

# Essays on Macroeconomic Implications of International Capital Flow and Fiscal Uncertainty

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# Abstract

This dissertation comprises three chapters that contribute to a broader and an ongoing discussion in the macroeconomics, international economics and development economics literature. Specifically, the first chapter focuses on understanding how shocks to long-term U.S debt held by foreign official institutions such as foreign central banks and foreign ministries of finance affect the U.S economy. In the context of a dynamic stochastic general equilibrium (DSGE) model with imperfect asset substitution between short and long-term government bonds, I find that shocks to long-term U.S debt held by foreign official institutions have expansionary effects on the economy—they lower the long-term interest rate and increase output, consumption and inflation. This result is supported by empirical findings from a structural vector autoregression model (SVAR). The second chapter advances the study of foreign aid fungibility by showing how subtle characteristics of household behavior interact with fungible aid and institutional factors to impact aid effectiveness. Specifically, I build a simple dynamic optimizing model and show that the way consumers internalize an aid induced increase in government spending can have very contrasting impacts on aid effectiveness— a feature absent in the extensive empirical literature. Finally, the third chapter studies how different discretionary government spending policy options impact the consequences of explosive government transfer payments. I employ a DSGE model with a fiscal limit— a point where higher taxation is no longer a feasible financing for this study.

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# Chapter 1

## Foreign Official Holdings of U.S Treasuries, Stock Effect and the Economy: A DSGE Approach

## Abstract

Previous studies focus on quantifying the effect of foreign official holdings of long-term U.S Treasuries (FOHL) on the long-term interest rate. The consensus is that FOHL has a large and negative effect on the long-term interest rate. Long-term interest rates matter in determining aggregate demand, but most studies discount the macroeconomic implications of FOHL on the U.S economy. This chapter extends the literature and studies the macroeconomic implications of FOHL shocks through their impact on the long-term interest rate in a dynamic stochastic general equilibrium (DSGE) model. The model treats short and long-term government bonds as imperfect substitutes through endogenous portfolio adjustment frictions(costs). Three main findings emerge from the baseline model: (1) a positive shock to FOHL impacts the long-term interest rate negatively through a stock effect channel— defined as persistent changes in interest rate as a result of movement along the Treasury demand curve. This result is consistent with the empirical literature; (2) the decline in the long-term interest rate creates favorable economic conditions that feed back into the economy and increases consumption, output and inflation through an endogenous term structure implied by the model and; (3) the monetary authority responds to the increase in inflation and output by raising the short-term interest rate. The simultaneous increase in the short-term interest rate and the fall in the long-term interest rate causes the term spread to fall. This last result sheds light on the decoupling of interest rates observed between 2004-2006, a phenomenon known as the “Greenspan Conundrum”.

**JEL:** E43, E52, E58, F21, G12

**Keywords:** Foreign Official Holdings of U.S Treasuries, DSGE, Aggregate Demand, Endogenous Term Structure, Near-Structural VARs

## 1.1 Introduction

The U.S bond market plays a vital role on the global economy as well as the daily lives of every American. Through debt issuance, the government uses the bond market to borrow internationally and domestically in order to carry out key governmental spending on highways, bridges, military spending amongst other government programs. These governmental programs in turn create thousands of new jobs for the unemployed.<sup>1</sup> Furthermore, for several forward looking households in the economy, the bond market plays an essential role in their economic planning decisions including consumption, investment and savings both in the short and long-run.

These points underscore the fact that it is naïve to discount the macroeconomic implications of the actions and interactions of major holders of U.S debt such as foreign official institutions on the U.S economy. It is worth mentioning that excluding the Federal Reserve holdings of long-term government bonds, the share of outstanding long-term U.S Treasuries held by foreign officials increased from 13 percent of outstanding long-term debt in January 1990 to about 50 percent by June 2011.<sup>2</sup> This paper develops a dynamic stochastic general equilibrium model to study and understand the macroeconomic implications of FOHL.

To further elucidate the importance of studying the macroeconomic implications of FOHL, consider figure 1.1 and figure 1.2. As shown in figure 1.1, between July 2004 and July 2006, the 1-year interest rate increased from 1.24 percent to 5.22 percent (approximately 320 percent increase) following the Federal reserve tightening of policy rates, however, the 10-year interest rate only increased from 3.89 percent to 5.09 percent (approximately 34 percent increase).<sup>3</sup> The term spread which is given by the gray line fell during this period and in some cases attained negative values.<sup>4</sup> More importantly, other key factors at the time that included rising energy prices and

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<sup>1</sup>See for instance [Morrison and Labonte \(2011\)](#) for the examination of the importance to the U.S economy of China's investments in U.S Treasuries.

<sup>2</sup> Foreign officials consists of foreign ministries of finance, foreign central banks such as Bank of Japan, Bank of England, Central Bank of Republic of China and other foreign governmental institutions. The percentage of FOHL debt when the Feds holdings are included in outstanding debt is about 33 percent. This is still a significant share.

<sup>3</sup>The Federal Funds Target rate was raised from 2 percent to 5.25 percent in 0.25 percent increments at seventeen consecutive meetings.

<sup>4</sup>The spread between the 10-year yield and 1-year yield is given by the 10-year yield minus the 1-year yield.

robust real economic activity that tend to impact long-term interest rates positively in the past made the slow response of long-interest rates to the increase in short-term interest rates more unusual (Rudesbusch et al. (2006)).

This situation presented a deviation from the conventional wisdom that long-term interest rates will normally move in the same direction as short-term interest rates after controlling for expectations and other risk factors.<sup>5</sup> The sluggish increase in long-term interest rates while short-term rates increased sharply was referred to as the “Greenspan Conundrum”. Large asset purchases by foreign official institutions have been shown to have significantly contributed to the Conundrum.<sup>6</sup> This point succinctly highlights the significant role of foreign official agents in pricing of assets in the U.S which are important determinants for intertemporal economic decisions of domestic households.

Moreover, in figure 1.2, monthly long-term bond holding of U.S Treasuries held by foreign official institutions is compared to the Federal Reserve’s holding over the period January 1990 to June 2011. It is clear that FOHL has consistently been higher than the Federal Reserve holdings. The striking observation from the figure is that, even at the time of the quantitative easing (specifically, QE2- from November 2010 to June 2011), FOHL was approximately two times the Fed holdings of long term bonds.

It is important to note that the quantitative easing and FOHL are both forms of large asset purchases of long and medium term bonds. However, while the quantitative easing was specifically used as an unconventional policy tool to lower long-term interest rates at the Zero Lower Bound with a goal of stimulating the economy, not much is known about how FOHL affects the macroeconomy.<sup>7</sup> Moreover, large asset purchases by foreign official institutions take place in the absence of monetary policy constraint such as Zero Lower Bound, hence they can have unpleasant implications– e.g. the Conundrum – given that monetary policy can be active during such large

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<sup>5</sup>See for instance, the Expectation Hypothesis Theorem.

<sup>6</sup>See Bernanke et al. (2004); Warnock and Warnock (2009); Sierra (2010); Bertaut et al. (2011); Beltran et al (2013) and Kaminska and Zinna (2014), Kohn (2015) for example. An exception to this finding is Rudesbusch et al. (2006) who find no effect of foreign official asset purchases of U.S Treasuries on the long-term interest rate.

<sup>7</sup>See for example Joyce et al. (2012), Gertler and Karadi (2011), Swanson (2011), Falagiarda (2014) for evidence of the effectiveness of Quantitative Easing policy in the U.S.

asset purchases.

This paper draws motivation from the aforementioned examples and examines the macroeconomic implications of large asset purchases by foreign official agents. Specifically, the paper focuses on investigating the impact of FOHL on major macroeconomic variables including consumption, output and inflation through FOHL effect on the long-term interest rate.<sup>8</sup> To see this connection, consider the example of inter-temporal decisions by households in an economy. Inter-temporal decisions on savings, investment, and consumption depend not only on the dynamics of short-term interest rates, but also on the long-term interest rate and the term spread.<sup>9</sup> That is, all other things being equal, low interest rates, both short and long, create favorable economic conditions that stimulate real economic activity.<sup>10</sup>

The key link to studying the implications of shocks to FOHL on the economy is through its impact on the long-term interest rate. Thus, in order to examine the effect of FOHL on the long-term interest rate and its subsequent implications on the macroeconomy, the standard DSGE model employed in this paper is modified to include these three key ingredients: (i) long-term government debt; (ii) endogenous financial cost term; and (iii) non-zero-exogenous foreign official holding of long term bonds.

The endogenous financial cost term allows for imperfect substitution between short and long-term government bonds. Specifically, following Tobin (1969,1982), Andrés et al.(2004) and more recently Falagiarda (2015), the paper introduces an endogenous financial cost term in the form of a portfolio adjustment costs into a DSGE model.<sup>11</sup>

Imperfect substitution exists between short and long-term bonds because households rationalize that they lose liquidity any time they hold long-term bonds relative to holding short term

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<sup>8</sup>FOHL forms 70 percent of total foreign official holdings, see figure 1.3.

<sup>9</sup>For example Andrés et al. (2004) shows that the long-term interest rate unambiguously affects aggregate demand. Moreover, Marzo et. al (2008) provides an empirical and theoretical support for the feedback channel from the term- structure to the macroeconomy. Rudesbush et al. (2007) instead find that although there is no reverse relationship from the term-premium to the economy structurally, reduced form empirical analysis suggests that falls in the term structure is usually associated with stimulus to real economic activity.

<sup>10</sup>Section 3 provides a simple graphical intuition on this example.

<sup>11</sup>For more on portfolio adjustment friction see for instance Marzo et al. (2008), Falagiarda and Marzo (2012), Harrison (2011), Falagiarda(2014). See also D'Amico and King (2013) for empirical evidence of imperfect substitution or segmentation within the Treasury market.

bonds. Hence, households perceive entering the long-term bond market as riskier because longer-term bonds are illiquid relative to the same investment in shorter term bonds. For this reason, households internalize the loss of liquidity by holding additional short-term bonds to compensate themselves of the loss of liquidity anytime they hold long-term bonds. Households therefore self-impose a reserve requirement on their long-term investment in the form of liquidity costs associated with holding them.<sup>12</sup>

The endogenous financial cost term then permits for simultaneous examination of (i) how shocks to FOHL affect the long-term interest through the stock effect channel, defined as persistent changes in price and hence interest rate that result from movements along the Treasury demand curve and include the market reaction due to changes in expectations about future withdraws of supply of Treasuries<sup>13</sup> (ii) the shocks implications on the macroeconomy through a feedback channel from the endogenous term-structure implied by the model.

FOHL is modelled so that it is an exogenous time varying share of long-term outstanding government debt thus they evolve independently of bond prices. This modelling stance on FOHL is in line with [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) who show that foreign officials' demand for U.S Treasuries is inelastic. Essentially, a foreign central bank accumulates more dollar reserves in response to receipt of a dollar capital inflow— buying Treasuries regardless of their prices relative to other assets. Moreover, these foreign officials demand for Treasuries are only slightly sensitive to risk-return considerations.<sup>14</sup>

The model is approximated to the first order and solved numerically using *Dynare*. The impulse response functions from shocks to FOHL in the model are then studied. The findings from the model show that FOHL plays an important role in the economy and their actions have

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<sup>12</sup>See [Andrés et al. \(2004\)](#)

<sup>13</sup>See [D'Amico and King \(2010\)](#) for more on the stock effect. I elaborate more on the stock effect channel in my model in section 4.3.

<sup>14</sup>In contrast, recent surveys of central banks show that most reserve managers do change their reserve portfolios in response to changes in Treasury prices and other macroeconomic variables. Specifically, foreign official institutions optimize their foreign reserve portfolio, hence they are in fact endogenous (See for instance, [Beltran et. al \(2013\)](#); [Borio, Galati and Heath \(2008\)](#); [Pringle and Carver \(2002\)](#)). The focus of this paper is not to study the factors that drive foreign official holding of U.S Treasuries. Thus to keep the model tractable without losing its ability to answer the main questions examined in this paper, I follow studies such as [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Warnock and Warnock \(2009\)](#) by treating FOHL as exogenous.

expansionary effects on the economy. In particular, in the baseline results, positive shocks from FOHL in the form of large purchases of U.S Treasuries affect the long-term interest rate negatively through the stock effect channel on impact. This negative impact on the long-term interest rate generates a feedback mechanism from the endogenous term structure to the economy which creates favorable economic conditions that stimulates the economy leading to an increase in consumption, output and inflation. Moreover, since the monetary authority responds to inflation hawkishly with some degree of policy inertia, short-term interest rates increase. The simultaneous fall in long-term interest rates and increase in short-term interest rate causes the term spread to fall. This last result sheds light on the mechanisms behind the interest rate Conundrum between 2004-2006.

Othe key findings are:

1. The degree of persistence of the FOHL shock demonstrates that the effect of FOHL on the long-term interest rate can range from no impact to a sizeable negative impact on the long-term interest rate. Particularly, when the persistence of FOHL is high, shocks to FOHL have no effect on the long-term interest rate. In contrast, when the persistence is low the shock has a significant and negative impact on the long-term interest rate on impact. This finding in the model is key to understanding the mixed result of the effect of FOHL on the long-term interest rate that exist in the literature. In all cases however, the model predicts a consistent negative effect of the shock on the term spread and the term premium.
2. Given different degrees of persistence of FOHL and imperfect asset substitution, FOHL shocks have similar effects on consumption and output as in the baseline results. However, high (low) degrees of persistence of FOHL and imperfect asset substitutions between the assets causes a longer(faster) return of the term spread to its steady-state generating higher (lower) feedback from the endogenous term structure over time. This yields higher (lower) peak values for consumption and output respectively.

The key assumptions and features in the model are incorporated into a five-variable structural near-VAR model to assess the empirical implication of the model. The empirical results from the near-VAR model complement the core results from the DSGE model.



## 1.2 Related Literature

There is an extensive empirical literature that employs different empirical models ranging from excess returns regression, term premium regressions, cointegrated vector autoregression models to no-arbitrage models to estimate the impact of FOHL on the long-term interest rate. The general consensus is that foreign official holdings have a significant and negative impact on the long term interest rate (Bernanke et al. (2004); Warnock and Warnock (2009); Sierra (2010); Bernanke et al. (2004); Bertaut et al. (2011); Beltran et al. (2013) and Kaminska and Zinna (2014)).<sup>15</sup> An equally important but exception to this finding is Rudesbusch et al. (2006). Employing an affine no-arbitrage macro-finance model, they find no effect of foreign official asset purchases of U.S Treasuries on the long-term interest rate.

In a literature that has predominantly focused on examining the empirical effects of FOHL on the long-term interest rate, the primary contribution of this paper is to examine the macroeconomic effects of FOHL on the U.S economy in the context of a DSGE model.<sup>16</sup> This is achieved by studying the effect of shocks to FOHL on the long-term interest rate through the stock effect channel and their consequent effects on the economy through a feedback mechanisms from the model implied endogenous term-structure. Both the stock effect and feedback mechanism is facilitated by the introduction of portfolio adjustment frictions (costs).

In addition, the paper is able to shed light on an explicit transmission channel– stock effect channel– of how FOHL impacts the long-term interest rate in a context of a DSGE model. In the baseline result, FOHL impacts the long-term interest rate negatively. This result is captured through the stock effect channel generated by the introduction of portfolio adjustment costs.<sup>17</sup>

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<sup>15</sup>See also Bernanke (2005), who argues that unconventional movements of the long term rates is as a result of a global savings glut (GSG) hypothesis. The GSG hypothesis explains that increased capital inflows from countries in which desired savings greatly exceeded desired investment including Asia emerging markets and commodity exporters were an important reason that US longer term interest rates during this period were lower than expected.

<sup>16</sup>See Favilukis et al. (2012) and Favilukis et al. (2014) for other macroeconomic outcomes (housing price and wealth effects) of large Asset Purchases by Foreign Officials.

<sup>17</sup>Kohn (2015) in a consumption based asset pricing model examines one transmission mechanism by assuming that foreign official purchases of U.S debt directly funds domestic consumption growth. Consumption growth is central in the pricing of these asset via the stochastic discount factor in their model. Hence, foreign official purchases affect the yields on long-term bonds through consumption growth. His model abstracts away from monetary policy.

Although the channel and results are not new, the approach employed in this paper to study the effect of FOHL shocks on the long-term interest rate is different from the empirical methodologies usually employed in existing studies. Hence, the baseline results from the model serve as a robustness check for the results in literature.<sup>18</sup>

Moreover, the flexibility of the model allows for a deeper understanding and a panoptic view of how certain characteristics of foreign official institutions, such as the persistence of their holdings of U.S Treasuries can impact the effect of FOHL on the long-term interest rate – a feature not readily observed in the studies above. The effects of such characteristics are studied through sensitivity analysis on the parameter that governs the degree of persistence of FOHL. The results from the sensitivity analysis lends another important contribution to the literature by unifying, in one framework, the contrasting effects of FOHL on the long-term interest rate found in the literature.

A low persistence of FOHL in the model shows that FOHL shocks can have a decently large and negative effect on the long-term interest rate. This is consistent with most of the results in the literature (Bernanke et al. (2004); Warnock and Warnock (2009); Sierra (2010); Bernanke et al. (2004); Bertaut et al. (2011); Beltran et al. (2013) and Kaminska and Zinna (2014)). However, high persistence of FOHL shows that shocks to FOHL have no effect on the long-term interest rate, a result similar to those found in Rudesbusch et al. (2006).

Lastly, unlike the other studies, this paper models monetary policy explicitly in a form of a reaction function and thus provides a different perspective on the interest rate Conundrum observed between 2004-2006 in the U.S. Specifically, monetary policy responds to the expansionary effects of FOHL by increasing short-term interest rates hawkishly.<sup>19</sup> Meanwhile, long-term interest rate are down and respond sluggishly to the increase in short-term interest rate due to the persistent negative stock effect of FOHL. This leads to a decoupling of long-term rate from short-term interest

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This paper instead models monetary policy explicitly and studies the transmission mechanism of FOHL through the stock effect channel generated by imperfect substitution between long and short term bonds.

<sup>18</sup>See Warnock and Warnock (2009) and Beltran et al. (2013).

<sup>19</sup>It is important to note that, during this period (the mid 2000s) monetary policy was “active” – it responded to increases in inflation and output by increasing policy rate.

rate causing the term spread to fall a result consistent with the *Greenspan Conundrum*.

### 1.3 Graphical Intuition

Before introducing the full model, a simple graphical exposition is employed to explain the mechanism through which large asset purchases by other agents apart from households, in this case foreign official agents can affect the economy. To do this, three basic economic relationships are employed: (i) demand and supply of long-term Treasuries; (ii) the inverse relationship between bond prices and their interest rates and (iii) the negative relationship between output and the real interest rate (*Long-term interest rate augmented- IS curve*).

Furthermore, for illustration purposes the following simplifying assumptions are made. The supply of long-term bonds,  $S^L$  is assumed to be inelastic so that foreign official purchases only affect the composition of outstanding supply. It is also assumed that inflation expectations are “well-anchored” – they can be taken as fixed and exogenous so that changes in the nominal and real interest rates are one in the same thing. Consider [figure A](#) below.

[Figure A.1](#) shows the demand and supply of long-term government bonds. Households demand for long-term bonds are negatively related to bond prices and given by  $D_L$ . Without foreign official purchases,  $\Delta F_P = 0$ , supply of long term bonds available to households is  $\bar{S}_L$  and the corresponding equilibrium price and quantity are  $P_D$  and  $B_L^D$  respectively. Now given that exogenous foreign officials increase their holdings by purchasing a positive amount of long-term bonds,  $\Delta F_P > 0$  at any given price, relative supply of long-term bonds available to households falls to  $S_L^{DF}$ . With demand high and supply of these assets low, the equilibrium price of long-term bonds increases from  $P_D$  to  $P_{DF}$  while equilibrium quantity for households falls to  $B_L^{DF}$ . Invoking the negative relationship between the price of bonds and interest rates depicted in [figure A.2](#), the increase in price of long-term bonds to  $P_{DF}$ , will in turn cause the nominal long-term interest to fall from  $i_L^D$  to  $i_L^{DF}$ .

It is important to note that in standard DSGE models, long-term interest rates and hence

Figure A: Simple Graphical Exposition of FOHL and the Economy

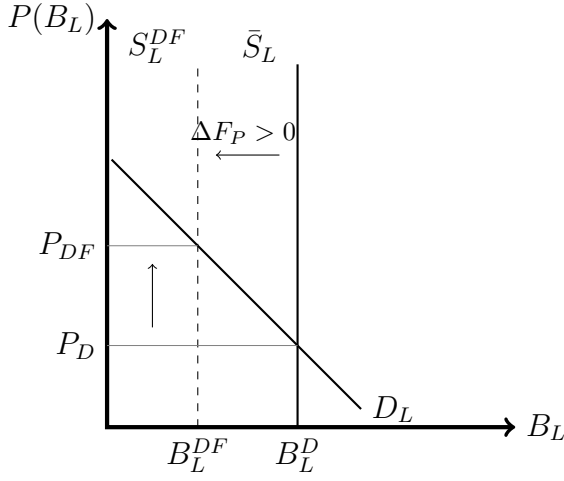


Figure A.1: Long-Term Bond Market with Exogenous Foreign Official Purchases ( $\Delta F_P$ )

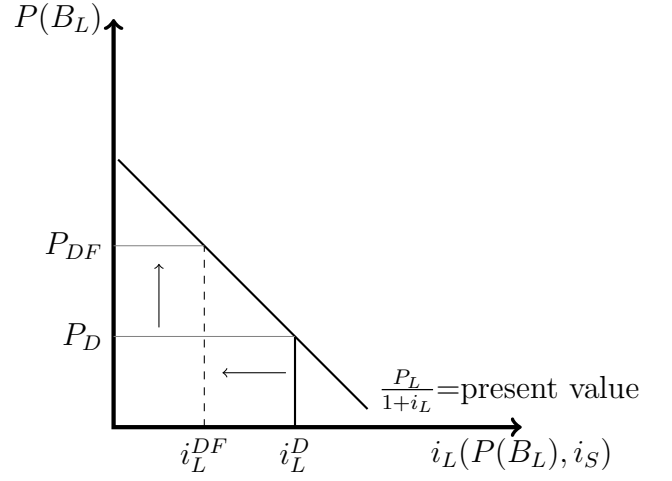


Figure A.2: Bond Price/Nominal Long-Term Interest Rate Negative Relationship

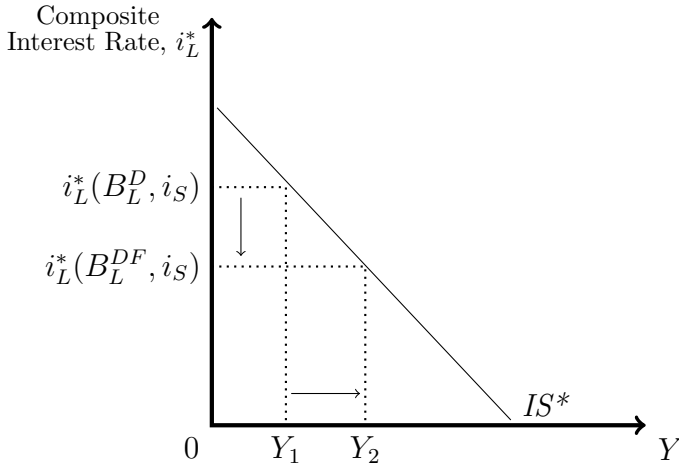


Figure A.3: Long-Term Interest Rate Augmented IS curve

relative bond supply do not play any explicit role in the determination of aggregate demand. Specifically, there is only one interest rate, the short-term rate and its expected path implicitly determines the long-term leaving no room for a separate role for the long-term interest rate and hence supply (quantity) of bonds. However, as shown in figures A.1 and A.2, relative supply of long-term bonds can impact the long-term interest rate independent of short-term interest rates. This implies that long-term interest rates are not simply a function of short-term interest rates but also a function of their relative quantity supplied. Hence without loss of generality, consider a representative interest rate of the economy,  $i_L(B_L, i_S)$ , a function of long-term bond supply and

short-term interest rate,  $i_S$ . Call this representative interest rate a composite interest rate (CIR).

