasnapp's (1983) results on predictive validity loss of composite constructs.

Conceptual and methodological adequacy of the real-ideal self discrepancy score (as an indicator of self-regard) also has been severely criticized on the grounds of both validity and reliability (Hoge & McCarthy, 1983; Wylie, 1970). Wylie argues, "Such two-part indices have been widely used without sufficient prior exploration of questions which are highly pertinent to their possible interpretation in terms of the construct which they purport to index" (p. 89). In her discussion Wylie (1974, p. 89) even distinguishes between two discrepancy scores, i.e., Ideal-real self and Ideal-other scores, regarding the methodological analysis. Wylie's (1974) argument stems from the fact that "...the self-ideal discrepancy is a phenomenal discrepancy, while the self-other discrepancy is not" (p. 89). This means in the former definition one is dealing with a discrepancy experienced by the reporting and reacted to directly by him.

- This implies that a subtractive procedure is not the only conceivable way of operationalizing such a discrepancy, that is, an alternate index might be taken in terms of S's reporting directly the experienced magnitude of the discrepancy. (p. 89)

Such an alternative form or model is known to be "self score." To calculate the "self score," the favorable end of the scale is selected as the "ideal self" reference point. One possible advantage of the self-score is that, since the ideal value is fixed for all Ss, the only source of unreliability in it is the self score itself. Further details about advantages as well as disadvantages of "self-score" can be found in Wylie (1970, p. 93-94). In summarizing her statement, Wylie (1974) contended,

- Many unwarranted conclusions can be reached if insufficient attention is paid to the interpretational pitfalls in the use of two-part indices and to the relationships of each component to other theoretically relevant behavior measures. Moreover, if one's interest is primarily in predicting behavior accurately (that is, without regard for the theoretical implications or explanations of such accuracy), it may turn out that self-report (as opposed to the self-ideal discrepancy report) should be used. (p. 74)

Interestingly enough, regarding the implication of the $y - x$ subtraction to real-ideal discrepancy, Wylie (1974) objected to Cronbach and Furby's (1970) suggestions that: (1) use of differences between two values to define individuals' scores (indicative of a construct) should be discontinued in favor of multivariate approaches; (2) or if researchers continue to use two-part subtractive scores, they should subtract the estimated "true" x_{∞} from estimated "true" y_{∞} ($y - ax$, where a could be any value) instead of $y - 1.00x$.

Wylie's (1974) point was that neither the discontinuation of the subtractive type of self-ideal index in favor of a multivariate approach nor weighting the x and y variables in the discrepancy is necessarily appropriate to the theoretical status of phenomenal self-ideal discrepancies. Wylie (1974) argues,

- Since the discrepancy is presumably something S can experience as a difference between his actual self-concept and his ideal for himself, there seems to be a theoretical reason to try to operationalize it by a subtractive score, as free as possible of irrelevant influences, of course (p. 91).

Researchers have also been concerned about the reliability and validity of self-ideal discrepancy scores. According to Stanley (1971), the attainment of a highly reliable difference score depends upon having a high average reliability of the two component scores and a low correlation between the component scores. Wylie's (1974) view is that "even if highly reliable self-ideal discrepancies are obtained, one must still hold open the question raised earlier: Do reported self-ideal discrepancies of various sizes merely indicate varying degrees of cognitivity experienced discrepancy, or are they valid indicators of degrees of self-regard as well? Does a large discrepancy from one part of the scale range necessarily indicate poorer self-regard than a smaller discrepancy from another part of the scale range?"(p. 93).

Wylie (1974) commented, "Ideal-self reports actually contribute relatively little to the variance in self-ideal discrepancies" (p. 93). Hoge and McCarthy (1983) reported, "The more the discrepancy score depends on the real-self score, the better it is at predicting global self-regard" (p. 1053). Hoge and McCarthy's (1983) general conclusion was that real-ideal discrepancy measures include many errors, which reduce their validity and reliability, and since their empirical findings indicated that the real-ideal discrepancy measures are poorer predictors of "global" self-regard than are self-evaluations (whether taken alone or summated in a scale) Hoge and McCarthy (1983) also called for discontinuation of the real-ideal discrepancy scores.

In summary the above findings indicate that methodological adequacies of the two-part indices, like raw gain scores, are questionable. Their problems are parallel to those discussed for change or "raw" gain scores earlier in this section. Overall, the merit of discrepancy scores, i.e., the raw gain scores and two part indices have come under heavy methodological criticisms in the literature, particularly in correlational studies. The proposed alternative models to measurement of change also have

shown to have their own methodological problems when used in correlational contexts. In the current project the extent to which different proposed discrepancy model estimators measures different underlying construct than the one measured by the original discrepancy model will be demonstrated.

Chapter 3

Method

The data for this project were generated through a numerical simulation program by using a random generation technique and input of parameter values into existing or derived mathematical formulas. That is, for a series of population parameters values (used as a set) the true and error components for targeted variables (x, y, and w) were generated. The observed scores were obtained from the summation of the corresponding true and error scores for x, y, and w variables. The true change scores were generated by substitution of true x and y into the change score formulas presented in Table 1.2, i.e., reliabilities were set at 1.00. The observed change scores (or estimator of change) were generated by substitution of the observed x and y into the same mathematical formulas as stated above.

3.1 Investigated Models

Six change score models (listed in Cronbach and Furby, 1970) were used to generate the desired data for this project. These models were:

Model 1. Raw gain score

$$Y - X$$

Model 2. Raw residual gain

$$Y - b_{xy}X$$

where

$$b_{xy} = \rho_{xy} \left(\frac{\sigma_y}{\sigma_x} \right)$$

Model 3. Estimated true gain, i.e., corrected for error in x

$$Y - \rho_{xx'} X$$

Model 4. Estimated true gain, i.e., corrected for errors in both x and y

$$\rho_{yy'} Y - \rho_{xx'} X$$

Model 5. Estimated true gain, i.e., regression method

$$\beta_1 X + \beta_2 Y$$

where

$$\beta_1 = \frac{(1 - \rho_{yy'}) \rho_{xy} \left(\frac{\sigma_x}{\sigma_y} \right) - \rho_{xx'} + \rho_{xy}^2}{1 - \rho_{xy}^2}$$

$$\beta_2 = \frac{\rho_{yy} - \rho_{xy}^2 - (1 - \rho_{xx'}) \rho_{xy} \left(\frac{\sigma_x}{\sigma_y} \right)}{1 - \rho_{xy}^2}$$

Model 6. Estimated true residual gain, i.e., base free measure of change

$$Y - b_{xy}^* X$$

where

$$b_{xy}^* = \rho_{xy} \left(\frac{\sigma_y}{\rho_{xx'} \sigma_x} \right)$$

3.2 Simulation Parameters

The independent variables or input parameters manipulated in this investigation were:

1. Reliability coefficients for the change score-components, i.e., $\rho_{xx'}$ and $\rho_{yy'}$;
2. Reliability coefficient for the third variable ($\rho_{ww'}$);
3. Variability in the x component (σ_x);
4. Variability in the y component (σ_y);
5. Validity coefficient for y component (ρ_{wy});
6. Validity coefficient for x component (ρ_{wx});
7. Correlation (colinearity) between x and y components (ρ_{xy}).

3.3 Selection of the Parameter Values

The standard deviations for x and y were set as follows: 1) equal (i.e., $\sigma_x = \sigma_y$) such that the ratio of $\frac{\sigma_x}{\sigma_y} = 1$ and 2) unequal (i.e., $\sigma_x \neq \sigma_y$ where $\frac{\sigma_x}{\sigma_y} = \frac{1}{2}$). The selected values for σ_x and σ_y were based on Zimmerman and Williams' (1982a,b) findings which demonstrated that when σ_x and σ_y are unequal, the raw change score can be highly reliable and valid in correlational contexts. Rogosa, Brandt, and Zimowski (1982) and Rogosa and Willett (1983) reached similar conclusions. Zimmerman and Williams' (1982) reliable change scores (where $\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$) result from a suppression condition (Glasnapp, 1984), and Rogosa et al.'s reliable change scores (where there exists high dispersion for true change and low stability coefficient for x and y variables) result from the domination of one variable over the other (i.e., y over x) in the regression context.

The coefficient of colinearity between x and y (ρ_{xy}) was set to be equal to: .75, .50, and .25 (where .75 equals a high correlation, .50 equals a moderate correlation, and .25 equals a low correlation). The selected values for ρ_{xy} were based on the findings of Kessler (1977) and Rogosa et al. (1982), who indicated that change correlates

highly with a third variable if the value of ρ_{xy} (or coefficient of stability) is not more than .50.

The validity coefficient of y was set to be equal to .70 and .50 (where .70 equals a high correlation and .50 equals a moderate correlation) and the validity coefficient for x was set to be equal to .50 and .30, where .30 was considered a low correlation coefficient relative to others. The combination of the stated parameter values, i.e., validity coefficients (ρ_{wx} and ρ_{wy}) variabilities for x and y ($\frac{\sigma_x}{\sigma_y} = \lambda$) and coefficient of colinearity (ρ_{xy}) generated underlying discrepancy change scores defined by redundant (additive model) and suppression conditions (negative suppression). These conditions were presented in Table 1.1. The nonzero correlation coefficients for ρ_{wx} also indicated the presence of colinearity between x and w in the correlation of change with the third variable (w). The colinearity was built into the current analysis based on the Kessler (1977) arguments about the merit of the use of residualized change in the correlational contexts. Kessler stated,

- Because the regression of y on x is determined before the gain score is regressed on other predictors it is not possible to consider the correlation between the initial score and these other predictors. The greater the correlation of w with the initial score, the greater is the underestimation of the effect of w on gain. (p. 55)

In sum, the parameter-value levels selected were:

1. $\rho_{xx'}$; $\rho_{yy'}$ (1.0, .90, .70)
2. $\rho_{ww'}$ (.80)
3. $\frac{\sigma_x}{\sigma_y} = \lambda$ (1, 1/2)
4. ρ_{wy} (.50, .70)
5. ρ_{wx} (.30, .50)

6. ρ_{xy} (.25, .50, .75)

From these parameter values, the variability and validity coefficient manipulations were combined to define these basic simulation conditions. The correlation between x and y (ρ_{xy}) and reliability coefficients for $\rho_{xx'}$ and $\rho_{yy'}$ were manipulated across all values for each of these three conditions. Table 3.1 summarizes the parameter values used in generating sets of scores for investigating the comparability of the six change score models across distinct populations.

3.4 Simulation of the True and Observed Change Scores

The true scores for x, y, and w were obtained by fixing the reliability coefficients of these variables at 1.00 in the input variance-covariance matrix where x and y were change score components and w was the third variable. To obtain the corresponding observed scores for x and y, two sets of error components were generated to be added to the true scores, one set when the reliability coefficients were fixed at $\rho_{xx'} = \rho_{yy'} = .90$ and a second set when $\rho_{xx'} = .70$ and $\rho_{yy'} = .90$. The observed w was generated by fixing its reliability at $\rho_{ww'} = .80$. Random true and error components were generated for separate sets of 2,000 cases to configure nine populations of data with the identified parameter values, i.e., one set of 2,000 cases for each condition (I,II,III) by ρ_{xy} combination. True change scores were generated by substituting the true x and y into the change score formulas presented above (Models 1 through 6). Observed change scores were generated by substituting each set of the observed x and y variables into the same mathematical formulas .

3.5 Data-Generation Procedures

The data for this investigation were generated by applying a subprogram written in FORTRAN which accessed the GGNSM subroutines from the I.M.S.L. "packages" (International Mathematical and Statistical Libraries, inc, 1984) available at the

Table 3.1

Simulation Design

Condition	Variabilities	Validity	$\rho_{xx'}$	$\rho_{yy'}$	$\rho_{ww'}$	Colinearity
		Coefficients				ρ_{xy}
I	$\sigma_x = \sigma_y$	$\rho_{wx} = \rho_{wy}$	1.00	1.00	1.00	0.25, 0.50, 0.75
		$\rho_{wx} = 0.5$	0.90	0.90	0.80	0.25, 0.50, 0.75
		$\rho_{wy} = 0.5$	0.70	0.90	0.80	0.25, 0.50, 0.75
II	$\sigma_x = \sigma_y$	$\rho_{wx} < \rho_{wy}$	1.00	1.00	1.00	0.25, 0.50, 0.75
		$\rho_{wx} = 0.3$	0.90	0.90	0.80	0.25, 0.50, 0.75
		$\rho_{wy} = 0.7$	0.70	0.90	0.80	0.25, 0.50, 0.75
III	$2\sigma_x = \sigma_y$	$\rho_{wx} < \rho_{wy}$	1.00	1.00	1.00	0.25, 0.50, 0.75
		$\rho_{wx} = 0.3$	0.90	0.90	0.80	0.25, 0.50, 0.75
		$\rho_{wy} = 0.7$	0.70	0.90	0.80	0.25, 0.50, 0.75

University of Kansas Computing Center. This subprogram is a program designed to generate a set of normally distributed variables with a mean of zero and standard deviation of 1.0 for a given covariance matrix. In order to activate the GGNSM subprogram, a seed was required as input into the program along with the population variance - covariance parameter matrix. This seed was variably assigned an integer value in the exclusive range 1.0 and 2147483647.0. Thus, for every condition a new seed and the corresponding variance - covariance matrix were fed into the program. Further details about the GGNSM subprogram are provided in Appendix A. In general, the simulation program performed the following statistical tasks:

1. Configured a matrix of input variances and covariances based on the selected population parameter values. Each matrix defined one of the nine parameter conditions for simulated data.

2. Using the parameter values as input to subroutine GGNSM, randomly generated nine sets of 2,000 true and error components for x, y, and w corresponding to each simulation set of conditions.
3. Formed observed scores for each case by combining true and error components.
4. Constructed "true" and "observed" change scores for each of the six models under each parameter value set for the 2,000 cases in that set.

In order to simulate the desired data for this project, the subprogram GGNSM was activated to generate the "seeds" for nine vectors of normally distributed random numbers, i.e., true x, y, and w and the corresponding error components (two sets of error vectors). Generated scores corresponded to three reliability conditions, i.e., 1) $\rho_{xx'} = \rho_{yy'} = \rho_{ww'} = 1.00$; 2) $\rho_{xx'} = \rho_{yy'} = .90$ and $\rho_{ww'} = .80$; and 3) $\rho_{xx'} = .70$, $\rho_{yy'} = .90$, and $\rho_{ww'} = .80$. True change scores were calculated by substituting true x and y into mathematical formulas presented in Table 1.2, where reliabilities were fixed at 1.0. Two sets of corresponding error vectors were combined with true scores (x, y, and w) to generate two sets of the observed scores (x, y, and w). Each set of observed x and y components was substituted in the mathematical formulas presented in Table 1.2 along with its corresponding reliabilities ($\rho_{xx'}$ and $\rho_{yy'}$) to generate the observed change scores.

To illustrate the procedures used, scores generated under Condition I ($\sigma_x = \sigma_y$; $\rho_{wx} = \rho_{wy} = .50$) and $\rho_{xy} = .25$ are provided as an example. The I.M.S.L. (GGNSM) subprogram was initialized by the elements of the 9x9 variance-covariance matrix presented in Table 3.2. The formula used to compute the error variance components was: Variance (error) = (variance observed score) - (1.0 - reliability). The covariances among the error components and the error and true score components were set at zero. The stated matrix was obtained by converting the elements of the 9x9 correlation matrix for true and error scores where TX, TY and TW represent true x, y and w and EX, EY and EW are the corresponding error scores for two levels of error of measurement for x, y and w (i.e., (1) $\rho_{xx'} = \rho_{yy'} = .90$ and $\rho_{ww'} = .80$; (2) $\rho_{xx'} =$

Table 3.2
Variance - Covariance Materix for Data in Condition 11

Parameters	σ_{TX11}	σ_{TY11}	σ_{TW11}	σ_{EX111}	σ_{EY111}	σ_{EW111}	σ_{EX112}	σ_{EY112}	σ_{EW112}
σ_{TX11}	1.00								
σ_{TY11}	0.25	1.00							
σ_{TW11}	0.50	0.50	1.00						
σ_{EX111}	0.00	0.00	0.00	0.10					
σ_{EY111}	0.00	0.00	0.00	0.00	0.10				
σ_{EW111}	0.00	0.00	0.00	0.00	0.00	0.20			
σ_{EX112}	0.00	0.00	0.00	0.00	0.00	0.00	0.30		
σ_{EY112}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	
σ_{EW112}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20

.70; $\rho_{yy'} = .90$ and $\rho_{ww'} = .80$). The above labeling remains constant throughout the entire document as well. The index "11" beside each of the stated variables represents condition number and ρ_{xy} value level. For example, for TX11, the first number from the left is the condition number (i.e., Condition I where, $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$) and the second number is the number of the ρ_{xy} level (i.e., $\rho_{xy} = 0.25$). For three values of ρ_{xy} (.25, .50 and .75) the index numbers were 1, 2 and 3, respectively, and they remain constant throughout the entire document. Condition I in combination with three ρ_{xy} levels had the following index: 11, 12 and 13. If the condition changed to II ($\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$) or III ($\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$), the index of the condition was changed to 2 or 3, respectively. Thus, the three levels of Conditions II and III were 21, 22 and 23 and 31, 32, and 33 respectively.

The nine data sets generated for this project had indices of 11, 12, 13, 21, 22, 23, 31, 32 and 33. From here on these indices will be used often by themselves or in combination with other indices. In any form of two or more digits in a raw score index, the first digit identifies the condition and the second identifies the ρ_{xy} level number. For error (EX, EY and EW) and observed scores (OX, OY, OW), a third index (1 or 2) is included to represent scores generated under one of the two imperfect reliability conditions. For example, the three digit indices beside the error variable (such as EX111 or EX112) in the above correlation matrix represent the condition number, the ρ_{xy} level and the reliability level for x and y. If the reliability of x and y were equal and less than perfect ($\rho_{xx'} = \rho_{yy'} = .90$), this index was 1 and if the reliability levels were unequal ($\rho_{xx'} = .70$ and $\rho_{yy'} = .90$), this index was set equal to 2. For ρ_{xy} levels, two ($\rho_{xy} = .50$) and three ($\rho_{xy} = .75$) of Condition I (i.e., Condition 12 and 13), the corresponding covariances of .50 and .75 for x and y replaced .25 in the variance-covariance matrix of Table 3.2 as input into the GGNSM subprogram. Because the x and y variances were set at 1.00, the covariances equal the values of ρ_{xy} . The rest of the elements of the stated variance-covariance matrix remained the same across all of the ρ_{xy} levels of this condition. Every time that the GGNSM

subprogram was activated, a new seed was implemented into the program. For the three levels of Condition II (Conditions 21, 22 and 23) and Condition III (Conditions 31, 32 and 33), their corresponding variance - covariance matrix served as input into the GGNSM program. That is, for the three levels of the stated conditions, ρ_{xy} was iterated from .25 to .75 as before and for any ρ_{xy} level a new seed was used to initiate generation of the data . Thus, three true and six error scores (for each of the two reliability conditions) were randomly generated for each of 2,000 cases within a condition corresponding to the x, y and w parameter values.

In total, nine different data sets of 2,000 cases were randomly generated to be used in this project. Corresponding true and error scores for each case were added to create observed scores(OX, OY, and OW) under each reliability condition. The stated true and observed x, y and w scores generated in each condition were substituted in the formulas presented in Table 1.2 to generate the true and observed change model scores for their corresponding conditions. The true change scores (total of six) for each of the nine conditions (condition 11-condition 33) were generated by inputting the true x and y scores into the mathematical formulas presented in Table 1.2 and setting reliabilities at 1.0 in these formulas. The observed change scores were generated by substituting observed x and y scores into the formulas in Table 1.2 (six change scores per each of the two sets of observed scores). A total of 18 change scores (six true and twelve observed) were generated for each of the 2,000 cases under each simulation condition. The true change scores are labelled as "GT" scores followed by appropriate index numbers to identify the conditions (1, 2 or 3), the level of ρ_{xy} (1, 2 or 3) and the model(1 through 6) number. The observed gain (change) scores for reliability condition 1 are labeled "GO1" and for reliability condition 2 "GO2" followed by the same index numbers as for the "GT" scores. Table 3.3 identifies the true scores, the observed scores, the true change scores and the 12 observed change scores generated for each of the 2,000 scores within each of the nine simulation conditions.

The input values used to generate the random true and error scores for each set of 2,000 cases resulted in simulation sets of 2,000 scores which under some conditions

Table 3.3
Guidance to the Notation Used
in This Document

Condition	Reliability Parameters	$\rho_{xy} = 0.25$	$\rho_{xy} = 0.50$	$\rho_{xy} = 0.75$
I	$\rho_{xx'} = 1.0$ $\rho_{yy'} = 1.0$ $\rho_{ww'} = 1.0$	TX11 TY11 TW11 GT111 GT112 GT113 GT114 GT115 GT116	TX12 TY12 TW12 GT121 GT122 GT123 GT124 GT125 GT126	TX13 TY13 TW13 GT131 GT132 GT133 GT134 GT135 GT136
	$\rho_{xx'} = 0.9$ $\rho_{yy'} = 0.9$ $\rho_{ww'} = 0.8$	OX111 OY111 OW111 GO1111 GO1112 GO1113 GO1114 GO1115 GO1116	OX121 OY121 OW121 GO1121 GO1122 GO1123 GO1124 GO1125 GO1126	OX131 OY131 OW131 GO1131 GO1132 GO1133 GO1134 GO1135 GO1136
	$\rho_{xx'} = 0.9$ $\rho_{yy'} = 0.9$ $\rho_{ww'} = 0.8$	OX112 OY112 OW112 GO2111 GO2112 GO2113 GO2114 GO2115 GO2116	OX122 OY122 OW122 GO2121 GO2122 GO2123 GO2124 GO2125 GO2126	OX132 OY132 OW132 GO2131 GO2132 GO2133 GO2134 GO2135 GO2136

Table 3.3 (continued)
 Guidance to the Notation Used
 in This Document

Condition	Reliability Parameters	$\rho_{xy} = 0.25$	$\rho_{xy} = 0.50$	$\rho_{xy} = 0.75$
II	$\rho_{xx'} = 1.0$	TX21 TY21 TW21	TX22 TY22 TW22	TX23 TY23 TW23
	$\rho_{yy'} = 1.0$	GT211 GT212 GT213 GT214	GT221 GT222 GT223 GT224	GT231 GT232 GT233 GT234
	$\rho_{ww'} = 1.0$	GT215 GT216	GT225 GT226	GT235 GT236
$\sigma_x = \sigma_y$	$\rho_{xx'} = 0.9$	OX211 OY211 OW211	OX221 OY221 OW221	OX231 OY231 OW231
	$\rho_{yy'} = 0.9$	GO1211 GO1212 GO1213 GO1214	GO1221 GO1222 GO1223 GO1224	GO1231 GO1232 GO1233 GO1234
$\rho_{wx} \neq \rho_{wy}$	$\rho_{ww'} = 0.8$	GO1215 GO1216	GO1225 GO1226	GO1235 GO1236
$\rho_{xx'} = 0.9$	$\rho_{yy'} = 0.9$	OX212 OY212 OW212	OX222 OY222 OW222	OX232 OY232 OW232
	$\rho_{ww'} = 0.8$	GO2211 GO2212 GO2213 GO2214 GO2215 GO2216	GO2221 GO2222 GO2223 GO2224 GO2225 GO2226	GO2231 GO2232 GO2233 GO2234 GO2235 GO2236

Table 3.3 (continued)
 Guidance to the Notation Used
 in This Document

Condition	Reliability Parameters	$\rho_{xy} = 0.25$	$\rho_{xy} = 0.50$	$\rho_{xy} = 0.75$
III	$\rho_{xx'} = 1.0$	TX31	TX32	TX33
	$\rho_{yy'} = 1.0$	TY31	TY32	TY33
	$\rho_{ww'} = 1.0$	TW31	TW32	TW33
		GT311	GT321	GT331
		GT312	GT322	GT332
		GT313	GT323	GT333
		GT314	GT324	GT334
		GT315	GT325	GT335
		GT316	GT326	GT336
	$\rho_{xx'} = 0.9$	OX311	OX321	OX331
		OY311	OY321	OY331
		OW311	OW321	OW331
$\sigma_x \neq \sigma_y$	$\rho_{yy'} = 0.9$	GO1311	GO1321	GO1331
		GO1312	GO1322	GO1332
		GO1313	GO1323	GO1333
		GO1314	GO1324	GO1334
$\rho_{wx} \neq \rho_{wy}$	$\rho_{ww'} = 0.8$	GO1315	GO1325	GO1335
		GO1316	GO1326	GO1336
	$\rho_{xx'} = 0.9$	OX312	OX322	OX332
		OY312	OY322	OY332
		OW312	OW322	OW332
	$\rho_{yy'} = 0.9$	GO2311	GO2321	GO2331
		GO2312	GO2322	GO2332
		GO2313	GO2323	GO2333
		GO2314	GO2324	GO2334
	$\rho_{ww'} = 0.8$	GO2315	GO2325	GO2335
		GO2316	GO2326	GO2336

had sample estimate values differing slightly from the defined initial parameter values. While the pattern and accuracy of the intercorrelations, variabilities and reliabilities among x , y and w adequately reflected the nine different simulated conditions, the values used in generating the 18 model change scores for each condition were those specific to the generated data rather than the fixed parameter values used to generate the scores. This maintained the internal validity of the change scores relative to the characteristics of the x , y , and w true and observed scores within conditions. Table 3.4 identifies the x , y intercorrelations, variability values, and reliabilities calculated for the 2,000 cases specific to each of the nine conditions of the study.

To adjust for the sampling fluctuation and reliability effect on the change score models, however, the initial values for ρ_{xy} , σ_x , σ_y , σ_w , $\rho_{xx'}$ and $\rho_{yy'}$ obtained from the analysis of the original data (see Table 3.4) were fed into the program along with their true or observed x and y scores to generate the corresponding change score models. An example of this program for Condition II is provided in Appendix C and D.

3.6 Analysis of the Data

3.6.1 Verification of Simulated Data Accuracy

Table 3.5 presents the formulas for correlating each model's change score with w . The formulas for correlating change with x and y were derived by substituting x or y in place of w in the stated formulas. The mathematical formulas for correlating the change scores with x , y and w to be used for verifying the data were generated by this writer and are presented in Appendix B. These correlations served as population base-line data for verification of the simulated data within condition sets. In addition, means, standard deviations and intercorrelations among generated true and error scores were examined to verify that the sample data exhibited expected characteristics.

Table 3.4
Sample Condition Values Used To Compute Change Scores

		Condition I			Condition II			Condition III		
		con11	con12	con13	con21	con22	con23	con31	con32	con33
		$\rho_{zy} = 0.25$	$\rho_{zy} = 0.50$	$\rho_{zy} = 0.75$	$\rho_{zy} = 0.25$	$\rho_{zy} = 0.50$	$\rho_{zy} = 0.75$	$\rho_{zy} = 0.25$	$\rho_{zy} = 0.50$	$\rho_{zy} = 0.75$
GT	ρ_{zy}	0.254	0.489	0.759	0.230	0.502	0.754	0.285	0.481	0.742
	SD_x	0.989	1.006	0.993	0.972	0.982	0.997	1.005	0.983	1.011
	SD_y	0.990	0.990	0.990	0.990	0.990	0.990	2.036	2.036	2.036
	$\rho_{zx'}$	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho_{yy'}$	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ_{xw}	0.476	0.472	0.497	0.281	0.304	0.308	0.310	0.285	0.281
	ρ_{yw}	0.486	0.471	0.505	0.708	0.687	0.702	0.687	0.699	0.687
GO1	ρ_{zy}	0.232	0.443	0.685	0.205	0.452	0.690	0.264	0.436	0.680
	SD_x	1.041	1.062	1.040	1.032	1.023	1.051	1.046	1.037	1.061
	SD_y	1.028	1.033	1.035	1.030	1.042	1.046	2.125	2.119	2.147
	$\rho_{zx'}$	0.906	0.909	0.908	0.908	0.902	0.907	0.910	0.907	0.915
	$\rho_{yy'}$	0.910	0.906	0.904	0.910	0.908	0.909	0.911	0.912	0.910
	ρ_{xw}	0.432	0.414	0.426	0.257	0.230	0.285	0.276	0.252	0.231
	ρ_{yw}	0.425	0.415	0.434	0.619	0.614	0.616	0.606	0.611	0.586
GO2	ρ_{zy}	0.220	0.434	0.616	0.186	0.406	0.640	0.252	0.408	0.622
	SD_x	1.134	1.133	1.147	1.111	1.126	1.137	1.150	1.116	1.152
	SD_y	1.037	1.030	1.035	1.044	1.038	1.046	2.138	2.129	2.145
	$\rho_{zx'}$	0.761	0.776	0.768	0.761	0.763	0.760	0.775	0.765	0.781
	$\rho_{yy'}$	0.908	0.906	0.910	0.910	0.908	0.914	0.908	0.911	0.915
	ρ_{xw}	0.389	0.383	0.3804	0.215	0.205	0.255	0.236	0.231	0.234
	ρ_{yw}	0.430	0.416	0.430	0.608	0.602	0.617	0.579	0.597	0.597

Table 3.5
Correlation of Change With The Third Variable

Name	Correlation	Formula
Raw change Scores	$\rho(y - x)w$	$\frac{\rho_{yw}\sigma_y - \rho_{wx}\sigma_x}{\sqrt{\sigma_y^2 + \sigma_x^2 - 2\rho_{xy}\sigma_x\sigma_y}}$
Residual Change Scores	$\rho(y \cdot x)w$	$\frac{\rho_{yw} - \rho_{wx}\rho_{xy}}{\sqrt{1 - \rho_{xy}^2}}$
	$\rho(wy - x)w$	$\frac{\rho_{yw} - \rho_{wx}\rho_{xy}}{\sqrt{\rho_{xx}' - \rho_{wx}^2} \sqrt{\rho_{yy}' - \rho_{xy}^2}}$
	$\rho(wy - x_T)w$	$\frac{\rho_{yw} - \rho_{wx}\rho_{xy}}{\sqrt{\rho_{xx}' - \rho_{wx}^2} \sqrt{\rho_{yy}' - \rho_{xy}^2}}$
	$\rho(w_t y_t \cdot x_t)$	$\frac{(\rho_{yw}' \rho_{yx} - \rho_{wx} \rho_{xy})}{\sqrt{\rho_{xx}' \rho_{yy}' - \rho_{xy}^2} \sqrt{\rho_{xx}' \rho_{yy}' - \rho_{xy}^2}}$
Estimated True Change Correction for Measurment Error in x & y	$\rho(y_t - x_t)w$	$\frac{(\rho_{yw}\sigma_y - \rho_{wx}\sigma_x)}{\sqrt{\rho_{xx}'\sigma_x^2 + \rho_{yy}'\sigma_y^2 - 2\sigma_x\sigma_y\rho_{xy}}}$
	$\rho(y_t - x_t)w_t$	$\frac{(\rho_{yw}\sigma_y - \rho_{wx}\sigma_x)}{\sqrt{\rho_{xx}' \sqrt{\rho_{xx}'\sigma_x^2 + \rho_{yy}'\sigma_y^2 - 2\sigma_x\sigma_y\rho_{xy}}}}$
Estimated True Change	$\rho(y - \rho_{zx}'x)w$	$\frac{\sigma_w(\rho_{yw}\sigma_y - \rho_{wx}\rho_{zx}'\sigma_x)}{\sqrt{\sigma_y^2 + \rho_{xx}'\sigma_x^2 - 2\rho_{xx}'\rho_{xy}\sigma_x\sigma_y}}$
	$\rho(\rho_{yy}'y - \rho_{zx}'x)w$	$\frac{\rho_{yy}'\rho_{yw}\sigma_y - \rho_{xx}'\rho_{wx}\sigma_x}{\sqrt{\rho_{yy}'\sigma_y^2 + 2\rho_{xx}'\sigma_x^2 - 2\rho_{yy}'\rho_{xx}'\rho_{xy}\sigma_x\sigma_y}}$
	$\rho(\beta_1x + \beta_2y)w$	$\frac{\beta_1\rho_{wx}\sigma_x + \beta_2\rho_{wy}\sigma_y}{\sqrt{\beta_1^2\sigma_x^2 + \beta_2^2\sigma_y^2 + 2\beta_1\beta_2\rho_{xy}\sigma_x\sigma_y}}$
Estimated true Residual Change:	$\rho(y - b_{yx}^*x)w$	$\frac{\rho_{xx}'\rho_{yw} - \rho_{wx}\rho_{xy}}{\sqrt{\rho_{xx}' + \rho_{yy}' - 2\rho_{xy}\rho_{xx}'}}$
	$\rho(\beta_{y_1 x_1}(y - \rho_{xy})(\frac{\sigma_x}{\sigma_y})x)$	$\frac{\sigma_w(\rho_{yw} - \rho_{wx}\rho_{xy})}{\sqrt{1 - \rho_{xy}^2}}$

3.6.2 Parameter Effects on the Change Score Models

In order to examine the parameter effects on models' change scores generated for this project, it was decided to keep the y scores constant across set of data ; thus, allowing for the merging of data across conditions. Within a condition 2,000 cases were sorted on true y. True y scores were subsequently used as the matching variable to combine data across conditions for analyses while still maintaining the relationships among x, y and w defined by the selected parameter values.

Since the y distribution in Condition III had twice the variability as in Conditions I and II (i.e., $\frac{\sigma_x}{\sigma_y} = \lambda = \frac{1}{2}$), it was not possible to replace the y distribution of this condition with the sorted true y from Condition I. Therefore, the generated data for three levels of Condition III were kept separated from the data of other conditions at this level. In this condition, the data were sorted on true y as before and true y scores subsequently were used as the matching variable to combine data across three levels of Condition III (i.e., Conditions 31, 32 and 33).

Consistency of y across different conditions was supposed to control for the parameter effects on the change score models.

Furthermore, to make the comparison of the change scores under Condition III with Conditions I and II possible, the data were simultaneously sorted on true x and y components . Parallel merging of data file records after sorting maintained the relationships among x, y and w defined by the selected parameter values .

3.6.3 Differences in the Underlying Construct of Change Score Models

To demonstrate similarities or differences in the underlying constructs measured by the estimators of change, the values derived from the mathematical formulas correlating change scores with x, y and w were analyzed and compared across the models, that is, the contribution of the x and y component were highlighted for each definition along with the change scores relationship with w . The observed variations in the

mathematical structure of these estimators were an indication of differences in the operational definition of the change score constructs defined by these models.

The obtained change score models from simulated data were correlated with x and y components (true and observed) to provide empirical support for the above statements. The squared correlation coefficient between change and each of the stated components were compared across different change score models. The stated correlation coefficient determined the domination of x or y or whether each contributed equally into the definition of the change score construct across the models.

