Recommended Provisions and Commentary on Development Length for High-Strength Reinforcement in Tension

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Samir Yasso

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THE UNIVERSITY OF KANSAS CENTER FOR RESEARCH, INC.
2385 Irving Hill Road, Lawrence, Kansas 66045-7563

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ABSTRACT

Design provisions on development length for straight reinforcing bars in tension are presented in code format and compared with those in ACI 318-14 (Building Code Requirements for Structural Concrete) and *fib* MC2010 (*fib* Model Code for Concrete Structures). The proposed provisions are based on a simplified version of the design equation in ACI 408R-03 (Bond and Development of Straight Reinforcing Bars in Tension) extended to apply to high-strength concrete up to 110 MPa (16,000 psi) and high-strength reinforcement up to 1070 MPa (155,000 psi). Compared with those in ACI 318-14 and *fib* MC2010, the recommended provisions produce designs with improved reliability and longer development lengths for conditions of low confinement or low concrete cover, but generally shorter development lengths for bars with higher degrees of confinement and wider spacing between bars. The recommended development length design equation gives values within 10% of those obtained from the design equation in ACI 408R-03.

Keywords: bond; deformed bars; high-strength concrete; high-strength steel; splice lengths.

INTRODUCTION

Since 1995, ACI Committee 318 has provided a practical formulation for calculating development length of straight bars based on 62 beam tests. Provisions in ACI 318-14 limit the reinforcement yield strength to 550 MPa (80,000 psi) and the value of $f_c'^{1/2}$ to 8.3 MPa (100 psi) for concrete with compressive strength in excess of 70 MPa (10,000 psi). Recommended design provisions in ACI 408R-03 (ACI Committee 408), based on 320 beam tests, account for the effects of a wider range of material strengths. The development length formulation in ACI 408R-03 is more elaborate than the one in ACI 318 and offers improved reliability. More recent tests (Seliem et al. 2009) with high-strength reinforcement were used to expand the ACI 408 database to 384 beam tests with concrete compressive strengths ranging from 14 to 110 MPa (2,000 to 16,000 psi) and reinforcement stresses between 280 and 1070 MPa (40,000 and 155,000 psi). The expanded database was used to derive a simplified version of the ACI 408R-03 development length equation with similar parameters to those in ACI 318-14.

DERIVATION OF DESIGN EQUATION

The design equation in ACI 408R-03 [Eq. (4-21)] for straight bar development length was derived from ACI 408R-03 [Eq. (4-11a)]:

$$\frac{\ell_d}{d_b} = \frac{\left(\dfrac{f_y}{\phi f_c'^{1/4}} - 57.4\omega\right)\alpha\beta\bar{\lambda}}{1.83\left(\dfrac{c\omega + K_{tr}}{d_b}\right)}, \text{ SI}$$

$$\frac{\ell_d}{d_b} = \frac{\left(\dfrac{f_y}{\phi f_c'^{1/4}} - 2400\omega\right)\alpha\beta\bar{\lambda}}{76.3\left(\dfrac{c\omega + K_{tr}}{d_b}\right)}, \text{ in.-lb} \qquad (4\text{-}11a)$$

where development length ℓ_d, bar diameter d_b, specified reinforcement yield strength f_y, and specified concrete compressive strength f_c' have identical definitions to those in ACI 318-14. Parameters α and β correspond, respectively, to the bar location and coating factors, ψ_t and ψ_e, in ACI 318-14. The parameter $\bar{\lambda}$ equals the inverse of λ, which is used in ACI 318-14 to represent the effect of lightweight concrete on development length. The term $(c_b\omega + K_{tr})/d_b$, which is limited to a maximum value of 4 in ACI 408R-03, represents the effect of confinement from concrete cover and transverse reinforcement to decrease the required development length. Parameters ψ_t, ψ_e, c_b, λ, ω, and K_{tr} are defined later in this paper. Note that Eq. (4-11a) does not use the bar size factor for small bars that appears in ACI 318-14 but incorporates the parameter ω to account for the beneficial effect of widely spaced bars. The value of ω varies between 1 and 1.25, approaching 1.25 where the clear spacing of reinforcement being developed exceeds 6 times the concrete clear cover (cover and spacing mutually perpendicular). Definitions of c and K_{tr} in ACI 408R-03 are more elaborate than in ACI 318-14. In an effort to use variables as defined in ACI 318-14, ACI 408R-03 Eq. (4-11a) is rewritten as

$$\frac{\ell_d}{d_b} = \frac{\left(\dfrac{f_y}{\phi f_c'^{1/4}} - m_2\right)\psi_t \psi_e}{m_1 \lambda \left(\dfrac{c_b \omega + K_{tr}}{d_b}\right)} \tag{1a}$$

where the values of m_1 and m_2 represent new constants and the parameter λ is now in the denominator as explained above. A simplified definition of ω is adopted here, with ω = 1.25 for cases where the clear spacing of reinforcement being developed is at least 6 times the concrete clear cover (cover and spacing mutually perpendicular), otherwise ω = 1. The upper limit on the confinement term $(c_b\omega + K_{tr})/d_b$ is set to 4, as in ACI 408R-03, with K_{tr} based on the definition in ACI 318-14.