The CIR suggests that both long-term and short term interest rates matter in the determination of aggregate demand (e.g. [Andrés et al.\(2004\)](#) shows that long-term interest rates unambiguously play a role in influencing aggregate demand).<sup>20</sup> Now employing the negative relationship between aggregate demand and the real interest rate, [figure A.3](#) shows the long-term interest rate augmented IS curve,  $IS^*$ .

The transmission mechanism of large asset purchases by foreign officials is as follows: In [figure 1.A](#), a large asset purchase by foreign officials reduces outstanding supply of long-term bonds available to households from  $\bar{S}_L$  to  $S_L^{DF}$ . This bids up the price of long-term bonds from  $P_D$  to  $P_{DF}$ . Given the negative relationship between bond prices and interest rate, the increase in price decreases the interest rates on long-term bonds from  $i_L^D$  to  $i_L^{DF}$  as shown in [figure 2.A](#). Holding inflation expectation and the short-term interest rate constant, the CIR falls from  $i_L^*(B_L^D, i_S)$  to  $i_L^*(B_L^{DF}, i_S)$  following the large asset purchase by foreign officials. The decrease in  $i_L^*$  stimulates the economy and leads to an increase in aggregate demand from  $Y_1$  to  $Y_2$  as depicted in [figure A.3](#).<sup>21</sup>

## 1.4 A DSGE Model with Foreign Official Holding of U.S Treasuries

This section presents the full model. The model comprises a representative agent who populates the economy and supplies labor inputs for firms; a monopolistically competitive firm that hires the labor to produce differentiated goods; a final good firm who purchases the intermediate goods to produce final goods; a government sector that conducts both monetary policy— by targeting inflation and the output gap with some degree of monetary policy inertia to stabilize economic

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<sup>20</sup>In this case we abstract from the short-term interest rate by assuming it to be constant. The full model the role of short-term interest rate.

<sup>21</sup> Notice that since supply of bonds are positively related to interest rate we have  $\frac{\partial i_L}{\partial B_L} > 0$  as demonstrated in [figure A.1](#) and [A.2](#) above. Moreover, from standard arbitrage conditions long-term interest rates are positively related to the short-term interest rate hence  $\frac{\partial i_L}{\partial i_S} > 0$ . The interaction effect of changes in bond supply and the substitute price, short-term interest rate,  $\frac{\partial^2 i_L}{\partial B_L \partial i_S}$ , on the long-term interest rate can be ambiguous and it depends on the relative magnitude of each effect as well as inflation expectations. For simplicity we assume this effect to be zero in the graphical exposition. The full model captures this interactive effect.

fluctuation- and fiscal policy by levying lump-sum taxes on households as well as issuing both short and long-term debt to generate revenue for government spending. Lastly, there is a foreign official agent whose demand for long-term government bonds is an exogenous evolving share of outstanding long-term government bonds.

### 1.4.1 Households

There is a representative agent who lives infinitely. The agent gains utility by choosing consumption bundle  $C_t$ , real money holdings  $M_t/P_t$  and labor hours  $N_t$  according to the utility function

$$u\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{(C_t - \theta C_{t-1})^{1-\gamma}}{1-\gamma} + \frac{\vartheta}{1-\eta} \left(\frac{M_t}{P_t}\right)^{1-\eta} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \quad (1.1)$$

where  $\chi > 0$ ,  $\vartheta > 0$ ;  $\gamma > 0$  is the coefficient of risk aversion,  $\eta > 0$  is the elasticity of money demand;  $\theta > 0$  is the habit formation parameter and  $\varphi \geq 0$  is the inverse of the Frisch elasticity of labor supply. The representative household thus maximizes her life-time utility

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, \frac{M_t}{P_t}, N_t) \quad (1.2)$$

with  $\beta \in (0, 1)$  as the discount factor. Since there is a continuum of consumption goods available for purchase,  $C_t$  corresponds to a Dixit-Stiglitz aggregate of consumption;

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (1.3)$$

where  $i \in (0, 1)$  represent the continuum of differentiated final goods and  $\varepsilon > 1$  governs the elasticity of substitution between different final goods.

The household's budget constraint which incorporates the secondary market for bond trading as [Ljungqvist and Sargent \(2004\)](#) is given by equation (2.4) where  $P_t$  is the aggregate price level in

the economy.

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + \rho_t) + \frac{M_t}{P_t} + T_t \leq \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} N_t - C_t - \frac{D_t}{P_t} \quad (1.4)$$

The household agent allocates wealth between money holding,  $M_t$  and two zero-coupon bonds which differ in maturity, these bonds are purchased at their nominal prices. The bonds are short term bonds and long term bonds denoted  $B_t$  and  $B_{L,t}^H$  respectively.  $B_t$  yields  $R_t$  and  $B_{L,t}^H$  yields  $R_{L,t}$ . The budget constraint of households reveals an active secondary market as proposed by [Ljungqvist and Sargent \(2004\)](#).

Particularly, the right hand side of the household budget constraint shows that long term bonds  $B_{L,t-1}^H$  are priced with short-term rates, that is, the agent carries over long term bonds purchased at time  $t-1$  and sells it on the secondary market at the rate  $1/R_t$ . However, at time  $t-1$ , an agent who buys long-term bonds and intends to sell them in period  $t$  faces price uncertainty as  $R_t$  is not known at time  $t-1$ .<sup>22</sup> This formulation of the budget constraint to incorporate secondary market allows for a straightforward modelling of assets of different maturities. Moreover, this helps to capture the active participation of foreign central banks on the secondary market.

In line with [Andrés et al. \(2004\)](#), [Falagiarda and Marzo \(2012\)](#), [Harrison \(2012\)](#) and [Falargiada \(2014\)](#), the paper assumes that intratemporal trading between bonds of different maturities is costly to agents thus they pay a cost whenever they shift the portfolio allocation between short and long-term bonds, the endogenous cost function is then modelled as :

$$\rho_t = \frac{\phi_L}{2} \left( \kappa_L \frac{B_t}{B_{L,t}^H} - 1 \right)^2 Y_t \quad (1.5)$$

where  $\phi_L > 0$  and  $\kappa_L = B_{L,t}^H/B_t$  is the inverse of steady state household holding of short-term to long-term bonds. This implies that  $\rho_t$  is zero at steady state. The financial friction term allows for imperfect substitutability between long and short term bonds.

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<sup>22</sup>As explained by [Ljungqvist and Sargent \(2004\)](#) the price  $R_t$  follows from a simple arbitrage arguments, in period  $t$ , these bonds represent identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period  $t$ . See also [Falagiarda \(2014\)](#) for a similar formulation of long-term bonds.

There are several motivation for including the transaction cost friction. However following [Andrés et al \(2004\)](#), its argued that households perceive entering the long term bond market as riskier, that is, they are illiquid relative to the same investment in shorter term bonds. Thus as they purchase long-term bonds, they hold additional short-term bonds to compensate themselves of the loss of liquidity. Specifically, households in effect self-impose a reserve requirement on their long-term investment.<sup>23</sup>

### 1.4.2 Optimality Conditions

The first order conditions for the optimizing consumer's problem is given as:

$$C_t : (C_t - \theta C_{t-1})^{-\gamma} - \beta \theta E_t (C_{t+1} - \theta C_t)^{-\gamma} = \lambda_t \quad (1.6)$$

$$N_t : \chi N_t^\varphi = \lambda_t \left( \frac{W_t}{P_t} \right) \quad (1.7)$$

$$M_t : \vartheta \left( \frac{M_t}{P_t} \right)^{-\eta} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \quad (1.8)$$

$$B_t : E_t \frac{\beta \lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_t} + \frac{\lambda_t \kappa_L \phi_L}{R_{L,t}} \left( \kappa_L \frac{b_t}{b_{L,t}^H} - 1 \right) Y_t \quad (1.9)$$

$$B_{L,t} : E_t \frac{\beta \lambda_{t+1}}{\pi_{t+1} R_{t+1}} = \frac{\lambda_t}{R_{L,t}} + \frac{\lambda_t \phi_L}{2 R_{L,t}} \left( \kappa_L \frac{b_t^H}{b_{L,t}^H} - 1 \right)^2 Y_t - \frac{\lambda_t \phi_L \kappa_L b_t}{R_{L,t} b_{L,t}^H} \left( \kappa_L \frac{b_t}{b_{L,t}^H} - 1 \right) Y_t \quad (1.10)$$

Equation (2.6) represents the marginal utility of wealth and it depends on the marginal utility

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<sup>23</sup>Other justifications for including the portfolio friction is the theory of preferred habitat by [Vayanos and Vila \(2009\)](#). Secondly, as in [Falagiarda \(2014\)](#), one can rationalize these costs as proxies for the shares of resources devoted to covering information costs or costs of managing bond portfolio.



of consumption today and the expected marginal utility of consumption tomorrow generated by the presence of habits in consumer preferences. Equation (1.7) relates real wage to the marginal rate of substitution between labor hours and consumption. Equation (2.6) and (2.9) can be combined to obtain an expression for money demand. Finally, equation (1.9) and (1.10) are the Euler equations for short and long term bond holdings respectively. As it is standard in the literature, we will show below that those two equations implicitly reveal a term structure relationship linking long and short term rates.<sup>24</sup>

### 1.4.3 Stock Effect Channel and Feedback Mechanism

To gain insight of the channel through which foreign official holdings affect the long term rate and hence the term spread in this model, I combine the log-linearized first order conditions of short and long term bond holdings, equations (1.9) and (1.10).<sup>25</sup> This yields:

$$\tilde{R}_{L,t} = \tilde{R}_t + \eta_1 E_t \tilde{R}_{t+1} + \eta_2 E_t (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) - \eta_3 (\tilde{b}_t - \tilde{b}_{L,t}^H) \quad (1.11)$$

or

$$\text{Term Spread} = \tilde{R}_{L,t} - \tilde{R}_t = \eta_1 E_t \tilde{R}_{t+1} + \eta_2 E_t (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) - \underbrace{\eta_3 (\tilde{b}_t - \tilde{b}_{L,t}^H)}_{\text{Stock Effect}} \quad (1.12)$$

where  $\eta_3 = \phi_L (1 + \frac{\kappa_L}{R}) > 0$  and  $\eta_1$  and  $\eta_2$  are convolutions of steady state values and structural parameters.<sup>26</sup> The stock effect is captured in the last term of equation (2.13), this is due to the imperfect substitutability between the bonds in this model. Equation (2.13) is consistent with Tobin's argument that relative supply of different assets affects the spreads of these assets. Notice that, the portfolio cost parameter,  $\phi_L$ , governs the degree to which relative bond holding movements along the Treasury demand curve affects the long-term rates. If  $\phi_L$  is equal to zero (i.e.  $\eta_3 = 0$ ), equation (2.12) reduces to a form of expectation hypothesis and the stock effect is absent.

<sup>24</sup>Similar results can be found in [Andrés et al \(2004\)](#), [Marzo et al \(2008\)](#) and [Falargiada \(2014\)](#)

<sup>25</sup>Throughout the model, variables with tildes represent deviations from their respective steady state.

<sup>26</sup>  $\eta_1 = \frac{\beta \bar{R}_L}{\bar{\pi} \bar{R}}$  and  $\eta_2 = \frac{\beta \bar{R}}{\bar{\pi}} - \frac{\beta \bar{R}_L}{\bar{\pi}}$ .

From equation (2.12), long-term rates depends positively on long term bond supply  $\tilde{b}_{L,t}^H$ . Short term bond supply on the other hand affects the long-term rate negatively. Hence persistent increase in long-term bond holdings by foreign official institutions reduces the relative supply of long-term bonds available to households. The long-term rate then falls given its positive relationship with long-term bond supply  $\tilde{b}_{L,t}^H$ .

An important feature which is central to studying the effects of shocks to foreign official holdings of long term bonds is the feedback channel from the model implied term structure to the macroeconomy. To see this feature, the log-linearized for equation for consumption and the first order condition of long-term bonds are combined to yield:

$$\tilde{C}_t = \eta_4 E_t \tilde{C}_{t+1} - \eta_5 \tilde{R}_{t+1} - \eta_5 \tilde{R}_{L,t} + \dots + \eta_7 E_t \tilde{\pi}_{t+1} \quad (1.13)$$

Equation (2.14) shows that both long-term rate and short-term rate are equally important in impacting current consumption. Moreover, in the case where policy rates are at the Zero Lower Bound (ZLB), it is clear that long-term rates play a much more direct role in impacting aggregate demand.<sup>27</sup> However, in this paper where focus is on the pre-ZLB period monetary policy is active. Hence, when long-term rates are not in tandem with short-term rates, as in the case of the Greenspan Conundrum, the two rates can have conflicting effects on current consumption.<sup>28</sup> Hence, the impact of a derivative of the two rates— in this case the simple spread between the long-term rates and the short-term rate— on current consumption needs to be considered. To see this, equation (2.14) can be rewritten such that:

$$\tilde{C}_t = \eta_4 E_t \tilde{C}_{t+1} - \eta_{5,6} \tilde{R}_t - \eta_6 \underbrace{(\tilde{R}_{L,t} - \tilde{R}_t)}_{\text{Term Spread}} + \dots + \eta_7 E_t \tilde{\pi}_{t+1} \quad (1.14)$$

Equation (1.14) shows that apart from future consumption, expected inflations and short-term

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<sup>27</sup>I refer to aggregate demand here because in a simple case where one assumes away government expenditure and investment, the market clearing condition will be  $Y_t = C_t$  in which case equation 2.14 become  $\tilde{Y}_t = \eta_4 E_t \tilde{Y}_t - \eta_5 \tilde{R}_{t+1} - \eta_5 \tilde{R}_{L,t} + \dots + \eta_7 E_t \tilde{\pi}_{t+1}$ . See Falagiarda (2014) for the treatment of the ZLB and Quantitative Easing in a similar context.

<sup>28</sup> This reiterates the importance of including not only short-term into the analysis of aggregate demand but also a separate role for long-term rates needs to be accounted for by including long-term bonds.

rates, falls in the term spread affects consumption positively and through the resource constraint and other general equilibrium forces can increase aggregate output and affect all the macroeconomic variable present in the model. The latter point can be elucidated as follows.

Suppose the economy is hit by a positive shock that initially stimulates consumption and hence increases inflation and output. Now since the monetary authority responds actively to inflation, it will increase policy rates. From equation (1.14), the increase in policy rates will decrease consumption. However, if long-term rates responds sluggishly (i.e.  $\tilde{R}_{L,t} < \tilde{R}_t$ ) to the increases in policy rates due to other factors such as the persistence of stock effect, then from equation (1.14), this can result in falls in the term spread. The fall in the term spread in turn can further lead to stimulating consumption.<sup>29</sup> In conclusion, monetary policy goals can be stifled by deviations of the long-term rates from short-term rates. This is the channel that shocks to FOHL affects the real economy in the model.

#### 1.4.4 Production of Intermediate Goods

Intermediate goods producing firm  $i$  has access to a constant returns to scale technology,

$$Y_t(i) = AN_t(i)$$

where  $Y_t(i)$  is output of the intermediate firm  $i$  and  $N_t(i)$  is the amount of labor the firm hires. The firm thus minimizes its total cost subject to the production technology.

#### 1.4.5 Price setting

A final goods producing firm purchases intermediate inputs at nominal price  $P_t(i)$  and produces the final composite good using the following constant returns to scale  $Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$  where  $\varepsilon > 0$  is the elasticity of substitution between goods. Profit-maximization by the final goods

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<sup>29</sup>Rudesbush et al. (2007) for instance finds that reduced form empirical analysis suggests that falls in the term structure is usually associated with stimulus to real economic activity.

producing firm yields a demand for each intermediate good given by

$$Y_t(i) = \frac{P_t(i)^{-\varepsilon}}{P_t} Y_t$$

Monopolistically competitive intermediate goods producing firm  $i$  chooses price  $P_t(i)$  to maximize the expected present value of profits:

$$E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \frac{D_{t+j}(i)}{P_{t+j}}$$

where  $Q_{t+j} = \frac{\lambda_{t+j}}{\lambda_t}$  is the household's stochastic discount factor,  $D_t(i)$  are nominal profits for firm  $i$  and  $P_t$  is the nominal aggregate price level in the economy. Real profit are therefore given by,

$$\frac{D_{t+j}(i)}{P_{t+j}} = \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \Psi_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - \frac{\psi}{2} \left( \frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1 \right)^2 Y_t$$

where  $\psi \geq 0$  governs adjustment costs,  $\Psi_t(i)$  is real marginal cost. Price adjustments are introduced through [Rotemberg \(1982\)](#) quadratic costs of adjustment reflecting the negative effect that price changes can have on firm-customer relationship. In a symmetric equilibrium, all firms make identical decisions and hence the first order condition is given as follows:

$$0 = (1 - \varepsilon) + \varepsilon \Psi_t - \psi \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} + \psi E_t \left[ Q_{t+1} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right] \quad (1.15)$$

#### 1.4.6 Foreign Official Holdings

As explained earlier, to keep the model simple and tractable without losing its ability to answer the main questions examined in this paper, FOHL are modelled as an exogenous time varying share of long-term outstanding government debt.<sup>30</sup> Particularly, I assume that long-term foreign official holding denoted by  $B_{L,t}^F$  is a share  $x_t$  of outstanding long term debt and  $x_{F,t}$  evolves exogenously

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<sup>30</sup> For example [Warnock and Warnock \(2009\)](#) assume foreign official holdings as exogenous when estimating its effect on interest rates. See also [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) for empirical evidence.

according to an AR(1) process. Hence,

$$B_{L,t}^F = x_{F,t} B_{L,t} \quad (1.16)$$

$$\log\left(\frac{x_{F,t}}{x_F}\right) = \rho_x \log\left(\frac{x_{F,t-1}}{x_F}\right) + \varepsilon_t^{x_F} \quad (1.17)$$

where  $x_F = B_L^F/B_L$  is the steady state values of  $x_t$ .

#### 1.4.7 Demand for Long-Term Bonds

Households and foreign officials demand outstanding long term bonds so that.

$$B_{L,t} = B_{L,t}^H + B_{L,t}^F \quad (1.18)$$

where  $B_{L,t}$  are outstanding government long-term bonds.

#### 1.4.8 The Government

Government expenditure is financed by seigniorage revenues, issuance of long-term, Lump sum taxes and short-term bonds. Thus the government budget constraint is given as

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{M_t}{P_t} + T_t = \frac{B_{t-1}}{P_t} + \frac{B_{L,t}}{P_t R_t} + G_t + \frac{M_{t-1}}{P_t} \quad (1.19)$$

Furthermore, I model the issuance of new long term bonds to follow an AR(1) process so that shocks to foreign official demand for long term bonds only affects the composition of outstanding government debt (See for instance [Marzo et al.\(2008\)](#) and [Falagiarda \(2014\)](#)).

$$\log\left(\frac{B_{L,t}}{B}\right) = \rho_{bL} \log\left(\frac{B_{L,t-1}}{B_L}\right) + \varepsilon_t^{bL} \quad (1.20)$$

Government expenditure  $G_t$  is set according to the AR(1) process:

$$\log\left(\frac{G_t}{G}\right) = \phi_G \log\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^G \quad (1.21)$$

where  $\phi_G \in (0, 1)$  and  $\varepsilon_t^G$  is an i.i.d shock with zero mean and standard deviation  $\sigma_G$

Lump sum taxes  $T_t$  is a function of the total government liabilities:

$$T_t = \zeta_0 + \zeta_1 \left( \frac{b_{t-1}}{\pi_t} - \frac{b}{\pi} \right) + \zeta_2 \left( \frac{b_{L,t-1}}{R_t \pi_t} - \frac{b_L}{R \pi} \right) \quad (1.22)$$

where  $\zeta_0$  is the steady-state level of  $T_t$ , and  $\zeta_1, \zeta_2$  have been set equal so taxes respond equally to short and long-term debt.

Finally, the central bank conducts monetary policy with a short-term interest rate feedback rule in the form specified by [Taylor \(1993\)](#) augmented to include interest rate smoothing:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_R)\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + (1 - \rho_R)\rho_Y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_t^R \quad (1.23)$$

hence  $R_t$  inflation and output through  $\rho_\pi$  and  $\rho_Y$  respectively with an interest rate smoothing component governed by  $\rho_R$ . The exogenous policy shifter in monetary policy,  $\varepsilon_t^R$  is assumed to be a white noise monetary policy disturbance.

#### 1.4.9 Resource Constraint

With the introduction of endogenous financial cost frictions, aggregate output of the economy is not simply allocated to consumption, government expenditure and price adjustment costs but also to a portfolio adjustment cost term which is priced in output. Thus the model is closed by a resource constraint given as:

$$Y_t = C_t + G_t + \frac{B_{L,t}}{P_t R_{L,t}} \rho_t - \frac{\psi}{2} \left( \frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1 \right)^2 Y_t \quad (1.24)$$

## 1.5 Results

This section presents the solution process and results of the model outlined in section 3. Simulations are conducted to study the impact of FOHL shocks on key macroeconomic variables using a calibrated version of the model. The model is log-linearized around its steady state and solved using *Dynare*. In what follows, the calibration of key parameters are discussed and then the results of the baseline model is analyzed. Finally, sensitivity analyses are carried out to examine the effects of varying the key parameters of the model, that is, the parameter governing portfolio costs ( $\phi_L$ ) and the persistence parameter for the share of FOHL ( $\rho_x$ ).

### 1.5.1 Calibration

The baseline model is calibrated at a quarterly frequency to match the behavior U.S data prior to the financial crisis in 2008.<sup>31</sup> A subset of the parameters are chosen based on previous studies and are standard in the literature. Specifically, following for instance [Fuhrer \(2000\)](#) the habit formation parameter  $\theta$  is set to 0.7. The discount factor is set at  $1.04^{-1/4}$ , which implies a steady-state annualized real interest rate of 4 percent. The implied steady-state real long-term interest rate is then given by  $R/\beta$ . Preferences over consumption are logarithmic, hence  $\gamma = 1$ . The Frisch labor supply elasticity is set to unity, so  $\varphi = 1$  and  $\chi$  is set such that the steady state share of time spent in employment is  $1/3$ . As mentioned earlier, intermediate goods-producing firms use a constant returns to scale production function. The common technology parameter,  $A$  is set to normalize the deterministic steady state level of output to 1.

The parameter that determines the interest elasticity of real money balances,  $\eta$  is set to 2.6 [[Mankiw and Summers \(1986\)](#), [Lucas\(1988\)](#), [Chari et al. \(2000\)](#)]. For real balances,  $\vartheta$  is set so that

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<sup>31</sup>See Table [B.1](#), [B.2](#) and [B.3](#) for the model calibration

the velocity in the deterministic steady state, defined as  $cP/M$  corresponds to a value of 2.4 as in Davig and Leeper (2006). The price elasticity of demand  $\varepsilon$  and the Rotemberg adjustment cost coefficient  $\psi$  are set to 6 and 100 respectively as in Schmitt-Grohé and Uribe (2004) and Ireland (2004) respectively. The parameter value of price elasticity of demand means firms markup the prices of their goods over marginal cost by 20 percent.

The parameters governing monetary and fiscal rules are calibrated in a standard way. Particularly, the interest rate smoothing parameter  $\rho_R$  is set to 0.75 while  $\rho_\pi$  and  $\rho_Y$  are set to 1.5 and 0.6 respectively. Adaptin a passive tax policy rule, the coefficients in the fiscal rule are set to  $\psi_b = \psi_{bl} = 0.15$ . The autoregressive coefficients and standard deviations of the shocks in the model are set to  $\phi_G = \rho_{bL} = 0.9$  while  $\sigma_R = 0.005$  and  $\sigma_G = 0.012$  [Kim (2000), Andrés et al. (2004), Altig et al. (2011), Falagiarda and Marzo (2012) and Zagaglia (2013)]. There is one free parameter which is the portfolio adjustment friction  $\phi_L \in [0.005, 0.1]$  which falls between values of Andrés et al (2004), Chen et al. (2012) and Falagiarda (2014). It is set to a value of 0.01 in the baseline case . Sensitivity analysis is conducted by perturbing the parameter to analyse its impact on the economy.