To demonstrate the noncongruency of the underlying construct measured by the stated change score, it is reasonable to show that different change scores rank the individuals differently. For this purpose the obtained change scores for each ρ_{xy} value level were correlated within each ρ_{xy} level to produce intercorrelation matrices among the models' change scores. The higher the correlation between the pairwise change scores were, the more congruent the underlying change scores were in ranking the individuals.

Furthermore, the obtained change scores were factor analyzed using a principal component technique. The factor analysis was done for each subset of parameter conditions, i.e., for each ρ_{xy} level (.25, .50, .75) as well as for combined ρ_{xy} levels within and across conditions (i.e., I and III). Since relationships to w were not considered in these analyses, Condition II's change scores replicated the change scores from condition I, and all of the results obtained from Condition I were generalized to Condition II as well.

The latter analyses tested the model differences within each ρ_{xy} level as well as measurement error effects. It was assumed that if all of the change scores are measuring the same underlying construct, the first principal component should account for a sizable portion of shared variance. Several principal component analyses conducted to examine the effects of variability as well as reliability on the change score models.

3.6.4 Differential Input of the Change Score Models within the Correlational Context

To demonstrate that different change score models have different input into the correlational context, correlations of the change score models with the third variable (w) were obtained. The observed variations in the stated correlation indicated the extent to which change score models differed in operation.

To examine the effect of w on the definition of the change score construct as they relate to w , w was partialled from the models' change scores and then the residual change scores were factor analyzed using a principal component technique. The comparison of the principal component results from this analysis and the ones where w was not considered determined the extent to which w could potentially affect the definition of the change score construct as it was related to the third variable (w). The reports on the results were descriptive.

3.6.5 Stability of the Change Score Models for Estimation of Change

To examine the degree of stability of each change score model for estimating change, correlations of the change scores from each model with w was compared across different conditions. The within model comparison on the stated correlation coefficients determined how consistent a model estimates or detects the change as the parameter condition changes from one to another thus identifying which parameter values are most likely to effect the change score construct resulting from a model's application . The observed variations on ρ_{Gw} (where G represented the change scores across different conditions) identify the models which were not stable in estimating the change across parameter conditions and those parameter values most likely to have an affect . The between-models comparisons in this context revealed how the models differ and which model was the most stable for estimation of change among the competing models. The reports on the results were descriptive.

Chapter 4

Results

The purpose of this project was to comparatively evaluate six proposed models for defining change or discrepancy scores in the correlational context. Data for evaluating these models were generated to reflect nine research situations, each defined by fixing parameter values for variables known to affect the reliability, validity and construct definition of raw change scores. The variables manipulated to define the nine different research contexts were the validity coefficients and variabilities for x and y and the intercorrelation between x and y. Manipulating the x and y variabilities and validity coefficients, three research conditions were defined: Condition I) $\sigma_x = \sigma_y$; $\rho_{wx} = \rho_{wy}$; Condition II) $\sigma_x = \sigma_y$; $\rho_{wx} \neq \rho_{wy}$; and Condition III) $\sigma_x \neq \sigma_y$; $\rho_{wx} \neq \rho_{wy}$. For interpretation purposes these conditions are referred to as research Conditions I, II, and III in this chapter. In each research condition, the magnitude of ρ_{xy} was iterated three times ($\rho_{xy} = .25, .50$ and $.75$). For notation purposes, these are referred to as ρ_{xy} levels. Combining the ρ_{xy} levels with the three research Conditions I, II, and III, nine parameter conditions were established (i.e., condition 11, 12, 13, 21, 22, 23, 31, 32, and 33 where, the first digit represents the condition number and the second the ρ_{xy} level). Within each of the nine parameter conditions, the reliability coefficient for x and y were iterated three times as well: 1) $\rho_{xx'} = \rho_{yy'} = 1.0$; 2) $\rho_{xx'} = \rho_{yy'} = .90$; and 3) $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = .70$ and $\rho_{yy'} = .90$).

The six change score models investigated were :

Model 1. Raw gain score

$$Y - X$$

Model 2. Raw residual gain

$$Y - b_{xy}X$$

where

$$b_{xy} = \rho_{xy} \left(\frac{\sigma_y}{\sigma_x} \right)$$

Model 3. Estimated true gain, i.e., corrected for error in

$$Y - \rho_{xx'}X$$

Model 4. Estimated true gain, i.e., corrected for errors in both x and y

$$\rho_{yy'}Y - \rho_{xx'}X$$

Model 5. Estimated true gain, i.e., regression method

$$\beta_1X + \beta_2Y$$

where

$$\beta_1 = \frac{(1 - \rho_{yy'})\rho_{xy} \left(\frac{\sigma_x}{\sigma_y} \right) - \rho_{xx'} + \rho_{xy}^2}{1 - \rho_{xy}^2}$$

$$\beta_2 = \frac{\rho_{yy} - \rho_{xy}^2 - (1 - \rho_{xx'})\rho_{xy} \left(\frac{\sigma_x}{\sigma_y} \right)}{1 - \rho_{xy}^2}$$

Model 6. Estimated true residual gain, i.e., base free measure of change

$$Y - b_{xy}^*X$$

where

$$b_{xy}^* = \rho_{xy} \left(\frac{\sigma_y}{\rho_{xx'}\sigma_x} \right)$$

These models were described in Table 1.2 . As is clear from the mathematical model of change, Models 1, 3, and 4 are similar in that they are discrepancy models (x has been subtracted from y). Likewise, Models 2 and 6 are based on residualization (i.e.,the linear relationship between x and y has been removed from y in one way or the other). In addition, the models differ in that the raw change score and raw residual gain models do not correct for errors in the x or y components, while the remaining models do. Model 5 (i.e., estimated true gain-regression method) is a model in between the two stated groups. It has a correction factor (for errors in the x and y components) as well as residualization of y by the x component. These differences highlight the mathematical differences of the change score models.

The change scores generated with perfect reliability for x and y are referred to as change (gain) scores (GT) and the change scores generated under imperfect reliability conditions are referred to as observed gain scores (GO1 and GO2 for the two reliability conditions). Each of the GT, GO1 and GO2 scores is accompanied by a three digit subscript number sequenced to explain the research condition (1-3), ρ_{xy} (1-3) level and the model number (1-6). For example GO2323 represents observed change scores for Model 3 (the last digit) generated for the second reliability condition ($\rho_{xx'} = 0.70, \rho_{yy'} = .90, \rho_{ww'} = 0.80$) under research Condition III ($\sigma_y = 2\sigma_x, \rho_{wx} = 0.30, \rho_{wy} = .70$) and the second level of ρ_{xy} (=0.50).

Using this notation system, the remainder of this chapter presents the results from data analyses addressing the following:

1. Verification of the accuracy of simulated data;
2. Congruency of change score models within each condition and parameter effects on the congruency of change score models;
3. Consistency of the change score models across nine different parameter conditions;
4. Correlation of change scores with w, i.e.,

- a. commonality of the models considering w
- b. commonality of the models partialing out w; and

5. Summary of the results.

4.1 Simulation Accuracy Verification

4.1.1 Mean, Standard Deviations and Intercorrelations

Means and standard deviations for true x, y, and w (TX, TY and TW) for the nine research conditions (11 - 33) are presented in Table 4.1. Means and standard deviations for the two sets of observed x, y and w (OX, OY and OW) also are reported in Table 4.1. As the findings in this table indicate, the values are in the expected range, i.e., means near zero for TX, TY and TW in Conditions I, II and III and standard deviations near 1.0 for TX, TY and TW in Conditions I and II. In Condition III, standard deviations were near 1.0 for TX and TW and near 2.0 for TY. The means of the corresponding change score (both true and observed) for each parameter condition are also presented in Table 4.11. Means for the change score models in all conditions were zero or near zero. Standard deviation of change score models in condition 11 ranged from 0.96 to 1.36 across the three reliability and ρ_{xy} levels. The corresponding standard deviation for change score models in the rest of the conditions (12 - 33) were 0.82-1.08 for condition 12, 0.55-0.85 for condition 13, 0.96-1.38 for condition 21, 0.85-1.18 for condition 22, 0.50-0.93 for condition 23, 1.83-2.16 for condition 31, 1.78-2.02 for condition 32, and 1.32-1.92 for condition 33.

Table 4.2 presents the correlation coefficients between TX, TY and TW and their corresponding error components for the nine parameter conditions (Conditions 11 - 33). The intercorrelations with and among generated observed scores also are given. The pattern of correlations for all nine conditions follow those expected given the parameter values serving as input conditions. For example, in condition 11 the correlation between TX and TY, TX and TW and TY and TW were .25, .48 and

Table 4.1

Mean and Standard Deviation of the Change Scores Models
and the Original Component of Change for Each Parameter Condition

condition 11			condition 12			condition 13		
Variable	Mean	SD	Variable	Mean	SD	Variable	Mean	SD
TX11	0.01	0.99	TX12	-0.03	1.01	TX13	-0.02	0.99
TY11	-0.01	0.99	TY12	-0.01	0.99	TY13	-0.01	0.99
TW11	0.00	0.99	TW12	-0.02	1.00	TW13	-0.03	0.99
OX111	0.01	1.04	OX121	-0.04	1.06	OX131	-0.02	1.04
OY111	-0.01	1.03	OY121	0.00	1.03	OY131	-0.01	1.03
OW111	0.01	1.08	OW121	-0.02	1.08	OW131	-0.02	1.09
OX112	0.02	1.13	OX122	-0.05	1.13	OX132	-0.02	1.15
OY112	-0.02	1.04	OY122	-0.01	1.03	OY132	0.00	1.03
OW112	0.00	1.07	OW122	-0.02	1.11	OW132	-0.03	1.08
GT111	-0.02	1.21	GT121	0.02	1.01	GT131	0.01	0.69
GT112	-0.01	0.96	GT122	0.00	0.86	GT132	0.01	0.64
GT113	-0.02	1.21	GT123	0.02	1.01	GT133	0.01	0.69
GT114	-0.02	1.21	GT124	0.02	1.01	GT134	0.01	0.69
GT115	-0.02	1.21	GT125	0.02	1.01	GT135	0.01	0.69
GT116	-0.01	0.96	GT126	0.00	0.86	GT136	0.01	0.64
GO1111	-0.02	1.28	GO1121	0.03	1.10	GO1131	0.01	0.82
GO1112	-0.01	1.00	GO1122	0.01	0.93	GO1132	0.00	0.75
GO1113	-0.02	1.22	GO1123	0.03	1.06	GO1133	0.01	0.79
GO1114	-0.02	1.16	GO1124	0.03	1.00	GO1134	0.01	0.75
GO1115	-0.02	1.13	GO1125	0.03	0.92	GO1135	0.01	0.58
GO1116	-0.01	1.00	GO1126	0.02	0.93	GO1136	0.01	0.77
GO2111	-0.04	1.36	GO2121	0.04	1.15	GO2131	0.02	0.96
GO2112	-0.02	1.01	GO2122	0.01	0.93	GO2132	0.01	0.81
GO2113	-0.04	1.19	GO2123	0.03	1.02	GO2133	0.02	0.85
GO2114	-0.04	1.13	GO2124	0.03	0.96	GO2134	0.02	0.80
GO2115	-0.03	1.06	GO2125	0.02	0.82	GO2135	0.01	0.55
GO2116	-0.03	1.02	GO2126	0.02	0.97	GO2136	0.02	0.93

Table 4.1 (continued)

Mean and Standard Deviation of the Change Scores Models
and the Original Component of Change for Each Parameter Condition

condition 21			condition 22			condition 23		
Variable	Mean	SD	Variable	Mean	SD	Variable	Mean	SD
TX21	0.00	0.97	TX22	-0.03	0.98	TX23	0.02	1.00
TY21	-0.01	0.99	TY22	-0.01	0.99	TY23	-0.01	0.99
TW21	0.00	1.01	TW22	0.03	1.00	TW23	0.01	1.00
OX211	0.01	1.03	OX221	-0.03	1.02	OX231	0.02	1.05
OY211	-0.01	1.03	OY221	-0.01	1.04	OY231	-0.02	1.05
OW211	0.00	1.10	OW221	0.02	1.10	OW231	0.00	1.08
OX212	0.04	1.11	OX222	-0.01	1.13	OX232	0.01	1.14
OY212	-0.01	1.04	OY222	-0.02	1.04	OY232	0.00	1.05
OW212	0.00	1.10	OW222	0.05	1.10	OW232	0.00	1.08
GT211	-0.02	1.22	GT221	0.02	0.98	GT231	-0.03	0.70
GT212	-0.01	0.96	GT222	0.00	0.86	GT232	-0.03	0.65
GT213	-0.02	1.22	GT223	0.02	0.98	GT233	-0.03	0.70
GT214	-0.02	1.22	GT224	0.02	0.98	GT234	-0.03	0.70
GT215	-0.02	1.22	GT225	0.02	0.98	GT235	-0.03	0.70
GT216	-0.01	0.96	GT226	0.00	0.86	GT236	-0.03	0.65
GO1211	-0.02	1.30	GO1221	0.02	1.08	GO1231	-0.04	0.83
GO1212	-0.01	1.01	GO1222	0.00	0.93	GO1232	-0.04	0.76
GO1213	-0.02	1.24	GO1223	0.02	1.03	GO1233	-0.04	0.79
GO1214	-0.02	1.18	GO1224	0.02	0.98	GO1234	-0.04	0.75
GO1215	-0.02	1.15	GO1225	0.02	0.89	GO1235	-0.03	0.58
GO1216	-0.02	1.01	GO1226	0.01	0.93	GO1236	-0.04	0.77
GO2211	-0.05	1.38	GO2221	0.00	1.18	GO2231	-0.01	0.93
GO2212	-0.02	1.03	GO2222	-0.01	0.95	GO2232	0.00	0.80
GO2213	-0.04	1.22	GO2223	0.00	1.05	GO2233	-0.01	0.83
GO2214	-0.04	1.15	GO2224	0.00	0.98	GO2234	-0.01	0.78
GO2215	-0.04	1.10	GO2225	0.00	0.85	GO2235	0.00	0.50
GO2216	-0.02	1.03	GO2226	-0.01	0.99	GO2236	-0.01	0.93

Table 4.1 (continued)

Mean and Standard Deviation of the Change Scores Models
and the Original Component of Change for Each Parameter Condition

condition 31			condition 32			condition 33		
Variable	Mean	SD	Variable	Mean	SD	Variable	Mean	SD
TX31	0.01	1.01	TX32	0.00	0.98	TX33	0.00	1.01
TY31	0.02	2.04	TY32	0.02	2.04	TY33	0.02	2.04
TW31	0.03	0.98	TW32	-0.01	1.00	TW33	0.05	1.00
OX311	0.00	1.05	OX321	0.00	1.04	OX331	0.00	1.06
OY311	0.05	2.13	OY321	0.01	2.12	OY331	0.02	2.15
OW311	0.03	1.08	OW321	0.00	1.10	OW331	0.05	1.09
OX312	0.00	1.15	OX322	0.01	1.12	OX332	-0.01	1.15
OY312	-0.01	2.14	OY322	0.04	2.13	OY332	0.00	2.15
OW312	0.01	1.07	OW322	0.01	1.08	OW332	0.05	1.10
GT311	0.01	2.00	GT321	0.01	1.78	GT331	0.02	1.45
GT312	0.02	1.95	GT322	0.01	1.78	GT332	0.02	1.36
GT313	0.01	2.00	GT323	0.01	1.78	GT333	0.02	1.45
GT314	0.01	2.00	GT324	0.01	1.78	GT334	0.02	1.45
GT315	0.01	2.00	GT325	0.01	1.78	GT335	0.02	1.45
GT316	0.02	1.95	GT326	0.01	1.78	GT336	0.02	1.36
GO1311	0.04	2.11	GO1321	0.01	1.91	GO1331	0.01	1.63
GO1312	0.04	2.05	GO1322	0.01	1.91	GO1332	0.01	1.57
GO1313	0.04	2.09	GO1323	0.01	1.91	GO1333	0.01	1.65
GO1314	0.04	1.92	GO1324	0.01	1.74	GO1334	0.01	1.48
GO1315	0.04	1.87	GO1325	0.01	1.65	GO1335	0.01	1.32
GO1316	0.04	2.05	GO1326	0.01	1.91	GO1336	0.01	1.61
GO2311	-0.02	2.16	GO2321	0.03	1.96	GO2331	0.01	1.69
GO2312	-0.02	2.07	GO2322	0.03	1.94	GO2332	0.01	1.68
GO2313	-0.02	2.10	GO2323	0.03	1.94	GO2333	0.01	1.74
GO2314	-0.01	1.92	GO2324	0.02	1.77	GO2334	0.01	1.57
GO2315	-0.01	1.83	GO2325	0.02	1.62	GO2335	0.00	1.32
GO2316	-0.02	2.10	GO2326	0.02	2.02	GO2336	0.02	1.92

Table 4.2
Correlation Among True, Error and Observed Scores
For Each Parameter Condition

Condition 11															
	TX11	TY11	TW11	EX111	EY111	EW111	EX112	EW112	EY112	OX111	OY111	OW111	OX112	OY112	OW112
TX11	1.00														
TY11	0.25	1.00													
TW11	0.48	0.49	1.00												
EX111	0.00	0.00	-0.01	1.00											
EY111	0.00	-0.03	-0.02	0.00	1.00										
EW111	0.06	0.00	0.00	-0.02	0.02	1.00									
EX112	0.00	-0.02	-0.02	-0.03	0.04	-0.03	1.00								
EY112	0.05	-0.01	0.03	0.00	0.00	0.02	0.04	1.00							
EW112	0.01	0.00	-0.04	0.02	-0.01	0.00	0.04	-0.05	1.00						
OX111	0.95	0.24	0.45	0.31	0.00	0.05	-0.01	0.04	0.02	1.00					
OY111	0.24	0.95	0.46	0.00	0.27	0.01	-0.01	-0.01	-0.01	0.23	1.00				
OW111	0.46	0.44	0.91	-0.01	-0.01	0.41	-0.02	0.04	-0.03	0.43	0.43	1.00			
OX112	0.87	0.21	0.41	-0.01	0.07	0.04	0.49	0.06	0.03	0.83	0.21	0.39	1.00		
OY112	0.26	0.95	0.47	0.00	-0.03	0.01	-0.01	0.36	-0.02	0.25	0.91	0.44	0.22	1.00	
OW112	0.45	0.45	0.91	0.00	-0.02	0.00	0.00	0.01	0.38	0.42	0.43	0.83	0.39	0.43	1.00
Condition 12															
	TX12	TY12	TW12	EX121	EY121	EW121	EX122	EY122	EW122	OX121	OY121	OW121	OX122	OY122	OW122
TX12	1.00														
TY12	0.49	1.00													
TW12	0.47	0.47	1.00												
EX121	0.02	0.00	0.01	1.00											
EY121	0.00	-0.02	0.00	0.00	1.00										
EW121	0.01	0.00	-0.02	-0.02	-0.02	1.00									
EX122	-0.02	0.02	0.00	0.00	-0.02	0.01	1.00								
EY122	0.01	-0.03	0.01	-0.04	-0.01	0.02	0.05	1.00							
EW122	0.01	0.02	0.04	0.04	-0.03	0.03	0.01	0.01	1.00						
OX121	0.95	0.46	0.45	0.32	0.00	0.00	-0.01	-0.01	0.02	1.00					
OY121	0.47	0.95	0.45	0.00	0.29	0.00	0.01	-0.03	0.01	0.44	1.00				
OW121	0.44	0.43	0.91	0.00	0.00	0.40	0.00	0.01	0.04	0.41	0.42	1.00			
OX122	0.88	0.44	0.42	0.02	-0.01	0.01	0.46	0.03	0.02	0.84	0.42	0.39	1.00		
OY122	0.47	0.95	0.45	-0.01	-0.02	0.01	0.03	0.28	0.02	0.44	0.90	0.42	0.43	1.00	
OW122	0.43	0.43	0.91	0.02	-0.01	0.00	0.00	0.01	0.44	0.41	0.41	0.84	0.38	0.42	1.00
Condition 13															
	TX13	TY13	TW13	EX131	EY131	EW131	EX132	EY132	EW132	OX131	OY131	OW131	OX132	OY132	OW132
TX13	1.00														
TY13	0.76	1.00													
TW13	0.50	0.50	1.00												
EX131	-0.01	0.00	-0.01	1.00											
EY131	-0.03	-0.02	-0.02	0.01	1.00										
EW131	0.00	0.01	0.02	-0.02	-0.01	1.00									
EX132	0.02	0.00	-0.01	0.01	0.00	0.00	1.00								
EY132	-0.04	-0.01	-0.01	0.01	0.01	0.01	-0.01	1.00							
EW132	-0.03	-0.03	0.00	0.03	-0.01	-0.01	0.02	0.02	1.00						
OX131	0.95	0.72	0.47	0.30	-0.03	0.00	0.02	-0.04	-0.02	1.00					
OY131	0.72	0.95	0.48	0.00	0.29	0.01	0.00	0.00	-0.03	0.68	1.00				
OW131	0.45	0.46	0.91	-0.02	-0.03	0.42	0.00	0.00	0.00	0.43	0.43	1.00			
OX132	0.88	0.66	0.43	0.00	-0.02	0.01	0.50	-0.04	-0.01	0.84	0.62	0.39	1.00		
OY132	0.71	0.95	0.48	0.00	-0.02	0.01	0.00	0.29	-0.02	0.68	0.91	0.44	0.62	1.00	
OW132	0.44	0.45	0.91	0.00	-0.03	0.01	0.00	0.00	0.41	0.42	0.42	0.83	0.38	0.43	1.00

Table 4.2 (continued)
Correlation Among True, Error and Observed Scores
For Each Parameter Condition

Condition 21															
TX21	TY21	TW21	EX211	EY211	EW211	EX212	EY212	EW212	OX211	OY211	OW211	OX212	OY212	OW212	
TX21	1.00														
TY21	0.23	1.00													
TW21	0.28	0.71	1.00												
EX211	0.03	0.00	0.02	1.00											
EY211	-0.02	-0.02	0.00	0.04	1.00										
EW211	0.01	-0.01	-0.01	0.04	0.01	1.00									
EX212	-0.01	-0.02	-0.01	0.01	0.01	-0.03	1.00								
EY212	0.00	0.02	0.01	0.00	-0.02	0.02	0.02	1.00							
EW212	0.00	-0.02	-0.01	0.04	0.01	-0.02	-0.01	-0.02	1.00						
OX211	0.95	0.22	0.27	0.34	-0.01	0.02	0.00	0.00	0.01	1.00					
OY211	0.21	0.95	0.68	0.01	0.28	0.00	-0.02	0.01	-0.01	0.20	1.00				
OW211	0.26	0.64	0.91	0.04	0.00	0.41	-0.03	0.01	-0.02	0.26	0.62	1.00			
OX212	0.87	0.19	0.24	0.04	-0.01	-0.01	0.48	0.02	-0.01	0.83	0.18	0.22	1.00		
OY212	0.22	0.95	0.67	0.00	-0.03	0.00	-0.01	0.32	-0.02	0.21	0.91	0.61	0.19	1.00	
OW212	0.26	0.64	0.91	0.04	0.00	-0.02	-0.02	0.00	0.40	0.25	0.62	0.83	0.22	0.61	1.00
Condition 22															
TX22	TY22	TW22	EX221	EY221	EW221	EX222	EY222	EW222	OX221	OY221	OW221	OX222	OY222	OW222	
TX22	1.00														
TY22	0.50	1.00													
TW22	0.27	0.70	1.00												
EX221	-0.03	-0.01	0.00	1.00											
EY221	-0.01	0.01	0.02	0.02	1.00										
EW221	-0.02	0.00	0.00	0.00	0.03	1.00									
EX222	0.00	-0.01	-0.03	0.02	-0.01	0.01	1.00								
EY222	0.00	0.00	-0.01	0.03	0.00	0.02	-0.04	1.00							
EW222	-0.00	0.00	0.00	-0.01	0.00	-0.03	-0.01	-0.03	1.00						
OX221	0.95	0.48	0.26	0.28	-0.01	-0.02	0.01	0.01	0.00	1.00					
OY221	0.47	0.95	0.67	0.00	0.31	0.01	-0.01	0.00	0.00	0.45	1.00				
OW221	0.24	0.64	0.91	-0.01	0.03	0.41	-0.02	0.00	-0.02	0.23	0.61	1.00			
OX222	0.87	0.43	0.23	-0.02	-0.02	-0.01	0.49	-0.02	0.00	0.83	0.41	0.20	1.00		
OY222	0.48	0.95	0.66	0.00	0.01	0.00	-0.02	0.30	-0.01	0.46	0.91	0.61	0.41	1.00	
OW222	0.25	0.64	0.91	-0.01	0.02	-0.01	-0.03	-0.03	0.41	0.24	0.61	0.83	0.20	0.60	1.00
Condition 23															
TX23	TY23	TW23	EX231	EY231	EW231	EX232	EY232	EW232	OX231	OY231	OW231	OX232	OY232	OW232	
TX23	1.00														
TY23	0.75	1.00													
TW23	0.31	0.70	1.00												
EX231	0.01	0.00	0.03	1.00											
EY231	0.04	0.02	0.01	0.01	1.00										
EW231	0.03	0.00	-0.02	-0.02	0.01	1.00									
EX232	-0.01	0.01	0.03	0.01	-0.04	0.05	1.00								
EY232	0.04	0.03	0.03	0.03	0.01	0.00	0.00	1.00							
EW232	-0.02	-0.02	-0.02	0.02	0.02	-0.01	0.01	0.05	1.00						
OX231	0.95	0.72	0.30	0.31	0.04	0.02	-0.01	0.05	-0.01	1.00					
OY231	0.73	0.95	0.67	0.00	0.32	0.01	0.00	0.03	-0.01	0.69	1.00				
OW231	0.29	0.65	0.91	0.02	0.01	0.39	0.04	0.02	-0.03	0.28	0.62	1.00			
OX232	0.87	0.66	0.28	0.01	0.02	0.05	0.48	0.04	-0.01	0.83	0.63	0.28	1.00		
OY232	0.73	0.96	0.67	0.01	0.02	0.00	0.01	0.32	0.00	0.69	0.91	0.62	0.64	1.00	
OW232	0.28	0.64	0.91	0.03	0.02	-0.02	0.03	0.04	0.39	0.27	0.61	0.83	0.25	0.62	1.00

Table 4.2 (continued)
 Correlation Among True, Error and Observed Scores
 For Each Parameter Condition

Condition 31															
	TX31	TY31	TW31	EW311	EY311	EX311	EX312	EY312	EW312	OX311	OY311	OW311	OX312	OY312	OW312
TX31	1.00														
TY31	0.29	1.00													
TW31	0.31	0.69	1.00												
EX311	-.02	-.01	0.00	1.00											
EY311	0.01	-.01	0.02	0.02	1.00										
EW311	0.01	0.00	0.01	0.02	0.02	1.00									
EX312	0.02	0.03	0.00	-.01	-.02	0.01	1.00								
EY312	-.01	0.00	0.00	0.04	-.01	0.03	0.04	1.00							
EW312	-.04	-.03	-.01	0.02	0.04	-.03	0.01	-.02	1.00						
OX311	0.95	0.27	0.30	0.28	0.02	0.01	0.01	0.00	-.03	1.00					
OY311	0.28	0.95	0.66	0.00	0.29	0.01	0.02	0.00	-.02	0.26	1.00				
OW311	0.28	0.62	0.91	0.01	0.03	0.42	0.00	0.01	-.02	0.28	0.61	1.00			
OX312	0.88	0.26	0.27	-.02	0.00	0.01	0.49	0.01	-.02	0.84	0.25	0.25	1.00		
OY312	0.27	0.95	0.65	0.00	-.01	0.01	0.04	0.31	-.03	0.26	0.91	0.60	0.25	1.00	
OW312	0.27	0.61	0.91	0.01	0.04	0.00	0.01	-.01	0.42	0.26	0.60	0.82	0.24	0.58	1.00
Condition 32															
	TX32	TY32	TW32	EX321	EY321	EW321	EX322	EY322	EW322	OX321	OY321	OW321	OX322	OY322	OW322
TX32	1.00														
TY32	0.48	1.00													
TW32	0.28	0.70	1.00												
EX321	0.01	-.01	-.02	1.00											
EY321	0.00	-.02	-.01	0.00	1.00										
EW321	0.03	0.00	0.00	0.00	0.01	1.00									
EX322	-.01	0.01	0.00	0.01	-.02	0.02	1.00								
EY322	0.00	-.01	0.00	0.01	-.01	0.02	-.03	1.00							
EW322	-.01	-.06	-.02	0.04	-.02	0.01	0.00	-.01	1.00						
OX321	0.95	0.45	0.26	0.32	0.00	0.03	-.01	0.00	0.00	1.00					
OY321	0.46	0.96	0.67	-.01	0.28	0.01	0.01	-.01	-.06	0.44	1.00				
OW321	0.27	0.64	0.91	-.02	-.01	0.41	0.01	0.01	-.02	0.25	0.61	1.00			
OX322	0.87	0.43	0.25	0.02	-.01	0.04	0.47	-.01	-.01	0.83	0.41	0.25	1.00		
OY322	0.46	0.95	0.67	0.00	-.02	0.01	0.01	0.29	-.06	0.44	0.91	0.61	0.41	1.00	
OW322	0.26	0.62	0.92	-.01	-.02	0.01	0.01	0.00	0.38	0.24	0.59	0.84	0.23	0.60	1.00
Condition 33															
	TX33	TY33	TW33	EX331	EY331	EW331	EX332	EY332	EW332	OX331	OY331	OW331	OX332	OY332	OW332
TX33	1.00														
TY33	0.74	1.00													
TW33	0.28	0.69	1.00												
EX331	0.01	0.01	0.00	1.00											
EY331	0.03	0.02	0.00	-.01	1.00										
EW331	-.04	-.02	0.00	0.00	0.01	1.00									
EX332	0.01	0.03	0.02	0.02	-.01	0.01	1.00								
EY332	-.03	0.03	0.01	0.02	-.01	0.00	-.01	1.00							
EW332	0.02	0.01	0.02	0.01	0.00	-.01	-.02	-.01	1.00						
OX331	0.96	0.71	0.27	0.31	0.02	-.03	0.02	-.02	0.02	1.00					
OY331	0.71	0.95	0.63	0.01	0.32	-.02	0.02	0.02	0.01	0.68	1.00				
OW331	0.24	0.62	0.91	0.00	0.00	0.41	0.02	0.01	0.01	0.23	0.59	1.00			
OX332	0.88	0.66	0.25	0.02	0.02	-.02	0.48	-.03	0.01	0.85	0.64	0.22	1.00		
OY332	0.70	0.96	0.66	0.02	0.01	-.02	0.02	0.32	0.01	0.67	0.91	0.59	0.62	1.00	
OW332	0.26	0.63	0.91	0.01	0.00	0.00	0.01	0.01	0.42	0.25	0.59	0.83	0.23	0.60	1.00

.49, respectively, where the expected population parameter values for these variables were .25, .50 and .50. Correlations between TX, TY, TW and their corresponding errors were 0.00, - 0.03 and 0.00, respectively, indicating the independence of the true and error components of the stated variables. The independence of the true and error components for x, y and w hold for all of the other parameter conditions as well. Decreases in the correlations among variables due to unreliability follow an expected pattern as demonstrated by the intercorrelations with observed scores.

4.1.2 Correlation of Change Score Models with X, Y and W

Table 4.3 represents zero order correlations between the change score models and the x and y components of change and w for the simulated data sets. The values are mathematically predictable using the formulas in Appendix B and were used to verify the accuracy of the procedures used to generate the change model scores. As Table 4.3 reveals, the magnitudes and pattern of the correlation coefficients follow that expected and predictable. For the true score conditions (GT scores) Models 3, 4 and 5 are identical to Model 1 as reliability is perfect and there is no correction; therefore, they have the same correlations with TX and TY as do Model 1 scores. Similarly, true scores generated under Model 6 are nearly identical to Model 2 true scores. Two other indications of the accuracy of the data are the zero correlations of x with Model 2 change scores and the increasing pattern of the correlation of raw change scores (Model 1) with w under the more favorable negative suppression parameter conditions. Because Model 2 change scores are residualized scores, x has been removed and correlation coefficients should be zero. Similarly, the correlation between Model 1 scores and w should be higher when $\rho_{wx} \neq \rho_{wy}$ (Conditions I vs II) and when $\sigma_x \neq \sigma_y$ (Conditions I vs III and II vs III). The pattern holds in all cases.