Equation (1a) may be further simplified by introducing modification factor ψ_y:

$$\psi_y = \left(1 - m_2 \frac{\phi f_c'^{1/4}}{f_y}\right) \tag{1b}$$

leading to Eq. (2a):

$$\frac{\ell_d}{d_b} = \frac{\dfrac{f_y}{\phi f_c'^{1/4}} \psi_t \psi_e \psi_y}{m_1 \lambda \left(\dfrac{c_b \omega + K_{tr}}{d_b}\right)} \tag{2a}$$

A final simplification is achieved by dropping the dependence of ψ_y on f_c':

$$\psi_y = \left(1 - m_3 \frac{\phi}{f_y}\right) \tag{2b}$$

Using the expanded ACI 408 database (with 384 beam tests), the values of m_1 and m_3 were derived by minimizing the square of the differences between the measured (f_{su}) and calculated bar stresses ($f_{s,calc}$), where $f_{s,calc}$ is solved after replacing f_y in Eq. (2a) and (2b). All 384 specimens in the database correspond to conditions where parameters ψ_t, ψ_e, and λ equal 1.0. The optimal values of m_1 and m_3, providing test-to-calculated stress ratios with a mean of 1.0, were m_1 = 1.81 (75.4) and m_3 = 178 (25,900), SI (in.-lb). ACI 408R-03 recommended the use of ϕ = 0.82 in Eq. (4-11a) after reliability analyses. To attain similar reliability with the use of Eq. (2a) and (2b), a value of ϕ = 0.79 is required given that a slightly higher coefficient of variation for test-to-calculated stress ratios was associated with the use of Eq. (2a) when compared with Eq. (4-11a). These values of m_1, m_3, and ϕ substituted in Eq. (2a) and (2b) give

$$\frac{\ell_d}{d_b} = \frac{\dfrac{f_y}{f_c'^{1/4}} \psi_t \psi_e \psi_y}{1.43 \lambda \left(\dfrac{c_b \omega + K_{tr}}{d_b}\right)}, \text{ SI}$$

$$\frac{\ell_d}{d_b} = \frac{\dfrac{f_y}{f_c'^{1/4}} \psi_t \psi_e \psi_y}{59.6 \lambda \left(\dfrac{c_b \omega + K_{tr}}{d_b}\right)}, \text{ in.-lb} \tag{3a}$$

$$\psi_y = \left(1 - \frac{141}{f_y}\right), \text{ SI}$$

$$\psi_y = \left(1 - \frac{20,400}{f_y}\right), \text{ in.-lb} \tag{3b}$$

If for convenience ψ_y is made equal to 1.0 for the case of f_y = 420 MPa (60,000 psi), then coefficients 1.43 (59.6), 1 (1), and 141 (20,400) become 2.14 (89.4), 1.5 (1.5), and 211 (30,600), which are simplified to 13/6 (90), 1.5 (1.5), and 210 (30,000). Additionally, a minimum value of 0.75 for ψ_y limits the reduction of ℓ_d/d_b for cases developing stresses at or below 280 MPa (40,000 psi). The final design equation becomes

$$\frac{\ell_d}{d_b} = \frac{6}{13}\frac{\dfrac{f_y}{f_c'^{\,1/4}}\psi_t\psi_e\psi_y}{\lambda\left(\dfrac{c_b\omega + K_{tr}}{d_b}\right)}, \text{ SI}$$

$$\frac{\ell_d}{d_b} = \frac{1}{90}\frac{\dfrac{f_y}{f_c'^{\,1/4}}\psi_t\psi_e\psi_y}{\lambda\left(\dfrac{c_b\omega + K_{tr}}{d_b}\right)}, \text{ in.-lb} \tag{4a}$$

$$\psi_y = \left(1.5 - \frac{210}{f_y}\right) \geq 0.75, \text{ SI}$$

$$\psi_y = \left(1.5 - \frac{30,000}{f_y}\right) \geq 0.75, \text{ in.-lb} \tag{4b}$$

where

$$\left(\frac{c_b\omega + K_{tr}}{d_b}\right) \leq 4 \text{ and } K_{tr} = \frac{40A_{tr}}{sn} \tag{4c}$$

All variables in Eq. (4a), (4b), and (4c), except ω, have the same definition as in ACI 318-14. Recommended changes to ACI 318-14 using the derived equations are presented next.

RECOMMENDED PROVISIONS AND COMMENTARY

1.0 *Notation*

A_{tr} = total cross-sectional area of all transverse reinforcement within spacing s that crosses the potential plane of splitting through the reinforcement being developed, mm^2 (in.^{2})

c_b = lesser of: (a) the distance from center of a bar to nearest concrete surface, and (b) one-half the center-to-center spacing of reinforcement being developed, mm (in.)

c_c = clear cover of reinforcement, mm (in.)

d_b = nominal diameter of bar or wire, mm (in.)

f_c' = specified compressive strength of concrete, MPa (psi)

f_{ct} = measured average splitting tensile strength of lightweight concrete, MPa (psi)

f_y = specified yield strength of reinforcement, MPa (psi)

K_{tr} = transverse reinforcement index, mm (in.)

ℓ_d = tension development length of deformed reinforcement, mm (in.)

n = number of bars or wires being developed or lap spliced along plane of splitting

s = maximum center-to-center spacing of transverse reinforcement within ℓ_d, mm (in.)

λ = lightweight concrete factor. Refer to Section 2.4

ψ_t = reinforcement location factor. Refer to Section 2.4

ψ_e = reinforcement coating factor. Refer to Section 2.4

ψ_y = reinforcement yield strength factor. Refer to Section 2.4
ω = reinforcement spacing factor. Refer to Section 2.4

2.0 *Development of deformed bars and deformed wires in tension*

2.1 Development length ℓ_d for deformed bars and deformed wires in tension shall be the longest of (a) through (c):

(a) Length calculated in accordance with Section 2.2 or 2.3 using the applicable modification factors of Section 2.4.

(b) $16d_b$

(c) 300 mm (12 in.)