Appendix 1.B derives the model implied parameters and steady state values see. Table B.3 reports the steady states values of bond holdings by households, foreign official institutions and total bond demand. The steady state values were computed from Betaut-Tyron measures of benchmark consistent positions. The steady-state total of debt to output ratio is 28 percent and this corresponds to a steady-state tax output ratio of 19.5 percent.<sup>32</sup>

The steady-state value for the share of foreign official holding of long term bonds  $x_t$  can be pinned down by equations (15),  $x = B_L^F/B_L$ . The persistence parameter,  $\rho_x$  that governs the AR(1) process for the share of FOHL in outstanding debt ( $x_t$ ) is set to 0.72 and the corresponding standard deviation is  $\sigma_x$  is set to 1.53.<sup>33</sup> Sensitivity analysis is conducted on  $\rho_x$  by setting it to high and low values away from the baseline value of 0.72.

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<sup>32</sup>The steady debt to output ratio is a little lower than the usual 33 percent value. This is because the analysis abstracts away from the Feds holdings of debt. This does not change the main results of the model

<sup>33</sup>An ARCH-in-Mean estimation is carried out for the AR(1) equation of  $x_t$  to obtain the baseline parameter values for  $\rho_x$  and  $\sigma_x$ .



### 1.5.2 Impact of foreign official holding shock

To examine the impact of shocks to foreign official holdings, figure 1.4 shows the equilibrium models impulse responses following a positive shock to foreign official purchases of long-term U.S Treasuries (i.e. a shock to  $x_t$ ). An average positive shock of  $\sigma_x = 1.53$  to long-term bond holdings by foreign officials reduces the relative supply of long-term bond supply and hence the amount of long-term bonds available to households. Through the stock effect channel shown in equation (2.13), the reduction of relative supply of long-term bonds available to households then reduces the long-term yield by an average of 13 basis points on impact. The shocks' negative impact on long-term interest rate is consistent with results found in the empirical literature (See for instance, Warnock and Warnock (2009), Bernanke et al. (2004), Beltran et al. (2013), Kaminska and Zinna (2014)).<sup>34</sup>

The effect of a on the macroeconomy occurs via the feedback mechanism from the endogenous term structure generated by the model. Through the feedback mechanism shown in equation (1.14), the fall in long-term interest rate creates favorable economic conditions that stimulates consumption. Consequently, through the resource constraint, output increases which in turn increases inflation via equilibrium forces. Monetary policy responds to the increase in output and inflation by increasing the short term rate ( $R_t$ ). Finally, the decoupling of long-term rates from short-term rate reduces the term spread defined as long-term rate minus short-term rate. The simultaneous rise in short-term interest rate, fall in long-term interest rate, and fall in the term spread is consistent with the “Greenspan Conundrum”, i.e. the decoupling of long-term interest rates from short term interest rates between 2004 to 2006.

It is important to conduct sensitivity analysis to gain insight of the principal mechanisms at work. Specifically, low and high parameter values are assigned to the parameter governing: (i) the persistence of FOHL shock,  $\rho_x$  and (ii) portfolio adjustment cost,  $\phi_L$ . The dynamics of the model following the variations of these parameters is then compared to the baseline model. Furthermore,

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<sup>34</sup>The quantitative impact of FOHL shock on long-term rates in the model is compared to selected empirical studies in Section 4.

the impact of FOHL shock on the term-premium is discussed in the sensitivity analysis.<sup>35</sup> The results from the sensitivity analysis is discussed in below.

### 1.5.3 Role of Persistence of Foreign Official Holding Shock, $\rho_x$

To investigate the role of the persistence of the FOHL shock, sensitivity analysis for the parameter governing the AR(1) process for the share of FOHL,  $\rho_x$  is carried out. The parameter is set to a low and high value away from the baseline value of 0.72. The corresponding low and high value for  $\rho_x$  are 0.52 and 0.83 respectively.

Figure 1.5 plots the impulse response functions when varying  $\rho_x$  to examine the role of the persistence of FOHL shock. The solid blue line is the baseline case, the dotted red line is the low persistence case and the dashed black line is the high persistence case. The mechanisms at work is the same as explained in the baseline case. However, on impact, a higher persistence value associated with FOHL shock (black dashed line) increases consumption, output, and hence inflation higher than the baseline case. This causes the monetary authority to raise the short-term rates more aggressively, which in turn offsets the negative effect of the shock on long-term interest rate.

This offsetting effect makes it appear that the FOHL shock has little or no impact on the nominal long-term interest rate when it hits. This outcome is in line with results in [Rudesbusch et al. \(2006\)](#) who find an insignificant effect of foreign official purchases on the long-term interest rate. It is important to note that long-term rates do not respond one-to-one to the aggressive increase in the short-term rates since it takes longer for demand of long-term bonds to return to its steady state— a persistent stock effect. Consequently, there is a persistent delay in the term-spread to return to its steady after it falls. This effect feeds back into the economy inducing higher peak values for consumption, output and inflation relative to baseline case. The opposite effect holds for the case of low low persistence value of  $\rho_x$  (blue line).

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<sup>35</sup>The term premium is computed as the deviation of the long-term interest rates from its expectation hypothesis component and it is given as  $TP_t^{(k)} \equiv R_t^{(k)} - \frac{1}{k} \sum_{j=0}^{k-1} E_t R_{t+j}$  where  $R_t^k$  is the yield of a  $k$  - period zero-coupon bond at time  $t$  proxied by constant maturity bond in the model;  $R_t^1 = R_t$  and  $k = 10$  in the model.

#### 1.5.4 Role of Portfolio Adjustment Costs, $\phi_L$

To examine the role of portfolio adjustment cost which reflects the degree of imperfect asset substitutability between short and long-term bonds, the parameter value governing portfolio adjustment costs  $\phi_L$  is varied according to low substitutability (dotted red line,  $\phi_L = 0.005$ ) and high substitutability (dashed green line,  $\phi_L = 0.02$ ) values. These variations in  $\phi_L$  are then compared to the baseline case (solid blue line with  $\phi_L = 0.01$ ). Notice that in the absence of portfolio adjustment costs ( $\phi_L = 0$  when short and long term bonds are perfect substitutes), reductions in relative supplies of the two bonds have no impact the interest rates and hence the economy. In this scenario, the stock effect is non-existent.

Figure 1.6 plots the impulse response functions when varying the parameter that governs imperfect asset substitution,  $\phi_L$ . Given an equal fall in long-term bond supply available to households following a positive shock to FOHL, a higher imperfect asset substitutability generates higher stock effect relative to the baseline value of portfolio adjustment cost. Specifically, in the case of higher portfolio adjustment cost (the dashed black line), the term spread falls more compared to the low and baseline cases of the portfolio adjustment cost which is given by the dotted red line and solid blue line respectively. Again, through the feedback mechanism from the endogenous term structure explained in equation (2.14), the peak effect of consumption, output and inflation are higher in the case of high portfolio adjustment costs due to a more severe fall in the term spread. The opposite holds in the case of low portfolio adjustment costs (dashed blue line).

#### 1.5.5 Model's Effect on Long-Term Interest Rate Compared to Other Studies

While investigating the macroeconomic implications of FOHL shock in the model, the long-term interest rate served as the key link connecting the dots on how FOHL affects the economy. Consequently, the paper directly studies the effect of FOHL on the long-term interest rate through the stock effect channel in the context of a DSGE model with portfolio adjustment costs. The methodology employed in this paper is fundamentally different from those in the existing literature that use a broad spectrum of fully-fledged empirical models to study the impact of FOHL

on the long-term interest rates. It is therefore necessary—after acknowledging all conceptual and methodological differences— to compare how well the model performs quantitatively on the impact of FOHL on the long-term rate to other studies.<sup>36</sup>

Table B.4 compares the quantitative effect of FOHL on the long-term interest rate and the term spread implied by the model to selected empirical studies. Overall, on impact, the model implied quantitative effect of FOHL on the long-term interest rate (level)—with the maximum and minimum effect of -22 and 0 basis points respectively— is rather low compared to the values from the selected studies. This disparity as explained earlier are due to methodological and conceptual issues which can include the choice of approximation technique selected to numerically solve the model.<sup>37</sup> Notice however that the zero impact effect of the shock on long-term interest rate is consistent with the no effect found in Rudesbusch et al. (2006).

The model implied effect of FOHL on the term premium is however comparable to the empirical studies. Particularly, on impact, the shock’s effect on the term premium ranges from -34 to -54 basis points which compares to a similar result, -46 to -50 basis points— found in Beltran et. al (2013) and -51 as in Kohn (2015). Lastly, the stock effect of FOHL in the model is -26 basis points, a value -6 higher than the result in Beltran et al. (2013) and -11 basis point higher than stock effect in Bertaut et al. (2011).<sup>38</sup>

## 1.6 The Model’s Empirical Implication

The goal of this section is to assess to empirical implications of the DSGE model explained above. An important feature of the modelled economy in section 4 is the fact that, FOHL shock impacts the economy through the endogenous term structure of interest rate, in this case captured through

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<sup>36</sup>A caveat to this comparison is that different methodologies, measures of foreign official holdings (e.g.6-month, 12-month flow measure) or the frequency of the data employed to study the effect of FOHL on the long-term interest rate are likely to lead to very different results. This point is well-emphasized in Beltran et al.(2013). They note that differences in their estimates compared to those from large-scale asset purchases (LSAP) can be attributed to conceptual and methodological issues. I take an agnostic stands on these issues by acknowledging these caveats and compare my results to other studies.

<sup>37</sup>Table B.5 compares model generated moments to the empirical moments from data.

<sup>38</sup>The stock effect ( $SE_x$ ) is computed as the impact effect  $I_{\sigma_x}$  divided by one minus the persistence of FOHL and normalized by 3 to achieve its monthly effect since the model is calibrated quarterly,  $SE_x = \frac{1}{3} \left( \frac{I_{\sigma_x}}{1-\rho_x} \right)$ .

the term-spread. Moreover, FOHL do not respond to any asset price or any macroeconomic variable. Therefore to assess the empirical implications of the DSGE model, these key features from the model are incorporated into a five variable structural near-VAR model. The effects of FOHL shock on the variables included in the near-VAR are then studied through impulse response functions from the near-VAR. Specifically, the paper does a Monte Carlo integration analysis of a combination of a near-VAR for the lag coefficients and a structural VAR for the covariance matrix.

### 1.6.1 The near-VAR model

Based on assumptions and implications of the DSGE model discussed above, quarterly data from the period 1986:1 to 2007:4 is used to estimate the near-VAR model below<sup>39</sup>:

$$\begin{bmatrix} y_t \\ \pi_t \\ R_t \\ spd_t \\ x_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) & A_{13}(L) & A_{14}(L) & A_{15}(L) \\ A_{21}(L) & A_{22}(L) & A_{23}(L) & A_{24}(L) & A_{25}(L) \\ A_{31}(L) & A_{32}(L) & A_{33}(L) & A_{34}(L) & A_{35}(L) \\ A_{41}(L) & A_{42}(L) & A_{43}(L) & A_{44}(L) & A_{45}(L) \\ 0 & 0 & 0 & 0 & A_{55}(L) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ R_{t-1} \\ spd_{t-1} \\ x_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{bmatrix}}_{e_t} \quad (1.25)$$

The estimated model is partitioned into two blocks. The first block includes the following four variables<sup>40</sup>: the cyclical component of real gross domestic product,  $y_t$ , which is obtained by applying the Hodrick-Prescott filter; the rate of inflation,  $\pi_t$ , computed from the GDP deflator; the effective Federal Funds rate,  $R_t$ ; the term spread,  $spd_t$ , computed as the 10-year interest rate minus the 3-month interest rate. The second block includes one variable, the 3-month average of 3-month flow measure of FOHL,  $x_t$ .<sup>41</sup> As in the DSGE model, the feedback mechanism from the

<sup>39</sup>The start date for the data is due to data availability and the end data is to avoid the nonlinearities posed by Zero Lower Bound and Quantitative Easing after the global financial crisis in 2008.

<sup>40</sup>This specification is standard in the structural vectorautoregression literature. See [Marzo et al. \(2008\)](#) for further discussion. In [Marzo et al. \(2008\)](#) the term structure is captured by including the 1-year, 5-year and 10-year interest rate. I instead summarize the term structure by including the term spread. This is discussed more in the identification section.

<sup>41</sup>The data on real GDP, GDP deflator, Federal Funds rate, 10-year and 3-month interest rate were obtained from the FRED database available at: <https://research.stlouisfed.org/fred2/>. Data concerning foreign official holding

term structure to the economy is capture by including the lag terms of the term spread in the output and inflation equation. Furthermore, the lags of FOHL,  $(x_t)$  is included in all the other equations, however, since FOHL does not respond to any macroeconomic variable, zero restrictions are placed on all the coefficients of the macroeconomic variables in the FOHL equation of the near-VAR model.

Due to the zero restrictions on the lag variables in the last equation, a Seemingly Unrelated Regression (SUR) is employed to estimate the system in (2.26). One can obtain consistent estimators from using OLS in the presence of a near-VAR, however, as explained in Zellner (1962) there are potential efficiency gains in using SUR. As far as the lag length selection goes, the Shwartz Information Criteria suggests one lag for the estimated VARs.

### 1.6.2 Identification

To summarize the identification strategy, let  $\mathbf{e}_t$  denote the  $5 \times 1$  vector that collects the reduced form near-VAR residuals  $(e_{it})$  and let  $\boldsymbol{\varepsilon}_t$  denote the  $5 \times 1$  vector that collects the structural shocks  $(\varepsilon_{it})$  for  $i = \{1, 2, 3, 4, 5\}$ . The structural shocks are therefore related to the reduced form residuals through the following equations:

$$\varepsilon_{1t} = e_{1t} \tag{1.26}$$

$$\varepsilon_{2t} + b_{21}\varepsilon_{1t} = e_{2t} \tag{1.27}$$

$$\varepsilon_{3t} + b_{32}\varepsilon_{2t} + b_{31}\varepsilon_{1t} = e_{3t} \tag{1.28}$$

$$\varepsilon_{4t} + b_{43}\varepsilon_{3t} + b_{42}\varepsilon_{2t} + b_{41}\varepsilon_{1t} + b_{45}\varepsilon_{5t} = e_{4t} \tag{1.29}$$

$$\varepsilon_{5t} = e_{5t} \tag{1.30}$$

Equation (2.27) - (1.30) can be written compactly as:

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of long-term U.S. Treasuries was obtained from the Bertaut-Tyron dataset available at:<http://www.federalreserve.gov/Pubs/ifdp/2007/910/default.htm>.

$$\mathbf{B}\boldsymbol{\epsilon}_t = \mathbf{e}_t, \quad \text{where} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ b_{21} & 1 & 0 & 0 & 0 \\ b_{31} & b_{32} & 1 & 0 & 0 \\ b_{41} & b_{42} & b_{43} & 1 & b_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.31)$$

The explanation for the ordering of the shocks in the  $\mathbf{B}$  matrix reveals a combination of two identification strategies:

1. The first block, equations (2.27) – (1.29) is ordered such that, inflation only responds to output shock, the policy rate shock responds contemporaneously to output and inflation while the term spread which is given by long-term rate minus the short-term rate respond to output, inflation and the monetary policy rate shocks. Specifically, the shocks  $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t})$  are ordered in a Choleski fashion which is consistent in the literature. In addition, the term spread shock responds to FOHL shock, in a way capturing the stock effect channel. In essence the term spread shock is impacted by all the shock variables in the VAR model. This is representative of the DSGE model discussed in section 4.<sup>42</sup>
2. The second is a single equation, equation (1.30) which describes the exogeneity of FOHL assumed in the DSGE model and imposes the restriction that the FOHL shock is not correlated to any other shock in the model. Hence, zero restrictions are imposed on all the coefficients of the other shocks as shown in the last row of  $\mathbf{B}$ .

As mentioned earlier, with the assumption that FOHL is invariant to the other shocks in the near-VAR, the structural covariance system in equation (2.27)-(1.30) is overidentified. Hence to obtain the impulse response functions with their corresponding confidence intervals, the paper employs a Monte Carlo integration and Gibbs sampling for the overidentified structural covariance model.<sup>43</sup>

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<sup>42</sup>In particular, see equation 2.13.

<sup>43</sup>The Gibbs sampler is a particular technique recently adopted to tackle instances in where it is impossible to make direct draws based on random Normals (See Doan (2010)). The MONTENEARSVAR.RPF provides an example for the implementation of MCMC analysis of a combination of a near VAR for the lag coefficients and a structural VAR for the covariance matrix.

### 1.6.3 Impulse Response Functions from near-VAR

To analyse the effects of FOHL shock on the variables in the near-VAR model, the impulse response functions which traces out the path of the variables in periods  $t = 0, 1, 2, \dots$  in response to FOHL are considered. Specifically, median responses are reported alongside the error bands in response to a one time structural disturbance in period  $t = 0$  to FOHL. In light of [Sims and Zha \(1999\)](#), the 16th and 84th percentiles are reported for the confidence bands.

Figure 1.8 shows that in response to a shock to FOHL, the term spread declines. Note that, the term spread comprises two components, the expectation components and the term-premium component.<sup>44</sup> With the identification strategy for restrictions on the lag coefficients and the structural shocks in the system of equations in (2.26) and (1.31), it is reasonable to infer that the first few quarters that the term spread declines can be attributed to fall in the term-premium component. This gradual decline in the term spread feeds back into the economy and in turn increases real output. This result is consistent with the predictions from the DSGE model as well as [Rudesbush et al. \(2007\)](#) who finds that falls in the term premium is usually associated with stimulus to real economic activity.

As output rises, inflation also increases. The monetary authority respond to the increase in inflation and output by raising policy rates. However, even with the increase in policy rates, the term spread assumes only a slight upward trajectory and still remains negative. This highlight an interest rate conundrum similar to the case in the DSGE model explained above. Specifically, this implicitly reveals that the long-term interest rate is not purely determined by the current and future path of short-term interest rate, hence breaking down the Expectation Hypothesis Theorem.

In summary, the results from the impulse response functions generated from the near-VAR above provide empirical evidence in support of the hypothesis that shocks to FOHL have expansionary macroeconomic effects on the U.S economy. This complements the core findings from the DSGE model in section 3.

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<sup>44</sup>For a comprehensive treatment of decomposition of the term spread see [Rosenberg and Maurer \(2008\)](#).



## 1.7 Conclusion

This paper has investigated the macroeconomic implications of FOHL on key economic variables including consumption, output and inflation through its impact on the long-term interest rate. Employing a DSGE model that treats short and long-term bonds as imperfect substitutes through portfolio adjustment costs, the paper finds that shocks to FOHL have expansionary macroeconomic effects on the U.S economy – FOHL shocks increase consumption, output and inflation. This result is complemented by empirical impulse response from a structural near-VAR model.

Although the primary contribution of this paper is to study the macroeconomic implications of FOHL shocks in a DSGE model, the core results help draw the following broad conclusions and policy implication:

1. The results show that it is naïve to discount the macroeconomic effects of the actions of major holders of U.S debt such as foreign official agents. In the context of this paper, these effects are expansionary– FOHL shocks increase consumption, output and inflation. This result is captured through the negative stock effect channel of FOHL shocks on the long-term interest rate which feeds back in an expansionary fashion into the economy from the endogenous term-structure. Both the stock effect and feedback from the endogenous term-structure are generated by the introduction of portfolio adjustment cost. This emphasizes the fact that unlike previous studies that focus on quantifying the impact of FOHL on the long-term interest rate, it important to extend studies to understand the macroeconomic implications FOHL shocks.
2. The characteristics of private agents and foreign official institutions are crucial to understanding the degree of the impact of FOHL on the macro economy. For instance, given the mechanisms at work in the model–stock effect channel and feedback mechanism from the endogenous term structure– if households do not treat short and long-term bonds as imperfect substitutes, the model shows that FOHL will have no impact on the economy. Meanwhile, with some degree of imperfect asset substitutability, a high (low) degree of persistence of FOHL shocks can lead to a high (moderate) expansionary macroeconomic effects. Moreover,

a low persistence of FOHL shows that on impact, shock to FOHL can have a decently large and negative effect on the long-term interest rate while a high persistence has no effect. This result contributes to the literature by unifying the mixed results in the existing literature.

3. Lastly, since the monetary authority responds to inflation and output in the model, short-term interest rate increases due to the expansionary effect of FOHL shock. However, FOHL shocks have a negative effect on long-term interest rates and hence there is a simultaneous fall in long-term interest rates and increase in short-term interest rate. This causes the term spread to fall similar to the Conundrum experienced between 2004-2006. This last results prompts attention to the fact that monetary policy must somehow acknowledge the actions of foreign official agents when making policy decisions as their actions can generate unpleasant macro-implications.

This paper may be extended in at least two ways. First, by examining the role of FOHL in the face of the Quantitative Easing at the Zero Lower Bound of interest rates. Second, by treating FOHL as an endogenous variable where FOHL respond to changes in the bond prices and other macro-factors. In the first case, following the global financial crisis in 2008, policy rates have been constrained at the Zero Lower Bound (ZLB) in the U.S until recently. Unprecedented large asset purchases by the Federal Reserve (i.e. Quantitative Easing) was employed as unconventional monetary policy tool at the ZLB to help stimulate the economy. It will be equally important to study the separate role of FOHL at the ZLB in the presence of Quantitative Easing. This will complement studies such as [Eggertsson and Woodford \(2003\)](#); [Gertler and Karadi \(2011\)](#); and [Falagiarda \(2014\)](#). In the second case, although this paper treats FOHL as an exogenous variable and is able to study their implications on the macroeconomy, an important extension will be to examine this same question but in the context of endogenous FOHL. This will be in line with empirical studies that treat foreign officials as endogenous agents (See for instance, [Beltran et al \(2013\)](#); [Sierra \(2010\)](#)).

# Bibliography

- [1] Andrés, J, López-Salido, J.D., and Nelson, E. (2004). Tobin's Imperfect Asset Substitution In Optimizing General Equilibrium. *Journal of Money, Credit and Banking*, 36(4):665-690.
- [2] Altig, D., Christiano, L. J., Eichenbaum, M., and Lind, J. (2011). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 14(2):225-247.
- [3] Beltran, D. O., Kretchmer, M., Marquez, J., and Thomas, C. P. (2013). Foreign holdings of US Treasuries and US Treasury yields. *Journal of International Money and Finance*, 32, 1120-1143.
- [4] Bernanke, B.S (2005) The Global Savings Glut and the U.S Current Account Deficit, Homer Jones Lecture, April 14, 2005. <http://www.federalreserve.gov/boarddocs/speeches/2005/20020414/default.htm>
- [5] Bernanke, B., Reinhart, V., and Sack, B. (2004). Monetary policy alternatives at the zero bound: An empirical assessment. *Brookings papers on economic activity*, 2004(2), 1-100.
- [6] Bertaut, C., DeMarco, L.P., Kamin, S., Tryon, R., (2011). ABS Inflows to the United States and the Global Financial Crisis. *International Finance Discussion Papers 1028*. Board of Governors of the Federal Reserve System
- [7] Bertaut, C.C and Tyron, R.W (2007). "Monthly Estimates of U.S. Cross-Border Positions." *International Finance Discussion Paper No. 910*, Board of Governors of the Federal Reserve system.
- [8] Blinder, A. S. (2010). Quantitative Easing: Entrance and Exit Strategies (Digest Summary). *Federal Reserve Bank of St. Louis Review*, 92(6), 465-479.
- [9] Borio, C., Galati, G. and Heath, A. (2008). FX Reserve Management: Trends and Challenges. *BIS Papers 40*, Bank for International Settlements.
- [10] Calvo, Guillermo (1983). Staggered Prices in a Utility-Maximization Framework. *Journal of Monetary Economics*, 12:3,383-98.
- [11] Chen, H., Curdia, V., and Ferrero, A. (2012). The macroeconomic effects of large-scale asset purchase programmes. *Economic Journal*, 122(564):F289-F315.
- [12] Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2000). Sticky price models of the business cycle: can the contract multiplier solve the persistence problem?. *Econometrica*, 68(5), 1151-1179.