Table 4.3
Validity Coefficient of Change Scores Models and Their Correlation
With X and Y Component For Nine Parameter Conditions

Condition 11	Condition 12	Condition 13																																																																																																																																																																																																																																																
<p>Redundant</p> <table border="1"> <tr><td>01111</td><td>0.31</td><td>0.41</td><td>0.31</td></tr> <tr><td>01112</td><td>0.40</td><td>0.57</td><td>0.41</td></tr> <tr><td>01113</td><td>0.41</td><td>0.41</td><td>0.31</td></tr> <tr><td>01114</td><td>-0.41</td><td>0.41</td><td>0.31</td></tr> <tr><td>01115</td><td>-0.41</td><td>0.41</td><td>0.31</td></tr> <tr><td>01116</td><td>0.01</td><td>0.31</td><td>0.31</td></tr> <tr><td>001111</td><td>-0.53</td><td>0.41</td><td>-0.31</td></tr> <tr><td>001112</td><td>0.00</td><td>0.57</td><td>0.31</td></tr> <tr><td>001113</td><td>-0.38</td><td>0.46</td><td>0.01</td></tr> <tr><td>001114</td><td>-0.53</td><td>0.42</td><td>-0.31</td></tr> <tr><td>001115</td><td>-0.53</td><td>0.42</td><td>-0.31</td></tr> <tr><td>001116</td><td>-0.53</td><td>0.56</td><td>-0.31</td></tr> <tr><td>001121</td><td>-0.57</td><td>0.58</td><td>0.00</td></tr> <tr><td>001122</td><td>0.00</td><td>0.58</td><td>0.31</td></tr> <tr><td>001123</td><td>-0.33</td><td>0.71</td><td>0.00</td></tr> <tr><td>001124</td><td>-0.38</td><td>0.47</td><td>0.00</td></tr> <tr><td>001125</td><td>-0.40</td><td>0.45</td><td>0.31</td></tr> <tr><td>001126</td><td>-0.31</td><td>0.51</td><td>0.31</td></tr> </table>	01111	0.31	0.41	0.31	01112	0.40	0.57	0.41	01113	0.41	0.41	0.31	01114	-0.41	0.41	0.31	01115	-0.41	0.41	0.31	01116	0.01	0.31	0.31	001111	-0.53	0.41	-0.31	001112	0.00	0.57	0.31	001113	-0.38	0.46	0.01	001114	-0.53	0.42	-0.31	001115	-0.53	0.42	-0.31	001116	-0.53	0.56	-0.31	001121	-0.57	0.58	0.00	001122	0.00	0.58	0.31	001123	-0.33	0.71	0.00	001124	-0.38	0.47	0.00	001125	-0.40	0.45	0.31	001126	-0.31	0.51	0.31	<p>Redundant</p> <table border="1"> <tr><td>02111</td><td>-0.31</td><td>0.49</td><td>-0.41</td></tr> <tr><td>02112</td><td>0.00</td><td>0.47</td><td>0.21</td></tr> <tr><td>02113</td><td>-0.32</td><td>0.49</td><td>-0.41</td></tr> <tr><td>02114</td><td>-0.32</td><td>0.49</td><td>-0.41</td></tr> <tr><td>02115</td><td>-0.32</td><td>0.49</td><td>-0.41</td></tr> <tr><td>02116</td><td>-0.01</td><td>0.47</td><td>0.21</td></tr> <tr><td>001121</td><td>-0.55</td><td>0.31</td><td>-0.41</td></tr> <tr><td>001122</td><td>0.00</td><td>0.36</td><td>0.26</td></tr> <tr><td>001123</td><td>-0.48</td><td>0.37</td><td>0.01</td></tr> <tr><td>001124</td><td>-0.55</td><td>0.31</td><td>-0.41</td></tr> <tr><td>001125</td><td>-0.55</td><td>0.36</td><td>-0.41</td></tr> <tr><td>001126</td><td>-0.12</td><td>0.44</td><td>0.21</td></tr> <tr><td>001131</td><td>-0.59</td><td>0.47</td><td>0.40</td></tr> <tr><td>001132</td><td>0.00</td><td>0.50</td><td>0.20</td></tr> <tr><td>001133</td><td>-0.42</td><td>0.43</td><td>0.00</td></tr> <tr><td>001134</td><td>-0.49</td><td>0.37</td><td>0.00</td></tr> <tr><td>001135</td><td>-0.51</td><td>0.31</td><td>0.00</td></tr> <tr><td>001136</td><td>-0.18</td><td>0.21</td><td>0.13</td></tr> </table>	02111	-0.31	0.49	-0.41	02112	0.00	0.47	0.21	02113	-0.32	0.49	-0.41	02114	-0.32	0.49	-0.41	02115	-0.32	0.49	-0.41	02116	-0.01	0.47	0.21	001121	-0.55	0.31	-0.41	001122	0.00	0.36	0.26	001123	-0.48	0.37	0.01	001124	-0.55	0.31	-0.41	001125	-0.55	0.36	-0.41	001126	-0.12	0.44	0.21	001131	-0.59	0.47	0.40	001132	0.00	0.50	0.20	001133	-0.42	0.43	0.00	001134	-0.49	0.37	0.00	001135	-0.51	0.31	0.00	001136	-0.18	0.21	0.13	<p>Redundant</p> <table border="1"> <tr><td>03111</td><td>-0.15</td><td>0.31</td><td>0.41</td></tr> <tr><td>03112</td><td>0.40</td><td>0.45</td><td>0.20</td></tr> <tr><td>03113</td><td>-0.31</td><td>0.34</td><td>0.41</td></tr> <tr><td>03114</td><td>-0.35</td><td>0.31</td><td>0.41</td></tr> <tr><td>03115</td><td>-0.35</td><td>0.31</td><td>0.41</td></tr> <tr><td>03116</td><td>0.02</td><td>0.44</td><td>0.30</td></tr> <tr><td>001131</td><td>-0.60</td><td>0.39</td><td>0.10</td></tr> <tr><td>001132</td><td>-0.30</td><td>0.73</td><td>0.10</td></tr> <tr><td>001133</td><td>-0.41</td><td>0.49</td><td>0.00</td></tr> <tr><td>001134</td><td>-0.41</td><td>0.39</td><td>0.00</td></tr> <tr><td>001135</td><td>-0.40</td><td>0.46</td><td>0.00</td></tr> <tr><td>001136</td><td>-0.10</td><td>0.46</td><td>0.10</td></tr> <tr><td>001141</td><td>0.00</td><td>0.70</td><td>0.10</td></tr> <tr><td>001142</td><td>-0.20</td><td>0.38</td><td>0.00</td></tr> <tr><td>001143</td><td>-0.25</td><td>0.38</td><td>0.00</td></tr> <tr><td>001144</td><td>-0.45</td><td>0.43</td><td>0.00</td></tr> <tr><td>001145</td><td>-0.45</td><td>0.43</td><td>0.00</td></tr> <tr><td>001146</td><td>-0.40</td><td>0.30</td><td>0.00</td></tr> </table>	03111	-0.15	0.31	0.41	03112	0.40	0.45	0.20	03113	-0.31	0.34	0.41	03114	-0.35	0.31	0.41	03115	-0.35	0.31	0.41	03116	0.02	0.44	0.30	001131	-0.60	0.39	0.10	001132	-0.30	0.73	0.10	001133	-0.41	0.49	0.00	001134	-0.41	0.39	0.00	001135	-0.40	0.46	0.00	001136	-0.10	0.46	0.10	001141	0.00	0.70	0.10	001142	-0.20	0.38	0.00	001143	-0.25	0.38	0.00	001144	-0.45	0.43	0.00	001145	-0.45	0.43	0.00	001146	-0.40	0.30	0.00																								
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<p>Redundant</p> <table border="1"> <tr><td>02211</td><td>0.41</td><td>0.47</td><td>0.34</td></tr> <tr><td>02212</td><td>0.00</td><td>0.57</td><td>0.44</td></tr> <tr><td>02213</td><td>-0.41</td><td>0.43</td><td>0.34</td></tr> <tr><td>02214</td><td>-0.41</td><td>0.43</td><td>0.34</td></tr> <tr><td>02215</td><td>-0.41</td><td>0.43</td><td>0.34</td></tr> <tr><td>02216</td><td>0.01</td><td>0.48</td><td>0.44</td></tr> <tr><td>001221</td><td>-0.63</td><td>0.63</td><td>0.20</td></tr> <tr><td>001222</td><td>0.00</td><td>0.50</td><td>0.50</td></tr> <tr><td>001223</td><td>-0.38</td><td>0.57</td><td>0.20</td></tr> <tr><td>001224</td><td>-0.43</td><td>0.63</td><td>0.20</td></tr> <tr><td>001225</td><td>-0.43</td><td>0.63</td><td>0.20</td></tr> <tr><td>001226</td><td>-0.41</td><td>0.57</td><td>0.57</td></tr> <tr><td>002221</td><td>-0.67</td><td>0.41</td><td>0.20</td></tr> <tr><td>002222</td><td>0.00</td><td>0.50</td><td>0.50</td></tr> <tr><td>002223</td><td>-0.34</td><td>0.73</td><td>0.21</td></tr> <tr><td>002224</td><td>-0.38</td><td>0.49</td><td>0.21</td></tr> <tr><td>002225</td><td>-0.38</td><td>0.49</td><td>0.21</td></tr> <tr><td>002226</td><td>-0.11</td><td>0.37</td><td>0.55</td></tr> </table>	02211	0.41	0.47	0.34	02212	0.00	0.57	0.44	02213	-0.41	0.43	0.34	02214	-0.41	0.43	0.34	02215	-0.41	0.43	0.34	02216	0.01	0.48	0.44	001221	-0.63	0.63	0.20	001222	0.00	0.50	0.50	001223	-0.38	0.57	0.20	001224	-0.43	0.63	0.20	001225	-0.43	0.63	0.20	001226	-0.41	0.57	0.57	002221	-0.67	0.41	0.20	002222	0.00	0.50	0.50	002223	-0.34	0.73	0.21	002224	-0.38	0.49	0.21	002225	-0.38	0.49	0.21	002226	-0.11	0.37	0.55	<p>Neg. Suppression</p> <table border="1"> <tr><td>03211</td><td>-0.35</td><td>0.58</td><td>0.43</td></tr> <tr><td>03212</td><td>0.00</td><td>0.46</td><td>0.43</td></tr> <tr><td>03213</td><td>-0.09</td><td>0.50</td><td>0.43</td></tr> <tr><td>03214</td><td>-0.09</td><td>0.50</td><td>0.43</td></tr> <tr><td>03215</td><td>-0.09</td><td>0.50</td><td>0.43</td></tr> <tr><td>03216</td><td>0.02</td><td>0.48</td><td>0.43</td></tr> <tr><td>001231</td><td>-0.31</td><td>0.31</td><td>0.37</td></tr> <tr><td>001232</td><td>0.00</td><td>0.39</td><td>0.51</td></tr> <tr><td>001233</td><td>-0.44</td><td>0.40</td><td>0.41</td></tr> <tr><td>001234</td><td>-0.31</td><td>0.31</td><td>0.37</td></tr> <tr><td>001235</td><td>-0.31</td><td>0.31</td><td>0.37</td></tr> <tr><td>001236</td><td>-0.01</td><td>0.35</td><td>0.37</td></tr> <tr><td>002231</td><td>0.40</td><td>0.43</td><td>0.37</td></tr> <tr><td>002232</td><td>0.00</td><td>0.41</td><td>0.51</td></tr> <tr><td>002233</td><td>-0.42</td><td>0.41</td><td>0.41</td></tr> <tr><td>002234</td><td>-0.44</td><td>0.40</td><td>0.41</td></tr> <tr><td>002235</td><td>-0.52</td><td>0.37</td><td>0.41</td></tr> <tr><td>002236</td><td>-0.20</td><td>0.36</td><td>0.41</td></tr> </table>	03211	-0.35	0.58	0.43	03212	0.00	0.46	0.43	03213	-0.09	0.50	0.43	03214	-0.09	0.50	0.43	03215	-0.09	0.50	0.43	03216	0.02	0.48	0.43	001231	-0.31	0.31	0.37	001232	0.00	0.39	0.51	001233	-0.44	0.40	0.41	001234	-0.31	0.31	0.37	001235	-0.31	0.31	0.37	001236	-0.01	0.35	0.37	002231	0.40	0.43	0.37	002232	0.00	0.41	0.51	002233	-0.42	0.41	0.41	002234	-0.44	0.40	0.41	002235	-0.52	0.37	0.41	002236	-0.20	0.36	0.41	<p>Neg. Suppression</p> <table border="1"> <tr><td>03311</td><td>-0.14</td><td>0.31</td><td>0.34</td></tr> <tr><td>03312</td><td>0.00</td><td>0.46</td><td>0.34</td></tr> <tr><td>03313</td><td>-0.36</td><td>0.31</td><td>0.34</td></tr> <tr><td>03314</td><td>-0.36</td><td>0.31</td><td>0.34</td></tr> <tr><td>03315</td><td>-0.36</td><td>0.31</td><td>0.34</td></tr> <tr><td>03316</td><td>0.01</td><td>0.46</td><td>0.34</td></tr> <tr><td>001331</td><td>-0.60</td><td>0.39</td><td>0.43</td></tr> <tr><td>001332</td><td>-0.30</td><td>0.73</td><td>0.43</td></tr> <tr><td>001333</td><td>-0.41</td><td>0.49</td><td>0.43</td></tr> <tr><td>001334</td><td>-0.41</td><td>0.39</td><td>0.43</td></tr> <tr><td>001335</td><td>-0.40</td><td>0.46</td><td>0.43</td></tr> <tr><td>001336</td><td>-0.10</td><td>0.46</td><td>0.43</td></tr> <tr><td>002331</td><td>0.40</td><td>0.39</td><td>0.43</td></tr> <tr><td>002332</td><td>0.00</td><td>0.40</td><td>0.51</td></tr> <tr><td>002333</td><td>-0.39</td><td>0.49</td><td>0.43</td></tr> <tr><td>002334</td><td>-0.40</td><td>0.39</td><td>0.43</td></tr> <tr><td>002335</td><td>-0.40</td><td>0.39</td><td>0.43</td></tr> <tr><td>002336</td><td>-0.10</td><td>0.37</td><td>0.51</td></tr> <tr><td>003331</td><td>0.40</td><td>0.31</td><td>0.34</td></tr> <tr><td>003332</td><td>0.00</td><td>0.40</td><td>0.51</td></tr> <tr><td>003333</td><td>-0.39</td><td>0.49</td><td>0.43</td></tr> <tr><td>003334</td><td>-0.39</td><td>0.49</td><td>0.43</td></tr> <tr><td>003335</td><td>-0.39</td><td>0.49</td><td>0.43</td></tr> <tr><td>003336</td><td>-0.10</td><td>0.37</td><td>0.51</td></tr> </table>	03311	-0.14	0.31	0.34	03312	0.00	0.46	0.34	03313	-0.36	0.31	0.34	03314	-0.36	0.31	0.34	03315	-0.36	0.31	0.34	03316	0.01	0.46	0.34	001331	-0.60	0.39	0.43	001332	-0.30	0.73	0.43	001333	-0.41	0.49	0.43	001334	-0.41	0.39	0.43	001335	-0.40	0.46	0.43	001336	-0.10	0.46	0.43	002331	0.40	0.39	0.43	002332	0.00	0.40	0.51	002333	-0.39	0.49	0.43	002334	-0.40	0.39	0.43	002335	-0.40	0.39	0.43	002336	-0.10	0.37	0.51	003331	0.40	0.31	0.34	003332	0.00	0.40	0.51	003333	-0.39	0.49	0.43	003334	-0.39	0.49	0.43	003335	-0.39	0.49	0.43	003336	-0.10	0.37	0.51
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4.2 Relative Contribution of X and Y Components into the Definition of Change

Using Table 4.3 as a referent, the correlation coefficient between x and y and each model's change scores may be viewed similar to structure coefficients within the context of canonical, discriminate or factor analyses. The magnitude and sign of the coefficient help define the weighted composite (resultant change score) in terms of its relationship to its components. As the magnitude of the correlations change or differ across models or conditions, evidence is provided that the construct defined by the change score model is changing or is different given the model or parameter conditions. With an exception for Model 6, the effect of the changing reliability conditions (GT, GO1 and GO2) on the models' correlations with x and y appear to be minimal within each of the nine major simulation conditions. The most drastic change for any of the Models 1 through 5 is for Model 3 under condition 23 in Table 4.3. The correlations for Model 3 GT scores (GT233) with x and y are $-.36$ and $.34$, respectively. For Model 3 GO2 scores, the correlations with x and y are $-.24$ and $.60$, respectively. For Model 6, changes occur to varying degrees, but becomes more evident as ρ_{xy} increases from $.25$ to $.50$ to $.75$. For example when $\rho_{xy} = .75$ under Condition I, Model 6 GT scores (GT136) correlate $.02$ and $.66$ with x and y , respectively and its GO2 scores (GO2136) correlate $-.49$ and $.39$, respectively. The construct defined by these two sets of scores would appear to be different, one set totally dominated by y (GT136) and the other defining a change score with x slightly dominating the definition.

When descriptively viewing the effects of the other parameter conditions on the change score models' correlations with x and y , the magnitude and pattern of effects appear to be the same for Models 1, 3, 4, and 5 which are different from Model 2 with the exception of under condition 32 where the correlations with x and y are nearly identical for all five models. Because of its mathematical definition, Model 2 scores are always defined by y . The extent to which the operational definition of Model 2 scores change is provided by the correlations with y across conditions. The

effect pattern is consistent, decreasing from approximately .97 to .87 to .66 as ρ_{xy} increases from .25 to .50 to .75, respectively. This pattern is identical for all three of the parameter Conditions I, II and III for Model 2.

For Models 1, 3, 4 and 5, the correlations with x and y are approximately equal with opposite signs and identical across Conditions I and II, but decreasing in magnitude as ρ_{xy} increase from .25 to .75. The definition of change scores for Models 1, 3, 4, and 5 as defined by the correlations with x and y under Condition III where $\sigma_y = 2\sigma_x$, $\rho_{wx} = .30$ and $\rho_{wy} = .70$ differ from Conditions I and II and change as a function of increases in ρ_{xy} . The y component dominates the definition of change for all three levels of ρ_{xy} levels, remaining stable at .88, but change scores' correlations with X change from approximately -.21 under $\rho_{xy} = .25$ to 0.00 under $\rho_{xy} = .50$ to .34 under $\rho_{xy} = .75$. It is under Condition II and $\rho_{xy} = .50$ that Models 1, 3, 4, and 5 are identical to Model 2.

The operational definition of Model 6 as defined by the correlations with x and y appears to be most variable and the parameter effects on Model 6 appear to be interactive. Model 6 always resembles Model 2 under true score conditions. Under Conditions I and II, Model 6 moves away from Model 2 looking more like Model 1 (raw change) as ρ_{xy} increases and reliabilities are lowered (GO1 and GO2). In fact, Model 6 and Model 1 GO2 scores are identical under condition 23 with a correlation of -.50 with X and .34 with Y. Like Model 2, however, Model 6's definition remains constant across Conditions I, II and III within reliability level, but Condition III changes the definition of Model 1 so Model 6 no longer resembles its scores.

In summary, the reliability manipulation appears to only affect Model 6 to any extent. The manipulation of ρ_{xy} affected all models, but was constant across other conditions only for Model 2. For Models 1, 3, 4, and 5 the effect of manipulating the magnitude of ρ_{xy} was the same under Conditions I and II, but differed for Condition III. When Model 6 was the focus, the interactive effects of ρ_{xy} and reliability levels were evident, but these effects were constant across condition levels I, II and III.

4.3 Congruency of Change Score Models

The relative relationships explored in the last section of the x and y components to the definition of the change scores provided evidence that the parameters investigated do have differential and, in some cases, interactive effects on the underlying definition of change resulting from application of a specific model. While the surface descriptive information indicates that some of the models differ from each other and from themselves individually across parameter conditions, the extent of the differences and similarities still needs to be explored. The remainder of this section and those that follow attempt to provide evidence documenting the magnitude of the similarities and differences among the models within and across the parameter conditions manipulated.

A complete intercorrelation matrix on which all of the following presentations are based is provided in Appendix E. This matrix is a 243 by 243 matrix with intercorrelations presented for generated scores both within and across the nine major parameter conditions. Within each of the nine conditions, the intercorrelations among 27 scores are presented. These 27 scores consist of the three true scores for x, y and w, the six observed scores (one each for x, y and w under two reliability conditions) and the 18 change model scores (one score for each of six models generated from each of the reliability conditions, i.e., true scores (GT) observed one scores (GO1) or observed two scores (GO2)). Correlating these 27 scores specific to a condition across the nine conditions resulted in a 243 by 243 intercorrelation matrix.

It should be noted that in this section, the congruency among models is explored without considering the relationship of x or y to the external criterion, w. Therefore, Conditions I and II produce identical sets of change scores. Condition III differs in that the variability of y is twice that of x. Thus, the following presents results only for Conditions I and III.

4.3.1 Measurement Error and Colinearity Effects—Comparison of Change Score Models Within and Across ρ_{xy} levels of the Same Condition

The intercorrelations among the 18 change scores within and across each of the three levels of ρ_{xy} for Conditions I and III are provided in Appendix E. Again, because Models 3, 4, and 5 include reliability adjustments, they function identically to Model 1 under the true score (GT) condition. Similarly, Model 6 is identical to Model 2. Inspection of the intercorrelations among the true change scores for Models 1 and 2 across the three ρ_{xy} levels of Condition I reveals that the relationship between the discrepancy (Model 1) and the residual (Model 2) models was 0.79, 0.85 and 0.94, respectively. As ρ_{xy} increased, the models became more similar. Squaring these correlations indicate that the models true scores shared 62%, 74% and 88% of their variance for ρ_{xy} values of 0.25, 0.50 and 0.75, respectively.

When measurement errors were taken into consideration (GO1 and GO2 scores) the adjustments for unreliability made by Models 3, 4 and 5 had little effect. The intercorrelations among these models and with Model 1 remained in the high 0.90's (a low of 0.96). In practice, there would be no difference in the functioning of Models 1, 3, 4 and 5 under the parameter conditions of Condition I across levels of ρ_{xy} . All results for Model 1 when compared to Models 2 and 6 hold also for Models 3, 4 and 5. The intercorrelations of GO1 and GO2 scores between Model 1 and Model 2 dropped slightly from the true score conditions. The correlations across ρ_{xy} levels were 0.78, 0.84 and 0.92 for GO1 scores and 0.75, 0.80 and 0.85 for GO2 scores. These are contrasted with GT score intercorrelations of 0.79, 0.86 and 0.94. The largest effect occurring for the high level of ρ_{xy} (equal to 0.75) and reliabilities for x of 0.70 and for y of 0.90. A loss of 16% (0.88 vs 0.72) in shared variance occurred under this most discrepant condition.

Under Condition I, the greatest instability across reliability levels and ρ_{xy} levels occurred for Model 6. Under the true score conditions, it is identical to Model

2. However, as measures became more unreliable and as ρ_{xy} increased, Model 6 became more discrepant from Model 2 until under the most unreliable conditions and $\rho_{xy} = 0.75$, Model 6 and 1 were identical with Models 6 and 2 correlating 0.87, a loss of 24% in shared variance. This complex change in Model 6 is illustrated by its intercorrelations with Models 1 and 2 across the reliability and ρ_{xy} conditions. When $\rho_{xy} = 0.25$, the intercorrelations with Model 6 GT, GO1 and GO2 scores for Model 1 were .79, 0.81 and .84, respectively. The same Model 6 intercorrelations with Model 2 scores were 1.00, 1.00 and 0.99, respectively. When $\rho_{xy} = 0.50$, the intercorrelations with Model 1 were 0.86, 0.90 and 0.95 and with Model 2 were 1.00, 0.99 and 0.95. Under this condition, Model 6 GO2 (GO2126) scores correlated equally (0.95) with Models 1 and 2. When $\rho_{xy} = 0.75$, the Model 6 intercorrelations with Model 1 were 0.93, 0.97 and 1.00 and with Model 2 were 1.00, 0.98 and 0.87.

Under Condition III, the larger variability of y relative to x ($\sigma_x = 2\sigma_y$) created change scores for the discrepancy models (1, 3, 4 and 5) dominated by y, thus making them more similar to Model 2. In condition 31, where $\rho_{xy} = 0.25$ the discrepancy models were perfectly correlated across three reliability levels and the correlations between discrepancy and residual models were perfect or nearly perfect (0.96 - 1.0). In Condition III, when ρ_{xy} was set to 0.50, all change score models were perfectly correlated under $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xx'} = \rho_{yy'} = 0.90$ reliability conditions. Only when the reliability coefficients for x and y were set to be unequal did Model 6 show minor signs of separating itself from the rest of the models (see Appendix E).

In condition 33, where $\rho_{xy} = 0.75$, Model 6 separated itself from both the residual and discrepancy models. The correlation of Model 6 with residual and discrepancy models showed a decreasing trend across three reliability levels, while the correlation of Model 2 with discrepancy models increased. Under unequal reliability coefficients, Model 2 had almost perfect correlation (0.99) with Models 1, 3 and 4 while Model 6 correlated 0.82, 0.73 and 0.76 with Models 1, 3 and 4 respectively.

In an attempt to quantitatively summarize the amount of common variance

defined by each change score model a principal component analysis was conducted on scores within each level of Conditions I and III. Condition's II change score models were excluded from this analysis because of their duplication with the change scores of Condition I (both condition used $\sigma_x = \sigma_y$). The change score models included in this analysis were: GT scores for Models 1 and 2 from the true condition (i.e., $\rho_{xx'} = \rho_{yy'} = 1.0$), GO1 scores for Models 1, 2, 3 and 6 (i.e., $\rho_{xx'} = \rho_{yy'} = 0.90$) and GO2 scores for Models 1, 2, 3 and 6 (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). A total of ten change scores were selected from each of the ρ_{xy} level of Conditions I and III to be used in these analyses. Models 4 and 5 were excluded from the analyses because of their perfect association with Model 1 scores. Correlations of Models 4 and 5 with Model 1 were perfect or nearly perfect across all of the included conditions (i.e., conditions 11 - 13 and conditions 31 - 33).

Table 4.4 presents the percentage of variance explained by the first factor when a principal component analysis was conducted separately within each of the six conditions 11 - 33. The findings in this table indicate specifically how much of the variance of the change score models was shared in common within each condition. In Condition I these percentages ranged from 80.1% to 85.2% across the three ρ_{xy} levels while in Condition III they ranged from 82.7% to 93.3%. Within Condition I and III, there appear to be little difference in the amount of commonality when ρ_{xy} equals 0.25 or 0.50 with the models sharing 7-8 percent more variance under Condition III. However, when x and y are highly correlated ($\rho_{xy} = 0.75$), the commonality among the models is slightly more affected, dropping approximately 5 percent under Condition I and 9 percent under Condition III.

The magnitude of the loadings of the change score models on the first principal component for different conditions are reported in Table 4.5. As is clear from the data in Table 4.5, the loadings of the change score models on the first principal component were very high. The range of these loadings for the included conditions were as follows: condition 11 (.88 - .95), condition 12 (.89 - .95), condition 13 (.86 - .92), condition 31 (.95 - .98), condition 32 (.93 - .97) and condition 33 (.76 - .94). As the

Table 4.4

Percentage of Variance Explained by the First Factor of the Principal Component Across Three ρ_{xy} Levels of Conditions I and III

ρ_{xy}	condition I	condition III
0.25	84.7	93.3
0.50	85.2	91.8
0.75	80.1	82.7

findings in Table 4.5 indicate, in condition 11 the highest loading (.95) belonged to Model 6 and the smallest (.88) to Model 1 under unequal reliability conditions. All of the models except Model 2 under the true and observed conditions had a loading of .90 or above. In condition 12 the patterns of the loading were the same as in condition 11. In condition 13 the highest loading (.92) belonged to Models 2 and 6 for the true and observed 1 conditions (see Table 4.5). The lowest loading (.86) belonged to Model 1 from the observed 1 condition (GO1) and Models 1, 2 and 6 from the observed 2 condition (GO2). In condition 31 all of the loadings were .95 or higher. The highest (.98) belonged to Model 2 from the GT condition. The smallest loading (.95) belonged to Model 1 from the observed 2 condition ($\rho_{xx'} = .70$ and $\rho_{yy'} = .90$). In condition 32 the highest loading (.97) belonged to Models 1 and 2 from the true condition and the smallest (.93) to Model 6 from the observed 2 condition. In condition 33 the highest loading (.94) belonged to Models 1 and 2 from the true condition and the lowest (.76) belonged to Model 6 from the observed 2 condition. Consistent with prior results when examining the intercorrelations among models,

Table 4.5

Magnitude of the Loadings of the Change Score
Models on the First Factor of the Principal
Component for Different Parameter Conditions

ρ_{xy}	Condition I change scores	Condition I loading	Condition III change scores	Condition III loading
0.25	GT111	0.91	GT311	0.97
	GT112	0.93	GT312	0.98
	GO1111	0.90	GO1311	0.96
	GO1112	0.92	GO1312	0.96
	GO1113	0.92	GO1313	0.97
	GO1116	0.93	GO1316	0.97
	GO2111	0.88	GO2311	0.95
	GO2112	0.91	GO2312	0.96
	GO2113	0.94	GO2313	0.97
	GO2116	0.95	GO2316	0.97
0.50	GT121	0.92	GT321	0.97
	GT122	0.93	GT322	0.97
	GO1121	0.91	GO1321	0.96
	GO1122	0.92	GO1322	0.96
	GO1123	0.93	GO1323	0.96
	GO1126	0.94	GO1326	0.96
	GO2121	0.89	GO2321	0.95
	GO2122	0.91	GO2322	0.96
	GO2123	0.94	GO2323	0.96
	GO2126	0.95	GO2326	0.93
0.75	GT131	0.91	GT331	0.94
	GT132	0.92	GT332	0.94
	GO1131	0.89	GO1331	0.93
	GO1132	0.90	GO1332	0.93
	GO1133	0.91	GO1333	0.93
	GO1136	0.92	GO1336	0.88
	GO2131	0.86	GO2331	0.92
	GO2132	0.87	GO2332	0.92
	GO2133	0.90	GO2333	0.92
	GO2136	0.87	GO2336	0.76

*Note: GT, GO1, and GO2 are true, observed 1, and observed 2 conditions respectively

Model 6 scores are most affected and different under conditions 33 when reliabilities are unequal and lower.

4.3.2 Variability Effect–Comparison of Data Across Conditions I and III:

A vertical comparison across Conditions I and III of the correlations in the Appendix E matrix reveals that when $\rho_{xy} = 0.25$ (i.e., condition 11 vs. condition 31), the correlations of discrepancy (Models 1, 3, 4 and 5) and residual (Models 2 and 6) models in Condition III were much higher than the correlation of residual and discrepancy models in Condition I. This holds true across all three reliability levels. For example, in condition 11 where $\rho_{xx'} = \rho_{yy'} = 1.0$, the correlations of the residual models (i.e., Models 2 and 6) with the discrepancy models was as high as 0.79, but in condition 31 the corresponding correlations were 0.98. When reliability coefficients for x and y were set at $\rho_{xx'} = \rho_{yy'} = 0.90$, however, in condition 11, correlations of Model 2 with the discrepancy models ranged from 0.78 to 0.82, but for Model 6 it ranged from 0.81 to 0.84. The corresponding correlations for condition 31 were 0.95 - 0.98 and 0.98 - 0.99, respectively. When reliability coefficients for x and y were set to be unequal (GO2 scores) in condition 11, correlations of Model 2 with the discrepancy models ranged from 0.75 to 0.85, while for Model 6 they ranged from 0.84 to 0.92. In condition 31, correlations of Models 2 and 6 with the discrepancy models were 0.96 - 0.99 and 0.99 - 1.0, respectively.

When $\rho_{xy} = 0.50$ in Condition I the correlations of discrepancy and residual change scores were as high as 0.86 for $\rho_{xx'} = \rho_{yy'} = 1.0$, and for $\rho_{xx'} = \rho_{yy'} = 0.90$, the correlations of Model 6 with the discrepancy models exceeded the correlation of Model 2 with the stated models. In this context, the range of correlations for Model 2 was 0.80 - 0.91 and for Model 6 was 0.95 - 0.99. In condition 32 the correlations of the residual and discrepancy models were perfect for both $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xx'} = \rho_{yy'} = 0.90$, and all models functioned identically. When reliability coefficients for x and y were again set to be unequal, the correlation of Model 6 with the discrepancy models

exceeded the correlations of Model 2 with the stated models in condition 12, but in condition 32 this order was reversed. In condition 32 the correlation of Model 6 with discrepancy and regression models ranged from 0.95 - 0.99, but Model 2's correlation ranged from 0.99 to 1.0. These findings reveal that correlations of discrepancy and residual models for condition 32 were higher in magnitude than for condition 12.

In condition 13 where $\rho_{xy} = 0.75$, when $\rho_{xx'} = \rho_{yy'} = 1.0$, the correlation of Models 2 and 6 with the discrepancy change score model were 0.94 and 0.93, and the same results were consistent for condition 31 as well. When reliability coefficients for x and y were set to be less than perfect in condition 13, the correlation of Model 6 with the discrepancy models showed an increasing trend across the stated reliability levels, but in condition 33, it followed a decreasing trend. In Condition 13 the correlations of Model 2 with Model 1 (raw gain score) decreased across the three reliability levels (i.e., 0.94 - 0.85), but in Condition 33 the correlations of Model 2 with raw gain scores had an increasing trend, i.e., 0.94 - 0.99.

In condition 13 when $\rho_{xx'} \neq \rho_{yy'}$, the correlations of Model 6 with the discrepancy models ranged from 0.98 to 1.0, but in condition 33 the correlations of Model 6 with the discrepancy models ranged from 0.67 to 0.87. Under given conditions, the correlation of Model 6 with the raw residual gain was 0.87 and its correlation with the rest of the models was diverse in magnitude. Its lowest correlation was with Model 5 (0.67), while in condition 13 the correlation of Model 6 with 5 was perfect under unequal reliability coefficients for x and y components.

Overall, these findings reveal that when the variability for y is doubled in size in Condition III, the change score models shared a higher portion of their variance with each other relative to Conditions I or II. This indicates that in Condition III there is a high degree of congruency among the change score models, especially in conditions 31 and 32. In condition 32 there actually was a single underlying change score explaining all of the change score models across all reliability levels except for $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{xx'} = 0.90$.) In Condition III, Model 6 was the one most affected by the reliability and ρ_{xy} levels. For example, for $\rho_{xy} = 0.75$, Model 6

separated itself from the rest of the change score models, but in Condition I, Model 6 increased its correlation with all of the models relative to $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xx'} = \rho_{yy'} = 0.90$ conditions, particularly with Model 1.

In further analyses using the principal component technique, the congruency/noncongruency of the change score models across Conditions I and III (i.e., 11, 12, 13, 31, 32 and 33) were simultaneously evaluated. The models included in the analysis were the same as before, i.e., ten model scores (each of six) per condition (two GT scores for Models 1 and 2, four GO1 and four GO2 scores for models 1, 2, 3 and 6). All sixty model scores were included in a single principal components analysis.

Eight factors extracted by the principal components had eigenvalues greater than 1.00. Two components (or factors) were selected as the most powerful for explaining the underlying variability of the change score models. The first factor extracted 70.7% of the variance of the change score models and the second extracted 7.6% of the stated variance. The two factors together extracted 78.3% of the underlying variability of the change score models. The shared variance of Models 1, 2, 3 and 6 with factor 1 and 2 across Conditions I and III are reported in Table 4.6.