R2.0 *Development of deformed bars and deformed wires in tension*

R2.1 This provision offers a two-tier approach for the calculation of tension development length. The user can either use the simplified provisions of Section 2.2 or the general development length equation [Eq. (2.3a)], which is derived from an equation developed by ACI Committee 408 (ACI 408R-03). In Table 2.2, ℓ_d is based on two preselected values of $(c_b\omega + K_{tr})/d_b$, whereas ℓ_d from Eq. (2.3a) is based on the actual value of $(c_b\omega + K_{tr})/d_b$, with an upper limit of 4. An additional minimum development length of $16d_b$ has been included based on the observation that low values of bond strength in development and splice tests, with respect to all predictive equations, were obtained in cases where $\ell_d < 16d_b$ (Darwin et al. 1996).

Development and splice failures tend to be brittle. Tests regularly show that transverse reinforcement improves the ductile behavior of straight bars anchored or spliced in tension.

2.2 For deformed bars or deformed wires, ℓ_d shall be calculated in accordance with Table 2.2.

Table 2.2 – Development length for deformed bars and deformed wires in tension

Spacing and cover	ℓ_d
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{4}{13}\frac{f_y\psi_t\psi_e\psi_y}{\lambda f_c'^{1/4}}\right)d_b$, SI $\left(\frac{f_y\psi_t\psi_e\psi_y}{135\,\lambda f_c'^{1/4}}\right)d_b$, in.-lb
Other cases	$\left(\frac{6}{13}\frac{f_y\psi_t\psi_e\psi_y}{\lambda f_c'^{1/4}}\right)d_b$, SI $\left(\frac{f_y\psi_t\psi_e\psi_y}{90\,\lambda f_c'^{1/4}}\right)d_b$, in.-lb

R2.2 This provision recognizes that many current practical construction cases use spacing and cover values along with confining reinforcement, such as stirrups or ties, that result in a value of $(c_b\omega + K_{tr})/d_b$ of at least 1.5. Examples include a minimum clear cover of d_b along with either minimum clear spacing of $2d_b$, or a combination of minimum clear spacing of d_b and minimum ties or stirrups. For these frequently occurring cases, the development length can be taken as ℓ_d = $4/13\, f_y\, \psi_t\, \psi_e\, \psi_y /(\lambda\, f_c'^{1/4})d_b$ SI, [$1/135\, f_y\, \psi_t\, \psi_e\, \psi_y /(\lambda\, f_c'^{1/4})d_b$] [in.-lb]. For "other cases", the values are based on using $(c_b\omega + K_{tr})/d_b$ = 1 in Eq. (2.3a).

The user may construct simple, useful expressions. For example, in members with normalweight concrete (λ = 1), uncoated reinforcement (ψ_e = 1), bottom bars (ψ_t = 1), f_c' = 28 MPa (4,000 psi), and Grade 420 (60) reinforcement (ψ_y = 1), the expressions in Table 2.2 (SI units) reduce to

$$\ell_d = \frac{4}{13}\frac{(420)(1.0)(1.0)(1.0)}{(1.0)(28)^{1/4}}d_b = 56d_b$$

and

$$\ell_d = \frac{6}{13}\frac{(420)(1.0)(1.0)(1.0)}{(1.0)(28)^{1/4}}d_b = 84d_b$$

For this example, if minimum cover of d_b is provided along with a minimum clear spacing of $2d_b$, or a minimum clear cover of d_b and a minimum clear spacing of d_b are provided along with minimum ties or stirrups, then $\ell_d = 56d_b$. The penalty for spacing bars closer or providing less cover results in $\ell_d = 84d_b$. These values of ℓ_d are nearly 20% longer than those required by previous editions of ACI 318, but the higher limit of $(c_b\omega + K_{tr})/d_b = 4$ allows considerably lower values of ℓ_d when Eq. (2.3a) is used to calculate ℓ_d. While equations for ℓ_d in previous editions of ACI 318 led to safe average values, the new equations improve the overall reliability of bar development length.

2.3 For deformed bars or deformed wires, ℓ_d shall be calculated by:

$$\ell_d = \left(\frac{6}{13} \frac{f_y}{\lambda f_c'^{\,1/4}} \frac{\psi_t \psi_e \psi_y}{\left(\dfrac{c_b\omega + K_{tr}}{d_b} \right)} \right) d_b \text{, SI}$$

$$\ell_d = \left(\frac{1}{90} \frac{f_y}{\lambda f_c'^{\,1/4}} \frac{\psi_t \psi_e \psi_y}{\left(\dfrac{c_b\omega + K_{tr}}{d_b} \right)} \right) d_b \text{, in.-lb} \qquad (2.3a)$$

in which the confinement term $(c_b\omega + K_{tr})/d_b$ shall not exceed 4, ω is determined in accordance with Table 2.4, and

$$K_{tr} = \frac{40A_{tr}}{sn} \qquad (2.3b)$$

where n is the number of bars or wires being developed or lap spliced along the plane of splitting. For f_y > 550 MPa (80,000 psi) and f_c' > 70 MPa (10,000 psi), transverse reinforcement shall be provided such that $K_{tr} \geq 0.5d_b$. It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

R2.3 Equation (2.3a) includes the effects of the main variables controlling development length. The equation is based on a design equation developed by ACI Committee 408 (ACI 408R-03) but differs from the development length equation that appeared in ACI 318-14 and earlier Code editions in three significant ways: (1) The contribution of concrete compressive strength is now represented by $f_c'^{\,1/4}$, rather than $f_c'^{\,1/2}$. The lower power of f_c' provides a more accurate representation of the concrete contribution to development and splice strength over a wider range of f_c', allowing its application up to 110 MPa (16,000 psi) (Darwin et al. 1996; Zuo and Darwin 2000); (2) to reflect the observation that development length is not proportional to the stress developed in a bar, the yield strength factor ψ_y has been added leading to development lengths that increase by a greater percentage than the increase in yield strength as the grade of reinforcement increases; and (3) the limit on the confinement term $(c_b\omega + K_{tr})/d_b$ has been increased from 2.5 to 4. An upper limit is retained for the same reason as in previous editions of ACI 318. If $(c_b\omega + K_{tr})/d_b \leq 4$, splitting failures are likely to occur, whereas for values above 4, pullout failures are likely (Darwin et al. 1996; Zuo and Darwin 2000).