- [13] Davig, T., and Leeper, E. M. (2006). “Fluctuating Macro Policies and the Fiscal Theory. In NBER *Macroeconomics Annual 2006*, ed. by D. Acemoglu, K. Rogoff and Woodford, pp. 247-316. MIT Press, Cambridge.
- [14] D’Amico, S., and King, T. B. (2013). Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics*, 108(2), 425-448.
- [15] Doan, T.A. (2010). RATS User’s Manual Version 8, Estima, 1560 Sherman Avenue, Suite 510, Evanston, IL 60201.
- [16] Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297308.
- [17] Eggertsson, G. B., and Woodford, M. (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 34(1), 139-235.
- [18] Engle, R. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives*, 157-168.
- [19] Falagiarda, M. (2014). Evaluating quantitative easing: a DSGE approach. *International Journal of Monetary Economics and Finance*, 7(4), 302-327.
- [20] Falagiarda, M. and Marzo, M. (2012). A DSGE Model With Endogenous Term Structure. Quaderni DSE Working Paper 830, Department of Economics, Università di Bologna.
- [21] Favilukis, J., Kohn, D., Ludvigson, S. C., and Van Nieuwerburgh, S. (2012). International capital flows and house prices: Theory and evidence. In *Housing and the Financial Crisis* (pp. 235-299). University of Chicago Press.
- [22] Favilukis, J., Ludvigson, S. C., and Van Nieuwerburgh, S. (2014). Foreign ownership of US safe assets: good or bad? (No. w19917). National Bureau of Economic Research.
- [23] Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, 367-390.
- [24] Gertler, M., and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1), 17-34.
- [25] Greenspan, A. (2005). Testimony of Chairman Alan Greenspan at the Federal Reserve Boards Semiannual Monetary Policy Report to the Congress before the Committee on Banking, Housing, and Urban Affairs, US Senate, February 16, 2005.
- [26] Harrison, R. (2011). Asset Purchase Policies and Portfolio Balance Effects: A DSGE Analysis.” In Chadha J. and Holly, S., editors, *Interest Rates, Prices and Liquidity: Lessons From The Financial Crisis*, chapter 5.
- [27] Harrison, R. (2012). Asset Purchases Policy at The Effective Lower Bound For Interest Rates. Bank of England Working Papers 444, Bank of England.
- [28] Ireland, P. N. (2004). Money’s role in the monetary business cycle. *Journal of Money, Credit and Banking*, 36(6):969-83.

- [29] Joyce, M., Miles, D., Scott, A., and Vayanos, D. (2012). Quantitative easing and unconventional monetary policy: an introduction. *The Economic Journal*, 122(564), F271-F288.
- [30] Kaminska, I., and Zinna, G. (2014). Official Demand for US Debt: Implications for US real interest rates (No. 14-66). International Monetary Fund.
- [31] Kim, J. (2000). Constructing and estimating a realistic optimizing model of monetary policy. *Journal of Monetary Economics*, 45(2):329-359.
- [32] King, T. B. (2010). Flow and Stock Effects of Large-Scale Treasury Purchases. In Finance and Economics Discussion Series 201052. Washington: Board of Governors of the Federal Reserve System.
- [33] Kohn, D. (2015). Addicted to Debt: Foreign Purchases of US Treasuries and the Term Premium. Department of Economics Working Papers 2015\_1, Universidad Torcuato Di Tella.
- [34] Krishnamurthy, A., and Vissing-Jorgensen, A. (2012). The demand for Treasury debt. *Journal of Political Economy*, 42(2), 215-265.
- [35] Ljungqvist, L. and Sargent, T (2004), *Recursive Macroeconomic Theory*. MIT Press.
- [36] Lucas, R. E. (1988, December). Money demand in the United States: A quantitative review. In *Carnegie-Rochester Conference Series on Public Policy* (Vol. 29, pp. 137-167). North-Holland.
- [37] Mankiw, N. G., and Summers, L. H. (1986). Money demand and the effects of fiscal policies. *Journal of Money, Credit and Banking*, 415-429.
- [38] Marzo, M., Söderström, U., and Zagaglia, P. (2008). The Term Structure of Interest Rates and the Monetary Transmission Mechanism., Unpublished Manuscript.
- [39] Morrison, W. M., and Labonte, M. (2011). China's holdings of US securities: implications for the US economy. *Current Politics and Economics of Northern and Western Asia*, 20(3), 507.
- [40] Pringle, R. and Carver, N (2002). How Countries Manage Reserve Assets. Chapter 1. London: Central Banking Publications.
- [41] Rosenberg, J. V., and Maurer, S. (2008). Signal or noise? Implications of the term premium for recession forecasting. *Federal Reserve Bank of New York-Economic Policy Review*, 14(1).
- [42] Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4):517-531. 24.
- [43] Rudesbush, G.D., E.T Swanson, and T. Wu (2006). The Bond Yield “Conundrum” from a Macro-Finance Perspective, *Monetary and Economic Studies*, 24(S1),83-109.
- [44] Rudebusch, G. D., Sack, B. P., and Swanson, E. T. (2007). Macroeconomic Implications of Changes in the Term Premium. *Federal Reserve Bank of St. Louis Review*, July/August 2007, 89(4), pp.241-69.
- [45] Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics*, 26(2):183-209.
- [46] Sierra, J. (2010). International Capital Flows and Bond Risk Premia. Working Papers 10-14,

Bank of Canada.

- [47] Sims, C.A. and Zha, T. (1999) Error bands for Impulse Response. *Econometrica* 67: 1113-1156.
- [48] Swanson, E. T. (2011). Let's twist again: a high-frequency event-study analysis of operation twist and its implications for QE2. *Brookings Papers on Economic Activity*, 2011(1), 151-188.
- [49] Taylor, J. B. (1993). Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy, 39:195-214.
- [50] Tobin, J. (1969). A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, 1(1):15-29.
- [51] Tobin, J. (1982). Money and Finance in the Macroeconomic process. *Journal of Money, Credit and Banking*, 14(2):171-204.
- [52] Vayanos, D., and Vila, J. L. (2009). A preferred-habitat model of the term structure of interest rates (No. w15487). National Bureau of Economic Research.
- [53] Warnock, F. E., and Warnock, V. C. (2009). International capital flows and US interest rates. *Journal of International Money and Finance*, 28(6), 903-919.
- [54] Zagaglia, P. (2013). Forecasting Long-Term Interest Rates with a General-Equilibrium Model of the Euro Area: What Role for Liquidity Services of Bonds?. *Asia-Pacific Financial Markets*, 20(4), 383-430.
- [55] Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggregation Bias. *Journal of American Statistical Association* 56, 500-509.

# Appendix A

## Appendices

### Appendix 1.A

#### Calibration of baseline values for $\rho_x$ and $\sigma_x$

An Autoregressive Conditionally Heteroskedastic ARCH(1)-in-mean model is used in estimating the parameters of the process for the share of foreign official holdings of long-term U.S Treasuries. The parameters  $\rho_x$  and the standard  $\sigma_x$  are estimated as follows:

$$foi_{L,t} = \beta_x + \rho_x foi_{L,t-1} + \alpha \sigma_{t,x}^2 + \varepsilon_t^x \quad (1.1A)$$

where  $foi_{L,t}$  is 3-month foreign official inflows of long term bonds official inflows computed from Bertaut-Tyron measures of foreign official holdings. The error term  $\varepsilon_t^x$  is modelled such that it follows an ARCH(1) process:

$$(\varepsilon_t^x)^2 = \alpha_0^x + \alpha_1^x (\varepsilon_{t-1}^x)^2 + v_t^x \quad (1.2A)$$

Table 1. below reports the estimation results.

Table A.1: Estimation results : ARCH-in-mean

Variable	Coefficient	(Std. Err.)
Equation 1.1A : $foi_{L,t}$		
$foi_{L,t-1}$	0.726***	(0.076)
$\sigma^2$	31.260***	(4.874)
Intercept	-72.794	(0.000)
Equation 1.2A: ARCH		
$\varepsilon_t^2$	0.005***	(0.001)
Intercept	2.340***	(0.365)

Significance levels : † : 10% \* : 5% \*\*\* : 1%

# Appendix B

## B

### Appendix 1.B: The Steady-State and Implied Parameters

Steady state values of the economic variables in the model are defined such that, for any time period  $t$ ,  $X_t = X_{t+1} = X$ . Hence, at steady-state, the variable  $X_t$  is time invariant so the time subscripts are dropped. Below are the equations defining steadystate values of the economic variables that have closed form solutions.

FOC Consumption:

$$\lambda = (C - \theta C)^\gamma (1 - \beta\theta) \quad (1.1B)$$

FOC Short Term Bond:

$$R = \frac{\pi}{\beta} \quad (1.2B)$$

FOC Long Term Bond

$$R_L = \frac{\pi R}{\beta} \quad (1.3B)$$

FOC Labor

$$\chi = \lambda \frac{w}{N^\varphi} \quad (1.4B)$$

Velocity of Money Definition:

$$m = \frac{C}{vel.} \quad (1.5B)$$

FOC Labor:

$$\vartheta = \lambda \left(1 - \frac{1}{R}\right) m^\eta \quad (1.6B)$$

Firm Pricing:

$$\Psi = \frac{\varepsilon - 1}{\varepsilon} \quad (1.7B)$$



Constant Technology:

$$A = \frac{Y}{N} \tag{1.8B}$$

Marginal Cost:

$$w = A\Psi \tag{1.9B}$$

Government Budget Constraint:

$$T = \frac{b}{\pi} + \frac{b_L}{R_L\pi} + G + \frac{m}{\pi} - m - \frac{b}{R} - \frac{b}{R_L} \tag{1.10B}$$

## Appendix 1.C: Full Log-Linearized Model

The dynamic economic problem presented in the paper takes on a system of non-linear difference equations. Since there are no closed form solutions, I employ a first order Taylor expansion to approximate the nonlinear model around the neighborhood of its steady-state and solve it numerically. Particularly, for a smooth arbitrary function  $h(x_t)$ , the function is approximated linearly as:

$$h(x_t) = h(x) + h'(x)(x_t - x)$$

**Below is the full log linearized model:**

FOC Consumption:

$$(\beta\theta\gamma(C\tilde{c}_{t+1} - \theta C\tilde{c}_t) - \gamma(C\tilde{c}_t - \theta C\tilde{c}_{t-1}))(C - \theta C)^{-\gamma-1} = \lambda\tilde{\lambda}_t \quad (1.1C)$$

FOC Real Money Balances:

$$\tilde{m}_t = \frac{1}{\eta} \left( \frac{\pi}{\pi - \beta} \tilde{\lambda}_t - \frac{\beta}{\pi - \beta} E_t(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) \right) \quad (1.2C)$$

FOC Labor:

$$\tilde{w}_t = \varphi\tilde{n}_t - \tilde{\lambda}_t \quad (1.3C)$$

FOC Short Term Bond:

$$E_t \frac{\beta}{\pi} (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) = \frac{\tilde{\lambda}_t}{R} - \frac{\tilde{R}_t}{R} - \frac{\kappa_L \phi_L}{R_L} (\tilde{b}_t^h - \tilde{b}_{L,t}^h) \quad (1.4C)$$

FOC Long Term Bond:

$$E_t \frac{\beta}{\pi R} (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1} - \tilde{R}_{t+1}) = \frac{\tilde{\lambda}_t}{R_L} - \frac{\tilde{R}_{L,t}}{R_L} - \frac{\phi_L}{R_L} (\tilde{b}_t^h - \tilde{b}_{L,t}^h) \quad (1.5C)$$

Household Budget Constraint:

$$\begin{aligned} \frac{b^h}{R} \tilde{b}_t^h - \frac{b^h}{R} \tilde{R}_t + \frac{b_L^h}{R_L} \tilde{b}_{L,t}^h - \frac{b_L^h}{R_L} \tilde{R}_{L,t} + m\tilde{m}_t &= \frac{b^h}{\pi} \tilde{b}_{t-1}^h - \frac{b^h}{\pi} \tilde{\pi}_t + \\ \frac{b_L^h}{\pi R} \tilde{b}_{L,t-1}^h - \frac{b_L^h}{\pi R} \tilde{R}_t - \frac{b_L^h}{\pi R} \tilde{\pi}_t + \frac{m}{\pi} \tilde{m}_{t-1} - \frac{m}{\pi} \tilde{\pi}_t + Y\tilde{y}_t - C\tilde{c}_t & \end{aligned} \quad (1.6C)$$

Production Technology:

$$\tilde{y}_t = \tilde{n}_t \quad (1.7C)$$

Supply of Long-term bonds available to households:

$$b_L^h \tilde{b}_{L,t}^h = b_L \tilde{b}_{L,t} - x b_L \tilde{b}_{L,t} - x \tilde{x}_t b_L \quad (1.8C)$$

Government Budget Constraint

$$\frac{b}{R}(\tilde{b}_t - \tilde{R}_t) + \frac{b_L}{R_L}(\tilde{b}_{L,t} - \tilde{R}_{L,t}) + m \tilde{m}_t = \frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}_t) + \frac{b_L}{\pi}(\tilde{b}_{L,t-1} - \tilde{\pi}_t - \tilde{R}_t) + \frac{m}{\pi}(\tilde{m}_{t-1} - \tilde{\pi}_t) + G \tilde{g}_t - T \tilde{t}_t \quad (1.10C)$$

Monetary Policy Rule:

$$\tilde{R} = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) \rho_\pi \tilde{\pi}_t + (1 - \rho_R) \rho_Y \tilde{y}_t + \varepsilon_t^R \quad (1.11C)$$

Tax Rule:

$$\tilde{T}_t T = \zeta_1 \frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}) + \zeta_1 \frac{b_L}{R \tilde{\pi}}(\tilde{b}_{L,t-1} - \tilde{\pi}_t) \quad (1.12C)$$

Firm Pricing:

$$\tilde{\pi}_t = E_t \beta \tilde{\pi}_{t+1} + \frac{\varepsilon - 1}{\psi} \tilde{\Psi}_t \quad (1.13C)$$

AR(1) process for share of FOHL:

$$\tilde{x}_{F,t} = \rho_x \tilde{x}_{F,t-1} + \varepsilon_t^x \quad (1.14C)$$

AR(1) process for Long-Term Bond Supply:

$$\tilde{b}_{L,t} = \rho_{bL} \tilde{b}_{L,t-1} + \varepsilon_t^l \quad (1.16D)$$

AR(1) process for Government Spending:

$$\tilde{g}_t = \phi_G \tilde{g}_{t-1} + \varepsilon_t^g \quad (1.17C)$$

Table B.1: **Calibrated steady-state values of some variables**

<b>Description</b>	<b>Notation</b>	<b>Value</b>
Output	$Y$	1.00
Taxes-output ratio	$T/Y$	0.195
Labor hours	$N$	1/3
Gross short-term rate, Annual	$R$	1.04
Steady-state inflation rate	$\pi$	1

Table B.2: **Baseline Parameter calibration**

<b>Description</b>	<b>Notation</b>	<b>Value</b>
Discount factor	$\beta$	0.995
Habit formation	$\theta$	0.7
Coefficient of risk aversion	$\gamma$	1.0
Elasticity of money demand	$\eta$	2.6
Inverse of elasticity of labor supply	$\varphi$	1
Elasticity of Demand	$\varepsilon$	6.0
Cost of Price Adjustment	$\psi$	100
Portfolio adjustment friction	$\phi_L$	0.01
<i>Monetary policy</i>		
Monetary policy response to output	$\rho_Y$	0.6
Monetary policy response to inflation	$\rho_\pi$	1.5
Monetary policy inertia	$\rho_R$	0.85
<i>Taxation policy</i>		
Steady-state Lump Sum Tax	$\zeta_0$	0.195
Tax response to short-term bonds	$\zeta_1$	0.15
Tax response to short-term bonds	$\zeta_2$	0.15
<i>Autoregressive Coefficients</i>		
Monetary Policy	$\phi_{\phi_R}$	0.85
Government spending	$\phi_G$	0.90
LT bonds shock	$\rho_{bL}$	0.90
<i>Standard Deviations</i>		
Monetary Policy Shock St. Dev.	$\sigma_R$	0.0025
Government Spending Shock St. Dev.	$\sigma_G$	0.012
LT bonds Shock St. Dev.	$\sigma_L$	0.01

Table B.3: Calibration of Key Parameter and Steady State Values

Description	Notation	Value
Total LT bonds Outstanding per GDP	$B_{L,t}$	0.220
Total ST bonds Outstanding per GDP	$B_t$	0.056
LT bonds held by households per GDP	$B_{L,t}^H$	0.151
LT bonds held by FOH per GDP	$B_{L,t}^F$	0.068
Share of FOH LT bonds( $x_t$ ) shock Coef.	$\rho_x$	0.72
Magnitude of FOHL shock	$\sigma_x$	1.53

Table B.4: Comparison of Model Results to Empirical Estimates of Foreign Official Purchases on Long Term Yield

Studies	On Impact	Methodology
<a href="#">My Model: Long-term Interest Rate</a>	No Effect to -22 bps	Calibrated DSGE model
<a href="#">My Model: Term Premium</a>	-34 to -54 bps	Calibrated DSGE model
Kohn (2015)	-51 bps	CBAPM (Term Premium)
Rudesbusch et. al (2006)	No Effect	ANM-F model
Beltran et al. (2013)	-39 to -62 bps	Excess returns regression
Beltran et al. (2013)	-46 to -50 bps	Term premium regression
Warnock & Warnock (2009)	-68 bps	OLS regression
Stock Effect (Max. Effect)		
<a href="#">My Model</a>	-26 bps	Calibrated DSGE model
Beltran et al. (2013)	-20 bps	Cointegration (Holdings(level))
Bertaut et al. (2011)	-15 bps	Regressions (Holdings(level))

Source: Beltran et al. (2013) and author's computation/compilation

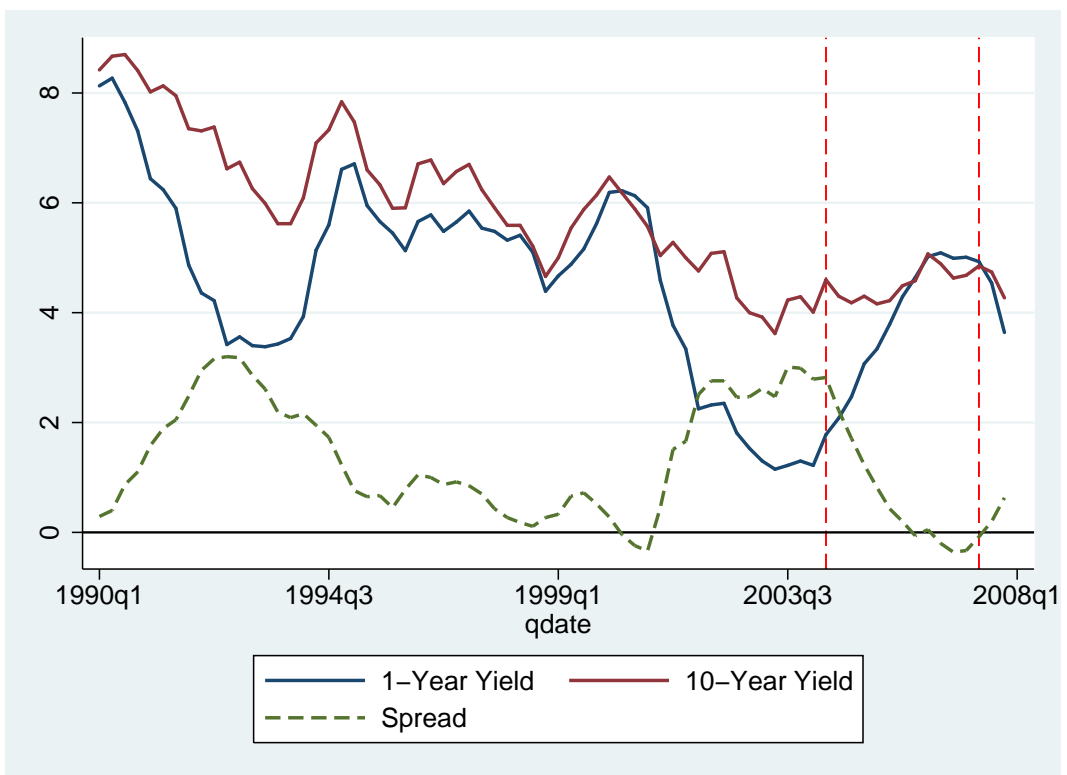
Notes: Consumption Based Asset Pricing Model (CBAPM); Affine no-arbitrage macro-finance (ANM-F)

Table B.5: Moment Comparison

Std. Dev. of variable	Data	Model ( $\phi_L = 0.005$ )	Model ( $\phi_L = 0.01$ )	Model ( $\phi_L = 0.02$ )
<u>Macro-variables</u>				
Output	0.46	0.40	0.42	0.44
Consumption	0.72	0.10	0.10	0.11
Inflation	0.32	0.62	0.91	1.10
<u>Financial variables</u>				
Short-term interest rate	0.30	0.50	0.67	0.81
Long-term interest rate	0.12	0.47	0.59	0.71
Term-Spread	0.59	0.32	0.36	0.40

The table compares empirical moments from data and theoretical moments implied by the model. The data is treated similar to the variable in the model and it is filtered using the Hodrick-Prescott filter with a smoothing parameter of  $\lambda = 1600$ . All values are in percent with the exception of the term spread. Theoretical moments from the model with low habits ( $\theta < 0.7$ ) show similar results to baseline model but with a much higher volatility in consumption. Similar results hold for the FOHL persistence parameter ( $\rho_x$ ).

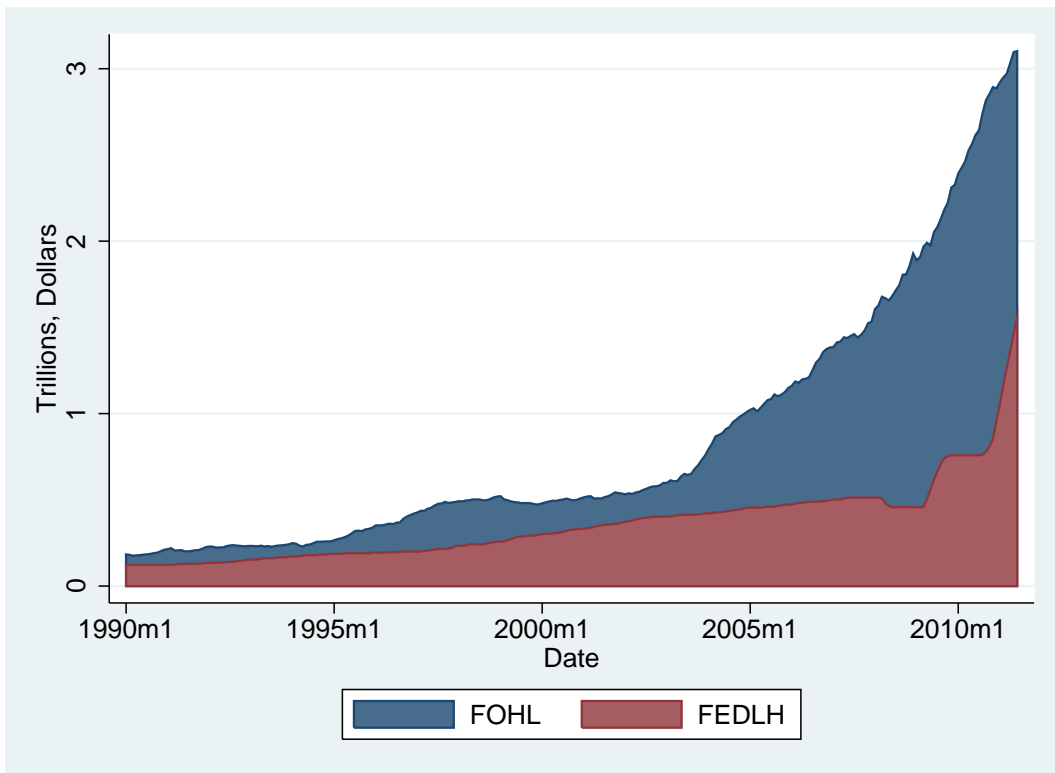
Figure 1.1: Yields and Spread of 1-year and 10-year Bond, 1990Q1-2007Q12



**Source:** FRED and author's calculation.

**Note:** The Spread is computed as the 10-year yield minus the 1-year yield

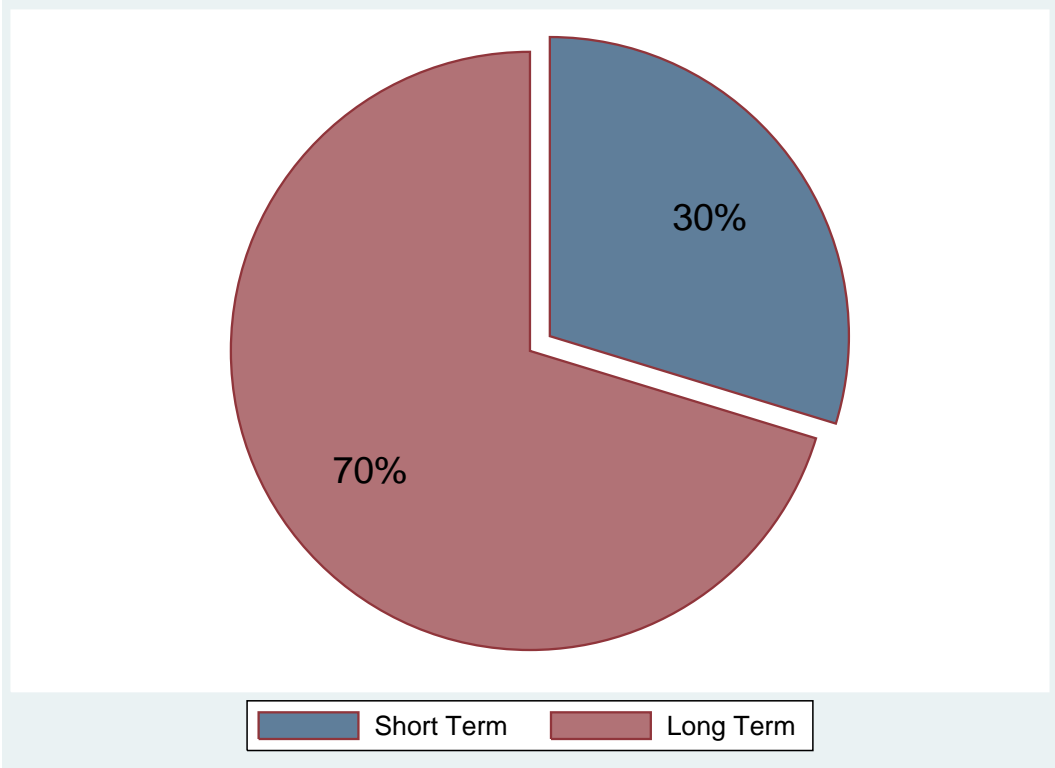
Figure 1.2: Long Term Bond Holdings, FED vs Foreign Official Institutions



Source: Author's illustration of Bertaut-Tyron Measure of Foreign Official Holdings and FRED



Figure 1.3: Shares of Short and Long Term Foreign Official Bond Holdings in Total Foreign Official Holding, January 1990 to June 2011



**Source:** Author’s illustration of Bertaut-Tyron Measure of Foreign Official Holding (Long-term bond holdings) and Treasury International Capital System (TIC), section A.2 (Short-term bond Holding).