As the findings in this table indicate, in general change score models in Condition III shared a higher portion of their variance with factor 1 than did the corresponding change score models from Condition I. This conclusion was especially true for discrepancy models 1 and 3 across the three reliability and ρ_{xy} levels. For example, Model 1 from Condition III, where $\rho_{xx'} = \rho_{yy'} = 1.0$ shared 90 - 93% of its variance with factor 1 relative to 64 - 66% which it shared with factor 1 in Condition I. The shared variance of Model 1 with the first factor decreased in magnitude in both Conditions I and III across the three reliability levels. For example, under $\rho_{xx'} = \rho_{yy'} = 0.90$, Model 1 from Condition III shares 74 - 80% of its variance with factor 1 while for the $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ condition, it shared 70 - 75% of its variance with factor 1. The corresponding variances for Model 1 in Condition I were 50 - 53% and 41 - 46%, respectively. The amount of shared variance of Model 1 with factor 1 decreased in magnitude by 3 - 6% in Condition III and 1 - 2% in Condition I across

Table 4.6

Shared Variance of the Change Score Models with
Factors 1 and 2 of the Principal Component
Combining Change Scores of Conditions I and III

			Condition I				Condition III			
Factor	ρ_{zy}	change scores	Model 1	Model 2	Model 3	Model 6	Model 1	Model 2	Model 3	Model 6
1	0.25	GT	0.66	0.83	-	-	0.93	0.87	-	-
		GO1	0.53	0.72	0.59	0.74	0.80	0.77	0.81	0.78
		GO2	0.46	0.70	0.61	0.74	0.76	0.75	0.78	0.78
	0.50	GT	0.64	0.93	-	-	0.93	0.93	-	-
		GO1	0.52	0.78	0.58	0.78	0.80	0.80	0.81	0.80
		GO2	0.43	0.75	0.59	0.67	0.74	0.76	0.76	0.69
	0.75	GT	0.66	0.87	-	-	0.90	0.85	-	-
		GO1	0.50	0.70	0.56	0.63	0.74	0.70	0.74	0.61
		GO2	0.41	0.69	0.57	0.44	0.70	0.67	0.72	0.40
2	0.25	GT	0.15	-0.11	-	-	-0.02	-0.02	-	-
		GO1	0.13	-0.11	0.09	-0.09	-0.02	-0.02	-0.03	-0.09
		GO2	0.09	-0.14	0.02	-0.07	-0.02	-0.13	-0.06	-0.05
	0.50	GT	0.21	-0.01	-	-	-0.02	-0.02	-	-
		GO1	0.14	-0.01	0.14	0.00	-0.02	-0.04	-0.03	-0.01
		GO2	0.18	-0.02	0.06	0.02	-0.02	-0.06	-0.07	0.00
	0.75	GT	0.23	0.05	-	-	-0.02	-0.02	-	-
		GO1	0.26	0.04	0.19	0.12	-0.02	0.00	-0.03	0.04
		GO2	0.25	0.01	0.10	0.22	-0.01	0.00	-0.04	0.11

Note: Square Loading

the three ρ_{xy} levels. The shared variance of Model 3 in Condition III with factor 1 was almost the same as the shared variance of Model 1 in this condition. However, the same relationship did not hold true for Models 1 and 3 in Condition I. Model 3 in Condition I had a higher shared variance with factor 1 than did Model 1. In condition III when $\rho_{xy} = 0.25$, the residual models shared variance with factor 1 was close to the discrepancy models and when $\rho_{xy} = 0.50$, the discrepancy and residual models shared the same degree of their variance with factor 1, but for $\rho_{xy} = 0.75$, the discrepancy models shared a higher portion of their variance with factor 1 specifically in comparison to Model 6. In Condition I, however, the residual models, particularly Model 2, consistently shared a higher portion of their variance with factor 1 than did the discrepancy models. In Condition I, Model 2 had the highest degree of shared variance with factor 1 while Model 1 had the lowest.

In Condition III, where $\rho_{xy} = 0.75$, Model 6 separates itself from the rest of the change scores by having the lowest shared variance with factor 1, particularly under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), but in Condition I for the same ρ_{xy} level, Model 6 is similar to model 1.

Overall, the comparison of the shared variance of Models 1, 2, 3 and 6 with the first factor of the principal component reveals that the change score models in Condition III were more congruent regarding their underlying change scores than in Condition I. In Condition I there was more discrepancy among the residual and discrepancy models, particularly under unequal reliability conditions, i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. The comparison of the shared variance of the change score models with factor 1 reveals that doubling the variability of y in condition III had a major impact on the loading of the discrepancy models only. That is, the increment of the y variance in Condition III increased the shared variance of Model 1 with factor 1 by 24 - 31% and Model 3 with factor 1 by 15 - 29% relative to the shared variance of the stated models in Condition I. For residual models, the increment of the shared variance with factor 1 in condition 31 ($\rho_{xy} = 0.25$) was small in magnitude (4 - 5%), in condition 32 was negligible (0 - 2%) and, in condition 33 decreased by 0 - 4%. In

this condition only, when $\rho_{xx'} \neq \rho_{yy'}$, the shared variance of Model 6 with factor 1 decreased by 4%.

Table 4.7 provides the summary of the differences in the shared variance of Models 1, 2, 3 and 6 across three ρ_{xy} levels of Conditions I versus III (changes due to variability effects). The negative signs indicate the shrinkage and the positive signs are increments in the shared variance of the models with factor 1. In general, the findings in Table 4.7 indicate that increasing the variability of y distribution only affected the underlying change score model of the discrepancy models and not the residual model. Whatever factor(s) account(s) for the underlying variability of the residual models in Condition I remained more or less stable in Condition III as well. Furthermore, the findings in Table 4.7 reveal that the shared variance of the change score models with factor 1 mainly diminished in magnitude as one moves from one reliability level to another, especially from $\rho_{xx'} = \rho_{yy'} = 1.0$ to $\rho_{xx'} = \rho_{yy'} = 0.90$.

Table 4.8 represents the amount of shrinkage in the shared variance of Models 1, 2, 3 and 6 with factor 1 across the three ρ_{xy} levels of Conditions I and III when moving from $\rho_{xx'} = \rho_{yy'} = 1.0$ (GT) to $\rho_{xx'} = \rho_{yy'} = 0.90$ (GO1) to $\rho_{xx'} = \rho_{yy'} = 0.70$ and $\rho_{yy'} = 0.90$ (GO2) conditions. The negative signs are indications of shrinkage in the shared variance of the change score models with factor 1 and positive signs represent the increment of the shared variance moving from one reliability level to another. The findings in this table indicate that the magnitude of shrinkage in the shared variance of Models 1, 2, 3, and 6 with factor 1 under both Condition I and III are higher under the GO1 condition than for the GO2 condition. Table 4.8 also reveals that when $\rho_{xy} = 0.75$ in both Conditions I and III the shared variance of Models 1, 2, 3, and 6 with the first principal component has a greater shrinkage in their magnitude than under $\rho_{xy} = 0.25$ and 0.50 under the GO1 condition. For all of the change score models excluding Model 6, shrinkage in their shared variance with factor 1 was less than 10% when the reliability of x and y was changed from $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). For Model 6 the patterns of the shrinkage in its shared variance with factor 1 across three ρ_{xy} levels were: 1) 0.0,

Table 4.7

Percent Differences in the Shared Variance of the
Change Score Models From Condition I and III

ρ_{xy}	change scores	model 1	model 2	model 3	model 6
0.25	GT	0.27	0.04	0.27	0.04
	GO1	0.27	0.05	0.22	0.04
	GO2	0.30	0.05	0.17	0.04
0.50	GT	0.29	0.00	0.29	0.00
	GO1	0.28	0.02	0.23	0.02
	GO2	0.31	0.01	0.17	0.02
0.75	GT	0.24	-.02	0.24	-.02
	GO1	0.24	0.00	0.18	0.02
	GO2	0.29	-.02	0.15	-.04

Table 4.8

The Magnitude of the Increment/Shrinkage in the Shared Variance of the Change Score Models With Factor 1 When Moving Across Three Reliability Levels From:

$\rho_{xx'} = \rho_{yy'} = 1.0$ to $\rho_{xx'} = \rho_{yy'} = 0.90$
and $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} = 0.70$ to $\rho_{yy'} = 0.90$

		Condition I				Condition III			
ρ_{xy}	change scores	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25	GT								
	GO1	-.13	-.11	-.07	-.08	-.13	-.10	-.12	-.09
	GO2	-.07	-.02	-.02	0.00	-.04	-.02	-.03	0.00
0.50	GT								
	GO1	-.12	-.05	-.06	-.05	-.13	-.13	-.12	-.13
	GO2	-.09	-.03	+.01	-.01	-.06	-.04	-.05	-.11
0.75	GT								
	GO1	-.16	-.17	-.10	-.24	-.16	-.15	-.16	-.24
	GO2	-.09	-.01	+.01	-.19	-.04	-.03	-.02	-.21

0.01 and 0.19, and 2) 0.0, 0.11 and 0.21 for Conditions I and III, respectively. Relative to the other models, the shared variance of Model 6 with factor 1 showed a sharp decrease in its magnitude, i.e., 19 - 14%, when moving across three reliability levels in both Conditions I and III where $\rho_{xy} = 0.75$ (i.e., conditions 13 and 33) particularly in Condition III. These latter results provides further evidence that the definition of change by Model 6 is less stable and is affected far more than the other models by a change in reliability.

Inspection of the data in Table 4.8 also reveals that the degree of shrinkage in the shared variance of Model 1 with factor 1 when going from $\rho_{xx'} = \rho_{yy'} = 1.0$ to $\rho_{xx'} = \rho_{yy'} = 0.90$ was the same across Conditions I and III, i.e., 13% when $\rho_{xy} = 0.25$ and 0.50 and 16% when $\rho_{xy} = 0.75$. Under unequal reliability conditions, i.e., moving from $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ however, shrinkage in the shared variance of Model 1 with factor 1 was less than 10% and across the three levels of Condition I such shrinkage was a little higher than in Condition III.

In condition 32, the degree of shrinkage in the shared variance of the change score models with factor 1 was the same for all of the models (13%) when we moved from $\rho_{xx'} = \rho_{yy'} = 1.0$ to $\rho_{xx'} = \rho_{yy'} = 0.90$. Obviously, such similarity was due to the presence of a single underlying change score for all of the models under given conditions. Only when we moved from $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ did Model 6 separate itself from the others by having a larger degree of shrinkage in its shared variance with factor 1.

In summary, the findings in Table 4.8 indicate that there are model differences regarding the measurement error effects on the change score models. Model 6 was the most susceptible to these effects among other models.

To analyze the effect of ρ_{xy} level (or colinearity effect) on the change score models the shared variance of Models 1, 2, 3 and 6 with factor 1 across three ρ_{xy} levels of Conditions I and III were compared and the results of the comparison are reported in Table 4.9. The findings in Table 4.9 determine the size as well as the direction of the changes in the shared variance with factor 1. As is clear from the results in this

Table 4.9

Difference in the Shared Variance of the Change Score
Models Due to Changes in ρ_{xy} Levels

		Condition I				Condition III			
ρ_{xy}	change scores	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25 vs 0.50	GT	-0.02	+0.10	-0.02	+0.10	0.00	+0.06	0.00	+0.06
	GO1	-0.01	+0.06	-0.01	+0.04	0.0	+0.03	0.00	+0.02
	GO2	-0.03	+0.05	-0.02	-0.07	-0.02	+0.01	-0.02	-0.09
0.50 vs 0.75	GT	+0.02	-0.06	+0.02	-0.06	-0.03	-0.08	-0.03	-0.08
	GO1	-0.02	-0.08	-0.02	-0.15	-0.06	-0.05	-0.07	-0.19
	GO2	-0.02	-0.06	-0.02	-0.23	-0.04	-0.09	-0.04	-0.19
0.25 vs 0.75	GT	0.00	+0.04	0.00	+0.04	-0.03	-0.02	-0.03	-0.02
	GO1	-0.03	-0.02	-0.03	-0.11	-0.06	-0.03	-0.07	-0.17
	GO2	-0.05	-0.01	-0.04	-0.30	-0.06	-0.08	-0.06	-0.25

table, the shared variance of the discrepancy models with factor 1 changed very little when ρ_{xy} was changed from 0.25 to 0.75. In Condition III for $\rho_{xx'} = \rho_{yy'} = 0.90$ when ρ_{xy} changed from 0.50 to 0.75, the shared variance of discrepancy models with factor 1 dropped by 6% to 71%. For residual models the size and direction of the changes in the shared variance of the models with factor 1 due to changes in ρ_{xy} level had a different pattern. First, the size of the changes in the shared variance of the residual models (Models 2 and 6) with factor 1 were larger relative to the size of changes in the shared variance of the discrepancy models, particularly for Model 6. Second, when ρ_{xy} value changed from 0.25 to 0.50, the shared variance of residual models with factor 1 increased in magnitude except for Model 6 under unequal reliability conditions ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). Under the latter conditions the shared variance of Model 6 with factor 1 decreased by 7% in Condition I and 9% in Condition III. When ρ_{xy} was changed from 0.50 to 0.75, however, the shared variance of residual models decreased in magnitude across all reliability levels. For Model 6 the amount of shrinkage in such variance were as high as 15 - 23% under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions.

The bottom part of Table 4.9 presents the differences in the shared variance of Models 1, 2, 3 and 6 with factor 1 for changes in ρ_{xy} from 0.25 to 0.75. Apparently the shrinkage in the shared variance of Model 6 with factor 1 is the largest among all the others and is extreme in magnitude. In summary, the findings in Table 4.9 reveal that ρ_{xy} has differential impact on residual and discrepancy models regarding their shared variance with factor 1 and, in the stated context, Model 6 separated itself from both discrepancy and residual models, especially under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions.

Overall, the findings in Tables 4.6, 4.7, 4.8 and 4.9 revealed that the different parameters (reliability, colinearity and variability) have an interactive as well as a differential impact on the definition of change by the change score models. The findings in Tables 4.6 - 4.9 also reveals that from the three parameters affecting the change score models, i.e., variability of x and y (σ_x and σ_y), coefficient of colinearity (ρ_{xy}) and

reliability coefficients ($\rho_{xx'}$, $\rho_{yy'}$), variability (or $\lambda = \frac{\sigma_x}{\sigma_y}$) was the one which made the largest impact in defining different underlying constructs for the discrepancy change score models. The coefficient of colinearity (ρ_{xy}) and the reliability coefficients ($\rho_{xx'}$, $\rho_{yy'}$) exerted their impact more on residual than on discrepancy models, particularly on Model 6. Under unequal reliability coefficient conditions, the shared variance of Model 6 with factor 1 decreased across all three ρ_{xy} levels. Increments to the coefficient of colinearity also resulted in the shrinkage of the shared variance of Model 6 with factor 1, particularly for $\rho_{xy} = 0.75$ under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$.

The findings in Tables 4.6, 4.7, 4.8 and 4.9 also indicate that there are model differences among the change score models. Discrepancy and residual models in Condition I ($\sigma_x = \sigma_y$) are farther apart than in Condition III ($\sigma_y > \sigma_x$). In Condition I for $\rho_{xy} = 0.25$ and 0.50 , Model 6 mostly behaves like a residual model (Model 2), while for $\rho_{xy} = 0.75$ it behaves like a discrepancy model (such as Model 1), particularly under the $\rho_{xx'} \neq \rho_{yy'}$ condition (see Table 4.6). In Condition III, however, Model 6 separated itself from both residual and discrepancy models regarding its shared variance with factor 1. Comparison of the shared variance of Models 1 and 3 in Table 4.6 reveals that Model 3 shared a larger portion of its variance with factor 1 in condition I than did Model 1 particularly for $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. In Condition III, however, these model differences disappeared.

Findings in Table 4.6 also reveal that in addition to 70.7% of the variance extracted by the first factor in the principal component analyses, factor 2 also extracted 7.6% of the underlying variability of the change score models across Conditions I and III. The findings on factor 2 in Table 4.6 indicate that the discrepancy models from Condition I are mainly positively loaded on this factor. Apparently, factor 2 is a factor separating the discrepancy and residual models of Condition I from each other. From Condition III only Model 6 under $\rho_{xy} = 0.75$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ was positively loaded on the stated factor and shared 11% of its variance with the second

factor. Model 1 (raw gain score) shared a larger portion of its variance with factor 2 than did Model 3 across the reliability ($\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xy} = 0.70$ and $\rho_{yy'} = 0.90$) and ρ_{xy} levels. In condition 13, however, residual models as well as discrepancy models were positively loaded on the second factor. The shared variance of Model 2 with this factor was small in magnitude, but Model 6 shared 12 - 22% of its variance with factor 2 under $\rho_{xx'} = \rho_{xx'} = 0.90$ and $\rho_{xy} = 0.70$ and $\rho_{xx'} = 0.90$ conditions. Similarity of the shared variance of Models 6 and 1 under $\rho_{xx'} \neq \rho_{yy'}$ condition is due to the fact that Model 6 behaves like a discrepancy model in condition 13. In summary, these findings reveal that the underlying construct of the second factor is differences in underlying dimension of discrepancy and residual models in Condition I.

4.4 Consistency of the Change Score Models Across Six Different Parameter Conditions

To examine the consistency of the change score models across six different parameter conditions, the data on each change score across six conditions (11, 12, 13, 31, 32 and 33) were simultaneously factor analyzed using a principal component technique. Table 4.10 represents the loadings of each model on the first principal component factor and the percent of the variance extracted by this factor for each model. The inspection of the loadings reported in Table 4.10 revealed that Models 1, 3, 4 and 5 were mainly affected by the variability and reliability coefficient for x and y components and less by the coefficient of colinearity for these variables. Reliability and coefficient of colinearity had a minor effect on Model 2 and variability had no effect on this model. Finally, Model 6 had no impact from the variability for x and y components but the coefficient of colinearity and reliability interacted to decrease the loadings systematically as ρ_{xy} is increased and reliability decreased.

Among all other models, Model 2 appeared was least affected by reliability coefficient, coefficient of colinearity and variability of x and y components across

Table 4.10

Magnitude of the Loadings of Each Change Score Model on the First Factor of the Principal Component Across Nine Parameter Conditions and the Percent of the Variance Extracted by Each Model

change scores	loadings	change scores	loadings	change scores	loadings
model 1		model 2		model 3	
GT111	0.91	GT112	0.96	GT113	0.89
GO1111	0.81	GO1112	0.90	GO1113	0.82
GO2111	0.75	GO2112	0.89	GO2113	0.80
GT121	0.90	GT122	0.97	GT123	0.88
GO1121	0.80	GO1122	0.89	GO1123	0.81
GO2121	0.73	GO2122	0.89	GO2123	0.78
GT131	0.91	GT132	0.89	GT133	0.89
GO1131	0.78	GO1132	0.80	GO1133	0.79
GO2131	0.70	GO2132	0.82	GO2133	0.76
GT311	0.90	GT312	0.97	GT313	0.93
GO1311	0.83	GO1312	0.91	GO1313	0.85
GO2311	0.81	GO2312	0.91	GO2313	0.83
GT321	0.89	GT322	0.97	GT323	0.93
GO1321	0.82	GO1322	0.90	GO1323	0.85
GO2321	0.79	GO2322	0.89	GO2323	0.80
GT331	0.86	GT332	0.89	GT333	0.90
GO1331	0.78	GO1332	0.81	GO1333	0.80
GO2331	0.76	GO2332	0.81	GO2333	0.78
percent variance extracted	0.67		0.80		0.71

Table 4.10 (continued)

Magnitude of the Loadings of Each Change Score Model on the First Factor of the Principal Component Across Nine Parameter Conditions and the Percent of the Variance Extracted by Each Model

change scores	loadings	change scores	loadings	change scores	loadings
model 4		model 5		model 6	
GT114	0.90	GT115	0.90	GT116	0.94
GO1114	0.81	GO1115	0.81	GO1116	0.88
GO2114	0.78	GO2115	0.78	GO2116	0.87
GT124	0.89	GT125	0.89	GT126	0.97
GO1124	0.79	GO1125	0.79	GO1126	0.88
GO2124	0.76	GO2125	0.75	GO2126	0.80
GT134	0.90	GT135	0.90	GT136	0.93
GO1134	0.77	GO1135	0.77	GO1136	0.77
GO2134	0.74	GO2135	0.72	GO2136	0.63
GT314	0.92	GT315	0.92	GT316	0.95
GO1314	0.84	GO1315	0.84	GO1316	0.90
GO2314	0.82	GO2315	0.81	GO2316	0.89
GT324	0.91	GT325	0.91	GT326	0.97
GO1324	0.83	GO1325	0.83	GO1326	0.89
GO2324	0.79	GO2325	0.78	GO2326	0.82
GT334	0.88	GT335	0.88	GT336	0.91
GO1334	0.79	GO1335	0.76	GO1336	0.76
GO2334	0.77	GO2335	0.74	GO2336	0.80
percent variance extracted	0.69		0.68		0.76

the six parameter conditions, and the percent of the variance extracted by the first principal component factor for this model was the highest (79.9%) among all others, indicating that Model 2 is the most stable model among all other competing models. After Model 2, the first principal component factor of Model 6 extracted the highest percent of the underlying variability across the nine parameter conditions (73.6%). The major drawback of this model, however, is that it is highly influenced by the reliability coefficient for x and y component. Yet relative to the discrepancy models, overall its scores have slightly more variance in common across the condition (73.6% versus 67.3%). Finally, Models 3, 4 and 5 are not that much different than Model 1 regarding the amount of variance extracted by their first principal component factors.

4.5 Correlation of Change Score Models with w

4.5.1 Commonality of the Models Considering w

Table 4.11 represents the change score models correlations with w across all reliability coefficient and ρ_{xy} level for Condition I, II and III. The inspection of data from Condition I ($\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$) in this table reveals that the magnitude of correlation of the change score models with w does not exceed 0.38 across three reliability and ρ_{xy} levels and that is only true for the raw residual gain (Model 2) and estimated true residual gain (Model 6) models under perfect reliability conditions and $\rho_{xy} = 0.25$. When reliability coefficients for x and y were set to be 1) $\rho_{xx'} = \rho_{yy'} = 0.90$ and 2) $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$, the correlation of residual change scores with w did not exceed 0.35 and had a decreasing trend as the ρ_{xy} level increased in magnitude. For example, the correlation of Models 2 and 6 with w reached 0.25 and 0.03, respectively, where $\rho_{xy} = 0.75$ and $\rho_{xx'} \neq \rho_{yy'}$. Model 2's correlation with w was unaffected by changes in reliabilities while Model 6's correlation with w decreased both as a function of ρ_{xy} level and reliability.

In Condition II where validity coefficients for x and y were set to be unequal

Table 4.11
Correlation of Change Score Models With W
Across Three Research Conditions

Condition	parameters	model	$\rho_{xy} = 0.25$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.75$		
			GT	GO1	GO2	GT	GO1	GO2	GT	GO1	GO2
I	$\sigma_x = \sigma_y$ $\rho_{wx} = \rho_{wy}$	1	0.01	-0.01	0.00	-0.01	-0.01	0.00	0.01	0.01	0.00
		2	0.38	0.33	0.35	0.28	0.26	0.28	0.20	0.19	0.25
		3	0.01	0.02	0.09	-0.01	0.03	0.09	0.01	0.06	0.12
		4	0.01	-0.01	0.06	-0.01	-0.01	0.05	0.00	0.00	0.08
		5	0.01	-0.01	0.05	-0.01	-0.01	0.03	0.01	0.00	0.05
		6	0.38	0.31	0.29	0.27	0.21	0.15	0.20	0.12	0.03
II	$\sigma_x = \sigma_y$ $\rho_{wx} \neq \rho_{wy}$	1	0.35	0.29	0.29	0.43	0.37	0.33	0.56	0.42	0.38
		2	0.66	0.58	0.58	0.65	0.57	0.57	0.72	0.58	0.59
		3	0.35	0.32	0.37	0.43	0.45	0.43	0.56	0.47	0.51
		4	0.35	0.29	0.34	0.43	0.38	0.40	0.56	0.42	0.48
		5	0.35	0.29	0.34	0.43	0.38	0.38	0.56	0.42	0.44
		6	0.66	0.57	0.55	0.66	0.55	0.49	0.72	0.51	0.38
III	$\sigma_x \neq \sigma_y$ $\rho_{wx} \neq \rho_{wy}$	1	0.54	0.47	0.45	0.64	0.54	0.52	0.77	0.62	0.60
		2	0.62	0.55	0.54	0.64	0.56	0.55	0.71	0.59	0.58
		3	0.54	0.49	0.49	0.64	0.56	0.55	0.77	0.63	0.62
		4	0.54	0.47	0.48	0.64	0.54	0.54	0.77	0.62	0.61
		5	0.54	0.48	0.48	0.64	0.55	0.56	0.77	0.63	0.62
		6	0.62	0.54	0.49	0.65	0.53	0.47	0.71	0.52	0.39

(i.e., $\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$, the correlation of the change score models with w showed a major improvement relative to Condition I and the correlation of the discrepancy change scores with w had an increasing trend as the ρ_{xy} level increased in magnitude. The correlation of Model 2 with w stayed the same across the three ρ_{xy} levels, but the correlation of Model 6 decreased across ρ_{xy} levels as well as across reliability levels. The correlation of Model 2 change scores with w was the highest across all three ρ_{xy} levels (see Table 4.11). Under unequal reliability coefficients where $\rho_{xy} = .25$, the correlation of Model 1 with w was the lowest among all models (0.29)

In Condition III, where variability and validity coefficients for x and y were set to be unequal ($\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$), the correlation of discrepancy change score models with w increased in magnitude relative to the correlations in Condition II ($\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$). The correlations for Models 2 and 6 remained the same as in condition II, indicating that increased variability of y had less impact on residual change scores, but considerable impact on discrepancy model scores. In this condition, as in Condition II, the correlation of discrepancy change scores with w increased and the correlations of Models 2 remained approximately the same across ρ_{xy} levels. The reliability effect is again most noticeable for Model 6.

Comparison of the magnitude of correlations of change score models with w reveals that in Condition III ($\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$), change score models show a high resemblance to each other regarding the correlation with the third variable w , except for Model 6, which separated itself from the rest of the models under unequal reliability coefficients where $\rho_{xy} = 0.50$ and 0.75 (correlation of Model 6 with w was the lowest among all others). The findings in Table 4.11 also reveal that in Conditions II and III, the correlation of change score model with w are reasonably high, especially when ρ_{xy} was equal to 0.50 and 0.75 , and these are the conditions in which the underlying change score model is, in fact, a discrepancy model and is defined by negative suppression condition (i.e., $\rho_{xy} > \frac{\rho_{wx}}{\rho_{wy}}$). In the rest of the conditions the underlying change score model was an additive model defined by redundancy

conditions.

4.5.2 Commonality of the Change Score Models Removing w :

To examine the effects of w on the definition of the change score models, the change scores across Conditions I, II and III (i.e., conditions 11, 12, 13, 21, 22, 23, 31, 32 and 33) were residualized by w . The residualized change scores were then subjected to 12 separate factor analyses using the principal component technique. The first nine analyses were conducted using the residualized model scores within each of the nine conditions separately. The remaining three factor analyses were conducted combining the change score models from 1) Conditions I and II; 2) I and III; and 3) II and III.

Table 4.12 presents the results from the first nine factor analyses in terms of the percentage of the total variance extracted by the first principal component factor for each separate condition (11 - 33). From Table 4.12, interactive effects would appear to be evident. The residualized models under Condition II for $\rho_{xy} = .25$ and $.50$ have systematically less variance in common than do the models under Conditions I and III. The most discrepant case is 7.2% less variance comparing II versus III for $\rho_{xy} = .25$. However, when $\rho_{xy} = .75$, the change in common variance among the models is substantial in Condition III (a drop of 14.4%), less in Condition II (8.2%) and less in Condition I (5.8%). The models have the most variance in common once w has been removed under Condition III and $\rho_{xy} = .25$ (90%) and the least variance in common under Conditions II and III when $\rho_{xy} = .75$ (73%). The loadings of the models on factor 1 are reported in Table 4.13. As Table 4.13 reveals in conditions 23 and 33, the loadings of the change score models do not exceed .89 for all of the models. In condition 13 the loadings for change scores from observed 2 condition (GO2) were in general smaller than for true (GT) or observed 1 (GO1) loading. This indicates the interaction of reliability and ρ_{xy} level on the change score models in condition 13. In this condition change score loading on factor 1 for condition 23 ranged from .79 to .89 and the lowest belonged to Model 2 under unequal reliability coefficient

Table 4.12

Percentage of the Total Variance Extracted by the First Factor of the Principle Component Across Three ρ_{xy} Levels of Condition I, II, and III by Residual Change Scores

ρ_{xy}	condition I	condition II	condition III
0.25	86.4	82.4	89.9
0.50	86.2	80.8	87.0
0.75	80.4	72.6	72.6

($\rho_{xx'} \neq \rho_{yy'}$). In condition 33 the range of the loadings for the change score models was .71 to .89 with the lowest belonging to Model 6 under unequal reliability coefficient ($\rho_{xx'} \neq \rho_{yy'}$), indicating the higher susceptibility of Model 6 to the measurement error effect relative to other models. Comparison of the loadings of the change score models on Factor 1 across the three research conditions in Table 4.13 reveals that the underlying construct of change varies depending on the parameter conditions. Table 4.13 clearly shows the susceptibility of the change score models to various parameters. Differences in the loadings of the change score models under the extreme conditions of colinearity ($\rho_{xy} = 0.75$) versus the loading of the models in other conditions (11, 12, 21, 22, 31, 32) determine the potential influence of colinearity coefficient on the change score models. Loadings of change score models under observed 2 condition (GO2) in condition 31 determines the potential influence of reliability coefficient on the change score models and smaller loading for Model 6 versus the other in condition 33 determines the model differences among the change score models explainable by the interactive effect of various parameters.

Tables 4.14, 4.15, and 4.16 present the shared variance of the residualized

Table 4.13

Magnitude of the Loadings of the Residual Change Score Models on the First Factor of the Principal Component for Different Parameter Conditions

ρ_{xy}	change scores	condition I	change scores	condition II	change scores	condition III
0.25	RGT111	0.93	RGT311	0.90	RGT311	0.94
	RGT112	0.94	RGT312	0.91	RGT312	0.94
	RGO1111	0.92	RGO1311	0.91	RGO1311	0.95
	RGO1112	0.93	RGO1312	0.90	RGO1312	0.95
	RGO1113	0.93	RGO1313	0.92	RGO1313	0.96
	RGO1116	0.94	RGO1316	0.91	RGO1316	0.95
	RGO2111	0.90	RGO2311	0.89	RGO2311	0.94
	RGO2112	0.91	RGO2312	0.89	RGO2312	0.94
	RGO2113	0.94	RGO2313	0.93	RGO2313	0.95
	RGO2116	0.95	RGO2316	0.93	RGO2316	0.95
0.50	RGT121	0.93	RGT221	0.89	RGT321	0.93
	RGT122	0.94	RGT222	0.90	RGT322	0.93
	RGO1121	0.92	RGO1221	0.90	RGO1321	0.94
	RGO1122	0.92	RGO1222	0.89	RGO1322	0.94
	RGO1123	0.93	RGO1223	0.92	RGO1323	0.94
	RGO1126	0.94	RGO1226	0.92	RGO1326	0.94
	RGO2121	0.90	RGO2221	0.88	RGO2321	0.93
	RGO2122	0.91	RGO2222	0.86	RGO2322	0.93
	RGO2123	0.94	RGO2223	0.91	RGO2323	0.93
	RGO2126	0.94	RGO2226	0.92	RGO2326	0.91
0.75	RGT131	0.92	RGT231	0.84	RGT331	0.85
	RGT132	0.92	RGT232	0.85	RGT332	0.84
	RGO1131	0.90	RGO1231	0.88	RGO1331	0.89
	RGO1132	0.90	RGO1232	0.85	RGO1332	0.89
	RGO1133	0.91	RGO1233	0.89	RGO1333	0.88
	RGO1136	0.92	RGO1236	0.89	RGO1336	0.83
	RGO2131	0.86	RGO2231	0.84	RGO2331	0.88
	RGO2132	0.87	RGO2232	0.79	RGO2332	0.88
	RGO2133	0.89	RGO2233	0.84	RGO2333	0.86
	RGO2136	0.87	RGO2236	0.84	RGO2336	0.71

Note: R in front of GT, GO1, and GO2 was used to separate the original from residualized change scores

Table 4.14

Shared Variance of the Residual Change Score Models
 (when w was partialled out)
 With Factor 1 and 2 of the Principle Component Analysis

			Condition I				Condition II			
Factor	ρ_{xy}	change scores	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
1	0.25	RGT	0.83	0.62	-	-	0.60	0.47	-	-
		RGO1	0.69	0.52	0.70	0.55	0.55	0.42	0.57	0.44
		RGO2	0.58	0.48	0.62	0.56	0.46	0.38	0.51	0.45
	0.50	RGT	0.82	0.76	-	-	0.58	0.52	-	-
		RGO1	0.62	0.62	0.70	0.67	0.51	0.44	0.52	0.45
		RGO2	0.57	0.57	0.62	0.63	0.40	0.36	0.43	0.43
	0.75	RGT	0.82	0.84	-	-	0.55	0.55	-	-
		RGO1	0.63	0.64	0.65	0.66	0.44	0.39	0.44	0.44
		RGO2	0.53	0.57	0.58	0.54	0.32	0.29	0.32	0.32
2	0.25	RGT	0.00	-0.17	-	-	0.04	-0.11	-	-
		RGO1	0.00	-0.11	0.00	-0.18	0.02	-0.14	0.00	-0.12
		RGO2	0.00	-0.22	0.02	-0.15	0.00	-0.17	0.00	-0.12
	0.50	RGT	0.00	-0.07	-	-	0.10	0.00	-	-
		RGO1	0.00	-0.08	0.00	0.05	0.07	-0.01	0.05	0.00
		RGO2	0.00	-0.11	-0.01	-0.03	0.07	-0.02	0.02	0.00
	0.75	RGT	0.01	0.00	-	-	0.27	0.14	-	-
		RGO1	0.01	-0.01	0.00	0.00	0.28	0.13	0.24	0.21
		RGO2	0.00	-0.03	0.00	0.04	0.32	0.11	0.21	0.32

Note: R in front of GT, GO1, and GO2 was used to separate the original from residualized change scores

Table 4.15

Shared Variance of the Residual Change Score Models
 (when w was partialled out)
 With Factor 1 and 2 of the Principle Component Analysis

			Condition I				Condition III			
Factor	ρ_{zy}	change scores	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
1	0.25	RGT	0.75	0.69	-	-	0.66	0.59	-	-
		RGO1	0.62	0.59	0.65	0.61	0.59	0.52	0.58	0.54
		RGO2	0.53	0.56	0.61	0.62	0.56	0.51	0.56	0.56
	0.50	RGT	0.75	0.81	-	-	0.57	0.57	-	-
		RGO1	0.62	0.68	0.65	0.70	0.52	0.50	0.50	0.53
		RGO2	0.51	0.63	0.60	0.64	0.46	0.45	0.45	0.46
	0.75	RGT	0.76	0.83	-	-	0.36	0.55	-	-
		RGO1	0.58	0.65	0.61	0.62	0.35	0.39	0.32	0.39
		RGO2	0.49	0.61	0.57	0.50	0.31	0.32	0.28	0.26
2	0.25	RGT	-0.01	0.00	-	-	-0.05	-0.05	-	-
		RGO1	-0.01	0.00	-0.01	0.00	-0.07	-0.06	-0.07	-0.07
		RGO2	0.00	0.00	0.00	0.00	-0.06	-0.05	-0.06	-0.06
	0.50	RGT	-0.01	0.00	-	-	0.00	0.00	-	-
		RGO1	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00
		RGO2	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00
	0.75	RGT	-0.01	0.00	-	-	0.32	0.19	-	-
		RGO1	-0.01	0.00	-0.01	0.00	0.42	0.38	0.43	0.30
		RGO2	-0.01	0.00	0.00	0.00	0.46	0.44	0.46	0.26

Note: R in front of GT, GO1, and GO2 was used to separate the original from residualized change scores

Table 4.16

Shared Variance of the Residual Change Score Models
 (when w was partialled out)
 With Factor 1 and 2 of the Principle Component Analysis

			Condition II				Condition III			
Factor	ρ_{zy}	change scores	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
1	0.25	RGT	0.53	0.48	-	-	0.65	0.55	-	-
		RGO1	0.50	0.44	0.52	0.46	0.57	0.48	0.56	0.50
		RGO2	0.42	0.40	0.48	0.45	0.56	0.47	0.54	0.54
	0.50	RGT	0.52	0.51	-	-	0.57	0.57	-	-
		RGO1	0.46	0.45	0.48	0.47	0.51	0.49	0.49	0.52
		RGO2	0.38	0.38	0.42	0.43	0.47	0.44	0.44	0.48
	0.75	RGT	0.48	0.51	-	-	0.35	0.56	-	-
		RGO1	0.41	0.40	0.42	0.42	0.33	0.40	0.30	0.42
		RGO2	0.30	0.30	0.32	0.30	0.30	0.32	0.26	0.31
2	0.25	RGT	0.13	-0.01	-	-	-0.01	0.06	-	-
		RGO1	0.11	-0.02	0.09	0.01	-0.03	-0.11	-0.04	-0.09
		RGO2	0.08	-0.03	0.03	-0.01	-0.02	-0.12	-0.05	-0.05
	0.50	RGT	0.18	-0.01	-	-	-0.03	-0.03	-	-
		RGO1	0.11	-0.02	0.09	-0.01	-0.06	-0.08	-0.08	-0.05
		RGO2	0.08	-0.03	0.03	-0.01	-0.07	-0.12	-0.13	-0.03
	0.75	RGT	0.29	0.16	-	-	-0.13	0.00	-	-
		RGO1	0.25	0.11	0.21	0.18	-0.14	-0.04	-0.17	0.00
		RGO2	0.26	0.08	0.17	0.26	-0.10	-0.06	-0.17	0.00

Note: R in front of GT, GO1, and GO2 was used to separate the original from residualized change scores

change scores with factors 1 and 2 for combined Conditions I and II; I and III; and II and III, respectively. The percent of variance extracted by the first principal component factor for residualized change score were 54.6, 55.5 and 44.9% for combined Conditions I and II, I and III and II and III, respectively. This indicates that residualized change scores of the given conditions had only 54.6, 55.5 and 44.9 percent of their variance in common and the rest of their variance, i.e., 53.6, 54.5 and 43.9%, respectively, remained unaccounted for by the first principal component factor. This indicated the moderate degree of commonality among the residualized change score models. Previously we found out that the first principal component factor on the change scores of Conditions I and III or II and III using the original change scores extracted 70.7% of the underlying variance of the change score models (note change scores of Conditions I and II were the same due to equality of variance for x and y components). The comparison of the latter commonality index(i.e.,70.7%) with the ones given in above, i.e., 54.4 and 44.90% for Conditions I and III and II and III, respectively, determines the magnitude of the shrinkage in the commonality of the change score model due to removal of w from the analysis. The differences in the extracted variances for original and corresponding combined change scores were 16.3 and 25.8%, respectively. The stated differences represent the degree of the impact of w on the construct definition of change. Contrast of the change scores of Condition II versus Condition I represents the validity coefficient effect while keeping the variability constant; II and III represent changes in the shared variance of the change score model while keeping the variability of x and y constant; and I versus III represents simultaneous effects of variability and validity coefficient on the change score models. The combining of original change scores from Conditions I and III was due to the fact that change scores of Conditions I and II were identical when w was not partialled out from the analysis.