In Eq. (2.3a), c_b is a factor that represents the least of the side cover, the concrete cover to the bar or wire (in both cases measured to the center of the bar or wire), or one-half the center-to-center spacing of the bars or wires. K_{tr} is a factor that represents the contribution of confining reinforcement across potential splitting planes. ψ_t is the reinforcement location factor to reflect the effect of the casting position, formerly denoted as "top bar effect". ψ_e is a coating factor reflecting the effects of epoxy coating. There is a limit on the product $\psi_t\psi_e$. The spacing factor ω reflects the more favorable performance of reinforcement that has a wide spacing compared to the concrete cover (Zuo and Darwin 2000).

Many practical combinations of side cover, clear cover, and confining reinforcement can be used with Section 2.3 to produce significantly shorter development lengths than allowed by Section 2.2. For example, Grade 420 (60) reinforcement with the term $(c_b\omega + K_{tr})/d_b = 4$ would

require a development length of only $21d_b$ for the example in R2.2 instead of $56d_b$ that would be obtained using the simplified expression.
Prior to ACI 318-08, Eq. (2.3b) for K_{tr} included the yield strength of transverse reinforcement. The current expression includes only the area and spacing of the transverse reinforcement and the number of wires or bars being developed or lap spliced because tests demonstrate that transverse reinforcement rarely yields during a bond failure (Azizinamini et al. 1995).
The required minimum value of $K_{tr} = 0.5d_b$ is to promote ductile behavior when developing bar stresses ≥ 550 MPa (80,000 psi) in high-strength concrete. Terms in Eq. (2.3a) may be disregarded if such omission results in longer (more conservative) development lengths.

2.4 For the calculation of ℓ_d, modification factors shall be in accordance with Table 2.4.

Table 2.4 – Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	$\frac{1.8f_{ct}}{f_{cm}^{1/2}}$ ≤ 1, SI $\frac{f_{ct}}{6.7f_{cm}^{1/2}}$ ≤ 1, in.-lb
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Casting position[1] ψ_t	More than 300 mm (12 in.) of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0
Yield strength ψ_y	All	$1.5-\frac{210}{f_y}$ ≥ 0.75, SI $1.5-\frac{30{,}000}{f_y}$ ≥ 0.75, in.-lb
Spacing[2] ω	Bars or wires satisfying (a) and (b): (a) Clear spacing between bars or wires (within a plane) not less than $6c_c$ (b) Clear side cover (within the plane) not less than $3c_c$ where c_c is measured perpendicular to the plane	1.25
	Other	1.0

[1]The product $\psi_t \psi_e$ need not exceed 1.7.
[2]It shall be permitted to use ω = 1.0 even if the clear spacing between bars ≥ $6c_c$.

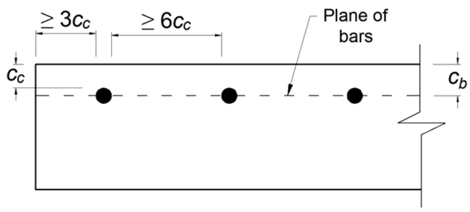
R2.4 The lightweight factor λ for calculating development length of deformed bars and deformed wires in tension is the same for all types of lightweight aggregate concrete. Research does not support the variations of this factor in Codes prior to 1989 for all-lightweight and sand-lightweight concrete. Section 2.4 allows a higher factor to be used when the splitting tensile strength f_{ct} of the lightweight concrete is specified.
The epoxy factor ψ_e is based on studies (Treece and Jirsa 1989; Johnston and Zia 1982; Mathey and Clifton 1976) of the anchorage of epoxy-coated bars that show bond strength is reduced because the coating prevents adhesion and lowers the coefficient of friction between the bar and the concrete. The factors reflect the type of anchorage failure likely to occur. If the

cover or spacing is small, a splitting failure can occur and the anchorage or bond strength is substantially reduced. If the cover and spacing between bars is large, a splitting failure is precluded and the effect of the epoxy coating on anchorage strength is not as large. Studies (Orangun et al. 1977; Darwin et al. 1996; Zuo and Darwin 2000) have shown that although the cover or spacing may be small, the anchorage strength may be increased by adding transverse reinforcement crossing the plane of splitting, and restraining the splitting crack.

Because the bond of epoxy-coated bars or zinc and epoxy dual-coated bars is already reduced due to the loss of adhesion and lower coefficient of friction between the bar and the concrete, an upper limit of 1.7 is established for the product of the factors for top reinforcement casting position and epoxy-coated reinforcement or zinc and epoxy dual-coated reinforcement.

The spacing factor ω reflects the more favorable performance of reinforcement with large clear spacing and side cover (Zuo and Darwin 2000). See Figure R2.4.

The reinforcement location or casting position factor ψ_t accounts for the position of the reinforcement in freshly placed concrete. The 1.3 factor is based on research (Jirsa and Breen 1981; Jeanty et al. 1988). The application of the casting position factor should be considered in determination of development lengths for inclined reinforcement.



Note: c_c is clear cover normal to plane of bars, $c_c = (c_b - d_b/2)$

Figure R2.4 – Requirements for applying ω according to Section 2.4.