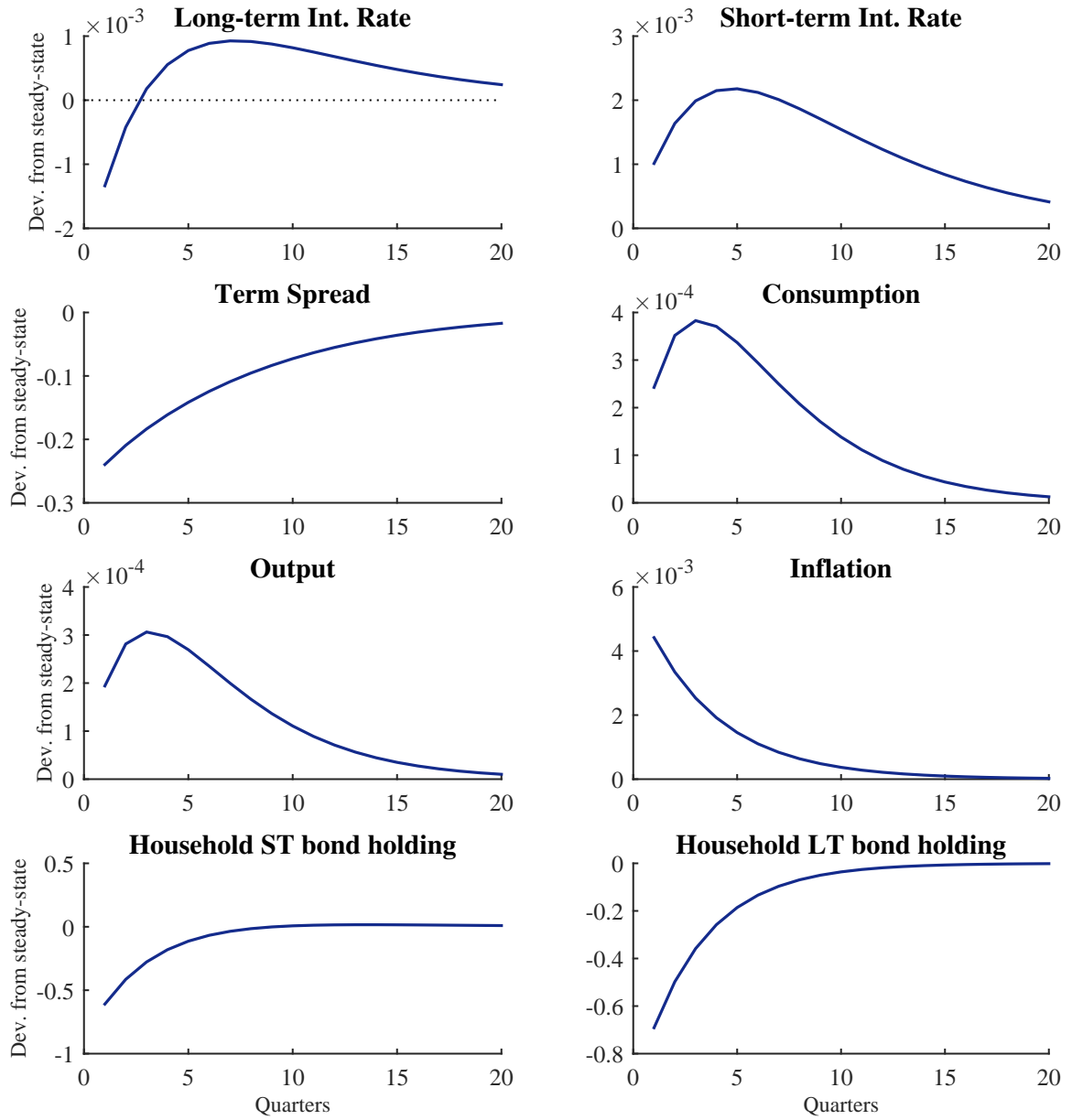


Figure 1.4: Selected impulse response functions from the equilibrium model following a shock to the share of long term bond holdings of foreign official Institutions ( $x_t$ )

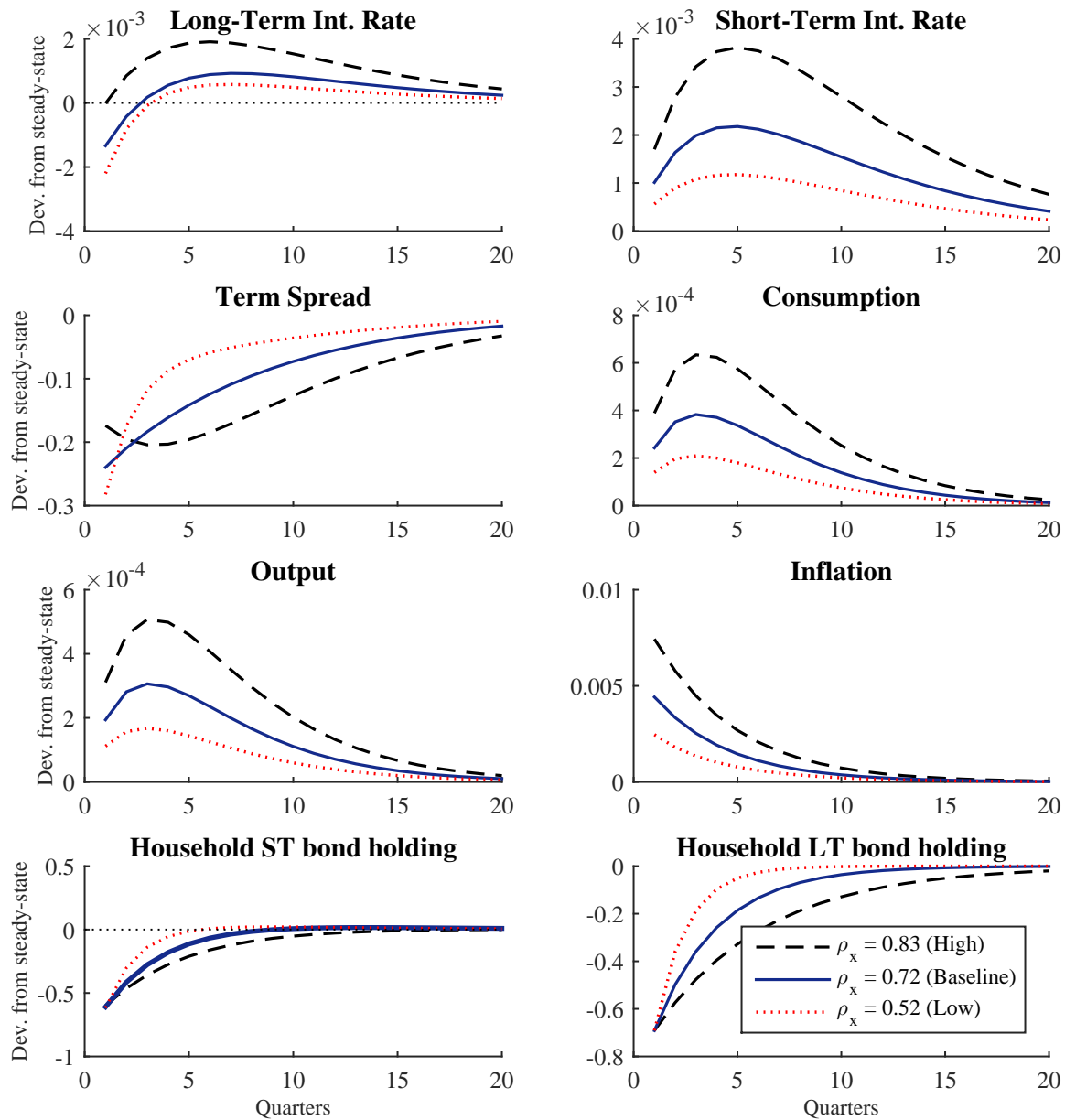


Figure 1.5: Selected impulse responses to a shock to the share of long term bond holdings of foreign official Institutions ( $x_t$ ) when varying the persistence parameter  $\rho_x$ . The black dashed line represents high persistence, the dotted red line represents low persistence and the solid blue line represents the baseline case.

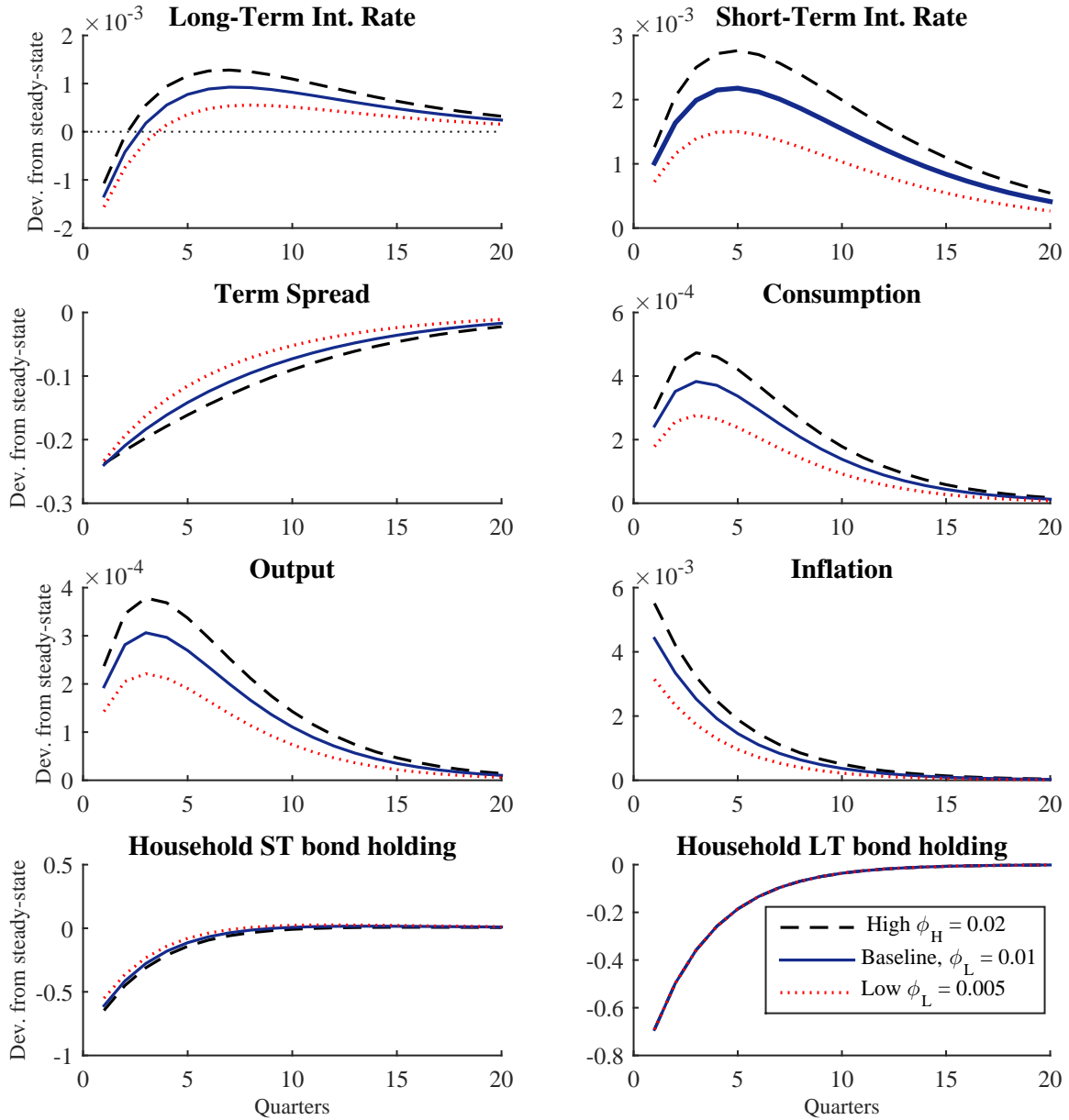


Figure 1.6: Selected impulse responses to a shock to the share of long-term bond holdings of foreign official Institutions ( $x_t$ ) when varying the portfolio adjustment cost parameter  $\phi_L$ . The black dashed line represents high portfolio adjustment costs, the dotted red line represents low portfolio adjustment costs and the solid blue line represents the baseline case.

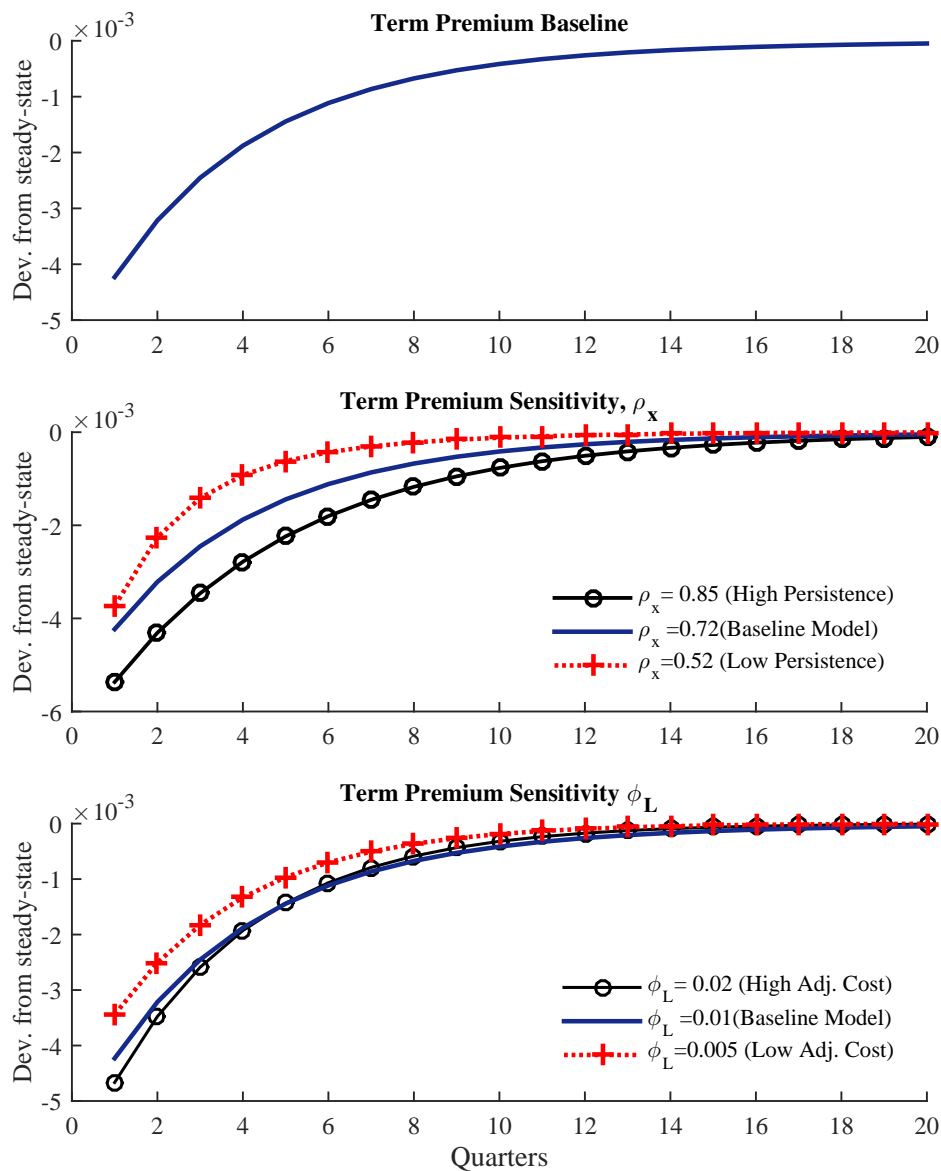
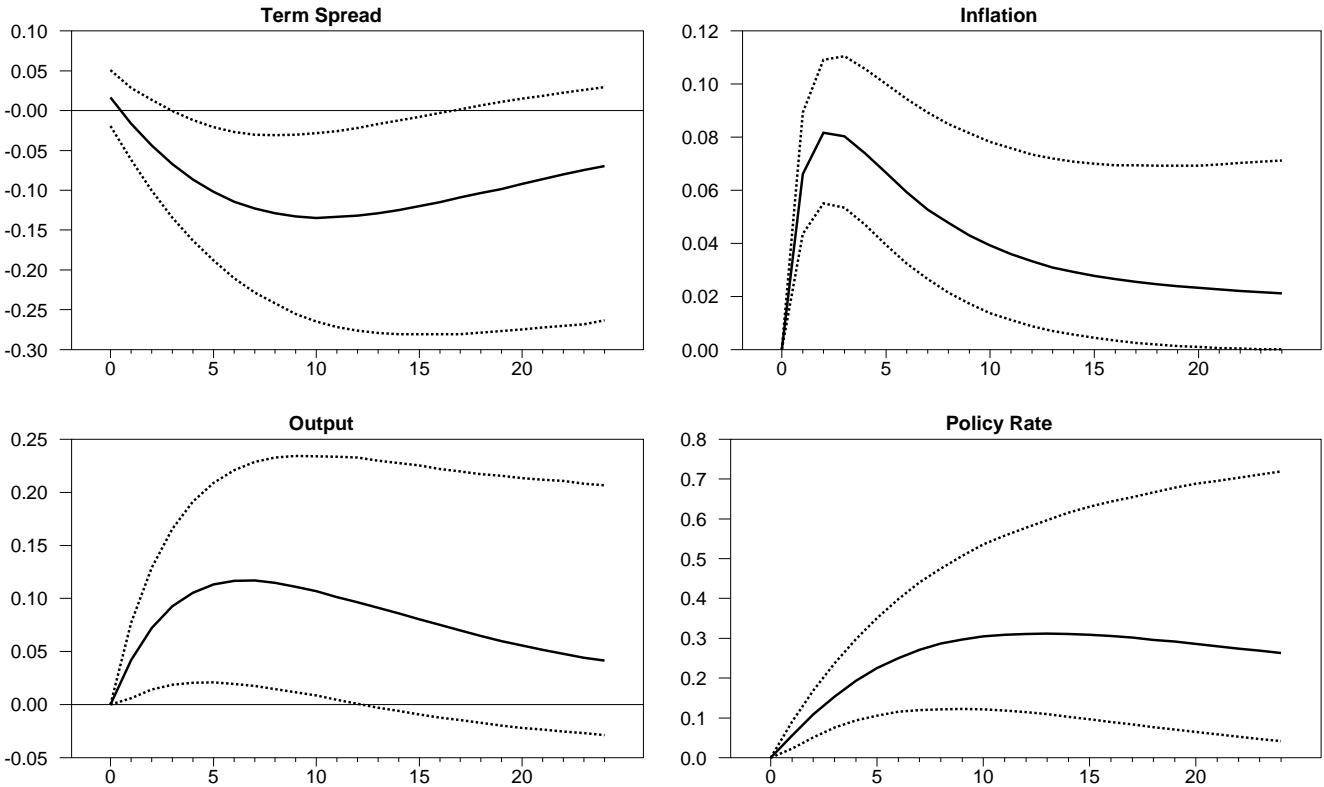


Figure 1.7: The figure shows the impulse responses of the Term Premium to a positive shock to the share of long-term bond holdings of U.S. Treasuries by foreign officials. The top figure depicts the baseline case; the middle figure show the response of the Term Premium when varying the persistence parameter for the AR(1) process for  $x_t$ , ( $\rho_x$ ) while the bottom figure shows the response of the Term Premium when varying the portfolio adjustment cost parameter,  $\phi_L$ . The thick blue line represents the baseline model. The circled black line and red plus lines represent high and low parameter values respectively.

Figure 1.8: Median impulse response functions for the term spread, output, inflation and Fedfunds rate are reported following a shock to foreign official holdings of long-term U.S. Treasuries. The corresponding confidence bounds are defined at 68% posterior bands.



Responses to Foreign Official Holding

**Graphical Appendix:** Sensitivity analysis on monetary policy inertia and habit parameter.

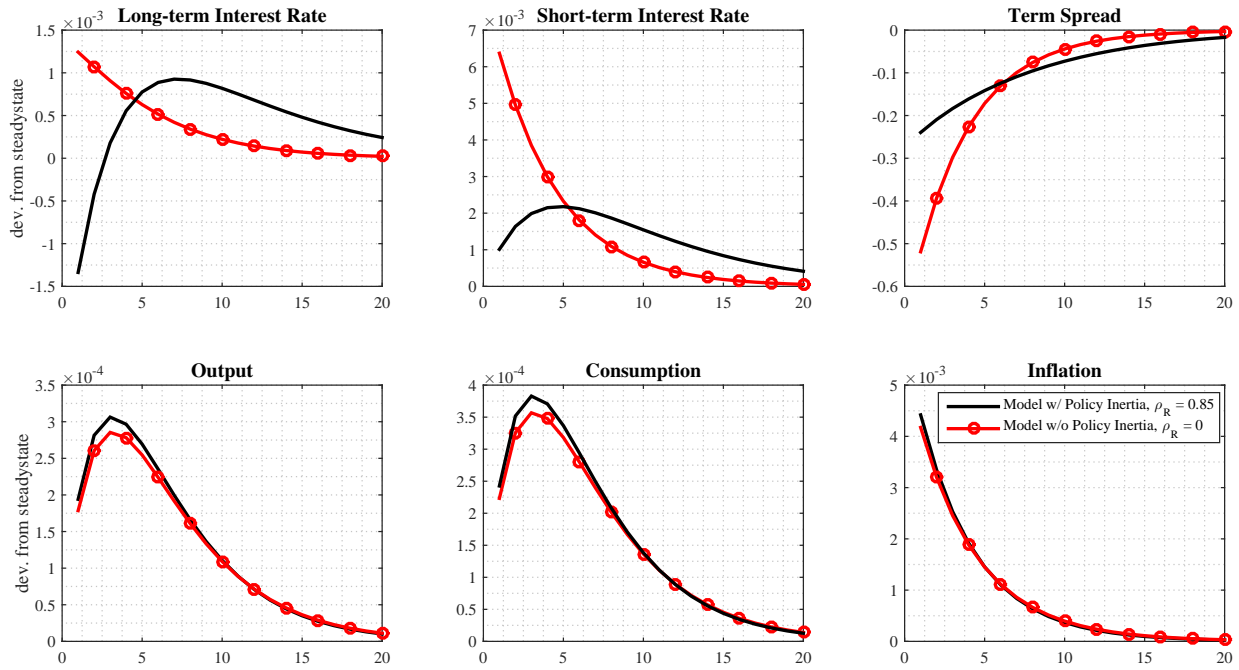


Figure 1.9: Effect of monetary policy inertia,  $\rho_R$ , following FOHL shock.