Comparison of the data in Tables 4.15 and 4.6 revealed that when the relationships with w removed from the analysis, the magnitude of the shared variance of the change score models was affected. Further support for the above findings was

provided when the original change scores obtained in Conditions I and III and the corresponding residualized change scores were simultaneously factor analyzed using a principal component technique. Tables 4.17, 4.18 and 4.19 present the shared variance of the combined original and residual change score models with factor 1 for Conditions I and II, I and III, and II and III, respectively. The data reported in the top part of Tables 4.17, 4.18 and 4.19 represent the shared variances of the residualized change scores with factor 1 and the bottom part of these tables represents the shared variances of the original change scores (i.e. when w is included in the data) with factor 1.

In order to facilitate the interpretation of data presented in Tables 4.17, 4.18 and 4.19 regarding the effect of w on the definition of the change as measured by different models, the residual change scores' shared variances with factor 1 (presented at the top of these three tables) were subtracted from the shared variances of the original change score shared variances within their corresponding tables. The magnitude of these differences for Condition I versus Condition II, Condition II versus Condition III, and Condition I versus Condition III are presented in Tables 4.20.a to 4.20.c. Comparison of the data of Condition I with that of Condition II determines the impact of w on the definition of change under equal/unequal validity coefficient for x and y components (Condition I: $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy} = .50$ and Condition II: $\sigma_x = \sigma_y = 1.0$ and $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$). Comparison of data from Conditions II and III determines the effect of w on the definition of change under equal/unequal variability for x and y components (Condition II: $\sigma_x = \sigma_y = 1.0$, $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$). Finally, comparison of data from Conditions I and III will determine the impact of w on the definition of change, while simultaneous effects of variability and validity coefficients for x and y components are taken under consideration. The findings in Table 4.20.a indicate that w has little effect on the definition of change in Condition I (equal variability, equal validity coefficient), while it has a considerable impact on the change score models defined in Condition II (equal variability, unequal validity coefficient for x and y components). In Condition II across the three variabil-

Table 4.17

Shared Variance of the Combined Residual Change Score
(when w was partialled out) and the original models
With Factor 1 of the Principle Component Analysis

Residual Change Scores									
		Condition I				Condition II			
ρ_{xy}	change score	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25	RGT	0.80	0.69	-	-	0.50	0.46	-	-
	RGO1	0.66	0.59	0.69	0.61	0.47	0.42	0.49	0.44
	RGO2	0.56	0.55	0.63	0.62	0.40	0.39	0.46	0.44
0.50	RGT	0.80	0.82	-	-	0.49	0.49	-	-
	RGO1	0.66	0.68	0.68	0.71	0.44	0.43	0.46	0.46
	RGO2	0.55	0.63	0.63	0.65	0.35	0.36	0.39	0.40
0.75	RGT	0.80	0.86	-	-	0.46	0.50	-	-
	RGO1	0.61	0.67	0.64	0.66	0.38	0.38	0.40	0.40
	RGO2	0.51	0.61	0.59	0.53	0.28	0.29	0.30	0.28
Original Change Scores									
		Condition I				Condition II			
0.25	GT	0.81	0.65	-	-	0.80	0.64	-	-
	GO1	0.66	0.55	0.69	0.58	0.68	0.57	0.71	0.60
	GO2	0.56	0.52	0.65	0.60	0.57	0.53	0.66	0.59
0.50	GT	0.80	0.82	-	-	0.80	0.82	-	-
	GO1	0.65	0.68	0.69	0.72	0.65	0.68	0.69	0.70
	GO2	0.55	0.63	0.64	0.67	0.52	0.59	0.61	0.64
0.75	GT	0.81	0.89	-	-	0.79	0.87	-	-
	GO1	0.61	0.69	0.65	0.68	0.61	0.68	0.65	0.67
	GO2	0.51	0.63	0.60	0.53	0.48	0.60	0.57	0.48

Table 4.18

Shared Variance of the Combined Residual Change Score Models
(when w was partialled out) and Original Change Score Models
With the First Factor of the Principle Component Analysis

Residual Change Scores									
		Condition I				Condition III			
ρ_{xy}	change score	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25	RGT	0.71	0.75	-	-	0.60	0.57	-	-
	RGO1	0.59	0.64	0.63	0.66	0.54	0.51	0.54	0.52
	RGO2	0.50	0.62	0.60	0.66	0.52	0.51	0.53	0.53
0.50	RGT	0.70	0.85	-	-	0.52	0.52	-	-
	RGO1	0.58	0.72	0.62	0.72	0.49	0.48	0.48	0.49
	RGO2	0.48	0.67	0.59	0.69	0.44	0.44	0.44	0.41
0.75	RGT	0.71	0.83	-	-	0.37	0.49	-	-
	RGO1	0.54	0.66	0.59	0.62	0.34	0.36	0.33	0.34
	RGO2	0.45	0.63	0.57	0.47	0.31	0.30	0.29	0.22
Original Change Scores									
		Condition I				Condition III			
0.25	GT	0.72	0.76	-	-	0.90	0.80	-	-
	GO1	0.58	0.66	0.63	0.68	0.78	0.70	0.77	0.73
	GO2	0.50	0.63	0.63	0.69	0.74	0.69	0.73	0.74
0.50	GT	0.70	0.90	-	-	0.89	0.89	-	-
	GO1	0.57	0.75	0.63	0.77	0.77	0.76	0.77	0.78
	GO2	0.48	0.71	0.62	0.68	0.72	0.71	0.71	0.69
0.75	GT	0.72	0.88	-	-	0.85	0.86	-	-
	GO1	0.54	0.71	0.60	0.66	0.71	0.70	0.70	0.63
	GO2	0.45	0.68	0.59	0.48	0.67	0.65	0.67	0.43

Table 4.19

Shared Variance of the Combined Residual Change Score Models
(when w was partialled out) and Original Change Score Models
With the First Factor of the Principle Component Analysis

Residual Change Scores									
		Condition II				Condition III			
ρ_{xy}	change score	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25	RGT	0.39	0.49	-	-	0.60	0.56	-	-
	RGO1	0.38	0.46	0.41	0.48	0.55	0.51	0.54	0.52
	RGO2	0.32	0.43	0.40	0.47	0.53	0.51	0.54	0.54
0.50	RGT	0.39	0.48	-	-	0.54	0.54	-	-
	RGO1	0.36	0.45	0.39	0.46	0.50	0.48	0.48	0.50
	RGO2	0.29	0.40	0.36	0.39	0.45	0.45	0.45	0.43
0.75	RGT	0.35	0.43	-	-	0.37	0.51	-	-
	RGO1	0.31	0.36	0.34	0.35	0.35	0.37	0.33	0.35
	RGO2	0.23	0.29	0.28	0.23	0.32	0.32	0.29	0.23
Original Change Scores									
		Condition II				Condition III			
0.25	GT	0.88	0.78	-	-	0.72	0.74	-	-
	GO1	0.77	0.69	0.76	0.71	0.60	0.67	0.65	0.69
	GO2	0.74	0.68	0.73	0.73	0.51	0.62	0.64	0.67
0.50	GT	0.88	0.88	-	-	0.73	0.89	-	-
	GO1	0.76	0.75	0.75	0.76	0.59	0.54	0.65	0.57
	GO2	0.71	0.70	0.70	0.69	0.48	0.68	0.61	0.67
0.75	GT	0.84	0.86	-	-	0.71	0.87	-	-
	GO1	0.70	0.70	0.69	0.64	0.56	0.70	0.61	0.65
	GO2	0.67	0.66	0.67	0.45	0.44	0.64	0.64	0.57

Table 4.20

Differences in the Shared Variance of the Original and Residualized
Change Scores With Factor 1 of the Principal Component

(a)									
		Condition I				Condition II			
ρ_{xy}	change score	model 1	model 2	model 3	model 6	model 1	model 2	model 3	model 6
0.25	GT	+01	-.04	+01	-.04	+030	+018	+030	+018
	GO1	0.00	-.04	0.00	-.03	+021	+015	+022	+016
	GO2	0.00	-.03	+02	-.02	+017	+014	+020	+015
0.50	GT	0.00	0.00	0.00	0.00	+031	+033	+031	+033
	GO1	-.01	0.00	+01	+01	+021	+025	+023	+024
	GO2	0.00	0.00	+01	+02	+017	+023	+022	+024
0.75	GT	+01	+03	+01	+03	+033	+037	+033	+037
	GO1	0.00	+02	+01	+02	+023	+030	+025	+027
	GO2	0.00	+02	+01	0.00	+020	+031	+027	+020
(b)									
		Condition II				Condition III			
0.25	GT	+049	+029	+049	+029	+012	+018	+012	+018
	GO1	+039	+023	+035	+023	+005	+016	+011	+017
	GO2	+042	+025	+033	+026	-.02	+011	+010	+013
0.50	GT	+049	+048	+049	+048	+019	+035	+019	+035
	GO1	+040	+030	+036	+030	+009	+026	+017	+025
	GO2	+042	+030	+034	+030	+003	+023	+016	+024
0.75	GT	+049	+043	+049	+043	+034	+036	+034	+036
	GO1	+039	+034	+035	+029	+021	+033	+028	+030
	GO2	+044	+037	+039	+022	+012	+032	+035	+034
(c)									
		Condition I				Condition III			
0.25	GT	+01	+01	+01	+01	+030	+023	+030	+023
	GO1	-.01	+02	0.00	+02	+024	+019	+023	+021
	GO2	0.00	+01	+03	+03	+022	+018	+020	+021
0.50	GT	0.00	+05	0.00	+05	+037	+037	+037	+037
	GO1	-.01	+03	+01	+05	+028	+028	+029	+029
	GO2	0.00	+04	+03	+04	+028	+027	+027	+028
0.75	GT	+01	+05	+01	+05	+048	+037	+048	+037
	GO1	0.00	+05	+01	+04	+037	+034	+037	+029
	GO2	0.00	+05	+02	+01	+036	+035	+038	+021

ity levels for x and y components, the ranges of the shrinkage in the shared variance of the change score models due to the removal of w were 17-31% for Model 1, 14-37% for Model 2, 20-33% for Model 3 and 15-37% for Model 6. In this condition, colinearity interacted with w effect on the change score models only for the residual models (2 and 6). As the colinearity coefficient (ρ_{xy}) increased in magnitude, the effect on Models 2 and 6 got larger. Under these three reliability levels these differences ranged from 4 - 18% for Model 2 and 4 to 12% for Model 6, indicating minor interaction effects between w and ρ_{xy} effect.

Table 14.20b represents the effect of w on the definition of change under equal validity coefficients but unequal variability for x and y components (i.e., Condition II: $\sigma_x = \sigma_y = 1.0$, $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$, and Condition III: $\sigma_y = 2 \sigma_x$ and $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$). The findings in this table indicate that the removal of w has a far greater impact on the definition of change defined under Condition II than the change score defined under Condition III. The shrinkage of the shared variance of the change score models in Condition II were: 39-49% for Model 1, 23-48% for Model 2, 33-49% for Model 3 and 22-48% for Model 6. The corresponding shrinkages for change score models in Condition III were: 2-34% for Model 1, 6-36% for Model 2, 10-35% for Model 3 and 7-35% for Model 6, indicating the greater susceptibility of the change score models in Condition II to the removal of w from the definition of change.

Table 4.20.c indicates that w had a considerable effect on the definition of change under unequal variability and validity coefficient (Condition III) relative to the equal variability and validity coefficient (Condition I). Under the latter condition, removal of w had no impact on the definition of change defined by different models. For Condition III, however, the ranges of shrinkage in the shared variance of change score models due to removal of w were 22-37% for Model 1, 18-37% for Model 2, 20-48% for Model 3 and 21-37% for Model 6. Apparently the high resemblance of the change score models to each other in Condition III (particularly Conditions 32 and 33, discovered previously) has resulted in the similarity of the shrinkages for the

shared variance of change score models due to removal of w . In summary, the findings in Tables 4.20.a-4.20.c reveal that w has a potential for exerting a considerable effect on the definition of change defined by different change score models depending on the combination of the parameter conditions. Only when variability and validity coefficients for x and y were equal, did w have no impact on the definition of change.

Combining the findings of this section with those of the previous section we can conclude that differences among the change score models do exist as they relate to the third variable, w . When w is partialled out from the change score models, it still differentially affects the definition of the change score models depending on the parameter condition.

Chapter 5

Discussion

The goal of the present study was to investigate the extent to which six selected change score models differ in operation. These models were 1) raw gain ; 2) raw residual gain; 3) estimated true gain corrected for error in x; 4) estimated true gain corrected for error in both x and y; 5) regression model; and 6) estimated true residual gain (base-free measure of change). Investigated differences focused on the operational definition of change as a construct and the extent to which different models had differential input into the correlational context. In order to conduct the above analyses, three simulation conditions were set up to vary the research context in which the change scores would be studied, i.e., I) $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$; II) $\sigma_x = \sigma_y$ and $\rho_{wy} \succ \rho_{xy}$; and III) $\sigma_y \succ \sigma_x$ and $\rho_{wy} \succ \rho_{wx}$. Then, within each stated condition, the degree of colinearity between x and y was iterated across three levels i.e., $\rho_{xy} = 0.25, 0.50$ and 0.75 . As a result, parameter conditions resulted in the simulation of nine distinct population sets of scores. Within each stated condition reliability coefficients for x and y components were also manipulated so that three sets of change scores (one true and two observed—six models per set) for each model were generated (total of 18 variables). The three reliability levels for x and y were 1) $\rho_{xx'} = \rho_{yy'} = 1.0$; 2) $\rho_{xx'} = \rho_{yy'} = 0.90$; and 3) $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. The values for the colinearity coefficients (ρ_{xy}), variability indices (σ_x and σ_y) and validity coefficients (ρ_{xy} and ρ_{wy}) were selected so that the underlying change score model

was defined by redundancy and suppression conditions within a context of a three variable regression model (Glasnapp, 1984). For example, in conditions 22, 23, 32 and 33, the underlying change score was a discrepancy model defined by negative suppression (i.e., $\rho_{xy} > \frac{\rho_{wx}}{\rho_{wy}}$), while in conditions 11, 12, 13, 21 and 31, it was an additive model defined by a redundancy condition (i.e., $\rho_{xy} < \frac{\rho_{wx}}{\rho_{wy}}$).

The data for this analysis were randomly generated using the corresponding variance-covariance matrix (explained in Chapter Three) for each parameter condition. In the remainder of this chapter, the findings reported in Chapter Four are summarized and discussed relative to the literature focusing on 1) input of x and y components into the definition of the underlying change ; 2) congruency/noncongruency of the change score models within each parameter condition; 3) effect of various parameter conditions on the change scores models; 4) commonality of the change score models as they relate to the third variable; and 5) commonality of the change score models removing w from the analysis.

5.1 Input of x and y Components into the Definition of the Change Score Models

5.1.1 Conclusions

The findings of this study indicated that the degree of contribution of x and y components into the definition of underlying change varied depending on the model and parameter conditions. For example, in Condition I ($\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$), the x and y components had equal intercorrelations with Models 1, 3, 4 and 5 under perfect reliability conditions, while by definition, x had no correlation with Models 2 and 6 and y had a consistently higher correlation with Model 2. When reliability of x and y were set to be less than perfect, the contributions of x and y into the definition of underlying change varied as a function of the model type and ρ_{xy} level.

In Condition III ($\sigma_y > \sigma_x$ and $\rho_{wy} > \rho_{wx}$) the contribution of y into the def-

initiation of the underlying change dominates the contribution of x for all of the ρ_{xy} levels except for Model 6 in condition 33 when reliability coefficients for the x and y components were set to be unequal. In this condition, the correlation of x with Model 6 exceeded the correlation of the y component. In condition 32, however, x had no correlation with any of the models and y was the only contributing factor into the definition of the underlying change under perfect reliability conditions. In Condition III in general, both the reliability and ρ_{xy} had an effect on the degree of contribution of the x and y components into the definition of the underlying change. The increased variability of y relative to x influenced the resulting definition of change for the discrepancy models (1, 3, 4 and 5) but not for Models 2 and 6. The pattern of correlations for Models 2 and 6 under Condition III was identical as that of Condition I. As expected because of its larger variance, y dominated the definition of its change under Condition III for Models 1, 3, 4 and 5. This fluctuation in the size and pattern of the correlations of x and y with the models' change scores demonstrate that the underlying constructs defined by the change scores vary as a function of the parameters manipulated. The magnitude of the differences/similarities among the defined underlying constructs was addressed by the principal component analysis summarized in a later section. Model 6 showed the most fluctuation as it moved from producing change scores that were identical to Model 2 under $\rho_{xy} = .25$ to scores that mirrored Model 1 under $\rho_{xy} = .75$.

5.1.2 Relation to Other Studies

The studies covering the concept of change as a definition of a construct in a comparative form, as it is used in this project, are very limited in number. Glasnapp (1984) and Raeissi and Glasnapp (1983), by correlating the raw gain score ($y-x$) to a third variable (w), demonstrated that the variation in $y-x$ related to variation in w was primarily dominated by either x or y except for very restrictive parameter conditions. Glasnapp and Raeissi (1985) investigated the differential construct definition of the five change score models (Models 1, 2, 3, 4 and 5) used in this project in correlational

context for limited parameter conditions. The findings of the present project were consistent with the findings of Glasnapp and Raeissi (1985) regarding the input of x and y components into the definition of the underlying change.

5.2 Congruency/Noncongruency of the Change Score Models

5.2.1 Conclusions

The findings of this study indicated that within each ρ_{xy} level the change scores models measured different underlying constructs to varying degrees. Across nine parameter conditions, Models 4 and 5 were indistinguishable from each other and they had a perfect or nearly perfect correlation with Model 1 (raw gain score) as well. This indicates that for all practical purposes Model 4 and 5 are not measuring a construct different than the raw gain score does. Model 3 also had a perfect or nearly perfect correlation with Model 1 across nine parameter conditions and across all reliability levels except in condition 13 for the $\rho_{xx'} \neq \rho_{yy'}$ condition. In this condition, the correlation of Model 3 with Model 1 was .95. Overall, minor differences among Models 1,3, 4 and 5 surfaced only under unequal reliability conditions for x and y components $\rho_{xx'} \neq \rho_{yy'}$. Model 5, for example, showed minor differences with discrepancy models only under condition 33, where $\rho_{xx'} \neq \rho_{yy'}$. In Condition III across all reliability and ρ_{xy} levels, Models 3 and 4 were almost identical to Model 1 (raw gain score). In Condition I for $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xx'} = \rho_{yy'} = 0.90$, Models 1, 3 and 4 had perfect or nearly perfect correlation across three reliability levels. In this condition only when $\rho_{xx'} \neq \rho_{yy'}$ and $\rho_{xy} = .50$ and 0.75 , did Model 3 show minor differences from the rest of the discrepancy models.

The residual change score Models (2 and 6) separated themselves from each other across three ρ_{xy} levels of conditions I and III as reliability coefficients, i.e., $\rho_{xx'}$ and $\rho_{yy'}$, were set to be less than perfect, particularly for the $\rho_{xx'} \neq \rho_{yy'}$ condition. The exception to these findings was for condition 32, where all of the change score

models were perfectly correlated under $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 conditions. For unequal reliability coefficients, i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$, the correlation of Models 2 and 6 dropped to .95. In condition III, where reliability was less than perfect ($\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), Model 6 showed a higher resemblance to discrepancy scores than did Model 2 across the three ρ_{xy} levels. In condition 31 Model 6 has a perfect correlation with Model 1 (raw gain score) and Model 5 (regression model) while in condition 33 Model 2 showed a higher resemblance to the discrepancy models than did Model 6. In condition 33, where $\rho_{xx'} \neq \rho_{yy'}$, Model 6 (estimated true residual gain) particularly separated itself from the rest of the models. For Condition III ($\sigma_y > \sigma_x$ and $\rho_{wy} > \rho_{wx}$) in general the change score models showed a higher resemblance to each other relative to the change scores in conditions I with the exception of when $\rho_{xy} = 0.75$. This later exception was due to Model 6 and its lower congruency with other models when $\rho_{xy} = 0.75$ under Condition III. The percent of shared variance among the models indicated by the first principal component was 84.7% for condition 11, 85.2% for condition 12, 80.1% for condition 13, 93.3% for condition 31, 91.8% for condition 32 and 82.7% for condition 33. In the later condition, Model 6 had loadings of 0.88 (77.4% shared variance) and 0.76 (57.8% shared variance) for the two reliability levels. Excluding Model 6 loadings, the percent of shared variance among the remaining change scores was 86.3% for condition 33.

In summary, the principal component analysis would indicate that Condition III parameters results in 6 to 8 percent on the average more congruency among the models' change scores than do the Condition I parameters due to the dominance of y . Within each condition the change of ρ_{xy} from 0.25 to 0.50 has little effect, but the increase of ρ_{xy} to 0.75 results in a reduction of five percent congruency in Condition I and a nine percent reduction in Condition III. Four percent of the latter reduction was due entirely to the change in the underlying construct defined by Model 6.

5.2.2 Relation to Other Studies

Among the available data in the literature regarding the commonality of the underlying construct measured by different models, only the findings of the Glasnapp and Raeissi (1985) study were directly comparable with the results of the present project. Glasnapp and Raeissi (1985) comparatively examined the degree of the commonality of the underlying construct defined by five change score models' raw change scores (Model 1), estimated true gain scores (Models 3 and 4) and residual gain scores (Models 2 and 6) as related to the third variable (w). Glasnapp and Raeissi (1985) illustrated the comparison of the models for three diverse situations: one in which the underlying predictive relationship between a linearly weighted x and y composite and the outside variable w was redundant and two where the underlying relationship specified a discrepancy composite for x and y as they related to w (suppression condition). The parameter values for the stated conditions in Glasnapp and Raeissi (1985) study were as follows: 1) reliability coefficient values in all conditions were $\rho_{xx'} = .80$, $\rho_{yy'} = .95$ and $\rho_{ww'} = 1.0$; 2) $\rho_{wy} = .75$ and $\rho_{wx} = .45$; 3) $\rho_{xy} = .30$ for condition 1 and 0.70 for conditions 2 and 3 (the combination of parameter conditions created situations where underlying models for x and y were defined either by redundant (condition 1) or suppression conditions as change score composite ($y-x$) was related to the third variable (w)); and 4) $\sigma_x = \sigma_y = \sigma_w$ for conditions 1 and 2 and $\sigma_y = 2\sigma_x = 2\sigma_w$ for condition 3.

Although parameter values selected by Glasnapp and Raeissi (1985) were somewhat different in magnitude and limited in number, yet to the extent that their study matches the investigated models and parameter conditions in the present project, their findings remained consistent with the findings of this project regarding the commonality of the underlying construct measured by the change score models.

Cronbach and Furby (1970) in discussing the methodological adequacy of the raw gain score and its alternative models seriously questioned the merit of the "base free measure of change" introduced by Tucker, Dumarin and Messick (1966). "A

base free measure of change” was primary intended for correlational work i.e., to be used as an intermediate step toward correlation. In the present study however, Model 6 (base free measure of change) changes and is similar to the raw gain model (Model 1) or raw residual gain model (Model 2) under different parameter conditions. Only other parameter conditions included in this study, Model 6 separates itself from both discrepancy or residual models indicating that Cronbach and Furby (1970) criticism are only valid under particular parameter conditions.

5.3 Effect of Various Parameter Conditions on the Change Score Models

5.3.1 Conclusions

The findings of this study presented in the previous chapter (Tables 4.7-4.9) indicate that different parameter conditions, i.e., variability (σ_x and σ_y), , reliability and colinearity coefficient (ρ_{xy}) for x and y components had differential impact on the change score models. In general, residual change scores were less susceptible to the changes in variability for x and y components than were the discrepancy models. For example, in Condition III (see Table 4.7), the magnitudes of the shared variances of Model 1 with the first factor of the principal component were 24 to 31% higher than the corresponding shared variances in Condition I across three reliability and ρ_{xy} levels. For Model 3 the stated variance range was 15-29% while these values for Models 2 and 6 ranged from 0.0 to 5% at the most.

Changes in reliability coefficient affected all of the models, but Model 6 was the most susceptible to these changes under the extreme conditions of ρ_{xy} level (i.e. $\rho_{xy} = 0.75$). Overall, the magnitudes of these differences were not large for all of the models (see Table 4.8). For example, in Condition I when moving from a perfect reliability coefficient ($\rho_{xx'} = \rho_{yy'} = 1.0$) to a less than perfect reliability coefficient but for equal reliability coefficient for x and y components ($\rho_{xx'} = \rho_{yy'} = .90$) the

variances of Model 1 with factor 1 decreased by 13-16% under the three ρ_{xy} levels. The corresponding variable range was 5 to 17% for Model 2, 6-10% for Model 3 and 5 to 24% for Model 5. Under unequal reliability coefficients for x and y components ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), these variance ranges got much smaller in magnitude across all of the models and ρ_{xy} levels in both Conditions I and III (0.0-9% for Models 1, 2 and 3). Model 6 separated itself from the other models by having different variance ranges. In Condition I the shrinkage in the shared variance of Model 6 when moving from $\rho_{xx'} = \rho_{yy'} = 0.90$ to $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ was 0.0, 1.0 and 19% across three ρ_{xy} levels. In Condition III the corresponding values were 0.0, 11 and 21%, respectively.

For the colinearity coefficient effect, Model 6 appeared to be more affected than the rest of the models (Models 1, 2 and 3). The effect of the changes in the colinearity coefficient on Models 1, 2 and 3 were very small or negligible (particularly for Models 1 and 3). Under the extreme conditions of colinearity, i.e., moving from $\rho_{xy} = 0.50$ to $\rho_{xy} = 0.75$, the shared variance of Model 6 with the first factor of the principal component shrunk by 15% and 23% in Condition I for observed 1 (GO1) and observed 2 (GO2) scores. In Condition III, these shrinkages were 19% for both GO1 and GO2 scores. For moving from $\rho_{xy} = 0.25$ to $\rho_{xy} = 0.75$, the shared variance of Model 6 with factor 1 shrunk by 11% and 30% for observed 1 (GO1) and observed 2 (GO2) scores in Condition I and in Condition III it shrunk by 17% and 25% for GO1 and GO2 scores, respectively. These findings indicate the differences of Model 6 with discrepancy models (Models 1 and 3) as well as with the raw residual model (Model 2) within the stated context.

5.3.2 Relation to Other Studies

Glasnapp (1984), Glasnapp and Raeissi (1985) and Raeissi and Glasnapp (1983), in correlating the raw gain score and/or alternate models of the change with a third variable, demonstrated that the underlying construct of change defined by the raw

gain score or its alternative model varied as a function of the parameter conditions. The findings of the present study were consistent with the those of the above researchers.

Corder-Bolz (1978), Labouvie (1980), Maxwell and Howard (1981) and Zimmerman and Williams (1982a,b), however, tried to identify those parameter conditions under which the use of raw change scores was useful, meaningful and methodologically sound. Zimmerman and Williams, in particular, demonstrated a range of parameter conditions under which raw gain scores were reliable and had high prediction potential. For example, they demonstrated that the potential ranges of the predictive validity and reliability of change scores are dependent on the ratio of the components of change, i.e., $\frac{\sigma_x}{\sigma_y} = \lambda$ and when λ is not equal to unity, i.e., $\sigma_x \neq \sigma_y$, the potential ranges of change score (y-x) validity and reliability coefficients increase in a reasonable way especially under unequal validity coefficients for x and y components ($\rho_{wx} \neq \rho_{wy}$). To evaluate the findings of the present study within the context of Zimmerman and Williams' (1982) study, reliability of the change score models ($\rho_{gg'}$) were calculated and are reported in Table 5.1. As the findings of Table 5.1 indicate, reliability of change score models vary as a result of parameter conditions. A summary of the findings of Table 5.1 are presented in Table 5.2 where, the upper row presents the range of reliability of change scores generated under $\rho_{xx'} = \rho_{yy'} = 0.90$ and the lower row represents the range of reliability of change score generated under $\rho_{xx'} \neq \rho_{yy'}$.

From the findings in Table 5.2 it is clear that in the three research conditions (I, II and III) when $\rho_{xy} < .50$, the reliability of the change score models is in a better condition than when $\rho_{xy} > .50$. The range of reliability of the raw change score (y-x) under $\sigma_x \neq \sigma_y$, $\rho_{wx} \neq \rho_{wy}$ and $\rho_{xx'} \neq \rho_{yy'}$ conditions in this study was also consistent with the findings of Zimmerman and Williams (1982a,b). Rogosa and Zimowski (1983) also discussed the reliability of the raw change scores (y-x) using the individual path line between time 1 and time 2 measurement of the same variable. These researchers illustrated that when $\sigma_x = \sigma_y$ and $\rho_{xx'} = \rho_{yy'}$ and the coefficient

Table 5.1
Reliability coefficient of the change score models within
each ρ_{xy} level across three research conditions
(I, II, and III)

condition parameter	model	$\rho_{xy} = 0.25$						$\rho_{xy} = 0.50$						$\rho_{xy} = 0.75$					
		1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
I: $\sigma_x = \sigma_y$ $\rho_{wx} = \rho_{wy}$	ρ_{GG_1}	0.88						0.83						0.71					
	ρ_{GG_2}	0.79	0.90	0.88	0.88	0.88	0.90	0.72	0.86	0.83	0.83	0.83	0.85	0.56	0.74	0.69	0.71	0.71	0.71
			0.88	0.79	0.79	0.79	0.86		0.83	0.72	0.72	0.72	0.76		0.66	0.56	0.58	0.58	0.50
II: $\sigma_x = \sigma_y$ $\rho_{wx} \neq \rho_{wy}$	ρ_{GG_1}	0.88						0.83						0.71					
	ρ_{GG_2}	0.79	0.90	0.88	0.88	0.88	0.90	0.71	0.86	0.83	0.83	0.85	0.76	0.53	0.74	0.69	0.71	0.71	0.71
			0.90	0.79	0.81	0.81	0.88		0.81	0.71	0.71	0.71	0.76		0.64	0.52	0.53	0.53	0.46
III: $\sigma_x \neq \sigma_y$ $\rho_{wx} \neq \rho_{wy}$	ρ_{GG_1}	0.88						0.86						0.81					
	ρ_{GG_2}	0.85	0.90	0.88	0.88	0.88	0.90	0.81	0.86	0.86	0.86	0.86	0.76	0.76	0.76	0.81	0.81	0.81	0.72
			0.88	0.86	0.86	0.86	0.85		0.83	0.83	0.83	0.83	0.76		0.69	0.79	0.79	0.81	0.52

Table 5.2

Range of the Reliability Coefficients for the Change Score Models Across Three Research Conditions

Conditions	change score	$\rho_{xy} = 0.25$	$\rho_{xy} = 0.50$	$\rho_{xy} = 0.75$
condition I	GO1	0.88-0.90	0.83-0.86	0.71-0.74
	GO2	0.79-0.86	0.72-0.83	0.50-0.66
condition II	GO1	0.88-0.90	0.83-0.86	0.69-0.74
	GO2	0.79-0.90	0.71-0.76	0.46-0.64
condition III	GO1	0.88-0.90	0.86-0.86	0.72-0.81
	GO2	0.85-0.88	0.76-0.83	0.52-0.81

Note: The first row represent the reliability coefficient for GO1 condition and second row represent the reliability of the change scores for GO2 condition

of stability (ρ_{xy}) is large, little individual differences are left to be detected and, as a result, raw gain score has a low reliability coefficient. Findings of the present study also were consistent with Rogosa and Zimowski's paper (1983). Rogosa and Zimowski, Zimmerman and Williams and other researchers, however, did not attend to the reliability of the various change scores as a function of parameter conditions as is illustrated in the present project. Thus, the comparable data in this regard were missing from the literature.