CONCLUSIONS

The recommended changes to development length provisions in ACI 318 (Building Code Requirements for Structural Concrete and Commentary) are supported by a practical and reliable design equation derived from the more elaborate formulation in ACI 408R-03 (Bond and Development of Straight Reinforcing Bars in Tension). The proposed changes allow the use of higher grade reinforcement and higher compressive strength of concrete than currently permitted. The changes also recognize the higher bond strength of widely spaced bars and the effect of higher levels of confinement.

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APPENDIX A: ALTERNATIVE DESIGN EQUATION

The proposed design equation for development length, Eq. (4a) expressed as a function of $f_c'^{\,1/4}$, was derived as a simplified version of ACI 408R-03 Eq. (4-11a) after incorporating the notation used in ACI 318-14. An alternative to Eq. (4a) is derived after substituting $f_c'^{\,1/4}$ with the combined use of $\kappa f_c'^{\,1/2}$ and an upper bound of 8.3 MPa (100 psi) on $f_c'^{\,1/2}$. The value of coefficient κ is derived after minimizing the differences between $\kappa f_c'^{\,1/2}$ and $f_c'^{\,1/4}$ for f_c' between 14 and 70 MPa (2,000 and 10,000 psi), which results in an optimal value of κ = 0.38 (0.11). The alternative design equation, Eq. (A.4a), is obtained after replacing $f_c'^{\,1/4}$ with 0.38 $f_c'^{\,1/2}$ (MPa) or 0.11 $f_c'^{\,1/2}$ (psi) in Eq. (4a) and rounding the resulting coefficient:

$$\frac{\ell_d}{d_b} = \frac{6}{5}\frac{\dfrac{f_y}{f_c'^{\,1/2}}\psi_t\psi_e\psi_y}{\lambda\left(\dfrac{c_b\omega + K_{tr}}{d_b}\right)}\text{, SI}$$

$$\frac{\ell_d}{d_b} = \frac{\dfrac{f_y}{f_c'^{\,1/2}}\psi_t\psi_e\psi_y}{10\lambda\left(\dfrac{c_b\omega + K_{tr}}{d_b}\right)}\text{, in.-lb} \qquad \text{(A.4a)}$$

where ψ_y and $(c_b\omega + K_{tr})/d_b$ are defined by Eq. (4b) and (4c).

Equation (A.4a) generally requires longer development lengths than Eq. (4a) for f_c' < 40 MPa (6,000 psi), see Appendix B for a detailed comparison.

APPENDIX B: COMPARISON OF EQUATIONS

A summary of the statistical data associated with the use of Eq. (4a), (A.4a), ACI 318-14 Eq. (25.4.2.3a), and ACI 408R-03 Eq. (4-21) is presented in Tables B.1 and B.2. The tables show test-to-calculated stress ratios $f_{su}/f_{s,calc}$, where f_{su} is the value based on the tests and $f_{s,calc}$ is the value based on the design equations. Table B.1 separates cases with and without confining reinforcement while Table B.2 further separates cases based on bar stress and bar size.

The bottom of Table B.1 shows that considering all 384 specimens, the mean of $f_{su}/f_{s,calc}$ resulted 1.27, 1.32, and 1.25 for Eq. (4a), (A.4a), and ACI 408R-03 Eq. (4-21), with a coefficient of variation (CV) of 0.15, 0.14, and 0.13. In contrast, for ACI 318-14 Eq. (25.4.2.3a), the mean and CV were 1.19 and 0.25, respectively, with $f_{su}/f_{s,calc}$ < 1 for 25% of the tests.

Table B.2 shows that for f_{su} > 550 MPa (80,000 psi), the use of ACI 318-14 resulted in $f_{su}/f_{s,calc}$ < 1 for 58% of specimens without confining reinforcement and 39% of specimens with confining reinforcement. In addition, 50% of the 72 specimens with bar sizes No. 19 (No. 6) or smaller resulted in $f_{su}/f_{s,calc}$ < 1, showing that the use of the bar size factor (ψ_s = 0.8) is not supported by the test data. Figure B.1 corresponds to the data in Table B.1 plotted as a function of f_c'.

Figures B.2 and B.3 compare the development length design equations presented above, Eq. (4a), (A.4a), ACI 318-14 Eq. (25.4.2.3a), and ACI 408R-03 Eq. (4-21), with the development length equation from the *fib* Model Code for Concrete Structures 2010 (*fib* MC2010):

$$\frac{\ell_d}{d_b} = \frac{1}{4}\frac{\sigma_{sd}}{f_{bd}} = \frac{1}{4}\frac{f_{yk}}{(f_{ck}/25)^{1/2}}\left(\frac{\gamma_c}{\gamma_s}\right)\left(\frac{1}{\eta_1\eta_2\eta_3\eta_4(\alpha_2+\alpha_3)}\right), \text{ SI} \qquad \text{(B.1)}$$

Eq. (B.1) results from *fib* MC2010 Eq. (6.1-25) after σ_{sd} (design reinforcement stress) is replaced with Eq. (6.1-24), and f_{bd} (design bond strength) with Eq. (6.1-20) and (6.1-21). Notation and units from *fib* MC2010 are retained in Eq. (B.1), except for ℓ_d/d_b replacing $\ell_b/\varnothing$.