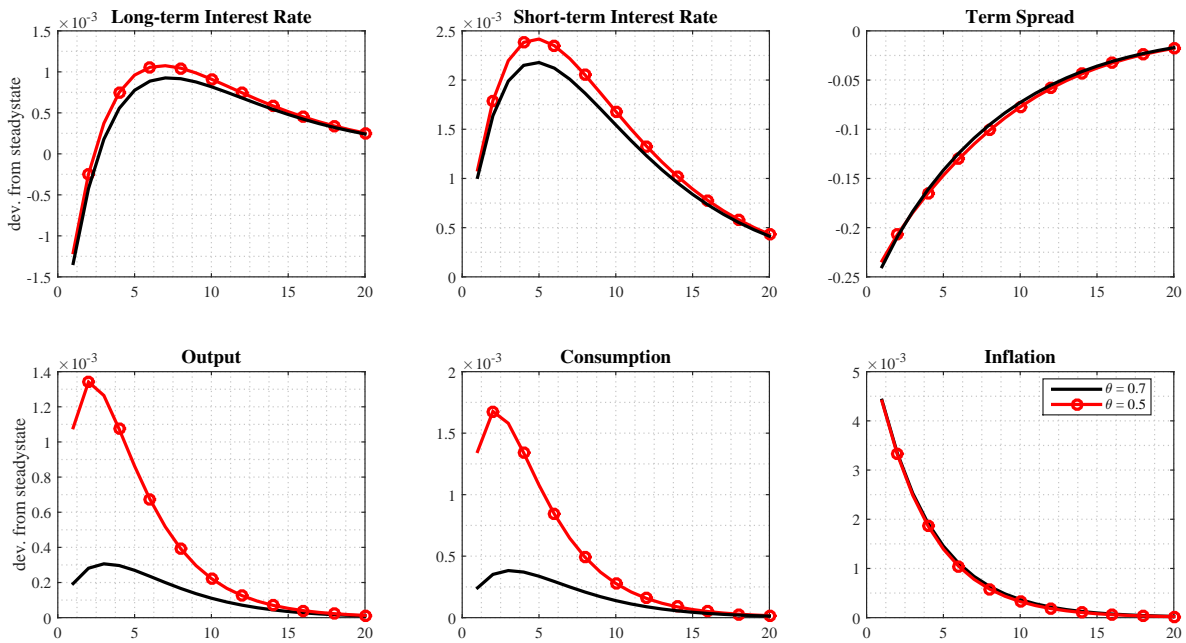


Figure 1.10: Effect of habits,  $\theta$ , following FOHL shock.

## Chapter 2

# On Foreign Aid Fungibility and Aid Effectiveness: The Role of Consumer Preferences over Government Spending



## Abstract

A persistent concern raised by policymakers and donors on foreign aid effectiveness is fungibility— the possibility that aid finances government spending in the recipient country in ways not intended by donors when disbursing the funds. In this chapter, we advance the study of aid fungibility by examining how the intra-temporal relationship between private and government consumption interact with fungible aid to impact aid effectiveness. Specifically, we provide empirical evidence of substitutability/complementarity between private and government consumption in aid dependent countries. We then incorporate this feature into a tractable dynamic general equilibrium model and show that the way consumers internalize an aid-induced government spending due to fungibility can have contrasting impacts on aid effectiveness. Three key findings emerge from analytical and numerical results: (1) when government spending is a complement or weak substitute to private consumption, inflow of fungible aid results in expansionary macroeconomic effects. These expansionary effects are stronger when the two goods are complements; (2) when government spending is a strong substitutes to private consumption, fungible aid has negative aggregate effects; (3) weak public institutional factors such as corruption and inefficient bureaucracy offsets the positive effects in (1) and mitigates the negative effects of fungible aid in (2). Our findings provide a new and additional channel that policymakers can exploit to improve aid effectiveness even when it is fungible.

**JEL:** D73, E02, F35, F40, O55

**Keywords:** Aid Effectiveness, Fungibility, Government Spending, Private Consumption, Intra-temporal Substitution

## 2.1 Introduction

A persistent concern raised by policymakers and donors on foreign aid effectiveness is fungibility—the possibility that aid is used in ways not intended by donors when disbursing the funds.<sup>1</sup> In a traditional sense, fungible aid can be rationalized as a situation where there is no “actual” diversion of aid funds, but instead, aid funds an activity that would have happened in the absence of aid. This in turn frees up resources to be used elsewhere in the economy.<sup>2</sup> In this sense, the actual effect of aid is directly tied to the “freed” up government funds which is ultimately spent in the recipient country.

In a “perfect world”, the increase in government spending due to foreign aid fungibility should not be a problem, because, at the very least, government spending contributes an immediate dollar to aggregate demand.<sup>3</sup> However, even in this loose scenario, classic macroeconomic problems such as crowding out of private consumption are always of concern. Moreover, as argued by [Bailey \(1971\)](#) the effectiveness of an increase in government spending crucially depends on the degree of substitutability between private and government consumption.<sup>4</sup> This implies that, when aid is fungible the relationship between private and government consumption directly impacts the aggregate effects of aid.

Recently, [Morrisey \(2015\)](#) finds that the extent to which aid is fungible is over-stated and even where evidence exists that aid is fungible, it does not appear to diminish the effectiveness of aid but adds that these conclusion are country-specific. Similarly, a number of studies have argued that fungibility is not necessarily a bad thing nor something donors need to be particularly concerned about with regard to aid effectiveness ([Hauck et al. \(2005\)](#); [Rothmann and ten Have \(2004\)](#); [Pettersson \(2007\)](#); [McGillivray and Morrissey \(2000\)](#); [McGillivray and Morrissey \(2004\)](#); [Morrissey \(2006\)](#); [Wagstaff \(2011\)](#)). These studies in turn propose that in countries where there are sound policies, appropriate allocations of expenditure, and effective services, donors can provide

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<sup>1</sup>Ample evidence of aid being fungible exists (See for instance [Marc \(2015\)](#); [Van de Sijpe \(2012\)](#); [Chatterjee et. al \(2012\)](#); [Outtara \(2006\)](#); [Feyzioglu et al. \(1998\)](#) and [Zampelli \(1986\)](#))

<sup>2</sup>[Singer \(1965\)](#), [Hjertholm et al. \(2000\)](#) and more recently [Leiderer \(2012\)](#) discuss into detail the issue of foreign aid fungibility

<sup>3</sup>This can be verified directly from the resource constraint, *i.e.*  $Y_t = C_t + G_t + I_t + X_t - M_t$ .

<sup>4</sup>See [Barro \(1981\)](#) and [Bailey \(1971\)](#) for a deeper discussion.

large amounts of assistance as general budget support, knowing that the resources will be well used.<sup>5</sup>

These studies, however, preclude the subtle but important relationship between private and government consumption from their evaluation of aid effectiveness when aid is fungible. In this paper we focus on the issue of foreign aid fungibility and study how the intra-temporal relationship between private consumption ( $C$ ) and public/government consumption ( $G$ ) plays a role in impacting the macroeconomic effects of foreign aid— a phenomenon absent in both the empirical and theoretical literature of foreign aid fungibility.<sup>6</sup>

A probing question that remain is, why is the relationship between private and government consumption relevant in understanding the macroeconomic effects of fungible aid? The following examples might help set this discussion in perspective. Consider major components of government spending in developing countries such as education, defense, and health expenditure. Increases in spending for these components can produce some externalities either, positive or negative, for private consumption. For instance, massive provision of public health services can reduce the need for private hospitals. Similarly, an increase in public schools can reduce the demand for private schools. Thus public health services and public schools substitutes for private health services and private schools respectively.

Meanwhile, education can serve as a complement for other components of private consumption. For instance, better educated people can contribute to an increase in the demand for intellectual books, or demand healthier foods.<sup>7</sup> These possible relationships between private consumption and different items of public spending makes government consumption, on aggregate, to be either a substitute or complement for private consumption.

Consequently, the type of relationship between private and public consumption, either as a complements or substitutes, can lead to nontrivial effects of government spending induced by inflow of fungible aid thereby impacting aid effectiveness. This substitution/complementarity channel

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<sup>5</sup>See also [Leiderer \(2012\)](#)

<sup>6</sup>In this paper we will interchange public spending with government spending often.

<sup>7</sup>Additionally, other composition of government spending for public order can reduce the need for a private policeman, while subsidies for purchasing computers can increase private expenditures for internet services and so on. [Ercolani\(2007\)](#) elaborates more on this and offers several examples of useful government spending.

between private and public consumption and its interaction with fungible aid remains a gap in the aid effectiveness literature.

We contribute and advance the literature on aid fungibility and aid effectiveness in the following ways. First, we take the country-specific approach and provide empirical evidence of substitutability/complementarity between government and private consumption by estimating a structural cointegration regression for three top aid-recipient economies— Egypt, Kenya, and Nigeria.<sup>8</sup> The results from the estimation suggest that, for the full sample, government consumption is complementary to private consumption in Egypt, a weak substitute to private consumption in Nigeria and wasteful in Kenya. However, for the estimates from sub-samples periods, we find that government and private consumption are strong substitutes in Nigeria but weak substitutes in Kenya. In contrast, there is no observed relationship between the two goods in Egypt. The results from our estimations imply that government spending on aggregate, is useful and utility enhancing in aid-recipient countries.

Second, with motivation from the empirical results, we develop a dynamic general equilibrium model that captures the existence of substitutability/complementarity between government and private consumption to explicitly study their role in impacting the macroeconomic effects of fungible aid. Additionally, we study the impact of institutional quality measured through accountability, transparency, and corruption in the public sector in our modelled economy.

In the model, complementarity/substitutability between public and private consumption is achieved by modeling a household agent who internalizes government spending as useful and gains direct utility from government spending. The complete model consists of a representative household agent; a benevolent foreign agent who disburses aid to support the general budget of the recipient country; a productive sector with intermediate and final goods firms and finally a government sector that conducts both monetary and fiscal policy.<sup>9</sup> In the model, foreign aid fungibility is captured in the fiscal policy block through a less than one-for-one increase in government spending following an inflow of foreign aid. We therefore evaluate the effect of fungible aid by

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<sup>8</sup>We discuss our choice of countries in Section 2.2.

<sup>9</sup>Leiderer (2012) and Koeberle et al. (2006) discuss into detail government budget support as an effective aid modality.

studying the effectiveness of the marginal increase in government spending given the degree of complementarity/substitutability between private and public consumption.

After log-linearizing and calibrating the model in a standard way, three key findings emerge from our analytical and numerical results<sup>10</sup>:

1. when government spending is a complement or weak substitute to private consumption, the marginal increase in government spending following a dollar inflow of fungible aid results in expansionary macroeconomic effects—increasing output growth of the recipient country. These expansionary effects are stronger when private and government consumption are complements;
2. in the case where government spending is a strong substitute to private consumption, the additional rise in government spending due to an inflow in fungible aid generates contractionary macroeconomic effects in the recipient country’s economy;
3. when we control for quality of institutions in the recipient country, we find that weak public institutional factors such as high corruption and inefficient bureaucracy reduces the positive effect of foreign aid in (1) and offsets the negative effects of foreign aid in (2). They however, do not change the impact of aid.

These results strongly suggest that, with fungibility in play, omitting *a priori* the substitutability or complementarity channel between private and government consumption when evaluating the macroeconomic effects of aid can bias the impact of aid. This conclusion is robust when we control for quality of institutions in the model. Policywise, the link between private consumption and government spending provides policymakers and donors alike an additional avenue that they can explore to tailor strategic policies that can greatly improve aid effectiveness even when it is fungible.

The rest of the paper is structured as follows. Section 2.2 provides an empirical evidence of substitutability/complementarity between private and public consumption; Section 2.3 describe the theoretical model with useful government spending and fungible aid; Section 2.4 discusses the analytical and numerical results from the model. Section 2.5 concludes.

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<sup>10</sup>We show that the analytical results hold with heterogenous households.

## 2.2 Empirical Evidence of Useful ‘G’ in Aid Dependent Countries

In this section we provide evidence of the degree of substitutability/complementarity between private consumption and government spending. As discussed earlier, when foreign aid is fungible, its macroeconomic effects in the recipient country’s economy depends critically on how government spending impacts the economy. Hence, investigating the usefulness of government spending and its relationship with private consumption in the long-run becomes central to understanding and evaluating the macroeconomic effects of fungible foreign aid.

We select three countries—Egypt, Kenya, and Nigeria— from the list of top aid receiving countries to provide evidence of substitutability/complementarity between government and private consumption.<sup>11</sup> These countries were selected primarily due to data availability.<sup>12</sup>

For our empirical analyses, we employ annual data for 1960 to 2015 from the World Development Indicators (World Bank (2016)) to estimate the reduced form estimation equation given in Eq.(2.1):

$$\ln(C_t/G_t) = \alpha + \beta \ln(P^g/P^c) + e_t \quad (2.1)$$

Since the focus is on the long-term relationship between private and government consumption, Eq.(2.1) represents a cointegration regression that relates the logarithm of private and government consumption ratio,  $\ln(C_t/G_t)$ , to their corresponding logarithm price ratio,  $\ln(P^g/P^c)$ . The gradient parameter  $\beta$  governs the elasticity of substitution between household and government consumption. Negative values of  $\beta$  represent complementarity between private and government consumption while a positive value means the two goods are substitutes. When  $\beta$  is not different from zero, then government spending is non-useful or wasteful.

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<sup>11</sup>Countries that rank in the top ten aid receiving countries using 3-year average (2012 - 2014) include, in descending order: Afghanistan; Vietnam; Egypt, Ethiopia; Syrian Arab Republic; Turkey; Tanzania; Kenya; DRC Congo; and Pakistan. Nigeria ranks 6th in the top 10 ODA receipts by recipient in Africa as of 2013.

<sup>12</sup>Alternatively, we can instead investigate the usefulness (complementarity/substitutability) of government spending by using panel data to expand our dataset. However, estimates obtained from the panel data provide global or regional information instead of country specific information which is more relevant from a policy perspective. Addison and Tarp (2015) for instance emphasize the need to carry out country-specific analyses in studying aid effectiveness.

Eq.(2.1) is a reduced form specification and do not have any structural interpretations. To lend structural interpretation to Eq.(2.1), we follow [Kwan \(2007\)](#), [Ogaki and Park \(1997\)](#), [Ogaki and Reinhart \(1992\)](#), and [Ogaki \(1992\)](#) and assume a representative consumer who gains utility from two goods, private and public. The agents expected lifetime utility function is governed by Eq.(2.2) and it is subject to stationary preference shocks:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^e) \quad (2.2)$$

where the effective consumption,  $C^e$  is given as

$$C^e = [\theta \varepsilon_t C_t^{1-(1/\eta)} + (1 - \theta) \epsilon_t G_t^{1-(1/\eta)}]^{1/(1-(1/\eta))} \quad (2.3)$$

The random preference shocks  $(\varepsilon_t, \epsilon_t)$  are strictly stationary, have unit means, and finite variances. The preference parameters  $\theta$  and  $\eta$  represent the relative weight assigned to private goods and the substitution parameter that measures the curvature of the indifference curves respectively. With the assumption that the agent's utility function is time separable, the optimal consumption bundle  $C^e$  satisfies the equality condition between marginal rate of substitution and relative price. Hence we obtain the condition:

$$\frac{\partial U_t / \partial G_t}{\partial U_t / \partial C_t} \equiv \frac{\epsilon_t \theta C_t^{1/\eta}}{\varepsilon_t (1 - \theta) G_t^{1/\eta}} = \frac{P_t^g}{P_t^c} \quad (2.4)$$

For ease of interpretation and estimation, we take the logarithm of Eq. (2.4) to obtain

$$\ln \left( \frac{C_t}{G_t} \right) = -\eta \ln \left( \frac{1 - \theta}{\theta} \right) + \eta \ln \left( \frac{P_t^g}{P_t^c} \right) - \eta \ln \left( \frac{\epsilon_t}{\varepsilon_t} \right) \quad (2.5)$$

The corresponding equation for the empirical study is:

$$\ln \left( \frac{C_t}{G_t} \right) = \beta_0 + \beta_1 \ln \left( \frac{P_t^g}{P_t^c} \right) + \nu_t \quad (2.6)$$

where,  $\beta_0 = -\eta \ln \left( \frac{1-\theta}{\theta} \right)$  and  $\beta_1 = \eta$ .

In this paper we estimate the reverse of Eq.(2.6) so that  $\ln\left(\frac{P_t^g}{P_t^c}\right)$  is the regressand. We discuss this more in Section 2.2.1

### 2.2.1 Data and Estimation

We employ annual data for 1960 to 2015 from the World Development Indicators (World Bank 2015) for three countries Egypt, Kenya, and Nigeria. To obtain the the consumption ratio,  $C_t/G_t$  we divide, household final consumption expenditure by general government final consumption expenditure, both in 2010 constant dollars. In light of Kwan (2007), the corresponding prices  $P_t^c$  and  $P_t^g$  are computed as the implicit price deflators which are constructed by dividing the nominal private and government consumption series by their respective constant price series.

As standard in the literature, we begin by studying the time series properties of the constructed series  $\ln(C_t/G_t)$  and  $\ln(P_t^g/P_t^c)$ . Figure 2.1 shows the log consumption and price ratio for Egypt, Kenya, and Nigeria. Individually, both series show strong persistence for all the three countries suggesting that they may be cointegrated and possibly I(1). Table 2.1 reports the formal unit root test results. It is evident from the table that, for all the countries, the unit root null hypothesis is not rejected for the level series however it is strongly rejected when we employ the first differenced series. Specifically, this shows that the log price and consumption ratio series are both I(1).

Additionally, the  $p$ -values obtained for the case of the level series suggests that the log price ratio has a weaker random walk component than that of its counterpart, the log consumption ratio. That is, the log price ratio is less integrated than the log consumption ratio. As proposed by Ng and Perron (1997), it is more desirable to put the more integrated series as the regressor and the less integrated series as the regressand. Applying the Ng and Perron rule to our case means that the cointegration regression we will be estimating the reverse of Eq(2.7). That is, the log price ratio,  $\ln(P_t^g/P_t^c)$ , will be the regressand and  $\ln(C_t/G_t)$  the regressor. The related cointegration regression then becomes,



$$\ln\left(\frac{P_t^g}{P_t^c}\right) = \alpha_0 + \alpha_1 \ln\left(\frac{C_t}{G_t}\right) + \nu_t \quad (2.7)$$

To interpret the results from Eq.(2.7), the parameter of interest will be  $1/\alpha_1$ . Indeed,  $1/\alpha_1$  should be equal to  $\beta_1$ , where  $\beta_1$  is the estimator from the direct regression in Eq.(2.6). However, with finite sample the estimates from the direct regression and the reverse regression have a tendency of being far from being reciprocal to each other and in certain cases have very stark different statistical properties.<sup>13</sup>

Table 2.1: Augmented Dickey-Fuller(ADF) Test Results for Unit Roots

	Level		First Difference	
	$\ln(C_t/G_t)$	$\ln(P_t^g/P_t^c)$	$\Delta \ln(C_t/G_t)$	$\Delta \ln(P_t^g/P_t^c)$
Egypt	-1.2338 (0.6506)	-1.6312 (0.4559)	-5.6590 (0.0000)	-3.3603 (0.0200)
Kenya	-2.3123 (0.4203)	-2.6086 (0.2783)	-6.8890 (0.0000)	-7.7397 (0.0000)
Nigeria	-1.8301 (0.6671)	-2.0456 (0.5557)	-6.6772 (0.0000)	-6.6164 (0.0000)

**Notes:** P-values are in parenthesis. The lags of the dependent variable used to obtain white-noise residuals are determined using Schwartz Information Criterion (SIC).  $H_0$ : Series has a unit root.  $H_1$ : Series does not have a unit root. Exogenous variables: Constant and Linear Trend. MacKinnon (1996) one-sided p-values.

Figure 2.2 provides preliminary correlation analysis of the two series for the full sample period and sub-period.<sup>14</sup> The figure depicts a consistent and negative correlation between the two series for Egypt. This implies private and government consumption may be complements in Egypt. The relationship for Kenya as shown in the second column is negative for the entire sample period. However, this relationship switches to a positive relationship when the sub-period

<sup>13</sup>Kwan (2007) offers a deeper discussion on this. We report the results from the direct estimation in Appendix C.1. The results from the direct estimation have the same statistical properties as the reverse estimation, however, the estimator  $\hat{\beta}_1$  is quantitatively different from  $\hat{\alpha}_1$

<sup>14</sup>The sub-periods are arbitrarily chosen and are different for each country due to data availability. More importantly, the idea of the sub-period analysis is to demonstrate that aside from country differences, the relationship between private consumption and government consumption for a particular country can change overtime.

1970-2015 is considered. Finally, in the third column, there appears to be a consistent positive correlation between the price and consumption ratios, suggesting a tendency of government and private consumption to be substitutes in Nigeria.

Two points are clear from the preliminary results from figure 2.2. First, the relationship (sign) and the degree of substitutability/complementarity between private and government consumption varies across countries. Second, within countries, the degree of substitutability can change over time and in some cases, the relationship between the two goods can switch sign as in the case of Kenya.

### 2.2.2 Estimation Results

Table C.1 reports the cointegrating regression results from estimating Eq.(2.7) for the three aid dependent countries selected. The table also reports results for the sub-samples periods. It is important that we check for robustness, hence we employ three different estimation methodologies—the fully modified ordinary least square (FM-OLS) by Phillips and Hansen (1990); the canonical cointegrating regression (CCR) by Park (1992); and Stock and Watson’s (1993) dynamic ordinary least square (DOLS). These estimation methods are all asymptotically efficient procedures for estimating cointegration regressions.

We estimated the cointegration equation with the log price ratio as the dependent variable, hence we are interested in the reciprocal of  $\hat{\alpha}_1$ . Specifically, to interpret the results from table C.1 we use the computed value,  $1/\hat{\alpha}_1$ , which governs the degree of substitution/complementarity between private and government consumption. Generally, the estimated parameter values for  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$  are stable across the estimation methods. The DOLS estimates yield the lowest values for  $\hat{\alpha}_1$  hence the highest values for  $1/\hat{\alpha}_1$ . Particularly, for the full sample, the degree of substitution for Egypt is negative and greater than 1, revealing strong complementarity between the two goods. In Kenya, although the values for  $\hat{\alpha}_1$  are negative, they are statistically insignificant (not different from zero) so is  $1/\hat{\alpha}_1$ . This indicates that government spending is “non-useful/wasteful” on aggregate for the full sample period in Kenya. In contrast to Kenya and Egypt,  $\hat{\alpha}_1$  values for the full sample for Nigeria is positive with and greater than 1 implying values of  $1/\hat{\alpha}_1$  less than 1.

Table 2.2: Cointegration Regressions Results

	FM-OLS		CCR		DOLS	
	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$
Egypt						
FS: 1975 - 2015	0.7485*** (0.0211)	-0.9621*** (0.4410)	0.7488*** (0.0218)	-0.9383*** (0.2925)	0.7462*** (0.0177)	-0.7183** (0.2763)
SS: 1985 - 2015	0.7757*** (0.0211)	-0.0765 (0.4410)	-0.7759*** (0.0197)	-0.0717 (0.3654)	0.7714*** (0.0207)	-0.0375 (0.4525)
Kenya						
FS: 1960 - 2015	-0.5974*** (0.2032)	-0.0704 (0.2469)	-0.5877*** (0.1719)	-0.0830 (0.2054)	-0.6092*** (0.1623)	-0.0222 (0.1996)
SS: 1970 - 2015	-1.4365*** (0.2950)	1.0602*** (0.3830)	-1.4041*** (0.2767)	1.0171*** (0.3580)	-1.2910*** (0.2236)	0.8760*** (0.2894)
Nigeria						
FS:1982 - 2015	-0.9645*** (0.1795)	1.0516*** (0.1136)	-0.9658*** (0.1853)	1.0517*** (0.1152)	-0.8967*** (0.1960)	1.0025*** (0.1213)
SS: 1990 - 2015	- 0.8307*** (0.0636)	0.9412*** (0.0944)	-0.8309*** (0.1466)	0.9410*** (0.0953)	-0.8108*** (0.1610)	0.9320*** (0.1033)

**Notes:** The dependent variable is  $\ln(P_t^g/P_t^p)$ . FS=Full Sample and SS= sub-Sample. Standard errors in parentheses. FM-OLS is Fully modified OLS; CCR is Canonical cointegrating regression; and DOLS is Dynamic OLS. FM-OLS and CCR use Andrew's automatic bandwidth selection method in computing the long-run variance matrix. DOLS includes one lead and one lag of the first difference of the regressors in the augmented regression.