5.4 Consistency of the Change Score Models Across Various Parameter Conditions

5.4.1 Conclusions

The findings of the present project revealed that Model 2 (raw residual gain) was the most stable model in defining change across various parameter conditions. Since Model 2 is only dependent on the y component, it is least affected by the various parameter conditions such as variability, coefficient of colinearity and reliability coefficient among all other models. Furthermore, in the principal component analysis, Model 2 extracted the highest variability across nine parameter conditions relative to other models. Model 6 (estimated true residual gain) was the next model, extracting the highest underlying variability of change across nine parameter conditions, but when the other factors are taken into consideration Model 6 turned out to be the least consistent model. Model 6 is not only affected by the changes in parameter values like other models (particularly the reliability effect), but it behaves in a way that is different from all other models. Model 6 is a residual change score which changes its identity as a residual model to a discrepancy model under specific parameter conditions such as condition 13 for $\rho_{xx'} \neq \rho_{yy'}$. Correlations of Model 6 with models 1 and 5 in the given conditions was perfect, but in condition 33, Model 6 separated itself from both discrepancy and residual models.

5.4.2 Relation to Other Studies

The comparable data regarding the consistency of the change score models in the estimation of the change across various parameter conditions were missing from the literature.

5.5 Commonality of the Change Score Models as they Relate to the Third Variable

5.5.1 Conclusions

The findings of this study indicated that change score models have differential input into the correlational context as a function of both model differences and parameter conditions. In condition I ($\sigma_x = \sigma_y$ and $\rho_{xy} = \rho_{wy}$), for example, the underlying change has no variability which can be detected in w by the discrepancy models. As a result, these models (1, 3, 4, and 5) have either no correlation or poor correlation with the third variable w . Residual models (2 and 6) had a moderate correlation with w , but the stated correlation decreased as the ρ_{xy} level increased in magnitude, particularly for Model 6 under unequal reliability conditions, i.e., $\rho_{xx'} \neq \rho_{yy'}$. In Condition II ($\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$) as in Condition I, the ability of the discrepancy models to detect a relationship with w in the correlational context varied more than the input of the residual models. The stated differences are greater in magnitude as the reliability coefficients for x and y were set to be less than perfect, particularly for unequal reliability coefficients ($\rho_{xx'} \neq \rho_{yy'}$). In condition III ($\sigma_y > \sigma_x$ and $\rho_{wy} > \rho_{wx}$), the input of discrepancy and residual models into the correlational context varied except when $\rho_{xy} = .50$ (condition 32). In the latter condition, only Model 6 separated itself from the rest of the models, particularly under the $\rho_{xx'} \neq \rho_{yy'}$ condition. In summary, the findings of this study indicated that only under rare parameter conditions such as condition 32 are the change score models measuring the same underlying construct as they relate to the third variable. Also, the change score models were better correlated

with the third variable (w) when the underlying model was a discrepancy model (defined by a suppression condition) versus an additive model (defined by a redundancy condition).

5.5.2 Relation to Other Studies

Glasnapp (1984) and Raeissi and Glasnapp (1983), by correlating the raw change score with the third variable (w), demonstrated that within the context of the correlational studies, when correlating raw change scores ($y-x$) with an outside variable, the variation in $y-x$ related to variation in w is primarily dominated by either the x or y variable, except for very restrictive situations. They argued that even though x and y may remain constant in definition, the resulting change construct that is defined in the relationship with w changes, depending on the intercorrelations among the x , y and w components. Further support for the findings of the present project was provided by Linn and Slinde (1977), who indicated that the “alternative approaches to measurement of change result in different correlations of changes with other variables. The different estimates have different theoretical and practical implications” (p. 128). Glasnapp and Raeissi’s findings (1985) also were consistent with the findings of the present project. These researchers, by correlating five change score models with w as stated before, concluded that change score models varied as a function of the parameter conditions and that they result in differential construct definitions. Unfortunately, Linn and Slinde (1977) have not provided readers with the empirical data regarding the extent to which the models were varied in relation to the third variable. The series of studies done by Glasnapp and Raeissi also was limited in scope regarding the number of models and parameter conditions investigated in their analyses. But, to the extent that their studies overlapped with the the context of the present project, their findings remain consistent with the findings of this study.

5.6 Commonality of the Change Score Models Removing W from the Analyses

5.6.1 Conclusions

In the present study, when w was partialled out from the models and the residualized change scores were factor analyzed, the shared variance of the change score models with the first factor of the principal component decreased in magnitude for both Conditions II and III.

Table 4.11 in previous chapter revealed that the change score models emerged under Condition I had a very small or no correlation with w . All discrepancy models shared 0.0% of their variance with w and residual change scores (Models 2 and 6) shared very small portion of their variance (up to 14%) with w only for $\rho_{xy} = 0.25$ and their shared variance with w decreased along incrementing ρ_{xy} .

The total variance extracted by the change score models before and after removing w from the analysis were reported in previous chapter in Tables 4.4 and 4.12 respectively. The amount of the shrinkage in the shared variance of the change score models extracted by the first factor of the principal component due to removal of w from the analysis were 2.3, 4.4 and 7.5% and 3.4, 4.8 and 10.1% for Conditions II and III respectively, indicating smaller commonality among the change score models after removal of w from the analysis. The loadings for the change score models on the first factor of the principal component before and after removal of w were reported in previous chapter in Tables 4.5 and 4.13 respectively. The comparison of the data in the stated tables revealed that the Loadings of the change score models for Conditions II and III were smaller after removal of w from the analyses than before removing w . To estimate the exact amount of w and the parameter values such as validity coefficient, variability, and interactive effects of validity coefficient and variability for x and y component on the commonality of the change score models, the combined original and residualized change scores for Conditions I and II, II and III and I and III were separately factor analyzed. The loadings of the original and

residualized change scores for the stated conditions were reported in Tables 4.17, 4.18, and 4.19 respectively. And the differences in the shared variance of the original and residualized scores in these three tables were reported in Tables 4.20a, 4.20b, and 4.20c respectively. Table 4.20a revealed that the removal of *w* from the analyses had no impact on the shared variance of the change score models from Condition I, but the amount of the shrinkage in the commonality of the change score models from Condition II were reasonable in size. For example the shrinkage in the shared variance of Models 1, 2, 3 and 6 were 17-33%, 14-37%, 20-33% and 15-37% respectively across three ρ_{xy} levels.

Table 4.20b revealed the magnitude of the shrinkage in the shared variance of the change score models from Condition II and III after removing *w* from the analyses. The effect of the removal of *w* on the change score models from Condition II were much higher than on the change score models from Condition III. For example the shrinkage in the shared variance of Models 1, 2, 3 and 6 from Condition II were 39-49%, 25-43%, 33-49% and 29-48% respectively across three ρ_{xy} levels, while in Condition III these shrinkages were 3-34%, 11-26%, 10-35% and 13-36% for Models 1, 2, 3 and 6 respectively.

Table 4.20c represented the amount of the shrinkage in the shared variance of the change score models from Conditions I and III due to the removal of *w* from the analyses. As stated previously removal of *w* from the analyses had no impact on the shared variance of the change scores from Condition I, but the shrinkage in the shared variance of Models 1, 2, 3 and 6 from Condition III were : 22-48%, 18-37%, 20-48% and 21-37% respectively.

In summary, the findings in Tables 4.20a, 4.20b and 4.20c revealed that the commonality of the change score models decreases in magnitude as the result of removing *w* from the analyses for Conditions II and III, particularly for condition II. Furthermore, *w* has differential impact on the discrepancy models (Models 1 and 3) and residual models (Models 2 and 6) depending to the interactive effects of variability, colinearity, validity and reliability

5.6.2 Relation to Other Studies

Data comparable with the findings of the present project in this section are missing from the literature.

5.7 General Conclusion and Implications of the Results

To facilitate the comparison and evaluation of the change score models from a measurement, methodological and practical point of view, the raw gain score ($y-x$) was selected to be the standard model in this analysis, and the comparison of each modified change score versus Model 1 is discussed separately in this section. For this comparison, the focus was on 1) the reliability coefficient; 2) variability; 3) validity coefficient of the change score models (i.e., relationship to w); and 4) the factor loadings on the first factor of the principal component, i.e., consistency of construct definition across conditions as indicated by a model's commonality when considering w and when ignoring it.

5.7.1 Residual Gain Scores Versus Raw Gain Score

a. Model 2 versus Model 1

Among the alternative models of change the raw residual gain (Model 2) primarily was proposed to be used in the correlational context (Dubois, 1957; Manning and Dubois, 1958, 1962) and theoretically, the change measured by the stated model is a purified change in that the effect of its initial measure has been removed. Now the question is, "how well this specific model behaves relative to the traditional model of change regarding its reliability, variability, correlation with the third variable, being affected by the changes made in different parameter conditions and the underlying construct measured or defined by the stated model?"

Findings in Table 5.1 indicated that Model 2 was as reliable as the raw gain score ($y-x$) in most of the parameter conditions (11-32) and even more reliable than

Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) conditions except in condition 33. In the latter condition, reliability coefficient of Model 2 was a little smaller than the reliability of Model 1 for both $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. Variability of Model 2, however, was smaller than the variability of Model 1 under all of the parameter conditions but condition 32, where $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 . Under the latter conditions, variabilities of these models were the same. In correlational context Model 2's validity coefficient (correlation with w) was higher than the validity coefficient of Model 1 in all parameter conditions but in condition 32 and 33.

Table 5.3 represents the shared variance of change score models with w across all three research conditions (Conditions I, II and III) and reliability levels. The findings in the table reveal that in Condition I for $\rho_{xy} = 0.25$, Model 1 shared 0% of its variance (across three reliability levels) with w while Model 2 shared 11-24%, 8% and 4-6% of its variance with the same variable across $\rho_{xy} = 0.25, 0.50$ and 0.75 , respectively. In Condition II, Model 2 shared a much greater portion of its variance with w than Model 1 did. For $\rho_{xy} = 0.25$ (Condition 21) across three reliability levels, differences in the shared variance of Model 2 with w were: 32, 25 and 25%, respectively. For $\rho_{xy} = 0.50$ (Condition 22) and $\rho_{xy} = 0.75$ (Condition 23), these differences were: 24, 18 and 21%) and 25, 16 and 21%), respectively. In Condition III, shared variances of Model 2 with w exceeded the shared variance of Model 1 by 8 to 10% under $\rho_{xy} = 0.25$ (Condition 31). For $\rho_{xy} = 0.50$ (Condition 32), the differences in the shared variance of Models 1 and 2 with w mainly disappeared. For $\rho_{xy} = 0.75$ (Condition 33), shared variance of Model 1 with w slightly exceeded the shared variance of Model 2 across three reliability levels (GT, GO1 and GO2). These differences were 9, 9 and 3%. In general, these findings indicated that the predictive power of Model 2 over that of Model 1 decreases in either Condition I ($\sigma_x = \sigma_y$ at $\rho_{wx} = \rho_{wy}$ or III ($\sigma_x = 2\sigma_y$ at $\rho_{wy} > \rho_{wx}$) for $\rho_{xy} = 0.50, 0.75$, particularly in Condition III. In Condition II, unequal validity coefficients for x and y ($\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$) drastically increased the validity coefficient of the resulting change

Table 5.3

Shared Variance of the Change Score Models With W
Across Three Research Conditions

Condition	parameter	model	$\rho_{xy} = 0.25$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.75$		
			GT	GO1	GO2	GT	GO1	GO2	GT	GO1	GO2
I	$\sigma_x = \sigma_y$	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		2	0.14	0.11	0.12	0.08	0.07	0.08	0.04	0.4	0.06
		3	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01
	$\rho_{wx} \neq \rho_{wy}$	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
		5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		6	0.14	0.10	0.08	0.07	0.04	0.02	0.04	0.02	0.00
II	$\sigma_x = \sigma_y$	1	0.12	0.08	0.08	0.18	0.14	0.11	0.31	0.18	0.14
		2	0.44	0.33	0.33	0.42	0.32	0.32	0.52	0.34	0.35
		3	0.12	0.10	0.14	0.18	0.17	0.18	0.31	0.22	0.26
	$\rho_{wx} \neq \rho_{wy}$	4	0.12	0.08	0.12	0.18	0.14	0.16	0.31	0.18	0.23
		5	0.12	0.08	0.12	0.18	0.14	0.14	0.31	0.18	0.19
		6	0.44	0.57	0.30	0.44	0.30	0.24	0.51	0.26	0.14
III	$\sigma_x \neq \sigma_y$	1	0.29	0.22	0.20	0.41	0.29	0.27	0.59	0.38	0.36
		2	0.38	0.30	0.29	0.41	0.31	0.30	0.50	0.34	0.33
		3	0.29	0.24	0.24	0.41	0.31	0.30	0.59	0.40	0.38
	$\rho_{wx} \neq \rho_{wy}$	4	0.29	0.22	0.23	0.41	0.29	0.29	0.59	0.38	0.37
		5	0.29	0.23	0.23	0.41	0.30	0.31	0.59	0.40	0.38
		6	0.38	0.29	0.29	0.42	0.28	0.22	0.50	0.27	0.15

scores, particularly for Model 2. In Condition III doubling the variability of y versus x had no impact on Model 2 but a major impact on discrepancy models including Model 1. Consequently, the validity coefficient of Model 1 increased but not that of Model 2. The similarity of the validity coefficients of Model 1 and Model 2 in Condition III resulted from a combination of both model differences and parameter effects on Models 1 and 2. In Condition II where we kept variability of x and y constant, unequal validity coefficients ($\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$) have a greater effect on Model 2 than on Model 1, while in Condition III where we chose to have unequal variability ($\sigma_y = 2\sigma_x$) and kept the validity coefficients of x and y constant, the validity coefficient of discrepancy models (such as Model 1) drastically increases, but the validity coefficient of a residual model (such as Model 2) does not.

Table 5.4, presents the increment or shrinkage in the shared variance of each change score model with w regarding the specific effects of validity coefficient, variability and simultaneous effect of both variability and validity coefficients. For example, in comparing Condition II versus Condition I, where variability for x and y were kept constant ($\sigma_x = \sigma_y$) but the validity coefficients of the x and y components changed from $\rho_{wx} = \rho_{wy} = 0.50$ to $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$, the shared variance of Model 1 with w increased in magnitude by 12, 8 and 8% across three reliability levels, while such increments for Model 2 were 29, 22 and 21% under the stated reliability conditions. For $\rho_{xy} = 0.75$, these increments were 25, 18 and 14% and 47, 30 and 29% for Models 1 and 2, respectively, indicating the higher predictive validity power of Model 2 over Model 1. Increment of variability of y component in Condition III versus Condition II increased the shared variance of Model 1 by 17, 13 and 11% across three reliability levels for $\rho_{xy} = 0.25$, while for Model 2 there was no increment, but a shrinkage. Its shared variance with w across three reliability levels was 5, 3 and 5%. The increments in the shared variance of Model 1 with w for $\rho_{xy} = 0.50$ and 0.75 across three reliability levels were (22, 15 and 16%) and (34, 20 and 22%), respectively. For Model 2 there was no increment as before, but minor shrinkage, indicating the lack of effect of variability on Model 2.

Table 5.4

Differences in the Shared Variance of the Change Score Models With W

Condition	model	$\rho_{xy} = 0.25$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.75$		
		GT	GO1	GO2	GT	GO1	GO2	GT	GO1	GO2
II VS I $\rho_{GW(II)}^2 - \rho_{GW(I)}^2$	1	+ .12	+ .08	+ .08	+ .18	+ .14	+ .11	+ .31	+ .18	+ .14
	2	+ .29	+ .22	+ .21	+ .34	+ .26	+ .24	+ .47	+ .30	+ .29
	3	+ .12	+ .10	+ .12	+ .18	+ .16	+ .17	+ .26	+ .22	+ .24
	4	+ .12	+ .08	+ .11	+ .18	+ .14	+ .16	+ .31	+ .18	+ .21
	5	+ .12	+ .08	+ .11	+ .18	+ .14	+ .14	+ .31	+ .18	+ .19
	6	+ .29	+ .22	+ .21	+ .36	+ .26	+ .21	+ .47	+ .25	+ .14
III VS II $\rho_{GW(III)}^2 - \rho_{GW(II)}^2$	1	+ .17	+ .13	+ .11	+ .22	+ .15	+ .16	+ .28	+ .20	+ .22
	2	- .05	- .03	- .04	- .01	- .01	- .02	- .01	+ .01	- .01
	3	+ .17	+ .13	+ .10	+ .22	+ .15	- .12	+ .28	+ .18	+ .13
	4	+ .17	+ .13	+ .11	+ .22	+ .15	+ .13	+ .28	+ .21	+ .16
	5	+ .17	+ .13	+ .11	+ .22	+ .16	+ .17	+ .28	+ .22	+ .19
	6	- .05	- .03	- .06	- .01	- .02	- .01	- .01	+ .01	+ .00
III VS I $\rho_{GW(III)}^2 - \rho_{GW(I)}^2$	1	+ .29	+ .22	+ .20	+ .41	+ .29	+ .27	+ .59	+ .38	+ .36
	2	+ .24	+ .19	+ .17	+ .33	+ .25	+ .22	+ .46	+ .31	+ .27
	3	+ .29	+ .24	+ .23	+ .41	+ .31	+ .29	+ .59	+ .39	+ .37
	4	+ .29	+ .22	+ .22	+ .41	+ .29	+ .29	+ .59	+ .38	+ .37
	5	+ .29	+ .22	+ .22	+ .41	+ .29	+ .31	+ .59	+ .40	+ .38
	6	+ .24	+ .20	+ .16	+ .35	+ .24	+ .20	+ .46	+ .26	+ .15

In comparison of Conditions I and III, that simultaneous effect of validity coefficient ($\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$ versus $\rho_{wx} = \rho_{wy} = 0.50$) and variability ($\sigma_y = 2 \sigma_x$ versus $\sigma_x = \sigma_y$) on the shared variance of change score model with w was assessed (see Table 5.5). Shared variance of Model 2 with w slightly lagged the shared variance of Model 1. In the given condition, the increments in the shared variance of Model 1 with w for $\rho_{xy} = 0.25$ were 29, 22 and 20%, while the corresponding values for Model 2 were 24, 19 and 17%. The observed increments in the shared variance of Model 2 with w are all due to unequal validity coefficients for x and y, while the effect of variability has been deducted from it. But for Model 1, the increments are due to both validity coefficient and variability effect. This is clear when comparing lines 1, 2 and 3 of Table 5.4. For $\rho_{xy} = 0.50$ and $\rho_{xy} = 0.75$, shared variance of Model 2 with w also lagged slightly behind the shared variance of Model 1 when the simultaneous effects of variability and validity coefficient were taken into consideration. The same interpretations were applicable to this part as for $\rho_{xy} = 0.25$ as well.

The factor loadings in Table 4.5 (reported in the previous chapter) were used to determine the consistency of the construct definition by Models 1 and 2 across three ρ_{xy} levels of Conditions I and III when the effect of w was not considered. As the findings in this table indicate, in Condition I where $\rho_{xy} = 0.25$, Models 2 and 1 shared 83-86% of their variance in common. The same was true for $\rho_{xy} = 0.50$ and 0.75 (the shared variance of Model 1 and for these conditions were (0.85 - 86%) and (83 - 85%) respectively). In Condition III however Models 2 and 1 shared 94-96%, 94 and 88 % of their variance in common for $\rho_{xy} = 0.25, 0.50$ and 0.75 , respectively. These findings indicate a greater commonality in the underlying construct measured by Models 1 and 2 in Condition III relative to Condition I. As is clear in Condition 33, commonality of Models 1 and 2 slightly decreased (6-8% shrinkage in their shared variance) relative to Conditions 31 and 32, indicating more model differences between discrepancy and residual models such as Models 1 and 2 in Condition 33. Overall, the findings in Table 4.5 reveal that across three ρ_{xy} levels of Condition I (conditions

11, 12 and 13) 14 to 17% of the variance was unaccounted for by Models 1 and 2, while in Condition III the unaccounted variance by these two models were 4-6% , 6% and 12% across three ρ_{xy} levels indicating the greater consistency for the underlying construct defined by the stated models under conditions 31 and 32. In Condition 33 these models shared a smaller portion of their variance with each other. Part of the reason for this shrinkage in the commonality of Models 1 and 2 with respect to condition 31 and 32 comes from the differential impact of various parameter values (variability, colinearity and/or reliability coefficient). Changes in variability have a major impact on Model 1 but not on Model 2. The magnitude of such impact is presented in Table 4.7. Doubling the size of the variability of y in Condition III versus Condition I increased the shared variance of Model 1 with factor 1 by 27-30% for $\rho_{xy} = 0.25$, 29-31% for $\rho_{xy} = 0.50$ and 24-29% for $\rho_{xy} = 0.75$, while for Model 2 such fluctuations ranged from 0-5% across all ρ_{xy} levels.

Table 4.8 presented the reliability effects on the shared variance of the change score models. The shrinkage in the shared variance of Models 1 and 2 due to reducing the reliability of x and/or y components across three ρ_{xy} levels of Conditions I and III were almost the same, particularly in Condition III, indicating no differential impact of the reliability level on these two models. Table 4.9 also revealed the magnitude of the impact of the colinearity coefficient on the change score construct definition. Within the stated context, colinearity had no differential impact on Models 1 and 2 across all ρ_{xy} levels. Only in Condition I for $\rho_{xy} = 0.25$ did the colinearity coefficient slightly increase the shared variance of Model 2 with factor 1 and as ρ_{xy} increased, the differential impact of colinearity coefficient on Model 2 disappeared. The findings of this study in the previous chapter and in Table 5.3 indicated that most of the time the effect of parameter conditions on the change score model are interactive effects rather than a single effect of a specific parameter. This is clear in Table 5.3 in Condition III. In summary, these findings reveal that when w is included in the analysis Models 1 and 2 define the construct of change differently under the following conditions: Condition I for $\rho_{xy} = 0.25, 0.50$ and 0.75 and Condition III for $\rho_{xy} = 0.75$. Only in

Condition III where $\rho_{xy} = 0.25$ and 0.50 , do Models 1 and 2 have a great resemblance to each other regarding defining of the construct of change.

Since the findings in Table 5.3 revealed that Model 2 has a different validity coefficient (correlation with w) than Model 1 does under some of the parameter conditions, it was reasonable to look at the commonality of the change score models when w is partialled out from the analysis as well. The results of the factor loading from residualized change scores were presented in Table 5.5. In Condition I for $\rho_{xy} = 0.25$ Models 1 and 2 had (80-88%) of their variance in common across three reliability levels. The same was true for $\rho_{xy} = 0.50$, but for $\rho_{xy} = 0.75$ under unequal reliability coefficient for x and y (GO2), the shared variances of Models 1 and 2 with factor 1 decreased in magnitude. Under perfect reliability coefficient these models had 85% of their variance in common. For the $\rho_{xx'} = \rho_{yy'} = 0.90$ condition they shared 81% in common and for $\rho_{xx'} = 0.70$ and $\rho_{xx'} = 0.90$ they shared 74% of their variance in common, reflecting a minor effect from the reliability coefficient on the change score models. In Condition II ($\sigma_x = \sigma_y$ and $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$) for $\rho_{xy} = 0.25$ under $\rho_{xx'} = \rho_{yy'} = 0.90$ and 1.0 , Models 1 and 2 shared 82-83% of their variance in common, but under $\rho_{xx'} \neq \rho_{yy'}$ they shared 79% of their variance. As we increased the colinearity coefficient to 0.50 and 0.75 , the commonality of Models 1 and 2 somehow decreased in magnitude across three reliability coefficients. For $\rho_{xy} = 0.50$ Models 1 and 2 shared 79-81% of their variance in common under $\rho_{xx'} = \rho_{yy'} = 0.90$ and 1.0 , but under $\rho_{xx'} \neq \rho_{yy'}$ condition they shared 74-77% of their variance. Again, this indicates the reliability effect on the commonality of change score models. For $\rho_{xy} = 0.75$ (Condition 23), commonality of all of the models decreased across all reliability levels. The range of shared variance of change score models with factor 1 of the principal component was 62-79%, indicating the impact of the colinearity coefficient on the change score models as well as the reliability level under the extreme conditions such as condition 23. For $\rho_{xx'} = \rho_{yy'} = 1.0$, Models 1 and 2 shared 71-72% of their variance in common. For $\rho_{xx'} = \rho_{yy'} = 0.90$ they shared 72-77% of their variance

Table 5.5
 Shared Variance of Models 1 and 2 When W Was Included
 in and Removed From the Analyses

Original change scores										
Condition	model	$\rho_{xy} = 0.25$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.75$		
		GT	GO1	GO2	GT	GO1	GO2	GT	GO1	GO2
I	1	83%	81%	77%	85%	83%	79%	83%	79%	74%
	2	86%	85%	83%	86%	85%	83%	85%	81%	76%
II	1	83%	81%	77%	85%	83%	79%	83%	79%	74%
	2	86%	85%	83%	86%	85%	83%	85%	81%	76%
III	1	94%	92%	90%	94%	92%	90%	88%	86%	85%
	2	96%	92%	92%	94%	92%	92%	88%	86%	85%
Residualized change scores										
I	1	86%	86%	81%	86%	85%	81%	85%	81%	74%
	2	88%	86%	83%	88%	85%	83%	85%	81%	76%
II	1	81%	83%	79%	79%	81%	77%	71%	77%	71%
	2	83%	81%	79%	81%	79%	74%	72%	72%	62%
III	1	88%	90%	88%	86%	88%	86%	72%	79%	77%
	2	88%	92%	88%	86%	88%	86%	71%	79%	77%

in common and for $\rho_{xx'} \neq \rho_{yy'}$ they shared 62-71% of their variance in common. The lower limit belonged to the shared variance of Model 2 with the first factor of the principal component, indicating the greater impact of the reliability coefficient on Model 2 than on Model 1. In Condition III for $\rho_{xy} = 0.25$, commonality of the change score models increased in magnitude, i.e., all selected change score models shared 88-90% of their variance in common with other models. But as we increased the colinearity coefficient the commonality of the change score models decreased in magnitude, indicating the effect of colinearity coefficient on the definition of the change score construct. For example, for $\rho_{xy} = 0.50$ in this condition, Models 1 and 2 shared 86-88% of their variance in common under three reliability levels. Under the extreme ρ_{xy} condition ($\rho_{xy} = 0.75$), however, commonality of the change score models decreased in magnitude (see Table 5.5) and Models 1 and 2 shared 71-72% of their variance in common under perfect reliability conditions. For $\rho_{xx'} = \rho_{yy'} = 0.90$ they shared 79% of their variance and for $\rho_{xx'} \neq \rho_{yy'}$ they shared 77% of their variance in common.

Overall, the findings on Model 2 indicated that from a psychometric point of view Model 2 is as strong as or stronger than Model 1 except under very extreme conditions such as condition 33. From a stability point of view Model 2 is more consistent (or stable) in estimation of change across various parameter conditions included in this project. From a predictive validity point of view, Model 2 is stronger model than Model 1 across all ρ_{xy} levels of conditions I and much stronger than Model 1 across all ρ_{xy} levels of Condition II. In Condition III when $\rho_{xy} = 0.25$, Model 2 is still a dominant model regarding the correlation with the third variable but for $\rho_{xy} = 0.50$ (condition 32), the validity coefficients of Models 2 and 1 are either the same or are very close and for $\rho_{xy} = 0.75$ Model 2 loses its strength over Model 1 and its validity coefficient is lower than the validity coefficient of Model 1.

b. Model 6 versus Model 1

Model 6 (the base free measure of change or estimated time residual gain) originally was proposed to eliminate both unreliability and the regression effect of the raw gain score by partialing out the true x rather than the observed x from the y component. In condition 11 Model 6 is a little more reliable than Model 1, particularly under $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition. From a variability point of view, however, Model 6 has less variability than Model 1 did. From predictive validity point of view Model 6 was not stronger than Model 1 under most of the reliability and ρ_{xy} levels of Condition I. Only when $\rho_{xy} = 0.25$ Model 6 shared 14, 10 and 8% of its variance with w under $\rho_{xx'} = \rho_{yy'} = 1.0$, 0.90 and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ respectively. The shared variance of Model 6 with w however, decreased in magnitude along incrementing the ρ_{xy} level so that for $\rho_{xy} = 0.75$ Model 6's shared variance with w reached zero under $\rho_{xx'} \neq \rho_{yy'}$ condition. The shared variance of Model 1 with w was zero percent across all ρ_{xy} and reliability levels.

In Condition II Model 6 shared a much greater portion of its variance with w than Model 1 did under $\rho_{xy} = 0.25$ and 0.50 (see Table 5.3) particularly for $\rho_{xy} = 0.25$ condition. For $\rho_{xy} = 0.75$ Model 6's shared variance with w was 20% larger than the shared variance of Model 1 with w under $\rho_{xx'} = \rho_{yy'} = 1.0$. This difference in the shared variance of the two Models 1 and 6 however, decrease to 8% and 0.0% under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ respectively. The fluctuation in the shared variance of Model 6 with w across the three ρ_{xy} levels of Condition II reflect the impact of colinearity and reliability coefficients on Model 6's shared variances with w . In condition 23 the interactive effect of colinearity and reliability coefficient influences Model 6 the predictive validity power of Model 6 over Model 1 diminishes and both models share only 14% of their variance with w (see Table 5.3).

In Condition III for $\rho_{xy} = 0.25$ Model 6 shared 9, 7 and 4% more of its variance with w than Model 1 did across three reliability levels. For $\rho_{xy} = 0.50$ Model 6 shared the same degree of its variance with w across all three reliability levels. For $\rho_{xy} =$

0.75 Model 6's shared variance with w shrank in size by 9, 11 and 21% across three reliability levels. The decreasing trend of shared variance of Model 6 with w across the three ρ_{xy} levels of this condition clearly reflect the differential impact of colinearity coefficients on Model 1 and 6 as they relate to the third variable (w). The shared variance of Model 1 with w had an increasing trend across three ρ_{xy} levels of this condition. Table 5.4 represents the magnitude of the specific effects of validity coefficient, variability for x and y components and interactive effects of these parameters on the change score models. The inspection of the findings from the comparison of the shared variance of Model 6 with w for Condition II versus I reveals that changes in the validity coefficient for x and y components had greater impact on the shared variance of Model 6 relative to Model 1. The findings in this table also reveals that changes in the variability of y had little influence on Model 6 shared variance with w. The comparison of the shared variance of Condition III versus Condition I reflects the interactive effect of validity coefficients and variability on Model 6 and 1's shared variance with w. Changing the validity coefficient i.e., from $\rho_{wx} = \rho_{wy} = 0.50$ to $\rho_{wx} = 0.30$ and $\rho_{wy} = 0.70$, increased the shared variance of Model 6 with w by 17, 14 and 13% greater than the shared variance of Model 1 with w across three reliability levels of $\rho_{xy} = 0.25$ condition. For $\rho_{xy} = 0.50$ the increment in the shared variance of Model 6 with w was 18, 12 and 10% across three reliability levels. For $\rho_{xy} = 0.75$ the shared variance for Model 6 over Model 1 was 16, 7 and 0.0% across three reliability levels. These findings reflect the interactive effect of colinearity and reliability coefficient with validity coefficients on the shared variance of Model 6 with w. That is under the extreme condition of ρ_{xy} level and $\rho_{xx'} \neq \rho_{yy'}$ differential impact of the validity coefficients of the x and y components disappeared. The increment of the variability of y in Condition III had little effect on shared variance of Model 6 but it increased the shared variance of Model 1 by 11-17%, 16-22% and 22-28% across reliability and ρ_{xy} levels.

From the stability point of view Model 6 appeared to be the least stable Model among all other change score models regarding its resemblance to Model 1

under various parameter conditions.

The consistency of the construct definition by Model 6 and 1 were investigated in a factor analytic context and the factor loadings on the first principal component were reported in the previous chapter in Tables 4.5 and 4.12. The data in the former table represents the results when w was considered in the analyses and the latter table's data represents the loadings of the change score when w was removed from the analyses. The shared variance of Models 1 and 6 are presented in Table 5.6. The findings in this table determines the degree of commonality of Models 1 and 6 for different parameter conditions for both when w was and was not included in the analyses.

In the factor analysis, loadings of Model 6 under both reliability conditions, i.e., $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$, were higher than the loadings of Model 1. These differences corresponded with the differences in the reliability coefficients for these two Models in Table 5.1.

In condition 12 Model 6 is a little more reliable than Model 1 and in factor analysis loading of Model 6 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ are a little larger than the loadings of Model 1. From a variability point of view, variability of Model 6 is less than variability of Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ conditions, but from a potential predictive validity point of view, Model 6 has much higher validity coefficient. From a construct definition point of view, however, these models were defined differently regarding the contribution of x and y components into the definition of the underlying change. In Model 6 y is mainly the contributing variable for defining the underlying change. Regardless of the way that Model 6 is defined, it seems Model 6 is doing a better job in a correlational context than Model 1.

In condition 13, Model 6 has higher reliability coefficient than Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ condition, but smaller reliability coefficient under $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). In factor analysis loading of Model 6 was a little higher than the loading of Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$. Variability of Model 6 was a

Table 5.6

Shared Variance of Models 1 and 6 When W Was Included
in and Removed From the Analyses

Original change scores										
Condition	model	$\rho_{xy} = 0.25$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.75$		
		GT	GO1	GO2	GT	GO1	GO2	GT	GO1	GO2
I	1	83%	81%	77%	85%	83%	79%	83%	79%	74%
	6	86%	86%	90%	86%	88%	90%	85%	85%	76%
II	1	83%	81%	77%	85%	83%	79%	83%	79%	74%
	2	86%	86%	90%	86%	88%	90%	85%	85%	76%
III	1	94%	92%	90%	94%	92%	90%	88%	86%	85%
	2	96%	94%	94%	94%	92%	86%	88%	77%	58%
Residualized change scores										
I	1	86%	85%	81%	86%	85%	81%	85%	81%	74%
	6	88%	88%	90%	88%	88%	88%	85%	85%	76%
II	1	81%	82%	79%	79%	81%	77%	71%	77%	71%
	6	82%	82%	86%	81%	84%	84%	72%	79%	71%
III	1	88%	90%	88%	86%	88%	86%	72%	79%	77%
	6	88%	90%	90%	86%	88%	83%	71%	69%	50%

little smaller than variability of Model 1 when reliability coefficients for x and y were less than perfect. Validity coefficient of Model 6 under $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xy} = 0.90$. The validity coefficient of Model 6 got close to the validity coefficient of Model 1 (0.03 versus 0.00) under $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. It seems that the predictive power of Model 6 over Model 1 decreases under extreme conditions such as condition 13 (where $\sigma_x = \sigma_y$, $\rho_{wx} = \rho_{wy}$ and $\rho_{xy} = 0.75$), particularly under unequal reliability conditions. Thus, for practical purposes in the correlational context, the gain from the application of Model 6 versus Model 1 may not be that much where ρ_{xy} is greater than 0.50 and $\rho_{xx'} \neq \rho_{yy'}$. Overall, in Condition I as colinearity (or coefficient of stability) increased, the contribution of x into the definition of the underlying change measured by Model 6 also increased in magnitude and reliability and coefficient of stability (colinearity) had an interactive effect on the base free measure of change (Model 6).