Table B.1 – Test-to-calculated stress ratios ($f_{su}/f_{s,calc}$) for specimens considered

	ACI 318-14 Eq. (25.4.2.3a)	Proposed Eq. (4a)	Proposed Eq. (A.4a)	ACI 408R-03 Eq. (4-21)
(No. of specimens)	Specimens without confining reinforcement (188)			
Maximum	2.369	1.980	1.969	1.575
Minimum	0.586	0.842	0.904	0.843
Mean	1.169	1.243	1.283	1.222
SD	0.299	0.175	0.177	0.139
CV	0.255	0.141	0.138	0.114
$f_{su}/f_{s,calc}$ < 1.0	27.7% (52)	5.9% (11)	5.9% (11)	6.9% (13)
(No. of specimens)	Specimens with confining reinforcement (196)			
Maximum	2.207	2.045	2.032	1.715
Minimum	0.650	0.913	0.988	0.903
Mean	1.215	1.305	1.347	1.279
SD	0.296	0.207	0.194	0.173
CV	0.244	0.158	0.144	0.135
$f_{su}/f_{s,calc}$ < 1.0	21.9% (43)	4.1% (8)	0.5% (1)	3.6% (7)
(No. of specimens)	All specimens (384)			
Maximum	2.369	2.045	2.032	1.715
Minimum	0.586	0.842	0.904	0.843
Mean	1.192	1.274	1.316	1.251
SD	0.298	0.194	0.188	0.160
CV	0.250	0.152	0.143	0.128
$f_{su}/f_{s,calc}$ < 1.0	24.7% (95)	4.9% (19)	3.1% (12)	5.2% (20)

Table B.2 – Test-to-calculated stress ratios ($f_{su}/f_{s,calc}$) based on bar stress and bar size

Specimens without confining reinforcement, cases with f_{su} ≤ 550 MPa (80,000 psi)									
	ACI 318-14 Eq. (25.4.2.3a)			Proposed Eq. (4a)			Proposed Eq. (A.4a)		
Bar size[1] (No. of specimens)	All (164)	≤ No. 19 (24)	≥ No. 22 (140)	All (164)	≤ No. 19 (24)	≥ No. 22 (140)	All (164)	≤ No. 19 (24)	≥ No. 22 (140)
Maximum	2.369	1.709	2.369	1.980	1.573	1.980	1.969	1.613	1.969
Minimum	0.586	0.684	0.586	0.842	1.067	0.842	0.904	1.120	0.904
Mean	1.201	1.104	1.218	1.239	1.253	1.237	1.278	1.305	1.274
SD	0.295	0.232	0.302	0.176	0.138	0.182	0.174	0.138	0.179
CV	0.246	0.211	0.248	0.142	0.110	0.147	0.136	0.105	0.141
$f_{su}/f_{s,calc}$ < 1.0	23.2% (38)	37.5% (9)	20.7% (29)	6.7% (11)	0.0% (0)	7.9% (11)	6.7% (11)	0.0% (0)	7.9% (11)
Specimens without confining reinforcement, cases with f_{su} > 550 MPa (80,000 psi)									
	ACI 318-14 Eq. (25.4.2.3a)			Proposed Eq. (4a)			Proposed Eq. (A.4a)		
Bar size[1] (No. of specimens)	All (24)	≤ No. 19 (8)	≥ No. 22 (16)	All (24)	≤ No. 19 (8)	≥ No. 22 (16)	All (24)	≤ No. 19 (8)	≥ No. 22 (16)
Maximum	1.359	1.285	1.359	1.552	1.552	1.396	1.617	1.616	1.617
Minimum	0.633	0.633	0.676	1.005	1.089	1.005	1.029	1.159	1.029
Mean	0.948	0.968	0.938	1.266	1.367	1.215	1.316	1.422	1.264
SD	0.225	0.235	0.227	0.170	0.183	0.143	0.198	0.178	0.190
CV	0.237	0.243	0.242	0.134	0.134	0.117	0.150	0.125	0.150
$f_{su}/f_{s,calc}$ < 1.0	58.3% (14)	50.0% (4)	62.5% (10)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)
Specimens with confining reinforcement, cases with f_{su} ≤ 550 MPa (80,000 psi)									
	ACI 318-14 Eq. (25.4.2.3a)			Proposed Eq. (4a)			Proposed Eq. (A.4a)		
Bar size[1] (No. of specimens)	All (130)	≤ No. 19 (15)	≥ No. 22 (115)	All (130)	≤ No. 19 (15)	≥ No. 22 (115)	All (130)	≤ No. 19 (15)	≥ No. 22 (115)
Maximum	2.207	1.444	2.207	2.045	1.489	2.045	2.032	1.601	2.032
Minimum	0.747	0.807	0.747	0.913	0.966	0.913	0.988	1.070	0.988
Mean	1.287	1.014	1.322	1.267	1.197	1.276	1.326	1.299	1.330
SD	0.300	0.161	0.296	0.220	0.161	0.226	0.210	0.150	0.217
CV	0.233	0.159	0.224	0.174	0.135	0.177	0.158	0.115	0.163
$f_{su}/f_{s,calc}$ < 1.0	13.1% (17)	53.3% (8)	7.8% (9)	6.2% (8)	13.3% (2)	5.2% (6)	0.8% (1)	0.0% (0)	0.9% (1)
Specimens with confining reinforcement, cases with f_{su} > 550 MPa (80,000 psi)									
	ACI 318-14 Eq. (25.4.2.3a)			Proposed Eq. (4a)			Proposed Eq. (A.4a)		
Bar size[1] (No. of specimens)	All (66)	≤ No. 19 (25)	≥ No. 22 (41)	All (66)	≤ No. 19 (25)	≥ No. 22 (41)	All (66)	≤ No. 19 (25)	≥ No. 22 (41)
Maximum	1.826	1.417	1.826	2.044	1.564	2.044	1.950	1.581	1.950
Minimum	0.650	0.704	0.650	1.072	1.072	1.086	1.005	1.005	1.045
Mean	1.073	0.973	1.134	1.380	1.385	1.376	1.388	1.384	1.391
SD	0.232	0.171	0.245	0.153	0.109	0.175	0.153	0.120	0.171
CV	0.216	0.176	0.216	0.111	0.079	0.127	0.110	0.087	0.123
$f_{su}/f_{s,calc}$ < 1.0	39.4% (26)	60.0% (15)	26.8% (11)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)	0.0% (0)

[1] Bar sizes No. 19 and No. 22 in millimeters are equivalent to bar sizes No. 6 and No. 7 in eighths of an inch.