This strongly suggests that government spending and private consumption are moderate or weak substitutes.

When the arbitrary sub-periods are considered, we find that although the sign of  $\hat{\alpha}_1$  remain positive for Nigeria, the degree of substitutability decreases, with  $\hat{\alpha}_1$  attaining values less than 1 which translates to  $1/\hat{\alpha}_1$  values greater than 1. This means private and government consumption become strong substitutes in Nigeria over the sub-sample. The sign for  $\hat{\alpha}_1$  switches from negative and insignificant for the full sample period to positive and statistically significant in Kenya. The related  $1/\hat{\alpha}_1$  values for the FM-OLS and CCR estimates of  $\hat{\alpha}_1$  is less than 1 implying that private and government consumption are weak substitutes. However, the corresponding  $1/\hat{\alpha}_1$  from the

DOLS estimate of  $\hat{\alpha}_1$  is greater than 1 which indicates strong substitutability between the two goods in Kenya. For Egypt, the results from the sub-sample means that government spending is non-useful/wasteful since  $\hat{\alpha}_1$  becomes positive but statistically insignificant.

The results from the table highlights two points. First, given the three countries selected for the analyses, it is clear that the degree of substitutability varies across countries. This has direct consequences on how a marginal increase in government spending due to an additional inflow of foreign aid may impact the economy in various countries. Second, within countries, the degree of substitutability/complementarity can change over time. This fact is captured via the estimates from the sub-period. The implication here is that, the dynamic effect of fungible can vary across time in a given country. For instance, in Nigeria, the degree of substitutability between the two goods gets stronger when the sub-period 1990-2015 is considered.

### 2.3 Dynamic Model with Fungible Aid and Useful ‘G’

In Section 2.2, we provided empirical evidence of intra-temporal relationship between private and government consumption revealing that government spending is useful and can either be a substitute or complement to private consumption in aid-dependent economies. The evidence of useful government spending and its variation across countries have direct implications on the macroeconomic effects of aid when aid fungibility. To study the macroeconomic implications of foreign aid fungibility when government spending is useful, we adopt a parsimonious dynamic stochastic general equilibrium model for our analysis.<sup>15</sup>

The model comprises the following economic agents: (i) A representative household agent who populates an aid-dependent economy. This agent supplies labor inputs for firms; (ii) a monopolistically competitive firm who hires this agent as labor to produce differentiated goods which is in turn demanded by final good firms to produce final goods; (iii) a government sector that conducts both fiscal policy through levying lump sum taxes and issuing of one-period bonds to fund public spending and (iv) a monetary authority that conducts monetary policy by targeting

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<sup>15</sup>To keep the model tractable and allow for analytical explanations, we assume a closed economy. We must add that extending our model to an open economy is straightforward.

inflation and output gap and finally (v) an altruistic foreign agent who disburses aid to support the recipient country's general budget. The breakdown of the economy is as follows:

### 2.3.1 Households

A typical household agent in the aid dependent country gains utility from government spending  $G_t$  in addition to their personal consumption  $C_t$ . Specifically, the way the agent internalizes a given  $G_t$  into their utility can serve as a complement, a substitute or purely wasteful. The instantaneous utility of the agent is therefore given by: <sup>16</sup>

$$u(C_t, N_t, G_t) = \frac{(C_t + \alpha_g G_t)^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + V(G_t) \quad (2.8)$$

where  $\chi > 0$ ;  $\gamma > 0$  is the coefficient of risk aversion;  $\eta \geq 0$  is the inverse of the Frish elasticity of labor supply;  $\alpha_g \geq 1$  means  $G_t$  is a strong substitute to  $C_t$  with perfect substitution when  $\alpha_g = 1$ .  $G_t$  is a complement to  $C_t$  if  $\alpha < 0$  while  $G_t$  is a weak substitute to  $C_t$  if  $0 < \alpha_g < 1$ . The case were  $\alpha_g = 0$  suggests that  $G_t$  is wasteful/non-useful.<sup>17</sup>  $V(G_t)$  ensures that when  $\alpha_g < 0$  the marginal utility of  $G_t$  do not attain negative values.<sup>18</sup> The representative household agent thus maximizes the life-time utility:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t, G_t) \quad (2.9)$$

with  $\beta \in (0, 1)$  as the discount factor. Since there is a continuum of consumption goods available for purchase,  $C_t$  corresponds to a Dixit-Stiglitz aggregate of consumption;

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.10)$$

<sup>16</sup> Karass (1994) uses this functional form of the utility function for his empirical study. See also Bailey (1971), Barro (1981), Kormendi (1983), and Aschauer(1985). However, this formulation is more empirically restrictive but offers more analytical tractability (Kwan (2007)). Hence, for analytical tractability we employ a monotonic transformation of the functional form employed by Karass (1994) for the theoretical anlysis. Our functional form is consistent with Fève et al. (2013) and Ganelli and Tervela (2009) to mention a few.

<sup>17</sup>The empirical studies in Section 2 covers all these scenarios.

<sup>18</sup>Since household takes  $G_t$  as given, they do not maximize it, hence  $V(G_t)$  do not appear in the FOC of household. However, for welfare evaluation,  $V(\cdot)$  plays a vital role. We do not study welfare analysis here.

where  $i \in (0, 1)$  represents the continuum of differentiated final goods and  $\varepsilon > 1$  governs the elasticity of substitution between different final goods.

The household maximizes Eq. (2.9) subject to the budget constraint is given as:

$$C_t + \frac{B_t}{P_t} + T_t \leq \frac{R_{t-1}B_{t-1}}{P_t} + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} \quad (2.11)$$

In the household budget constraint,  $B_t$  denotes the quantity of one-period nominal bond held by the household,  $R_{t-1}$  is the nominal return to bonds,  $T_t$  corresponds to lump-sum taxes levied on households, and  $D_t$  is profits.

### 2.3.2 Optimality Conditions

The first order conditions for the optimizing agent's problem is given as:

$$\text{Intratemporal Euler :} \quad \chi N_t^\eta = (C_t + \alpha_g G_t)^{-\gamma} w_t \quad (2.12)$$

$$\text{Intertemporal Euler :} \quad \beta E_t \left( \frac{C_{t+1} + \alpha_g G_{t+1}}{C_t + \alpha_g G_t} \right)^{-\gamma} \frac{1}{\pi_{t+1}} = \frac{1}{R_t} \quad (2.13)$$

Eq.(2.12) relates the marginal utility gained from effective consumption to labor supply. Eq.(2.13) on the other hand defines the intertemporal Euler equation for consumption by relating present consumption to future consumption. The presence of useful government spending means the path of government spending policy is relevant in determining the optimal path of consumption.

### 2.3.3 Firms

There are two types of firms in the economy, monopolistically competitive intermediate goods producers who produce a continuum of differentiated goods and competitive final goods producers.

## Production of Intermediate Goods

Intermediate goods producing firm  $i$  has access to a constant returns to scale technology,

$$Y_t(i) = AN_t(i) \quad (2.14)$$

where  $Y_t(i)$  is output of the intermediate firm  $i$ ,  $A$  is a constant technology common to all firms and  $N_t(i)$  is the amount of labor the firm hires. The firm thus minimizes its total cost subject to the production technology.

## Price setting

A final goods producing firm purchases intermediate inputs at nominal price  $P_t(i)$  and produces the final composite good using the following constant returns to scale  $Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$  where  $\varepsilon > 0$  is the elasticity of substitution between goods. Profit-maximization by the final goods producing firm yields a demand for each intermediate good given by

$$Y_t(i) = \frac{P_t(i)^{-\varepsilon}}{P_t} Y_t$$

Monopolistically competitive intermediate goods producing firm  $i$  chooses price  $P_t(i)$  to maximize the expected present value of profits:

$$E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \frac{D_{t+j}(i)}{P_{t+j}}$$

where  $Q_{t+j} = \frac{\lambda_{t+j}}{\lambda_t}$  is the household's stochastic discount factor,  $D_t(i)$  are nominal profits for firm  $i$  and  $P_t$  is the nominal aggregate price level in the economy. Real profit are therefore given by,

$$\frac{D_{t+j}(i)}{P_{t+j}} = \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \Psi_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - \frac{\psi}{2} \left( \frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1 \right)^2 Y_t$$

where  $\psi \geq 0$  governs adjustment costs,  $\Psi_t(i)$  is real marginal cost. Price adjustments are introduced through [Rotemberg \(1982\)](#) quadratic costs of adjustment reflecting the negative effect that

price changes can have on firm-customer relationship. In a symmetric equilibrium, all firms make identical decisions and hence the first order condition is given as follows:

$$0 = (1 - \varepsilon) + \varepsilon\Psi_t - \psi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} + \psi E_t \left[ Q_{t+1} \left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right] \quad (2.15)$$

### 2.3.4 Government Policy and Aid Modality

Foreign aid flows  $A_t$  supports the general budget of the recipient government who levies lump-sum taxes and issues government bonds to finance its spending. The government budget constraint is given as:

$$A_t = \frac{R_{t-1}B_{t-1}}{P_t} + G_t - \frac{B_t}{P_t} - T_t \quad (2.16)$$

Aid takes the form of unconditional budget support and hence the benovolent foreign agent disburses foreign aid exogenously according the AR(1) process:

$$\frac{A_t}{A} = \left(\frac{A_{t-1}}{A}\right)^{\rho^A} \exp(\varepsilon_t^A) \quad (2.17)$$

where  $0 < \rho^A < 1$  and the exogenous shifter in official development assistance,  $\varepsilon_t^A$  is an i.i.d with zero mean and standard deviation  $\sigma_A$ .

Tax policy follows the fiscal rule:

$$\frac{T_t}{T} = \left(\frac{(B_{t-1}/P_{t-1})}{b}\right)^{\zeta_1} \quad (2.18)$$

where  $T$  is the steady-state level of  $T_t$  and  $b = \frac{B}{P}$  is steady-state real debt, and  $\zeta_1$  governs the degree to which taxes respond to real debt.

The central bank sets the interest rate  $R_t$  according to the Taylor Rule

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi} \exp(\varepsilon_t^R) \quad (2.19)$$



hence  $R_t$  responds to inflation through  $\rho_\pi$ . The monetary policy shock  $\varepsilon_t^R$  is an i.i.d with zero mean and standard deviation  $\sigma_R$ .  $R$  and  $\pi$  are steady state values for the nominal interest rate and inflation respectively.

Finally, since aid is fungible, to close the model we specify government expenditure  $G_t$  so that it follows the following policy rule:

$$\frac{G_t}{G} = \left(\frac{G_{t-1}}{G}\right)^{\phi^G} \left(\frac{A_{t-1}}{A}\right)^{\phi^F} \left(\frac{b_{t-1}}{b}\right)^{\phi^B} \left(\frac{Y_t}{Y}\right)^{\phi^Y} \exp(\varepsilon_t^G) \quad (2.20)$$

where  $0 \leq \phi^G < 1$  is the AR(1) coefficient of  $G_t$ ,  $1 > \phi^F \geq 0$  is the parameter that governs the degree of aid fungibility,  $\phi^B \leq 0$  captures the response of government spending to the countries debt,  $\phi^Y \geq 0$  governs government spending response to output in a procyclical fashion as in [Alesina et al. \(2005\)](#) and  $\varepsilon_t^G$  captures the exogenous government spending policy shifter and is assumed i.i.d shock with zero mean and standard deviation  $\sigma_G$ .<sup>19</sup>

### 2.3.5 Resource Constraint

Aggregate output of the economy is not only allocated to consumption and government expenditure but also to price adjustment costs which is in the unit of output. Thus the model is completed by a resource constraint given as:

$$Y_t = C_t + G_t + \frac{\psi}{2} \left(1 - \frac{P_t(i)}{\bar{\pi} P_{t-1}}\right)^2 Y_t \quad (2.21)$$

## 2.4 Analytical and Numerical Results

### 2.4.1 Interplay of Useful Government Spending and Fungible Aid

Here, we provide an analytical intuition on how the relationship between private and government consumption plays a role in impacting aid effectiveness when aid is fungible. To achieve this, we

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<sup>19</sup>For our analytical model we set  $\phi^G = \phi^B = \phi^Y = 0$  while for the numerical analysis, we only set  $\phi^G = \phi^Y = 0$  to generate convenient impulse response functions. Switching on these parameters for the numerical analyses do not change the qualitative results of our model.

employ the log-linearized version of the of the following set of equations: Eq.(2.12), Eq.(2.14), Eq.(2.21) and the policy rule for government spending, Eq.(2.20). Additionally, we make the following simplifying assumptions: (i) we assume that the steady state level of government spending,  $\bar{G}$  is zero, which is a special case of  $\bar{G} \geq 0$  and (ii) we assume  $w_t = \bar{w}$ . Employing these assumptions Eq (2.12) yields:

$$\eta(\hat{c}_t + \hat{g}_t) = -(\hat{c}_t + \alpha_g \hat{g}_t) \Rightarrow (1 + \eta)\hat{c}_t = -(\alpha + \eta)\hat{g}_t \quad (2.22)$$

solving out for  $\hat{c}_t$  we obtain

$$\hat{c}_t = -\left(\frac{\eta + \alpha_g}{1 + \eta}\right) \hat{g}_t \quad (2.23)$$

also employing the production function  $\hat{n}_t = \hat{y}_t$ , the resource constraint  $\hat{y}_t = \hat{c}_t + \hat{g}_t$ , and Eq. (2.23) we obtain,

$$\hat{y}_t = \left(\frac{1 - \alpha_g}{1 + \eta}\right) \hat{g}_t \quad (2.24)$$

Critical to Eq.(2.24) is the fact that foreign aid is fungible. Hence combining the government spending policy rule described in Eq. (2.20) and Eq. (2.24) we obtain:

$$\hat{y}_t = \left(\frac{1 - \alpha_g}{1 + \eta}\right) (\phi^G \hat{g}_{t-1} + \phi^F a_t + \phi^B \hat{b}_{t-1} + \phi^Y \hat{y}_t + \epsilon_t^G) \quad (2.25)$$

with focus on foreign aid, we can conveniently drop the exogenous shifter ( $\epsilon_t^G$ ) and the other variable in the policy rule by setting  $\phi^G = \phi^B = \phi^Y = 0$  to obtain:<sup>20</sup>

$$\hat{y}_t = \left(\frac{1 - \alpha_g}{1 + \eta}\right) \phi^F a_t \quad (2.26)$$

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<sup>20</sup>Gannelli and Tervala (2009) show that as long as  $\eta < -\alpha_g$  consumption's response to government spending is positive. Which suggests that the effectiveness of government spending on consumption depends on  $\eta$ . Our focus is on aid's impact on total output hence we focus on Eq. (2.26).

taking derivative with respect to aid,  $a_t$  we arrive at:

$$\frac{\partial y_t}{\partial a_t} = \phi^F \left( \frac{1 - \alpha_g}{1 + \eta} \right) \quad (2.27)$$

It is clear that, with fixed values of  $\eta \geq 0$  the effectiveness of aid on output (sign of  $\partial y_t / \partial a_t$ ) when aid is fungible (i.e.  $\phi^F > 0$ ) ultimately depends on sign of the preference parameter  $\alpha_g$ . Thus setting  $\phi^F > 0$  and  $\eta \geq 0$  it follows that  $\frac{\phi^F}{1+\eta} > 0$ . Now considering Eq.(2.27) the following propositions emerge:

**PROPOSITION 1:** *If government spending is complementary to private consumption  $\alpha_g < 0$  then foreign aid  $a_t$  has a positive impact on output  $y_t$ .*

PROOF. Let  $\frac{\phi^F}{(1+\eta)} > 0$  and suppose  $\alpha_g < 0$  this implies  $1 - \alpha_g > 0$  hence  $\frac{\partial y_t}{\partial a_t} = \phi^F \left( \frac{1 - \alpha_g}{1 + \eta} \right) > 0$

**PROPOSITION 2:** *If government spending is a weak substitute to private consumption,  $0 \leq \alpha < 1$  then foreign aid  $a_t$  has a positive impact on output  $y_t$ .*

PROOF. Let  $\frac{\phi^F}{(1+\eta)} > 0$  and suppose  $0 \leq \alpha_g < 1$  implying  $1 > 1 - \alpha_g > 0$  thus  $\frac{\phi^F}{(1+\eta)} > \phi^F \left( \frac{1 - \alpha_g}{1 + \eta} \right) > 0$

**COROLLARY 1:** *It follows from proposition 1 and 2 that if  $\alpha < 1$  then aid has a positive effect on output and  $\frac{\partial y_t}{\partial a_t} |_{\alpha_g < 0} > \frac{\partial y_t}{\partial a_t} |_{0 \leq \alpha_g < 1}$ .*

**PROPOSITION 3:** *If government spending is a strong substitute to private consumption and  $\alpha > 1$  then foreign aid  $a_t$  has a negative impact on output  $y_t$ .*

PROOF. Suppose  $\alpha_g > 1 \Leftrightarrow 1 - \alpha_g < 0$  since  $\forall \phi^F > 0$  and  $\forall \eta \geq 0$  it implies  $\frac{\phi^F}{(1+\eta)} > 0$  hence  $\phi^F \left( \frac{1 - \alpha_g}{1 + \eta} \right) < 0$

## Quality of Institutions, Fungibility, and Aid Effectiveness

Active arguments still remain following the work by [Burnside and Dollar \(2000\)](#) on whether foreign aid contributes to an already existing poor policy environment and institutions in developing nations or whether poor quality institutions do in fact reduce the effectiveness of foreign aid.<sup>21</sup> In

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<sup>21</sup>See for instance [Dalgaard and Hansen \(2001\)](#), [Knack \(2001\)](#), [Kolstad et al. \(2008\)](#), [Easterly \(2003\)](#), [Okada and Samreth \(2012\)](#), [Asongu \(2013\)](#), [Bräutigam et al. \(2004\)](#). The importance of institutional quality as a key

this paper and as part of our contribution to the extensive literature on aid and institutions, we control for the quality of institutions in the recipient country and investigate the hypothesis that poor quality institutions negatively impact aid effectiveness.

Certain types of aid are particularly vulnerable to corruption. The large and complex nature of donor-funded projects tend to provide scope for bribery, plundering, and fraud. Fungible aid transfers such as general budget support which may be less easy to trace are also susceptible to corruption.<sup>22</sup> In the baseline model we assumed the economy is free from any institutional inefficiencies that can lead to “plundering/wastefulness of public funds” through lack of accountability, transparency and corruption. To account for the role of the quality of institutions, we assume that some level of institutional inefficiency exists. To capture this we assume that public funds in the form of government spending generated by the fungible aid are mismanaged, lost, and/or plundered at a rate  $\zeta \in (0, 1)$ .

Thus to analyse the effect of institutional inefficiency we modify the government spending policy as follows<sup>23</sup>:

$$g_t = (1 - \zeta)\phi^F a_t + \varepsilon_t^G \quad (2.28)$$

It is therefore straight-forward to show that with the introduction of  $\zeta \in (0, 1)$ , Eq. (2.27) reduces to,

$$\frac{\partial y_t}{\partial a_t} = \phi^F \left( \frac{1 - \alpha_g}{1 + \eta} \right) - \underbrace{\zeta \left( \frac{1 - \alpha_g}{1 + \eta} \right) \phi^F}_{\text{Effect of Institutional Inefficiencies}} \quad (2.29)$$

The following results emerge from Eq. (2.29):

**COROLLARY 3:** *If government spending is a complement or a weak substitute to private consumption with  $\alpha < 1$  then institutional inefficiencies (e.g. corruption and inefficient bureaucracy) offsets the positive impact of foreign aid on output,  $(dy_t/da_t)$  when aid is fungible.*

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factor to aid effectiveness is highlighted by [Alesina and Dollar \(2000\)](#) who find that some donors actually do take institutional quality seriously and in turn disburse aid based on the recipient country’s quality of institution.

<sup>22</sup>See for instance [Morrissey \(2006\)](#) and [Leiderer \(2012\)](#)

<sup>23</sup>The modelling of institutional inefficiencies follows similar modelling style in [d’Agostino, Dunne and Pieroni \(2016\)](#)

PROOF: From *Corollary 1* when  $\alpha_g < 1$ ,  $\phi^F \left( \frac{1-\alpha_g}{1+\eta} \right) > 0$ . It follows that with  $0 < \zeta < 1$ ,  $0 < \phi^F \left( \frac{1-\alpha_g}{1+\eta} \right) - \zeta \left( \frac{1-\alpha_g}{1+\eta} \right) \phi^F < \phi^F \left( \frac{1-\alpha_g}{1+\eta} \right)$ .

**COROLLARY 4:** *If government spending is a strong substitute to private consumption and  $\alpha > 1$  then institutional inefficiencies (e.g. inefficient bureaucracy) mitigates the negative effect of foreign aid on output,  $dy_t/da_t$ . when aid is fungible.*

PROOF. From *Proposition 3* when  $\alpha_g > 0$  then  $\phi^F \left( \frac{1-\alpha_g}{1+\eta} \right) < 0$ . It is straight forward to show that since  $0 < \zeta < 1$ ,  $\phi^F \left( \frac{1-\alpha_g}{1+\eta} \right) < \phi^F \left( \frac{1-\alpha_g}{1+\eta} \right) - \zeta \left( \frac{1-\alpha_g}{1+\eta} \right) \phi^F < 0$ .

### Caveat: Heterogenous Agents

Our model assumes a representative agent in the economy. However, in most developing countries that depend on foreign aid, there is evidence that a fraction of agents are credit constrained and cannot participate in the credit market, bond market and/or do not own firms.<sup>24</sup> These households are termed as rule-of-thumb household– households who simply consume their current income.<sup>25</sup> To ensure that the analytical results from the propositions are robust to these modelling assumptions, we incorporate into our standard model two types of household. A household that is not credit constrained and one who is credit constrained and thus consume their current income. Following some simplifying assumption we show that the main results obtained earlier still hold. To facilitate discussion the proof for the case of heterogenous agents is left to the appendix. See Appendix C.2 for proof.

#### 2.4.2 Discussion

We now discuss the main results from the propositions. Recall that foreign aid fungibility refers to the marginal increase in government spending in response to the additional aid that flow into to the recipient countries. In this sense, the macroeconomic effects of aid, when it is fungible, has to be evaluated through the lenses of the impact of government spending on economy. Our focus will therefore be on the impact of the marginal increase in government spending in the economy

<sup>24</sup>Deaton (1992); Rasmussen (2002); Duflo (2003); Banerjee and Duflo (2005); Edmonds (2006) and Berg (2012) provide evidence of credit constraint households in developing countries.

<sup>25</sup>See Campbell and Mankiw (1989).

following an inflow of aid given complementarity/substitutability between private and government consumption.

To facilitate discussion, we begin our analysis from the baseline scenario of nonuseful/wasteful government spending ( $\alpha_g = 0$ ). When government spending is wasteful, an increase in it, crowds consumption due to standard wealth effects due to increase in taxes.<sup>26</sup> This in turn influences households to cut down consumption and increase work hours as leisure falls. Although consumption falls, the increases in labor hours causes output rises. Essentially, if government spending is wasteful, the standard negative wealth effects resulting from an increase in government spending will reduce consumption, increase labor hours and output. This is the standard transmission mechanism of government spending in the economy when it not useful. Consequently, an increase in fungible aid is increases the output of the recipient country when government spending is wasteful.