In condition 21, Model 6 had higher reliability coefficient than Model 1, particularly under $\rho_{xx'} \neq \rho_{yy'}$ condition. In factor analysis, the magnitude of the loading for Model 6 on the first factor of the principal component exceeded the magnitude of the loading for Model 1. These differences correspond to the differences in the reliability level of Models 1 and 6. Variability of Model 6, however, was smaller than variability of Model 1 (under three reliability levels, i.e., $\rho_{xx'} = \rho_{yy'} = 1.0$; $\rho_{xx'} = \rho_{yy'} = 0.90$; and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). The predictive validity of Model 6 was about twice as high than the predictive validity of Model 1 and from a construct definition point of view, Model 6 was defined differently than Model 1 as in condition 11. The large variations in the validity coefficient of Model 6 versus validity coefficient of Model 1 were attributable to both unequal validity coefficients for x and y ($\rho_{wx} \neq \rho_{wy}$) and differences in the underlying construct measured by the stated model. This was confirmed by comparison of the results from conditions 11 and 21. Overall, in the given condition ($\sigma_x = \sigma_y$, $\rho_{wx} = \rho_{wy}$ and $\rho_{xy} = 0.25$), Model 6 is as powerful as Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and more powerful than Model 1 under $\rho_{xx'} = 0.70$ and $\rho_{yy'}$

= 0.90 from the psychometric point of view. From the predictive validity point of view, Model 6 turned out to be twice as strong as Model 1, indicating the advantage of using Model 6 over Model 1 under the given parameter conditions.

In condition 22 ($\sigma_x = \sigma_y$, $\rho_{wx} \neq \rho_{wy}$ and $\rho_{xy} = 0.50$), Model 6 is a little more reliable than Model 1 under both $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions. In the factor analysis loading of Model 6 was higher than the loading of Model 1 on the first factor of the principal component, variability of Model 6. Variability of Model 6, however, was lower than the variability of Model 1 under three reliability conditions. From a predictive validity point of view, Model 6 was much stronger than Model 1. For these two Models, however, x and y had different contributions into the definition of change. In Model 1, x and y contributions into the definition of change were close in magnitude, but in Model 6 y was the main contributing component into the definition of change, particularly under $\rho_{xx'} = \rho_{yy'} = 1.0$ and $\rho_{xx'} = \rho_{yy'} = 0.90$ conditions. Under unequal reliability conditions ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), the contribution of x into the definition of change in Model 1 dominated contribution of y, while in Model 6 this order was reversed and the contribution of y was much much higher than the contribution of x. Overall, differences in the underlying dimension defining the two models and the variations in the validity coefficients of x and y had an interactive effect on the validity coefficient of Models 1 and 6. Regardless of differences in the underlying construct defined by these two models, Model 6 is a stronger model from the psychometric and predictive validity points of view. Differences in the validity coefficient of Model 6 and Model 1 were mainly due to differences in their underlying construct.

In condition 23 ($\sigma_x = \sigma_y$; $\rho_{wx} \neq \rho_{wy}$ and $\rho_{xy} = 0.75$) mode Model 6 was as reliable as Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ but less reliable than Model 1 under $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. In principal component analysis, however, Model 6 had the same loadings as Model 1 under three reliability conditions ($\rho_{xx'} = \rho_{yy'} = 1.0$; $\rho_{xx'} = \rho_{yy'} = 0.90$; and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). Variability of Model 6 was smaller than variability of Model 1 under three stated reliability conditions. Predictive validity

of Model 6, however, exceeded the predictive validity of Model 1 under $\rho_{xx'} = \rho_{yy'}$ = 1.0 and 0.90, but under unequal reliability conditions, i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$, validity coefficients of Model 6 and Model 1 were identical, indicating that under given parameter conditions Model 6 is a more powerful predictor only when $\rho_{xx'} = \rho_{yy'}$. From a construct definition point of view, Models 6 and 1 were defined differently only under $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 reliability conditions, but they are identical under $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions. Under $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90, Model 1 was defined by almost equal conditions of x and y while Model 6 was defined mainly by the contribution of the y component. Under unequal reliability coefficient ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), however, x and y contributions to the definition of the underlying change measured by Models 1 and 6 were identical but x contribution dominated the y's contribution. This indicates that only when $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) Models 1 and 6 measure the same underlying construct, but their validity coefficient is not as high as if $\rho_{xx'} = \rho_{yy'} = 0.90$. Overall, under given conditions some benefit can be gained from applying Model 6 versus Model 1 regarding predictive validity of the change score only under $\rho_{xx'} = \rho_{yy'}$ conditions.

In condition 31, Model 6 is a little more reliable than Model 1 under $\rho_{xx'} = \rho_{yy'}$ = 0.90 and it is as reliable as Model 1 under $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$. The loadings of Model 6 under stated reliability conditions ($\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) were the same as the loadings of Model 1. Variability of Model 6 was slightly smaller than variability of Model 1 under various reliability levels. From the predictive validity point of view, however, Model 6 was a stronger predictor than Model 1 under three stated reliability conditions. From a construct definition point of view Models 1 and 6 were defined differently. In both models the contribution of y into a definition of change dominated the contribution of x by various degrees.

In condition 32 Model 6 was as reliable as Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$, but it was a little less reliable than Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ conditions. In factor analysis the loading of Model 6 on the first factor of the principal component also was the same

as the loading of Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ condition but a little smaller than the loading of Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ condition. Variability of Model 6 was the same as variability of Model 1 under $\rho_{xx'} = \rho_{yy'}$ condition, but under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) variability of Model 6 was larger than variability of Model 1. From a predictive validity point of view Model 6 had almost the same validity coefficient as Model 1, but lower validity coefficient under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition. This indicates that excess variability in Model 6 versus Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ condition is only error variance that has been added to Model 6 and not the true individual changes. In other words, under given parameter conditions a correction made on Model 1 did not generate a more reliable change score or a stronger predictor in correlational context. From a construct definition point of view, Model 6 was defined differently than Model 1 (regarding the contributions of x and y into the definition of the underlying change) only under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition (i.e., x and y components had various degrees of contribution into the definition of Models 1 and 6). Overall, in condition 32 Model 6 is as strong as Model 1 only under $\rho_{xx'} = \rho_{yy'}$ reliability conditions.

In condition 33, Model 6 was a little less reliable than Model 1 under both $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions. In principal component analysis, Model 6 had smaller loading on the first factor of principal component than Model 1 under both $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ condition. Variability of Model 6 was smaller than variability of Model 1 under $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 conditions, but larger than variability of Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ ($\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition. The validity coefficient of Model 6, however, was smaller than the validity coefficient of Model 1 under three reliability levels ($\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$), indicating that excess variability added to Model 6 is only error variance and not a true individual change. In the stated conditions Models 6 and 1 are also defined differently. In Model 1 the contribution of y dominated the contribution of x under three reliability conditions. For Model 6

contribution of y into the definition of change also dominated the contribution of x, but with various degrees except under $\rho_{xx'} \neq \rho_{yy'}$ condition that x contribution into the definition of change dominated the y contribution.

The findings in Table 5.6 on the original change scores for Condition I revealed that Model 6 shared slightly higher portion of its variance in common with other models than Model 1 did. The excess shared variance of Model 6 over the shared variance of Model 1 was 3, 5 and 13% for $\rho_{xy} = 0.25$, 1, 5 and 11% for $\rho_{xy} = 0.50$, and 2, 6 and 2% for $\rho_{xy} = 0.75$. Clearly Model 6 is affected by both reliability and ρ_{xy} levels more than Model 1 does. When we look at the shared variance of Models 1 and 6 with the first factor of the principal component as a commonality index it is clear that Models 1 and 6 do not share a very large portion of their variance in common across three ρ_{xy} levels of Condition I. Indicating the inconsistency in which the two stated models defining the construct of change.

In Condition III however, for $\rho_{xy} = 0.25$ and 0.50 Models 6 and 1 had 90% or more of their variance in common except under $\rho_{xy} = 0.50$ for $\rho_{xx'} \neq \rho_{yy'}$ condition that Model 6 shared 86% of its variance with other change scores. Under the extreme ρ_{xy} condition (i.e., $\rho_{xy} = 0.75$) Models 6 and 1 shared 88% of their variance in common under $\rho_{xx'} = \rho_{yy'} = 1.0$ condition. But as the measurement error were introduced to the data the two models became far apart regarding their consistency for defining the construct of change. That is for $\rho_{xx'} = \rho_{yy'} = .90$ shared variance of Model 6 with the first factor of the principal component lagged the shared variance of Model 1 by 9% and for $\rho_{xx'} \neq \rho_{yy'}$ by 27%. The low commonality of Models 6 and 1 in condition 23 was due to greater effect of both reliability and colinearity on Model 6. In summary, the findings on the original change scores (i.e., when w was included in the analysis) for Models 1 and 6 in Table 5.6 indicate that the only time that the change score construct defined by Models 1 and 6 are highly consistent is when $\sigma_y = 2\sigma_x$, $\rho_{wy} \succ \rho_{wx}$, $\rho_{xy} \preceq 0.50$ and $\rho_{xx'} = \rho_{yy'} = 1.0$ or 0.90 (conditions 31 and 32). Thus for the practical purposes the researches are advice to pay attention to the

combination of their parameter sets and avoid the assumption of the consistency of the definition of change score construct by Models 1 and 6 unless it is confirmed by their data sets.

The lower part of Table 5.6 provides the shared variance of Models 6 and 1 for when w was removed from the analysis. The comparison of these variances with the variance of Models 1 and 6 from the original scores in the upper part of Table 5.6 determines the size of the impact of w on the definition of the change score construct by Models 6 and 1. The findings on the residualized change scores for Condition I reveal that the removal of w from the analyses do not have any effect on the Models 1 and 6. The reason was that Model 1 and 6 appeared to have a very small or no correlation with the third variable (w) in Table 5.4.

In Condition II removal of w from the analyses slightly decreased the shared variance of Model 6 with the first factor of the principal component. The maximum shrinkage in the shared variance of Model 6 due to removal of w effect from the analyses was 4, 6 and 13% across the three ρ_{xy} and reliability levels. Apparently, removal of w from the analyses interact with colinearity effect and ρ_{xy} levels. Under the extreme condition for ρ_{xy} level (condition 23) the commonality of Models 1 and 6 decreased relative to the commonality of these Models for condition 21, indicating the colinearity effect on the two models.

In Condition III removal of w from the analyses decreased the shared variance of both Models 1 and 6 (see Table 5.6). In this condition the commonality of Models 1 and 6 have been decreased due to removal of w as well as due to effect of other parameter conditions such as variability, colinearity, reliability and validity coefficients. That is as ρ_{xy} increased in magnitude the commonality of Models 1 and 6 decreased in size, indicating less consistency in the way in which the two models define the underlying construct of change. Reliability coefficient also had a differential effect on the shared variance of Models 1 and 6 with the first factor of the principal component (see condition 33). For example in condition 33 shared variance of Model 6 with the first factor of the principal component lagged the shared variance of Model 1 by 10

and 27% for $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ respectively. After removal of w from the analyses the high resemblance of Models 1 and 6 for defining the construct of change previously observed in conditions 31 and 32 for the original scores were less observable for the residualized Models 1 and 6. After removal of w from the analyses Models 1 and 6 shared at most 90% of their variances in common across three ρ_{xy} levels of Condition III. In general, after removal of w effect from the analyses the shared variance of Model 1 with the first factor of the principal component ranged 77 - 90% and for Model 6 ranged 50 - 90% across three ρ_{xy} and all reliability levels. As the ρ_{xy} level increased in size the shared variance of both Models 1 and 6 showed a decreasing trend (see the lower part of Table 5.6)

Finally, the findings from the residualized scores on Models 1 and 6 for Conditions I and III showed a high resemblance to each other, but the readers should not interpret their similarity as the consistency in the way in which Models 1 and 6 define the construct of change under Condition I and III. The reason is that the variances reported for Conditions I and III are influenced by different parameter values included in this analysis and any assumption regarding the similarity of the constructs defined by the Models 1 and 6 under the stated conditions is misleading and results in faulty conclusions.

In summary, the findings on the residualized scores for Models 1 and 6 reveal that w has differential impact on the definition of the change score construct depending to the combination of the parameter condition. Under the conditions such as $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$ (Condition I) the impact of w on the change score construct is absent or negligible. Only when $\sigma_x = \sigma_y$ and $\rho_{wy} \succ \rho_{wx}$ (Condition II) and $\sigma_y \succ \sigma_x$ and $\rho_{wy} \succ \rho_{wx}$ (Condition III) the impact of w on the change score construct should be part of the researchers' concern.

Overall, under the extreme parameter conditions such as condition 33 ($\sigma_y \succ \sigma_x$; $\rho_{wy} \succ \rho_{wx}$ and $\rho_{xy} = 0.75$), Model 6 turned out to be a weaker Model for the estimation of change than Model 1 from both a psychometric and a predictive validity point of view. For practical purposes, Model 6 is a poor choice with respect to

Model 1 under the extreme conditions such as conditions 13, 23, 32 and 33 where $\rho_{xx'} \neq \rho_{yy'}$. In condition 13 both Model 1 and 6 had extremely poor predictive validity and by applying Model 6 versus Model 1 practitioners are not going to gain that much regarding the predictive validity of the change score. In other words, neither of the models is a good predictor. Model 6 is as strong as Model 1 in condition 23 where $\rho_{xx'} \neq \rho_{yy'}$, but a weaker model in conditions 32 and 33. Under all of the conditions included in this study, Model 6 was defined differently regarding the contribution of x and y components into the definition of change except in condition 23 where $\rho_{xx'} \neq \rho_{yy'}$ and condition 32 where $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90. In the latter condition, definition of change score construct was close in the two stated models. Regardless of the differences in defining the underlying change scores measured by Models 1 and 6, if there is any actual gain to be obtained from application of Model 6 versus Model 1, it is in a correlational context under Condition I ($\sigma_x \succ \sigma_y$ and $\rho_{wx} \succ \rho_{wy}$) and II ($\sigma_x = \sigma_y$ and $\rho_{wy} \succ \rho_{wx}$) where $\rho_{xy} = 0.25$ and 0.50. In Condition III (where $\sigma_y \succ \sigma_x$ and $\rho_{wy} \succ \rho_{wx}$) where $\rho_{xy} = 0.25$, Model 6 is a stronger predictor than Model 1 but not as strong as in conditions I or II where $\rho_{xy} = 0.25$ and 0.50.

5.7.2 Estimated “true” gain scores versus Model 1

a. Model 3 versus Model 1

Model 3 is an estimated “true” change score obtained via correction by simple regression for error in x to be substituted for the raw change score or gain in the change score analysis. How well this model improves the measurement of change for practical purposes will be answered in the following discussion using the general criteria used in this project.

From the psychometric point of view, Model 3’s reliability was not that much different from the reliability of Model 1, but in the principal component analysis the loadings of Model 3 on the first factor of the principal component was higher than the loading of Model 1 in conditions I and II. In Condition III the loadings of models

1 and 3 are the same across three levels of Condition I (i.e., 11, 12 and 13). Model 3 had a little smaller variability under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ condition). The same conclusion held true for the three levels of Condition II as well. In Condition III, however, variability of Model 3 with respect to variability of Model 1 had a different pattern across three ρ_{xy} levels. In condition 31 Model 3's variability is smaller than the variability of Model 1, but in condition 32 the variability of Model 3 is a little smaller than the variability of Model 1 only under $\rho_{xx'} \neq \rho_{yy'}$ condition and in condition 33 Model 3's variability is a little larger than that of Model 1.

From a predictive validity point of view, Model 3 is doing a better job with respect to Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$) condition. Recall that across three ρ_{xy} levels of Condition I, the correlation of Model 1 with w was zero. In the stated condition, the correlation of Model 3 with w ranged from 0.02 to 0.06 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and from .09 to .12 under $\rho_{xx'} \neq \rho_{yy'}$ condition. In Condition II the validity coefficient of Model 3 also was a little higher than the validity coefficient of Model 1 under both $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ conditions across three ρ_{xy} levels. The range of the stated differences were close to the ranges stated in above. In Condition III the validity of Model 3 was a little larger than the validity coefficient of Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} \neq \rho_{yy'}$ conditions. The range of these differences was 0.01 to 0.02 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and 0.02 to 0.04 under $\rho_{xx'} \neq \rho_{yy'}$ conditions. Overall, the findings on Model 3 indicate that from a psychometric point of view Model 3 is not that much different from Model 1 and the gain in reliability coefficient of Model 3 relative to the reliability coefficient of Model 1 was very small. From a validity point of view, however, Model 3 seems to be a little stronger than Model 1, particularly under $\sigma_x = \sigma_y$ and $\rho_{xx'} \neq \rho_{yy'}$ conditions. In conditions with $\sigma_y > \sigma_x$, the gains in the validity coefficient of Model 3 versus the validity coefficient of Model 1 are not large in magnitude. In summary, corrections made on Model 3 improve the measurement of change from a predictive validity point of view, but overall gains are not large in magnitude. If there is any actual gain from

the application of this model it is under the condition that $\sigma_x = \sigma_y$ and $\rho_{xx'} \neq \rho_{yy'}$.

Overall, the gains from the application of estimated "true" gain score (Models 3, 4 and 5) regarding reliability or predictive validity of a change are not large in magnitude. However, if the practitioners want to select any of the three stated modified change scores, Model 3 should be their choice.

b. Model 4 versus Model 1

Model 4 is another estimated "true" gain score obtained through correction by simple regression for error in x and in y components (Cronbach & Furby, 1970). How well this model improves the measurement of change relative to Model will be discussed in the following section. From a psychometric point of view Model 4 was as reliable as Model 1, and in Condition III only under $\rho_{xx'} \neq \rho_{yy'}$ condition did the reliability coefficient of Model 4 slightly exceed the reliability coefficient of Model 1. In zero-order correlation matrix, since correlation of Model 4 with Model 1 was perfect or near to perfect, Model 4 was eliminated from the factor analysis. From the variability point of view, Model 4 had smaller variability than Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions across all parameter conditions. From a predictive validity point of view, the validity coefficient of Model 4 only exceeded the validity coefficient of Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ (i.e., $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$). These differences ranged from 0.5 to 0.8 across three ρ_{xy} levels of condition I; in Condition II they ranged from 0.5 to .10 and in Condition III they ranged from .01 to .03. In summary, corrections made on Model 4 somehow improved reliability and validity coefficients of Model 4 with respect to Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ conditions, but the overall gains in this adjustment were not large in magnitude. The predictive power of Model 4 over Model 1 was mainly observable in Condition II where $\rho_{xx'} \neq \rho_{yy'}$. If the practitioners prefer such a small amount of gain, then Model 1 should be their choice. Under those conditions, i.e., $\rho_{xx'} \neq \rho_{yy'}$, Model 4 has minor differences from Model 1 regarding its correlation with the third variable w which are due to differences in the contribution of x and y variables to the change score models.

c. Model 5 versus Model 1

The Lord (1963) procedure was primarily designed to estimate a true difference or gain score for each individual via regression technique. In this procedure Lord used observed pre- and post-measurement to estimate the true scores with the assumption of equality of error variance for initial and final scores. How well this model measures the change with respect to the raw gain score is going to be discovered as follows using the general criteria of this project. Model 5 is as reliable as Model 1 in all parameter conditions under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ except in conditions 13, 21, 31, 32 and 33 where $\rho_{xx'} \neq \rho_{yy'}$. In the latter conditions Model 5 is a little more reliable than Model 1.

In the zero-order correlation matrix Model 5 had a perfect or nearly perfect correlation with Model 1 under all the parameter conditions and for this reason Model 5 was eliminated from the factor analysis. The factor loadings of Model 5 are not going to be compared against the factor loadings of Model 1 in this discussion.

From a variability point of view, Model 5 had the same degree of variability as Model 1 under $\rho_{xx'} = \rho_{yy'} = 1.0$ condition and smaller variability than Model 1 under $\rho_{xx'} = \rho_{yy'} = 0.90$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$ conditions. In all of the parameter conditions (11-33) from a predictive validity point of view Model 5 had slightly higher correlation with w across three ρ_{xy} levels of Condition I (conditions 11, 12, 13) only under $\rho_{xx'} \neq \rho_{yy'}$ condition. Across three ρ_{xy} levels of Condition II (i.e., conditions 21, 22 and 23), Model 5 also had the same degree of correlation with w except when $\rho_{xx'} \neq \rho_{yy'}$ and $\rho_{xy} > 0.50$. In the latter condition, the validity coefficient of Model 5 is higher than the validity coefficient of Model 1. Across three levels of Condition III Model 5's validity coefficient only exceeded the validity coefficient of Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ condition. These differences, however, were not large in magnitude across conditions 31, 32 and 33. Overall, for practical purposes, Model 5 is as strong as Model 1 and a little stronger than Model 1 under $\rho_{xx'} \neq \rho_{yy'}$ conditions, mainly in an extreme condition such as condition 33. From a predictive validity point of view, Model 5 turned out to be a stronger model only in conditions 22 and 23 where

$\rho_{xx'} \neq \rho_{yy'}$. In conditions 32 and 33, where $\rho_{xx'} \neq \rho_{yy'}$, Model 5's predictive validity is a little larger than the validity coefficient of Model 1, but these differences are not large in magnitude. In summary, the gains from Model 5 relative to Model 1 are not that great practice and practitioners may as well adopt raw gain score (Model 1) for measurement of change. There were minor differences in the way that models 1 and 5 were defined by the contributions of x and y components, mainly under $\rho_{xx'} \neq \rho_{yy'}$ condition.

5.8 Summary

Overall, the following conclusions can be drawn from the findings of this study:

1. Model differences do exist between the change scores under some of the parameter conditions, mainly in Conditions I and II for $\rho_{xy} = 0.25, 0.50$ and in condition III for $\rho_{xy} = 0.75$, particularly under unequal reliability coefficients for x and y variables.
2. Estimated true residual gain (Model 6) is as good as raw gain score (Model 1) or raw residual gain (Model 2), depending on the combination of different parameter conditions such as variability, reliability and coefficient of colinearity.
3. Only under rare parameter conditions such as condition 32 (where $\rho_{xx'} = \rho_{yy'}$) do all of the change score models measure the same underlying change score.
4. Changes in variability of x and y affect the underlying dimensions of the discrepancy models and not the residual ones.
5. Under equal variability conditions $\sigma_x = \sigma_y$, residual models are more powerful models in explaining the underlying variability of the change than the discrepancy change score models while under unequal variability condition $\sigma_y \succ \sigma_x$ this order was reversed.

6. Changes in reliability and colinearity coefficients for x and y components mainly affected the residual models rather than the discrepancy models. Changes in ρ_{xy} level (or colinearity coefficient) had a differential impact on residual models. For example, Model 6 (Base free measure of change) is more susceptible to these changes than Model 2 (raw residual gain).
7. In addition to their single effects variability, reliability and colinearity have differential interactive effects on the definition of the underlying change depending on the combination of the parameter models.
8. Among the change score models included in this study, Model 2 was the most stable model for the estimation of change across various parameter conditions.
9. While the congruency of the change score models as it is measured by Pearson correlations and factor loadings can be very high, the input of the change score model into the correlational context can vary depending on the magnitude of the validity coefficients for x and y components.
10. Change score models have high correlation with w under suppression conditions where the underlying model of change is defined by a discrepancy model, yet, x and y components have different contribution into the definition of underlying change.
11. Removal of the effect of w from the change score models can have a considerable effect on the definition of the underlying change depending on the combination of parameter values. Under equal variability and validity coefficient for x and y components the effect of w on the definition of the change are negligible .

5.9 Implications and recommendations

If we generalize the findings of this project on the raw gain score model (y-x) to the construct definition concept regarding the use of “discrepancy” or “difference

score” composites (i.e., two-part indices) in a correlational context, it is clear that the definition of the underlying construct measured by difference score such as (y-x) will vary depending on the parameter conditions. For example, in Table 4.3 we observed that the contributions of x and y into the definition of underlying construct measured by Model 1 (y-x) varied from one condition to another. Furthermore, the degree of x’s contribution to the definition of the difference score (y-x) versus y’s contribution varied within each parameter condition so that x’s contribution ranged from less than to greater than y’s contribution, and the lower tail of the stated range was at zero. To relate these findings to two-part indices constructs such as Self-regard = Real - Ideal self where Real self can be treated as y and Ideal self as x, we realize how the definition of the stated construct will vary from one parameter condition to another. In condition 11, for example, where $\sigma_x = \sigma_y$; $\rho_{xy} = 0.25$ and $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.90 and $\rho_{xx'} \text{ self} = 0.70$ and $\rho_{yy'} = 0.90$, contribution of real and ideal self into a definition of self-regard are similar in magnitude, while in condition 32 where $\sigma_y \succ \sigma_x$ and $\rho_{xy} = 0.75$, contribution of Ideal self (x) into the definition of self-regard construct is zero or near to zero under the following reliability conditions: $\rho_{xx'} = \rho_{yy'} = 1.0$ and 0.9 .

In conditions 13 and 23, where $\sigma_x = \sigma_y$; $\rho_{xy} = 0.75$ and $\rho_{xx'} = 0.70$ and $\rho_{yy'} = 0.90$, contribution of Ideal self (x) into the definition of the self-regard construct exceeds the contribution of the real-self (y). Furthermore, when we use the composite score of the self-regard as a predictor or criterion in a correlational context with given parameter conditions such as:

Condition I) $\sigma_x = \sigma_y$ and $\rho_{wx} = \rho_{wy}$

Condition II) $\sigma_x = \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$

Condition III) $\sigma_x \neq \sigma_y$ and $\rho_{wx} \neq \rho_{wy}$

where $\rho_{xy} = 0.25, 0.50$ and 0.75 (i.e., $\rho_{\text{Real, Ideal self}} = 0.25, 0.50$ and 0.75)

there is not going to be any meaningful relationship between the self-regard construct and the outside variable (w) in Condition I stated above (recall that all the

correlations of Model 1 with w were zero or near zero in Table 4.3).

In conditions II and III where $\rho_{xy} = 0.25, 0.50$ and 0.75 , correlation of self-regard with w is reasonable but from a regression point of view the only time that the construct of self-regard is defined as a discrepancy score and not an additive model is the time that the construct is defined by a suppression condition (negative suppression), i.e., one of the components acts as a suppressor for the other in the regression model. These parameter conditions were conditions II and III where $\rho_{xy} = 0.50$ and 0.75 . In conditions II and III where $\rho_{xy} = 0.25$, the self-regard concept is actually defined as an additive model and not a discrepancy one in the regression model.

Overall, the findings of the current project are consistent with the findings from Raeissi and Glasnapp (1985) and Glasnapp (1984). When practitioners select Model 1 (raw gain score) for the measurement of change, they will be faced with three potential problems: 1) misspecification of the model (loss of the relative importance of individual variables forming the composite); 2) predictive information loss; or 3) presence the moderate or extreme suppression conditions in the in the data.

Results from Conditions I and II indicate that raw gain scores are defined by equal contributions of x and y , but the effective weights of x and y are not 1 and -1.

The equality of the contributions of x and y into the definition of change (to whatever degree) is only guaranteed under the $\sigma_x = \sigma_y$ condition.

In condition III where $\sigma_y > \sigma_x$, y always dominates the contribution of x except under rare conditions for Model 6.

Thus, in Condition III, assignment of equality of weights (1 and -1) to the components of change result in misspecification of model and loss of predictive information.

The only time that the underlying model of change is a discrepancy model is when the parameter conditions define regression suppression condition.

Under these suppression conditions, x and y may or may not have equal contributions into the definition of change.

Once again, the results of this study lead to a conclusion that change scores in the form of any of the models are not sufficiently stable across research conditions to provide confidence in their use. It is recommended that the researchers examine their data in light of the parameter conditions studied to decide if use of a particular change score model has any potential utility in correlations with a third variable. In any event, those conditions most favorable to change scores are rare in practice and use of a single variable (y) will result in an equal amount of information. In less favorable conditions, an information increase can only be obtained by allowing both variables (x and y) to operate freely in a regression context to define the dominating linear composite in the data when relating to a third variable.

5.10 Limitation of the Study

The results obtained from the current project are only applicable to the situations similar to the parameter values selected for this project. While the values of ρ_{xy} were manipulated for a limited number of points, the values did span the possible range at appropriate intervals. This was not true for the manipulation of the reliability coefficients which were in a restricted range, particularly for y . The reliability effects may have been more pronounced if lower reliabilities were used for both x and y .

In addition, the distribution shape (normal) of the simulated score distributions presents another limitation. In real data, distribution shapes are often non-normal, thus potentially affecting the resulting observed change score distributions. What effect this might have on the results is unanswered in this study. How change score models behave under other parameter conditions besides what has been investigated in this document is a question that needs to be investigated. The high cost of the simulation restricted the choice of this researcher for the time being.

APPENDICES

APPENDIX A

IMSL ROUTINE NAME	GGNSM
PURPOSE	- MULTIVARIATE NORMAL RANDOM DEVIATE GENERATOR WITH GIVEN COVARIANCE MATRIX
USAGE	- CALL GGNSM (DSEED, NR, K, SIGMA, IR, RVEC, WKVEC, IER)
ARGUMENTS	<p>DSEED - INPUT/OUTPUT DOUBLE PRECISION VARIABLE ASSIGNED AN INTEGER VALUE IN THE EXCLUSIVE RANGE (1.00, 2147483647.00). DSEED IS REPLACED BY A NEW VALUE TO BE USED IN A SUBSEQUENT CALL.</p> <p>NR - INPUT. NUMBER OF K-DEVIATE VECTORS TO BE GENERATED.</p> <p>K - INPUT. NUMBER OF RANDOM DEVIATES PER VECTOR.</p> <p>SIGMA - INPUT/OUTPUT VECTOR OF LENGTH $K(K+1)/2$. ON INPUT SIGMA CONTAINS THE VARIANCE-COVARIANCE VALUES. SIGMA IS A POSITIVE DEFINITE MATRIX STORED IN SYMMETRIC STORAGE MODE. AFTER THE FIRST CALL TO GGNSM, SIGMA IS REPLACED BY ITS FACTOR (SQUARE ROOT) ON OUTPUT. (SEE REMARKS)</p> <p>IR - INPUT. ROW DIMENSION OF MATRIX RVEC EXACTLY AS SPECIFIED IN THE DIMENSION STATEMENT IN THE CALLING PROGRAM.</p> <p>RVEC - OUTPUT. NR BY K MATRIX OF MULTIVARIATE NORMAL DEVIATES.</p> <p>WKVEC - INPUT WORK VECTOR OF LENGTH K. WKVEC(1) SHOULD BE SET TO 0.0 ON THE FIRST OF A SERIES OF CALLS TO GGNSM. FOR ALL SUBSEQUENT CALLS, WKVEC(1) SHOULD BE NONZERO. IF ONLY ONE CALL IS REQUIRED, SET WKVEC(1) TO 0.0. (SEE REMARKS) THE REMAINDER OF WKVEC IS USED AS WORK AREA FOR THE NORMAL DEVIATE GENERATION.</p> <p>IER - ERROR PARAMETER. (OUTPUT) TERMINAL ERROR IER = 129 INDICATES THAT INPUT MATRIX SIGMA IS ALGORITHMICALLY NOT POSITIVE DEFINITE.</p>
PRECISION/HARDWARE	- SINGLE/ALL
REQD. IMSL ROUTINES	- GGNML, GGUBS, MDNRIS, MERFI, UERTST, UGETIO
NOTATION	- INFORMATION ON SPECIAL NOTATION AND CONVENTIONS IS AVAILABLE IN THE MANUAL INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
REMARKS	IF THE USER WISHES TO CONTINUE GENERATING MULTIVARIATE NORMAL DEVIATE VECTORS DISTRIBUTED WITH THE SAME SIGMA, THEN MULTIPLE CALLS MAY BE MADE TO GGNSM WITH WKVEC(1) NONZERO ON INPUT. WKVEC(1) SET TO 0.0 ON INPUT TRIGGERS THE CALCULATION OF THE FACTOR (SQUARE ROOT) OF SIGMA.

June, 1980

GGNSM-1

APPENDIX A (continued)

Algorithm

GGNSM generates NR vectors of K multivariate normal deviates, distributed with zero mean and covariance matrix SIGMA.

The triangular factorization method is used in generating a matrix of deviates, RVEC, of dimension NR by K. The symmetric matrix SIGMA is factored into LL^T using subroutine LUDECP which is coded in-line. Subroutine GGNML generates NR normal (0,1) deviates, in say, matrix X. Then RVEC is set to XL^T .

Programming Notes

GGNSM should be called initially with WKVEC(1)=0.0 on input so that the SIGMA matrix can be properly initialized. For all subsequent calls to GGNSM, WKVEC(1) should be nonzero and the user need not worry about setting WKVEC(1) unless a new SIGMA matrix is to be introduced. (In which case WKVEC(1) must be set to 0.0.) Otherwise, SIGMA should not be altered between calls.

Example

In this example GGNSM is called to generate ten vectors of length 2.