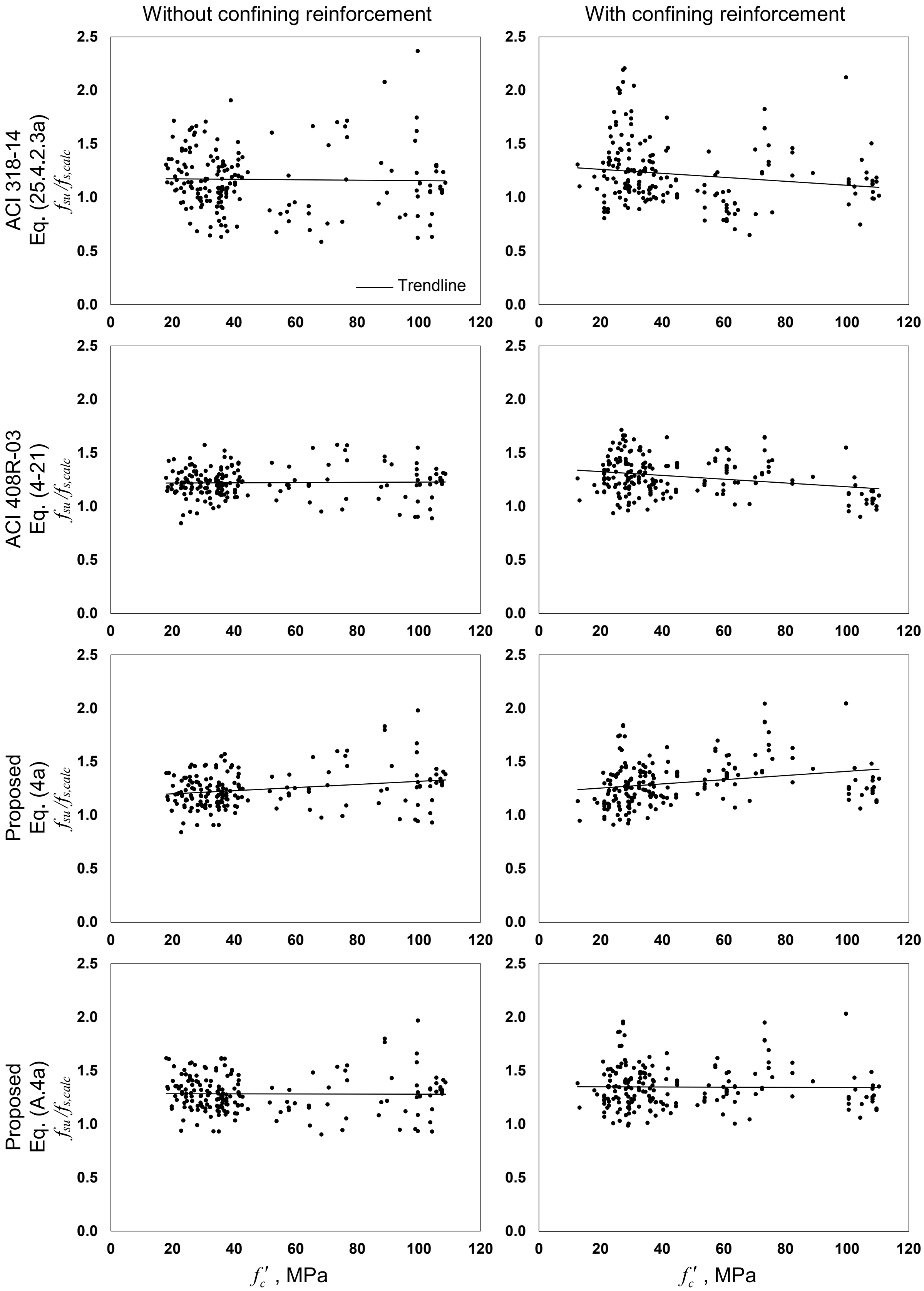


Figure B.1 – Test-to-calculated stress ratios ($f_{su}/f_{s,calc}$) versus compressive strength of concrete (f_c') for all specimens (1 MPa = 145.04 psi)

Characteristic-to-design strength ratios, γ_s for steel reinforcement and γ_c for concrete, may be taken as 1.15 and 1.5, respectively. Additionally, assuming straight bars in tension, deformed and uncoated (η_1 = 1.75), bottom bar location (η_2 = 1), No. 25 (No. 8) or smaller bars (η_3 = 1), $\eta_4 = (500/f_{yk})^{0.82}$, and replacing f_{yk} with f_y, and f_{ck} with f_c', simplifies Eq. (B.1) to