Input:

```

INTEGER          NR,K,IR,IER
REAL             SIGMA(3),RVEC(10,2),WKVEC(2)
DOUBLE PRECISION DSEED
DSEED = 466364003.D0
NR = 10
K = 2
SIGMA = (0.5,0.375,0.5)
IR = 10
WKVEC(1) = 0.0
CALL GGNSM (DSEED,NR,K,SIGMA,IR,RVEC,WKVEC,IER)
:
END
  
```

Output:

```

DSEED = 984906825.000
SIGMA = (0.70711,0.53033,0.46771)
RVECT = [ 1.081  .4702  .0120  .0664  -1.167  -1.499  -.6824  -.8090  -.2361  .3686
          .7216  .3530  .1637  .5430  -.8438  -1.123  -.0113  -.5437  .5396  .2278 ]
IER = 0
  
```

APPENDIX B

CORRELATION BETWEEN THE CHANGE SCORE MODELS

No	Corelation	Formula
1	$\rho(y - x)(y - x)$	$\frac{\sigma_y \sqrt{1 - \rho_{2y}^2}}{\sqrt{\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y}}$
2	$\rho(y - x)(y - \rho_{zx}x)$	$\frac{\sigma_y^2 - \rho_{zy}\sigma_z\sigma_y(1 + \rho_{zx}) - \rho_{zx}\sigma_z^2}{\sqrt{(\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y)(\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{zx}\rho_{zy}\sigma_z\sigma_y)}}$
3	$\rho(y \cdot x)(y - \rho_{zx}x)$	$\frac{\sigma_y \sqrt{1 - \rho_{2y}^2}}{\sqrt{\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{zx}\rho_{zy}\sigma_z\sigma_y}}$
4	$\rho(y - x)(y - \rho_{yy'}y - \rho_{zx}x)$	$\frac{\rho_{yy'}(\sigma_y^2 - \rho_{zy}\sigma_z\sigma_y) + \rho_{zx}(\sigma_z^2 - \rho_{zy}^2\sigma_y\sigma_z)}{\sqrt{(\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y)(\rho_{yy'}^2\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{yy'}\rho_{zx}\rho_{zy}\sigma_z\sigma_y)}}$
5	$\rho(y - b_{yx})(\rho_{yy'}y - \rho_{zx}x)$	$\frac{\sigma_y \rho_{yy'} \sqrt{1 - \rho_{2y}^2}}{\sqrt{\rho_{yy'}^2\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{yy'}\rho_{zx}\rho_{zy}\sigma_z\sigma_y}}$
6	$\rho(y - b_{zx})(\rho_{yy'}y - \rho_{zx}x - \rho_{zx}x)$	$\frac{\rho_{yy'}\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - \rho_{zx}\rho_{zy}(\sigma_z\sigma_y(1 + \rho_{yy'}(1 + \rho_{yy'})))}{\sqrt{(\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - \rho_{zx}\rho_{zy}\sigma_z\sigma_y)(\rho_{yy'}^2\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{yy'}\rho_{zx}\rho_{zy}\sigma_z\sigma_y)}}$
7	$\rho(y - x)(\beta_1 + \beta_2y)$	$\frac{\beta_1(\rho_{zy}\sigma_z\sigma_y - \rho_{zx}\sigma_z^2) - \beta_2(\rho_{zx}\rho_{zy}\sigma_z\sigma_y - \rho_y^2)}{\sqrt{(\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y)(\beta_1^2\sigma_z^2 + \beta_2^2\sigma_y^2 + 2\beta_1\beta_2\rho_{zy}\sigma_z\sigma_y)}}$
8	$\rho(y - b_{yx})(\beta_1x - \beta_2y)$ where $\beta_1 = \beta_{Dx-y} = \frac{(1 - \rho_{yy'})\rho_{zy}(\sigma_y) - \rho_{zx}\sigma_z + \rho_{zy}}{\sigma_x \sqrt{1 - \rho_{2y}^2}}$ $\beta_2 = \beta_{Dy-x} = \frac{\rho_{yy} - \rho_{zy}^2 - (1 - \rho_{zx})\rho_{zy}(\frac{\sigma_x}{\sigma_y})}{1 - \rho_{2y}^2}$	$\frac{\beta_1(\sigma_y - \rho_{zy}\sigma_z)}{\sqrt{1 - \rho_{2y}^2} \sqrt{\beta_1^2\sigma_z^2 + \beta_2^2\sigma_y^2 + 2\beta_1\beta_2\rho_{zy}\sigma_z\sigma_y}}$
9	$\rho(y - \rho_{zx}x)(\beta_1x + \beta_2y)$	$\frac{\beta_1(\rho_{zy}\sigma_z\sigma_y - \rho_{zx}\sigma_z^2) - \beta_2(\rho_{zx}\rho_{zy}\sigma_z\sigma_y - \rho_y^2)}{\sqrt{(\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - 2\rho_{zx}\rho_{zy}\sigma_z\sigma_y)(\beta_1^2\sigma_z^2 + \beta_2^2\sigma_y^2 + 2\beta_1\beta_2\rho_{zy}\sigma_z\sigma_y)}}$
10	$\rho(\rho_{yy'}y - \rho_{zx}x)(\beta_1x - \beta_2y)$ where $\beta_1 = \beta_{Dx \cdot y} = \frac{(1 - \rho_{yy'})\rho_{zy}(\sigma_y) - \rho_{zx}\sigma_z + \rho_{zy}}{\sigma_x \sqrt{1 - \rho_{2y}^2}}$ $\beta_2 = \beta_{Dy \cdot x} = \frac{\rho_{yy} - \rho_{zy}^2 - (1 - \rho_{zx})\rho_{zy}(\frac{\sigma_x}{\sigma_y})}{1 - \rho_{2y}^2}$	$\frac{\beta_1(\rho_{yy'}\rho_{zy}\sigma_y\sigma_z - \rho_{zx}\sigma_z^2) - \beta_2(\rho_{yy'}\sigma_y^2 - \rho_{zx}\rho_{zy}\sigma_z\sigma_y)}{\sqrt{(\rho_{yy'}^2\sigma_y^2 + \rho_{zx}^2\sigma_z^2 - \rho_{yy'}\rho_{zx}\rho_{zy}\sigma_z\sigma_y)(\beta_1^2\sigma_z^2 + \beta_2^2\sigma_y^2 + 2\beta_1\beta_2\rho_{zy}\sigma_z\sigma_y)}}$
11	$\rho(y - x)(y_t - \beta_{yt \cdot xt}x_t)$	$\frac{\sigma_{yt} \sqrt{1 - \rho_{yt \cdot xt}^2}}{\sqrt{\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y}}$
12	$\rho(y - b_{xy \cdot z})(y_t - \rho_{zt \cdot yt}(\frac{\sigma_{yt}}{\sigma_{zt}})x_t)$	$\frac{\sigma_{yt} \sqrt{1 - \rho_{yt \cdot xt}^2}}{\sigma_y \sqrt{1 - \rho_{2y}^2}}$
13	$\rho(y - \rho_{zx}x)(y_t - \beta_{yt \cdot xt}x_t)$	$\frac{\sigma_{yt} \sqrt{1 - \rho_{yt \cdot xt}^2}}{\sqrt{(\sigma_y^2 + \sigma_z^2 - 2\rho_{zy}\sigma_z\sigma_y)(\sigma_{zt}^2 - 2\rho_{zy}\sigma_z\sigma_{yt})}}$
14	$\rho(\rho_{yy'}y - \rho_{zx}x)(y_t - \beta_{yt \cdot xt}x_t)$	$\frac{\rho_{zx}\sigma_{yt} \sqrt{1 - \rho_{yt \cdot xt}^2}}{\sqrt{(\sigma_y^2 \rho_{yy'}^2 + \sigma_z^2 \rho_{zx}^2 - 2\rho_{zy}\sigma_z\sigma_y \rho_{zx}\rho_{yy'})}}$

APPENDIX B (continued)

CORRELATION BETWEEN THE CHANGE SCORE MODELS

No	Correlation	Formula
15	$\rho(\beta_{1x} - \beta_{2y})(y_t - \beta_{y_t, x_t} x_t)$	$\frac{\beta_{xy} \sqrt{1 - \rho_{y_t, x_t}^2}}{\sqrt{\beta_1^2 \sigma_x^2 + \beta_2^2 \sigma_y^2 - 2\beta_1 \beta_2 \rho_{xy} \sigma_x \sigma_y}}$
16	$\rho(y - x)(y - b_{y, x}^*)$	$\frac{\sigma_y (1 - \frac{\rho_{xy}}{\rho_{xx'}}) - \rho_{xy} \sigma_x (1 - \frac{1}{\rho_{xx'}})}{\sqrt{\sigma_y^2 + \sigma_x^2 - 2\rho_{xy} \sigma_x \sigma_y} \sqrt{\rho_{xx'}^2 + \rho_{yy'}^2 (1 - \rho_{xx'})}}$
17	$\rho(y - b_{y, x}^*)(y - b_{y, x}^*)$	$\frac{(1 - \frac{\rho_{xy}}{\rho_{xx'}})}{\sqrt{1 - \rho_{xy}^2} \sqrt{\rho_{xx'}^2 + \rho_{yy'}^2 (1 - \rho_{xx'})}}$
18	$\rho(y - \rho_{xx'})(y - b_{y, x}^*)$	$\frac{\sigma_y (1 - \frac{\rho_{xy}}{\rho_{xx'}}) + \rho_{xy} \sigma_x (1 - \rho_{xx'})}{\sqrt{\sigma_y^2 + \sigma_x^2 \rho_{xx'}^2 - 2\rho_{xy} \sigma_x \sigma_y \rho_{xx'}} \sqrt{\rho_{xx'}^2 + \rho_{yy'}^2 (1 - \rho_{xx'})}}$
19	$\rho(\rho_{yy'} - \rho_{xx'})(y - b_{y, x}^*)$	$\frac{\rho_{yy'} \sigma_y (\rho_{xx'} - \rho_{yy}') + \rho_{xx'} \rho_{xy} \sigma_x (1 - \rho_{xx'})}{\sqrt{\sigma_y^2 \rho_{yy'}^2 + \sigma_x^2 \rho_{xx'}^2 - 2\sigma_y \rho_{xy} \rho_{xx'} \rho_{yy'}} \sqrt{\rho_{xx'}^2 + \rho_{yy'}^2 - \rho_{xy}^2 \rho_{xx'}}$
20	$\rho(\beta_{1x} - \beta_{2y})(y - b_{y, x}^*)$	$\frac{\beta_x (\rho_{xx'} \rho_{xy} \sigma_x - \rho_{xy} \sigma_x) - \beta_2 \sigma_y (\rho_{xy} - \rho_{xx'})}{\sqrt{\beta_1^2 \sigma_x^2 + \beta_2^2 \sigma_y^2 - 2\beta_1 \beta_2 \rho_{xy} \sigma_x \sigma_y} \sqrt{\rho_{xx'}^2 + \rho_{yy'}^2 - \rho_{xy}^2 \rho_{xx'}}$
21	$\rho(y - b_{y, x}^*)(y_t - \beta_{y_t, x_t} x_t)$ where $b_{y, x}^* = \rho_{xy} (\frac{\sigma_y}{\rho_{xx'} \sigma_x})$	$\frac{\sigma_y \sqrt{(1 - \rho_{y_t, x_t}^2)}}{\sigma_y \sqrt{(\rho_{xx'}^2 + \rho_{yy'}^2 - \rho_{xy}^2 \rho_{xx'})}}$
22	$\rho(y - x) (\frac{\rho_{xx'} \rho_{yy'} - \rho_{xy}^2}{\rho_{xx'} \rho_{yy'} (1 - \rho_{xx'})})$	$\frac{\sigma_y (1 + \rho_{xy}^2)}{\sqrt{(\sigma_y^2 + \sigma_x^2 - 2\rho_{xy} \sigma_x \sigma_y)} \sqrt{1 - \rho_{xy}^2}}$
23	$\rho(y_t - x_t)(y - x)$	$\frac{\beta_{y_t, x_t} \sigma_y^2 (1 - \rho_{xy}^2)}{\beta_{y_t, x_t} \sigma_y^2 (1 - \rho_{xy}^2)} = 1$
24	$\rho(\beta_{y_t, x_t} (y - \rho_{xy} (\frac{\sigma_x}{\sigma_y}) x) (y - \rho_{xx'} x))$	$\frac{\sigma_y \sqrt{1 - \rho_{xy}^2}}{\sqrt{\sigma_y^2 + \sigma_x^2 \rho_{xx'}^2 - 2\rho_{xx'} \rho_{xy} \sigma_x \sigma_y}}$
25	$\rho(\beta_{y_t, x_t} (y - \rho_{xy} (\frac{\sigma_x}{\sigma_y}) x) (\rho_{yy'} - \rho_{xx'}))$	$\frac{\rho_{yy'} \sigma_y \sqrt{1 - \rho_{xy}^2}}{\sqrt{\rho_{yy'}^2 \sigma_y^2 + \sigma_x^2 \rho_{xx'}^2 - 2\rho_{yy'} \rho_{xx'} \rho_{xy} \sigma_x \sigma_y}}$
26	$\rho(\beta_{y_t, x_t} (y - \rho_{xy} (\frac{\sigma_x}{\sigma_y}) x) (\beta_{1x} - \beta_{2y}))$	$\frac{\beta_2 \sigma_y \sqrt{1 - \rho_{xy}^2}}{\sqrt{\beta_1^2 \sigma_x^2 + \beta_2^2 \sigma_y^2 + 2\beta_1 \beta_2 \rho_{xy} \sigma_x \sigma_y}}$
27	$\rho(\beta_{y_t, x_t} (y - \rho_{xy} (\frac{\sigma_x}{\sigma_y}) x) (y - b_{y, x}^*))$	$\frac{\sqrt{1 - \rho_{xy}^2}}{\sqrt{\sigma_y^2 + \sigma_x^2 \rho_{xx'}^2 - 2\beta_{y_t, x_t} \sigma_x \sigma_y}}$
28	$\rho(\beta_{y_t, x_t} (y - \rho_{xy} (\frac{\sigma_x}{\sigma_y}) x) (y_t - \rho_{y_t, x_t} (\frac{\sigma_{y_t}}{\sigma_{x_t}}) x_t))$	$\frac{\sigma_y \sqrt{1 - \rho_{y_t, x_t}^2}}{\sigma_y \sqrt{1 - \rho_{xy}^2}}$

APPENDIX C

```
INTEGER NR,K,IR,IER
REAL SIGMA(45),RVEC(2000,9),wkvec(9)
DOUBLE PRECISION DSEED
DATA SIGMA/1.0,0.25,1.0,0.50,0.50,1.0,0.0,0.0,0.0,0.0,0.10,
&0.0,0.0,0.0,0.0,0.10,0.0,0.0,0.0,0.0,0.0,0.0,0.2,
&0.0,0.0,0.0,0.0,0.0,0.0,0.30,0.0,0.0,0.0,0.0,0.0,
&0.0,0.0,0.0,0.10,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
&0.20/
DSEED = 383764937.00
NR = 2000
K = 9
IR = 2000
WKVEC(1)=0.0
CALL GGNSM (DSEED,NR,K,SIGMA,IR,RVEC,WKVEC,IER)
DO 20 I=1,NR
WRITE(12,30)(RVEC(I,J),J=1,9)
30 FORMAT(1X,9F7.3)
20 CONTINUE
STOP
END
```

APPENDIX D

```

FILE HANDLE G11/NAME=*MERGE11 POURAN A*
GET FILE=G11/DROP=GT111 TO GO2117
COMPUTE RXY1=.254
COMPUTE SDX1=.989
COMPUTE SDY1=.990
COMPUTE RXX1=1.0
COMPUTE RYY1=1.0
COMPUTE GT111=TY11-TX11
COMPUTE GT112=TY11-(RXY1*(SDY1/SDX1)*TX11)
COMPUTE GT113=TY11-RXX1*TX11
COMPUTE GT114=RYY1*TY11-RXX1*TX11
COMPUTE B11=((1-RYY1)*(RXY1*(SDY1/SDX1))-RXX1+RXY1**2)/(1-RXY1**2)
COMPUTE B12=(RYY1-RXY1**2-(1-RXX1)*(RXY1*(SDY1/SDX1)))/(1-RXY1**2)
COMPUTE GT115=B11*TX11+B12*TY11
COMPUTE B13=RXY1*(SDY1/RXX1*SDX1)
COMPUTE GT116=TY11-B13*TX11
COMPUTE B14=(RXX1*RYY1-RXY1**2)/((RXX1*RXY1)*(1-RXY1**2))
COMPUTE GT117=B14*(TY11-(RXY1*(SDY1/SDX1)*TX11))
COMPUTE RXY2=.232
COMPUTE SDX2=1.041
COMPUTE SDY2=1.028
COMPUTE RXX2=.906
COMPUTE RYY2=.910
COMPUTE GO1111=OY111-OX111
COMPUTE GO1112=OY111-(RXY2*(SDY2/SDX2)*OX111)
COMPUTE GO1113=OY111-RXX2*OX111
COMPUTE GO1114=RYY2*OY111-RXX2*OX111
COMPUTE B21=((1-RYY2)*(RXY2*(SDY2/SDX2))-RXX2+RXY2**2)/(1-RXY2**2)
COMPUTE B22=(RYY2-RXY2**2-(1-RXX2)*(RXY2*(SDY2/SDX2)))/(1-RXY2**2)
COMPUTE GO1115=B21*OX111+B22*OY111
COMPUTE B23=RXY2*(SDY2/RXX2*SDX2)
COMPUTE GO1116=OY111-B23*OX111
COMPUTE B24=(RXX2*RYY2-RXY2**2)/((RXX2*RXY2)*(1-RXY2**2))
COMPUTE GO1117=B24*(OY111-(RXY2*(SDY2/SDX2)*OX111))
COMPUTE RXY3=.220
COMPUTE SDX3=1.134
COMPUTE SDY3=1.037
COMPUTE RXX3=.761
COMPUTE RYY3=.908
COMPUTE GO2111=OY112-OX112
COMPUTE GO2112=OY112-(RXY3*(SDY3/SDX3)*OX112)
COMPUTE GO2113=OY112-RXX3*OX112
COMPUTE GO2114=RYY3*OY112-RXX3*OX112
COMPUTE B31=((1-RYY3)*(RXY3*(SDY3/SDX3))-RXX3+RXY3**2)/(1-RXY3**2)
COMPUTE B32=(RYY3-RXY3**2-(1-RXX3)*(RXY3*(SDY3/SDX3)))/(1-RXY3**2)
COMPUTE GO2115=B31*OX112+B32*OY112
COMPUTE B33=RXY3*(SDY3/RXX3*SDX3)
COMPUTE GO2116=OY112-B33*OX112
COMPUTE B34=(RXX3*RYY3-RXY3**2)/((RXX3*RXY3)*(1-RXY3**2))
COMPUTE GO2117=B34*(OY112-(RXY3*(SDY3/SDX3)*OX112))
FILE HANDLE MG1/NAME=*MER11 P R*
SAVE OUTFILE=MG1/DROP=RXY1 SDX1 SDY1 RXX1 RYY1 B11 B12 B13 B14
RXY2 SDX2 SCY2 RXX2 RYY2 B21 B22 B23 B24
RXY3 SDX3 SDY3 RXX3 RYY3 B31 B32 B33 B34

```


GO1121	GO1122	GO1123	GO1124	GO1125	GO1126	GO2121	GO2122	GO2123	GO2124	GO2125	GO2126	TX13	TY13	TW13	OX131	OY131	OW131	OZ132	GT132	GW132																																			
1.00	0.84 1.00	1.00 0.88 1.00	1.00 0.84 1.00 1.00	1.00 0.83 1.00 1.00 1.00	0.90 0.99 0.93 0.89 0.89 1.00	0.78 0.65 0.78 0.78 0.78 0.70 1.00	0.67 0.85 0.71 0.66 0.66 0.84 0.80 1.00	0.78 0.75 0.79 0.78 0.78 0.78 0.98 0.91 1.00	0.78 0.72 0.79 0.78 0.78 0.75 0.99 0.87 1.00 1.00	0.78 0.70 0.79 0.78 0.78 0.74 1.00 0.85 0.99 1.00 1.00	0.76 0.60 0.78 0.76 0.76 0.61 0.95 0.95 0.99 0.98 0.97 1.00	1.00	0.76 1.00	0.50 0.50 1.00	0.95 0.72 0.47 1.00	0.72 0.95 0.48 0.68 1.00	0.45 0.46 0.91 0.43 0.43 1.00	0.88 0.66 0.43 0.84 0.62 0.39 1.00	0.71 0.95 0.48 0.68 0.91 0.44 0.62 1.00	0.44 0.45 0.91 0.42 0.42 0.83 0.38 0.43 1.00	0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	0.00 0.65 0.20 0.00 0.62 0.18 -0.01 0.63 0.17	-0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	-0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	0.74 0.61 0.74 0.74 0.74 0.66 0.67 0.58 0.68 0.68 0.68 0.67	-0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	0.74 0.61 0.74 0.74 0.74 0.66 0.67 0.58 0.68 0.68 0.68 0.67	-0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	0.74 0.61 0.74 0.74 0.74 0.66 0.67 0.58 0.68 0.68 0.68 0.67	-0.35 0.34 0.01 -0.33 0.33 0.01 -0.32 0.34 0.01	0.70 0.78 0.73 0.70 0.70 0.79 0.64 0.76 0.72 0.70 0.68 0.76	0.02 0.66 0.20 0.02 0.64 0.19 0.00 0.65 0.18	0.62 0.51 0.63 0.62 0.62 0.56 0.59 0.50 0.60 0.60 0.60 0.58	-0.30 0.28 0.00 -0.40 0.39 0.01 -0.28 0.28 0.00	0.59 0.69 0.62 0.59 0.59 0.70 0.56 0.69 0.65 0.62 0.60 0.68	0.09 0.62 0.21 0.00 0.73 0.19 0.06 0.61 0.18	0.62 0.57 0.63 0.62 0.62 0.60 0.59 0.56 0.62 0.61 0.61 0.62	-0.20 0.38 0.06 -0.30 0.49 0.06 -0.19 0.37 0.05	0.62 0.51 0.63 0.62 0.62 0.55 0.59 0.50 0.60 0.60 0.60 0.58	-0.31 0.28 0.00 -0.41 0.39 0.00 -0.28 0.28 -0.01	0.62 0.63 0.63 0.61 0.61 0.65 0.59 0.62 0.64 0.62 0.61 0.65	-0.31 0.27 0.00 -0.41 0.39 0.00 -0.28 0.27 -0.01	0.57 0.47 0.58 0.57 0.57 0.51 0.52 0.44 0.32 0.53 0.52 0.51	-0.08 0.49 0.12 -0.18 0.60 0.12 -0.08 0.47 0.10	0.54 0.72 0.58 0.54 0.54 0.71 0.49 0.71 0.60 0.56 0.54 0.65	-0.28 0.24 0.01 -0.27 0.24 0.01 -0.53 0.34 0.00	0.58 0.61 0.60 0.58 0.58 0.62 0.52 0.59 0.58 0.56 0.55 0.60	0.22 0.70 0.28 0.21 0.67 0.25 0.00 0.79 0.25	0.58 0.56 0.59 0.58 0.58 0.59 0.53 0.54 0.56 0.55 0.55 0.57	-0.04 0.48 0.14 -0.04 0.46 0.13 -0.29 0.58 0.12	0.58 0.52 0.59 0.58 0.58 0.55 0.53 0.50 0.55 0.54 0.54 0.55	-0.13 0.40 0.10 -0.12 0.38 0.09 -0.38 0.50 0.08	0.58 0.50 0.58 0.58 0.58 0.51 0.52 0.47 0.54 0.54 0.53 0.53	-0.19 0.33 0.06 -0.19 0.32 0.05 -0.45 0.43 0.65	-0.23 0.29 0.03 -0.22 0.28 0.03 -0.49 0.39 0.63

GT131	GT132	GT133	GT134	GT135	GT136	G01131	G01132	G01133	G01134	G01135	G01136	G02131	G02132	G02133	G02134	G02135	G02136	TX21	TZ21	TW21
-------	-------	-------	-------	-------	-------	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------	------	------	------

GT131	1.00																			
GT132	0.94	1.00																		
GT133	1.00	0.94	1.00																	
GT134	1.00	0.94	1.00	1.00																
GT135	1.00	0.94	1.00	1.00	1.00															
GT136	0.93	1.00	0.93	0.93	0.93	1.00														
G01131	0.84	0.78	0.84	0.84	0.84	0.78	1.00													
G01132	0.77	0.86	0.77	0.77	0.77	0.86	0.92	1.00												
G01133	0.83	0.82	0.83	0.83	0.83	0.81	0.99	0.95	1.00											
G01134	0.84	0.78	0.84	0.84	0.84	0.78	1.00	0.91	0.99	1.00										
G01135	0.84	0.78	0.84	0.84	0.84	0.78	1.00	0.91	0.99	1.00	1.00									
G01136	0.82	0.84	0.82	0.82	0.82	0.84	0.97	0.98	0.99	0.97	0.97	1.00								
G02131	0.75	0.70	0.75	0.75	0.75	0.69	0.63	0.57	0.63	0.63	0.63	0.61	1.00							
G02132	0.69	0.81	0.69	0.69	0.69	0.82	0.57	0.72	0.62	0.57	0.57	0.67	0.85	1.00						
G02133	0.75	0.78	0.75	0.75	0.75	0.78	0.63	0.67	0.65	0.63	0.63	0.67	0.96	0.96	1.00					
G02134	0.76	0.76	0.76	0.76	0.76	0.76	0.63	0.64	0.65	0.63	0.63	0.65	0.99	0.93	1.00	1.00				
G02135	0.76	0.74	0.76	0.76	0.76	0.73	0.64	0.61	0.64	0.64	0.64	0.64	1.00	0.90	0.99	1.00	1.00			
G02136	0.75	0.72	0.75	0.75	0.75	0.71	0.64	0.60	0.64	0.64	0.63	0.63	1.00	0.87	0.98	0.99	1.00	1.00		

Table with columns: GT131, GT132, GT133, GT134, GT135, GT136, GO1131, GO1132, GO1133, GO1134, GO1135, GO1136, GO1137, GO2132, GO2133, GO2134, GO2135, GO2136, TX21, TX22, TX23, TX24. Rows include TX11 through GO2136.

G01224	G01225	G01226	G02221	G02222	G02223	G02224	G02225	G02226	TX23	TY23	TW23	OX231	OY231	OW231	OX232	OY232	OW232	GT231	GT232	GT233
--------	--------	--------	--------	--------	--------	--------	--------	--------	------	------	------	-------	-------	-------	-------	-------	-------	-------	-------	-------

G01224	1.00
G01225	1.00 1.00
G01226	0.90 0.90 1.00
G02221	0.76 0.76 0.69 1.00
G02222	0.66 0.66 0.83 0.80 1.00
G02223	0.76 0.76 0.77 0.98 0.91 1.00
G02224	0.77 0.77 0.74 0.99 0.88 1.00 1.00
G02225	0.77 0.77 0.73 1.00 0.86 0.99 1.00 1.00
G02226	0.74 0.74 0.81 0.94 0.96 0.99 0.98 0.97 1.00

TX23	-0.25	-0.25	0.21	-0.22	0.38	-0.02	-0.09	-0.13	0.12	1.00											
TY23	0.32	0.33	0.73	0.28	0.81	0.49	0.42	0.38	0.62	0.75	1.00										
TW23	0.44	0.45	0.64	0.40	0.66	0.52	0.48	0.46	0.59	0.31	0.70	1.00									
OX231	-0.25	-0.24	0.19	-0.21	0.36	-0.02	-0.09	-0.12	0.12	0.95	0.72	0.30	1.00								
OY231	0.30	0.30	0.68	0.26	0.77	0.46	0.39	0.36	0.58	0.73	0.95	0.67	0.69	1.00							
OW231	0.41	0.41	0.59	0.35	0.60	0.46	0.43	0.41	0.52	0.29	0.65	0.91	0.28	0.62	1.00						
OX232	-0.21	-0.21	0.19	-0.20	0.32	-0.03	-0.09	-0.12	0.10	0.87	0.66	0.28	0.83	0.63	0.28	1.00					
OY232	0.31	0.31	0.69	0.27	0.77	0.47	0.40	0.37	0.59	0.73	0.96	0.67	0.69	0.91	0.62	0.64	1.00				
OW232	0.41	0.41	0.58	0.36	0.60	0.47	0.44	0.42	0.53	0.28	0.64	0.91	0.27	0.61	0.83	0.25	0.62	1.00			
GT231	0.75	0.75	0.66	0.66	0.55	0.67	0.67	0.67	0.64	-0.36	0.34	0.56	-0.35	0.32	0.50	-0.30	0.32	0.51	1.00		
GT232	0.73	0.73	0.79	0.64	0.74	0.72	0.70	0.68	0.75	0.00	0.66	0.72	0.00	0.62	0.65	0.01	0.62	0.65	0.93	1.00	
GT233	0.75	0.75	0.66	0.66	0.55	0.67	0.67	0.67	0.64	-0.36	0.34	0.56	-0.35	0.32	0.50	-0.30	0.32	0.51	1.00	0.93	1.00
GT234	0.75	0.75	0.66	0.66	0.55	0.67	0.67	0.67	0.64	-0.36	0.34	0.56	-0.35	0.32	0.50	-0.30	0.32	0.51	1.00	0.93	1.00
GT235	0.75	0.75	0.66	0.66	0.55	0.67	0.67	0.67	0.64	-0.36	0.34	0.56	-0.35	0.32	0.50	-0.30	0.32	0.51	1.00	0.93	1.00
GT236	0.72	0.72	0.79	0.64	0.74	0.72	0.70	0.68	0.75	0.01	0.66	0.72	0.00	0.62	0.65	0.02	0.63	0.66	0.93	1.00	0.93
G01231	0.64	0.64	0.57	0.56	0.48	0.56	0.57	0.57	0.55	-0.29	0.30	0.46	-0.40	0.39	0.42	-0.26	0.27	0.42	0.84	0.79	0.84
G01232	0.61	0.61	0.71	0.53	0.67	0.62	0.59	0.58	0.66	0.10	0.63	0.63	0.00	0.72	0.58	0.08	0.60	0.58	0.76	0.86	0.76
G01233	0.64	0.64	0.62	0.56	0.54	0.59	0.58	0.58	0.59	-0.19	0.40	0.52	-0.29	0.49	0.47	-0.16	0.37	0.48	0.83	0.82	0.83
G01234	0.64	0.64	0.57	0.56	0.48	0.57	0.57	0.57	0.55	-0.29	0.30	0.46	-0.40	0.39	0.42	-0.25	0.28	0.43	0.84	0.79	0.84
G01235	0.64	0.64	0.57	0.56	0.48	0.56	0.57	0.57	0.55	-0.29	0.30	0.46	-0.40	0.39	0.42	-0.25	0.27	0.42	0.84	0.79	0.84
G01236	0.64	0.64	0.65	0.56	0.58	0.60	0.59	0.58	0.61	-0.10	0.48	0.56	-0.20	0.57	0.51	-0.09	0.45	0.51	0.82	0.84	0.82
G02231	0.57	0.57	0.50	0.52	0.43	0.52	0.52	0.52	0.50	-0.25	0.26	0.41	-0.24	0.25	0.35	-0.50	0.34	0.38	0.73	0.69	0.73
G02232	0.55	0.55	0.70	0.50	0.70	0.60	0.57	0.55	0.65	0.22	0.69	0.64	0.21	0.66	0.57	0.00	0.77	0.59	0.67	0.80	0.67
G02233	0.58	0.58	0.62	0.52	0.59	0.58	0.57	0.56	0.60	-0.01	0.51	0.56	0.01	0.49	0.49	-0.24	0.60	0.51	0.72	0.78	0.72
G02234	0.58	0.58	0.59	0.53	0.55	0.57	0.56	0.55	0.58	-0.08	0.44	0.51	-0.07	0.42	0.45	-0.32	0.52	0.48	0.73	0.75	0.73
G02235	0.58	0.58	0.56	0.53	0.51	0.55	0.55	0.54	0.55	-0.14	0.38	0.48	-0.13	0.36	0.42	-0.19	0.46	0.44	0.73	0.73	0.73
G02236	0.57	0.57	0.49	0.52	0.43	0.52	0.52	0.52	0.50	-0.25	0.26	0.41	-0.24	0.25	0.35	-0.50	0.34	0.38	0.73	0.69	0.73

GT234	GT235	GT236	GO1231	GO1232	GO1233	GO1234	GO1235	GO1236	GO2231	GO2232	GO2233	GO2234	GO2235	GO2236	TX31	TY31	TW31	OX31	OY31	OW31
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GT231	1.00																			
GT235	1.00	1.00																		
GT236	0.93	0.93	1.00																	
GO1231	0.84	0.84	0.78	1.00																
GO1232	0.76	0.76	0.86	0.92	1.00															
GO1233	0.83	0.83	0.82	0.99	0.96	1.00														
GO1234	0.84	0.84	0.79	1.00	0.92	0.99	1.00													
GO1235	0.84	0.84	0.78	1.00	0.92	0.99	1.00	1.00												
GO1236	0.82	0.82	0.84	0.98	0.98	1.00	0.98	0.98	1.00											
GO2231	0.73	0.73	0.68	0.62	0.57	0.62	0.62	0.62	0.61	1.00										
GO2232	0.67	0.67	0.80	0.57	0.71	0.62	0.57	0.57	0.66	0.86	1.00									
GO2233	0.72	0.72	0.78	0.61	0.67	0.64	0.62	0.61	0.66	0.96	0.97	1.00								
GO2234	0.73	0.73	0.75	0.62	0.65	0.64	0.62	0.62	0.65	0.98	0.95	1.00	1.00							
GO2235	0.71	0.73	0.73	0.62	0.62	0.63	0.63	0.62	0.64	0.99	0.92	0.99	1.00	1.00						
GO2236	0.73	0.73	0.68	0.62	0.57	0.62	0.62	0.62	0.61	1.00	0.86	0.96	0.98	0.99	1.00					

Table with columns OX312, OY312, OW312, GT311, GT312, GT313, GT314, GT315, GT316, GO1311, GO1312, GO1313, GO1314, GO1315, GO1316, GO2311, GO2312, GO2313, GO2314, GO2315, GO2316. The table contains numerical data for various categories, with some values underlined to highlight specific data points.

G01311	G01312	G01313	G01314	G01315	G01316	G02331	G02332	G02333	G02334	G02335	G02336
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G01311	1.00												
G01312	0.97	1.00											
G01313	1.00	0.75	1.00										
G01314	1.00	0.97	1.00	1.00									
G01315	0.99	0.93	1.00	0.99	1.00								
G01316	0.90	0.98	0.87	0.90	0.81	1.00							
G02331	0.78	0.74	0.78	0.78	0.78	0.64	1.00						
G02332	0.75	0.73	0.75	0.75	0.75	0.64	0.99	1.00					
G02333	0.80	0.73	0.81	0.80	0.82	0.64	0.99	0.97	1.00				
G02334	0.80	0.73	0.80	0.80	0.81	0.64	1.00	0.98	1.00	1.00			
G02335	0.81	0.72	0.82	0.80	0.83	0.64	0.97	0.94	1.00	0.99	1.00		
G02336	0.52	0.60	0.50	0.52	0.46	0.64	0.82	0.87	0.73	0.76	0.67	1.00	

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