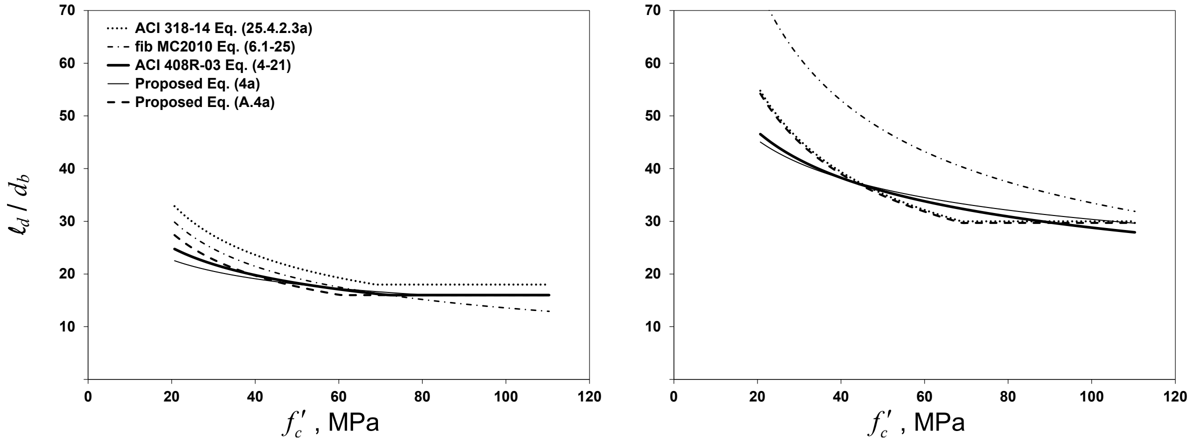
$$\frac{\ell_d}{d_b} = \frac{1}{175}\frac{f_y^{1.82}}{f_c'^{\,1/2}}\frac{1}{(\alpha_2 + \alpha_3)}, \text{ SI} \quad \text{(B.2)}$$

where α_2 and α_3 represent the influence of passive confinement from cover (α_2) and from transverse reinforcement (α_3). The term ($\alpha_2 + \alpha_3$) is limited to a maximum of 2.5, similar to the limitations of the confinement term $[(c_b + K_{tr})/d_b]$ in ACI 318-14.

Figures B.2 and B.3 assume that modification factors for epoxy-coated bar, casting position, bar size, bar spacing, and lightweight concrete, are all equal to 1 (i.e., $\psi_e = \psi_t = \psi_s = \omega = \lambda = 1$).

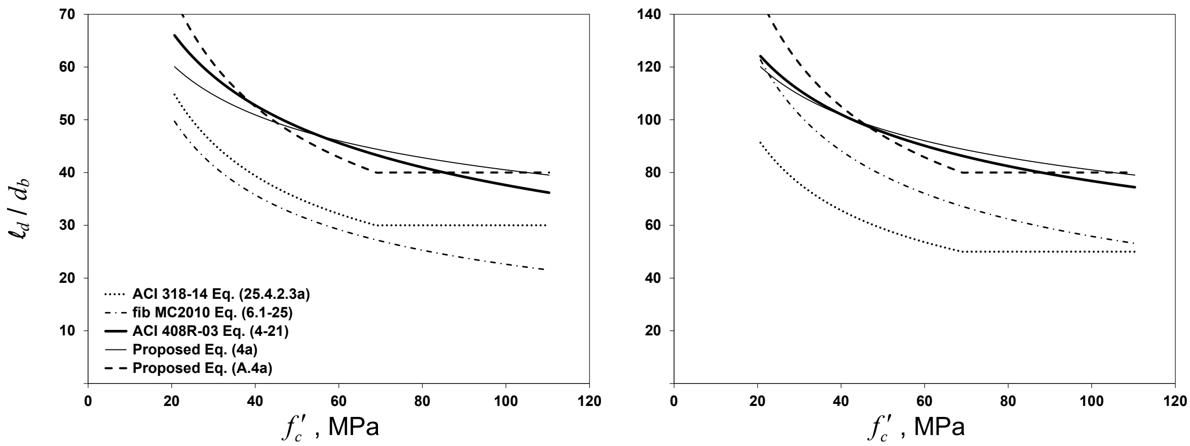
The curves in Figure B.2 show that for cases controlled by pullout (with high values of confinement), ACI 318-14 Eq. (25.4.2.3a) requires longer development length for reinforcement Grade 420 (60), while for Grade 690 (100) longer development length is required by *fib* MC2010 Eq. (6.1-25) [or Eq. (B.2) above]. This is mostly due to the limitation on the maximum value of the confinement term to 2.5 in both ACI 318-14 and *fib* MC2010. Figure B.2 also shows that the effect on ℓ_d/d_b of increasing f_y from 420 to 690 MPa (60,000 to 100,000 psi) is overestimated by η_4.

For cases controlled by splitting (with low values of confinement), Figure B.3 shows that both ACI 318-14 Eq. (25.4.2.3a) and *fib* MC2010 Eq. (6.1-25) [or Eq. (B.2) above] require significant shorter development length than the other equations, indicating that for cases without transverse reinforcement both ACI 318-14 and *fib* MC2010 are not as safe as the other equations (note that in Table B.1, ACI 318-14 shows $f_{su}/f_{s,calc}$ < 1 for 27.7% of cases without transverse reinforcement).



(a) f_y = 420 MPa (60,000 psi) (b) f_y = 690 MPa (100,000 psi)

Figure B.2 – Calculated development length versus compressive strength of concrete for $(c_b + K_{tr})/d_b$ = 4 (2.5 for ACI 318-14), ($\alpha_2 + \alpha_3$) = 2.5, and $\psi_e = \psi_t = \psi_s = \omega = \lambda = 1$ (1 MPa = 145.04 psi)



(a) f_y = 420 MPa (60,000 psi) (b) f_y = 690 MPa (100,000 psi)

Figure B.3 – Calculated development length versus compressive strength of concrete for $(c_b + K_{tr})/d_b$ = ($\alpha_2 + \alpha_3$) = 1.5, and $\psi_e = \psi_t = \psi_s = \omega = \lambda = 1$ (1 MPa = 145.04 psi)