We now turn to the case where government spending in useful. We focus on the preference parameters  $\alpha_g$  that governs the degree of substitutability/complementarity and fix the preference parameter that governs labor supply,  $\eta$  to unity to facilitate discussion.

When government spending and private consumption are complements ( $\alpha_g < 0$ ), an increase in government spending results in an increase of the marginal utility of consumption (Gannelli and Tervela(2009)). This in turn causes households to substitute leisure for consumption. Additionally, if government spending is complementary to private consumption, from *Proposition 1*, increase in government spending due to aid fungibility enhances labor effort by reducing the marginal disutility of giving up leisure. Basically, the leisure consumption substitution offsets the standard wealth effect generated in the case where  $\alpha_g = 0$ . This simultaneous increase in labor hours and consumption drives an increase in output. Thus if households internalize government spending as a complement to private consumption, our model predicts that fungible foreign aid will increase output and induce growth.

The transmission mechanism for the case where government spending and private consumption are weak substitutes ( $0 \leq \alpha_g < 1$ ) substitutes is the same as the case of complementarity.

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<sup>26</sup>In our model, since taxes are lump-sum, Ricardian Equivalence is extent, hence it does not matter whether government purchases are financed with current taxes or deficit spending.

That is, from *Proposition 2*, if the two goods are weak substitutes, then aid even when it is fungible, still has positive impacts on output. However, this positive effect is smaller for the case of weak substitutes compared to the case where the two goods are complements. This results is evident in Corollary 1

In contrast, from *Proposition 3*, when private and government consumption are strong substitutes (i.e.  $\alpha_g > 1$ ), increase in fungible foreign aid flow has the exact opposite effects on output. Specifically, in addition to the standard negative wealth effects, an aid induced government spending crowds out private consumption since the two goods are strong substitutes. Moreover, a government spending that is a substitute to private consumption generates strong disutility for labor effort hence discouraging work and making leisure relatively attractive. This can be thought of as a behavioral subsidy cost of government spending.<sup>27</sup> Thus, the increase in government spending due to fungible aid, results in a simultaneous fall in labor hours and consumption leading to a fall in output.

Tying these theoretical results to the empirical findings in Section 2.2, all other things being equal, a country such as Egypt where government and private consumption are complements or Kenya, where the two goods are weak substitutes, aid, when it is fungible is not a problem since the marginal increase in government spending due to an inflow of fungible aid have a positive aggregates effects on the economy. This is not the case in Nigeria where the two goods are strong substitutes— fungible aid will lead to contractionary macroeconomic effects.

On the quality of institutions, conditional on the relationship between private and government consumption, our model predicts that, institutional quality only plays an offsetting or enhancing role in aid effectiveness when aid is fungible. Particularly, the positive impact of fungible foreign aid on output when  $\alpha_g$  is less than 1 diminishes (enhances) as public sector inefficiencies— governed by  $\zeta$ — increases (decreases). On the other hand, weak institutional factor, including corruption

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<sup>27</sup>The behavioral subsidy cost: Government spending encourages destructive choices. Many government programs subsidize economically undesirable decisions. welfare programs encourage people to choose leisure over work. Unemployment insurance programs provide an incentive to remain unemployed. Flood insurance programs encourage construction in flood plains. These are all examples of government programs that reduce economic growth and diminish national output because they promote misallocation or underutilization of resources (Mitchell (2005)). In relation to an aid induced government spending, this mechanism or channel can be rationalized along the lines that, foreign aid affects a recipient country's psyche, making them lazy and indolent, .

offsets the negative macroeconomic effects generated by fungible aid when government and private consumption are strong substitutes (i.e.  $\alpha_g$  is greater than 1).

In the context of this paper, this last results can be explained as follows: (i) Since government spending is a strong substitute to private consumption and in turn crowds out consumption and reduces labor effort, the fact that not all government spending induced by fungible aid enters the economy due to corruption reduces the negative impacts of fungible aid. Also, our model assumes a closed economy hence also although public funds end up in “private pockets”, they are in turn spent in the economy, offsetting any original lose in output caused by institutional inefficiencies.<sup>28</sup> However, if we assume illicit capital flight—illegal government funds moving across borders to private accounts—this result has the tendency of changing.<sup>29</sup>

In summary, from the analytical results, aid fungibility does not appear to be a problem when government spending is purely wasteful, complementary, or a weak substitute to private consumption. This conclusion, however, reverses course when private and government consumption are strong substitutes. In such a case, fungible aid have detrimental effects on output. Finally, institutional factors such transparency, accountability, and corruption playing a mitigating or enhancing role on these effect of aid when is fungible.

### 2.4.3 Numerical Example

Calibration offers the quickest way to assess the usefulness of successive extension or modification of a model (DeJong and Dave (2007)). Thus to a provide a panoptic view of our analytical results, we calibrate the parameters of the model and simulate the model to compare the dynamics the key macroeconomic variables following a foreign aid shock. Specifically, guided by our empirical results in section 2, we simulate the model for three imaginary aid dependent economies that have the same macroeconomic environment. That is, these economies have the same level of fungibility, policy environment (fiscal, and monetary) and institutional factors— but differ in the degrees

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<sup>28</sup>Other channels where institutional inefficiencies such as corruption can be a “positive” includes for instance, Heckelman and Powell (2010) who find that corruption can facilitate growth by permitting private agents, particular, entrepreneurs avoid inefficient policies and regulations when economic freedom is limited. Although our model does not study economic freedom and corruption, it lays out another channel to which some institutional inefficiencies can lead to efficiencies.

<sup>29</sup>This case is left to future work.



of substitutability/complementarity between government spending and private consumption governed by  $\alpha_g$  to study the impact of foreign aid when it is fungible. This strategy allows us to isolate the true role of the relationship between private and government consumption when assessing the macroeconomic impact of aid when it is fungible. Moreover, it permits us to borrow parameter values from the existing literature without the restraint of estimating parameter values for specific economies.<sup>30</sup> Hence, for the simulation, the model is log-linearized, calibrated in a standard way, and solved numerically using *Dynare*. We then compare the impulse response function of the key macroeconomic variables following a foreign aid shock.

#### 2.4.4 Parameter Calibration

The model is calibrated at an annual frequency since foreign aid flows are usually disbursed in annual basis. Table 3.2 provides the calibrated parameter values. A subset of the parameters are chosen based on previous studies and are standard in the literature. Specifically, preferences over consumption are logarithmic, hence  $\gamma = 1$ . The Frisch labor supply elasticity is set to unity, so  $\eta = 1$  and  $\chi$  is set such that the steady state share of time spent in employment is 1/3. With guidance from our estimation of the degree of substitutability between private and government consumption, we set  $\alpha_g$  such that it lies in the interval, [-2,2]. In particular, for the three imaginary economies, we set  $\alpha_g$  to -1.5, 0.75, and 1.5. Nominal interest rates in aid dependent economies are generally high, we therefore set the subjective discount factor  $\beta$  to 0.927 to reflect a moderately high steady state gross nominal interest rate of 7.87 percent. The common technology parameter,  $A$  is set to normalize the deterministic steady state level of output to 1. Prices are more flexible at annual frequency (Davig et al (2011) and Richter (2015)). Thus, the adjustment cost parameter  $\psi$  is set to 10 to reflect evidence of low price rigidity in developing economies (see for instance Urama et al. (2015)). The parameter that governs the degree of fungibility  $\phi^F$  is set to 0.5. This means for every \$1 inflow of aid, government spending goes up by 50 cents.<sup>31</sup> The parameter governing

<sup>30</sup>Developing countries have a long standing history with data availability. In an ongoing work, we are constructing data for some of these countries to allow for country specific model estimation.

<sup>31</sup>For instance Njeru (2003) finds that a shilling increase in ODA leads to 88 cents increase in government spending.

institutional quality  $\zeta \in [0, 1]$  but it is set to zero in the baseline simulation.<sup>32</sup> The steady-state values of the variables in the model set to follow previous studies or are model implied. For the model implied steady state values see Appendix C.3.

Table 2.3: Baseline Calibration

Parameter Name	Symbol	Value	Parameter Name	Symbol	Value
Discount factor	$\beta$	0.927	Steady-state output	$Y$	1
Substitutability b/n $C_t$ & $G_t$	$\alpha_g$	$[-2, 2]$	Steady-state tax-to-GDP ratio	$T/Y$	0.175
Frisch elasticity of labor supply	$1/\eta$	1	Steady-state gov. spending share	$G/Y$	0.20
Elasticity of Intertemporal substitution	$1/\gamma$	1	Steady-state debt-to-GDP ratio	$b/Y$	0.40
Price elasticity of demand	$\varepsilon$	7.666	Steady-state gross inflation rate	$\pi$	1.00
Rotemberg Adjustment Cost Coef.	$\psi$	10	Steady-state aid-to-GDP ratio	$Aid/Y$	0.0565
Inflation coef.: monetary rule	$\phi_\pi$	1.5	Degree of aid fungibility	$\phi^F$	0.50
Debt coefficient: tax rule	$\alpha$	0.15	Debt coefficient: gov. spending rule	$\phi^B$	0.05
Steady-state labor	$N$	0.33	AR(1) Coef. in Aid process	$\rho^A$	0.85
Steady-state gross nominal interest rate	$R$	7.87	Standard deviation of the aid shock	$\sigma_A$	0.01

## 2.4.5 Impact of fungible foreign aid shock with different of degrees of substitutability,

$\alpha_g$

Figure 2.3 shows the impulse response functions of selected macroeconomic variables following a 1 percent increase in foreign aid (official development assistance (ODA)) to GDP ratio when aid is fungible and there exist complementarity/substitutability between consumption and public spending. Given the different degrees of substitutability between private and government consumption, it is clear that when is aid fungible it has very contrasting impact on key macroeconomic variables.

Particularly, from the red line in the figure, when government and private consumption are complements ( $\alpha_g = -1.5$ ), a foreign aid shock, will induce an increase in output, consumption, labor hours worked, and inflation on impact when aid is fungible. The mechanism is as follows: an increase in foreign aid inflow increases public spending since aid is fungible. Here, an aid induced increase in government spending when  $\alpha_g < 0$ , generates a consumption leisure effect that is strong enough to reverse the standard wealth effects that will usually cause a fall in consumption

<sup>32</sup>Appendix C.5 shows the construction of  $\zeta$  from the World Banks Country Policy and Institutional Assessment transparency, accountability, and corruption in the public sector rating.

following a rise in government spending. Specifically, since government and private consumption are complements, an increase in government consumption as a result of aid inflows increases the marginal utility of consumption and reduces the marginal disutility of work effort. This effect then stimulates consumption, increases labor effort and in turn leads to a rise in output.

Consequently, the increase in consumption and output causes inflation to rise through demand pull factors. This results and mechanism are similar for the case where government consumption is a weak substitute to private consumption,  $0 < \alpha_g < 1$  (i.e.  $\alpha_g = 0.75$ ) as given by the blue dashed line. However, consistent with *Corollary 1*, when  $0 < \alpha_g < 1$  and aid is fungible, the positive effect of foreign aid on output, is smaller compared to the case when government and private consumption are complements,  $\alpha_g < 0$ . Moreover, since government spending is a weak substitute to private consumption, the increase in public spending causes consumption to fall.

In contrast, when government and private consumption are strong substitutes, ( $\alpha_g = 1.5$ ), the increase in government spending due to fungibility reinforces the standard wealth effects and induces a fall in consumption. However, the fall in consumption does not translate into an increase in labor hours as in the case where government spending is wasteful  $\alpha_g = 0$ . This is because in this case where  $\alpha_g > 1$ , there is an additional disutility to work since government consumption enters the private agents utility as a substitute. This additional disutility along side the standard wealth effects on consumption leads to contractionary effects on output. Hence, the impact of aid induces negative macroeconomic effects on consumption, labor hours, output, and inflation—results consistent with *Proposition 3*.

#### 2.4.6 Role of Institutional Factors

With a given level of substitutability we now assess how institutional inefficiencies such as low accountability, corruption, and inefficient bureaucracy in the public sector, governed by the parameter,  $\zeta$ , in the recipient impacts the effects of fungible aid.  $\zeta$  is set to a moderate and high values of 0.2 and 0.5 respectively and compared to the case where there are no institutional inefficiencies  $\zeta = 0$ .

Figure 2.4 depicts the effects of institutional inefficiencies for the case where private and government consumption are complements with set at  $\alpha_g = 1.5$ . From the figure, inefficiencies in the public domain including corruption, low accountability and transparency as well as inefficient bureaucracy reduces the positive effects of aid when it is fungible. Particularly, when government and private consumption are complements, high institutional inefficiencies that lead to waste in public funds or plundering of public funds in the recipient country negatively impacts the positive macroeconomic effects of aid when is fungible and vice versa. This results is consistent with *Corollary 3*

As shown in figure 2.5, similar to the case of complementarity, the effect of institutional inefficiencies on the impact of fungible aid on the economy is similar for the case where private and government consumption are weak substitutes  $0 < \alpha_g < 1$ . Specifically, if inefficiencies are low, fungible aid has a stronger positive effect on labor hours worked and on output following an increase in government spending. The opposite effect occurs when institutional inefficiencies are high. High inefficiencies however, mitigates the fall in consumption as there is less increase of government spending to crowd out/substitute private consumption.

Finally, figure 2.6 depicts the impulse response from a shock to foreign aid in the case where government and private consumption are strong substitutes and  $\alpha_g > 1$ . The results show that higher the public inefficiency mitigates the negative effects of fungible aid. This results seems counterintuitive, however as explained in *Corollary 4* there are two ways to rationalize this: First, aid that directly/indirectly funds government spending that do not benefit the people will be more beneficial if they never enter the economy– the less is better argument. In this case, public inefficiencies reduces the degree of foreign aid fungibility which in tend lowers how much government spending is increased. Second, since our model is a closed economy, we can rationalize that the plundered funds somehow returns into the economy in a useful way that mitigates the original negative impacts of fungible aid when the two goods are strong substitutes.

In summary, all else equal, when government and private consumption are either complements or weak substitute (i.e.  $\alpha_g < 1$ ), foreign aid inflows has expansionary effects on the re-

recipient country's output even when aid is fungible. In contrast, fungibility becomes a genuine problem when government and private consumption are strong substitutes since the impact of the marginal increase in government spending as a result of foreign aid flows has severe negative and contractionary effects on the macroeconomy of the recipient country.

Inefficiencies in the public domain reduces the positive effects of foreign aid when government consumption is a complement/weak substitute to private consumption while mitigating the negative effects of foreign aid fungibility when strong substitutability exists between the two goods. These findings are consistent with the analytical results in Section 4.1.

## 2.5 Conclusion

The discussion on the impacts of fungible aid is very active. In this paper, we have studied how the intratemporal relationship between private and government consumption impacts the macroeconomic effects of foreign aid when it is fungible. Employing data from Egypt, Kenya, and Nigeria we estimate a structural cointegration regression of the ratio of the two goods and their relative prices and study the degree of complementarity/substitutability between consumption and public spending. The results for the full sample show that in Egypt the two goods are complement, in Kenya there appears to be no relationship between the two goods, and in Nigeria, the two goods are weak substitutes. However, estimates from subsample periods show that private and government consumption are strong substitutes in Nigeria and weak substitutes in Kenya. The results for Egypt show no relationship between the two goods. The empirical results shows that on aggregate government spending is useful and its relationship with private consumption varies across countries and time in a given country.

With motivation from the empirical results we develop a tractable macroeconomic model of fungible aid that explicitly incorporates the intratemporal relationship between private and government consumption into consumer utility. We then investigate the role of the degree of substitutability/complementary between the two goods on impacting the macroeconomic effects of fungible aid. Three key findings emerge from our analytical and numerical results:

1. when government spending is a complement or weak substitute to private consumption, the marginal increase in government spending due to a rise in fungible aid results in expansionary macroeconomic effects—increasing aggregate output in the recipient country. These expansionary effects are stronger when the two goods are complements;
2. in the case where government spending is a strong substitutes to private consumption, the additional rise in government spending due an inflow in fungible aid generates contractionary macroeconomic effects in the recipient country's economy;
3. when we control for quality of institutions in the recipient country, we find that weak public institutional factors such as high corruption and inefficient bureaucracy reduces the positive effect of foreign aid in (1) and offsets the negative effects of foreign aid in (2). They however, do not change the impact of aid.

Despite our modelling assumptions we can draw the following broad conclusions. With fungibility in play omitting *a priori* the substitutability/complementarity channel between the two goods when evaluating the macroeconomic effects of aid can bias the impact of aid. The latter is robust even after controlling for institutional inefficiencies.

From a policy perspective, the results suggests that in the presence of fungibility, the direct substitution between private and public consumption is equally if not more important in improving aid effectiveness in developing countries. This channel provides policymakers and donors alike an additional avenue that can be explored to tailor strategic policies that can greatly improve aid effectiveness even when it is fungible.

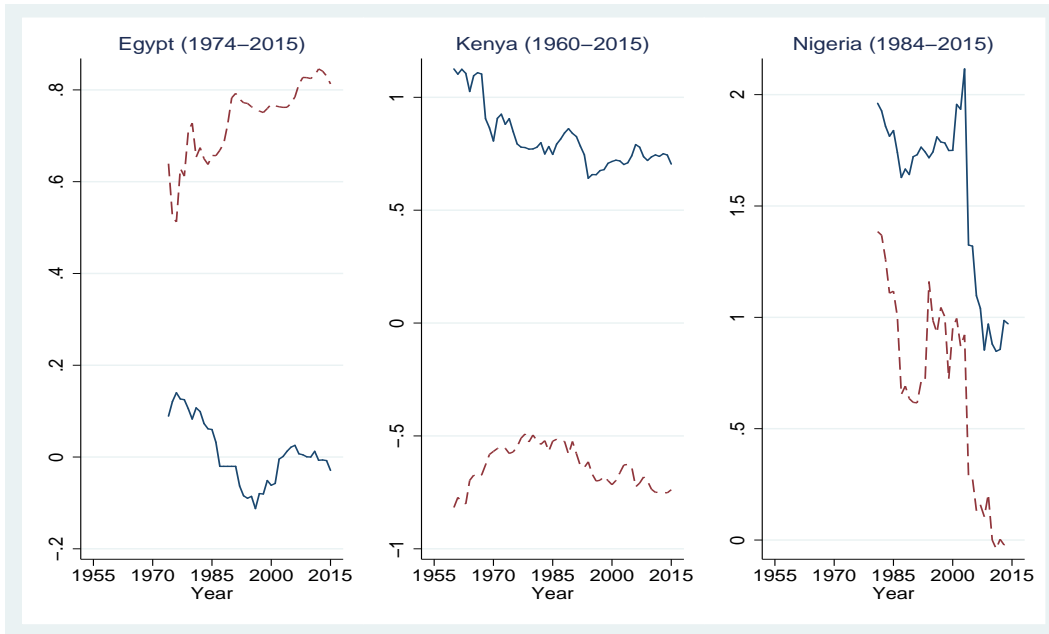


Figure 2.1: Private and government consumption ratio and relative price

Source: World Development Indicators (2016) and author's computations.

Notes: Solid = log consumption ratio; dashed= log consumption ratio.

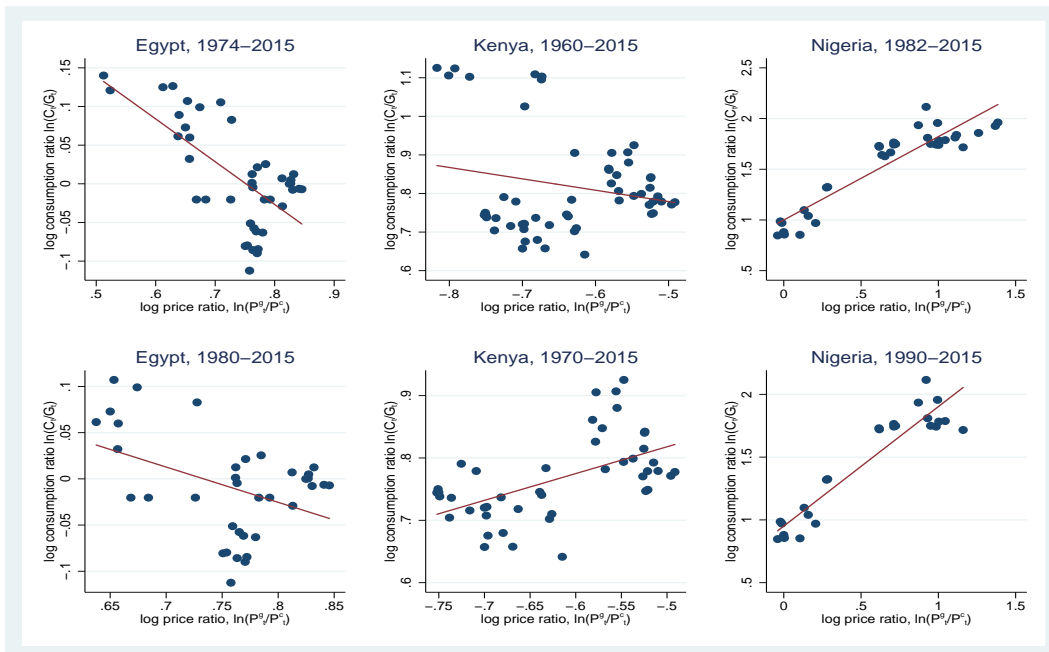


Figure 2.2: Correlation between log consumption ratio and log price ratio

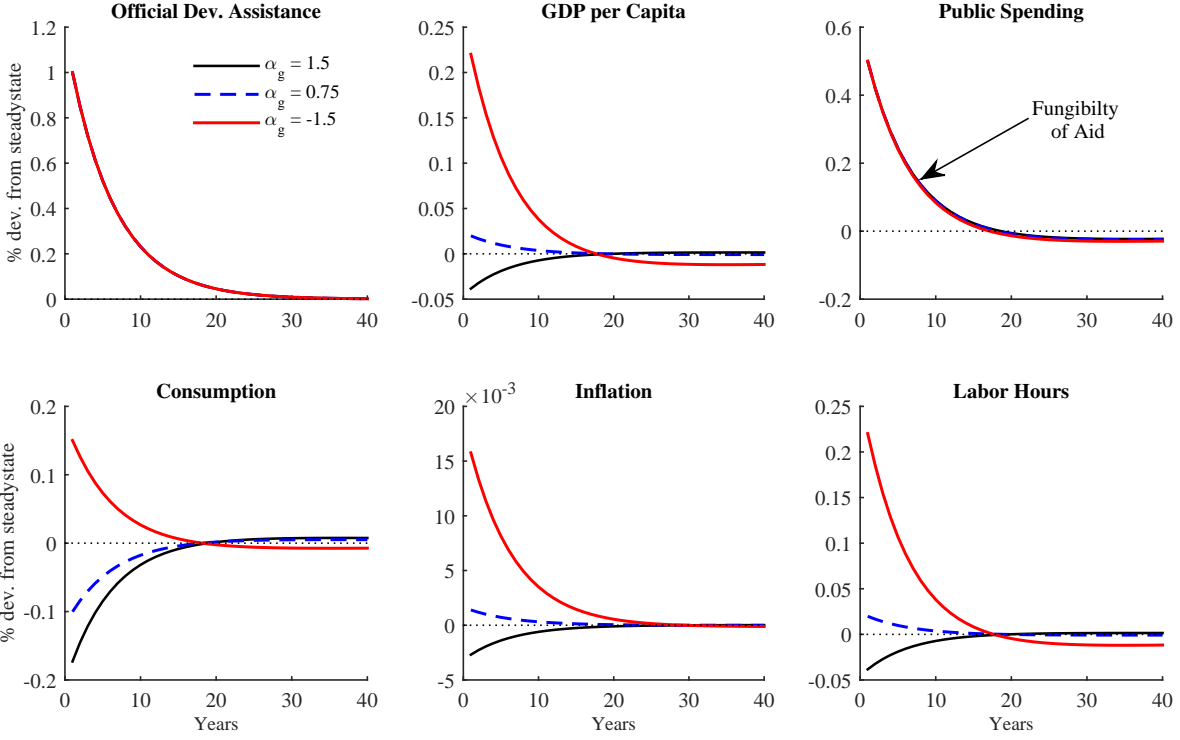


Figure 2.3: Comparison of the macroeconomic effects of an increase in fungible aid (official development assistance) following a positive shock to official development assistance given complementarity ( $\alpha_g = -1.5$ ), weak substitutability ( $\alpha_g = 0.75$ ), and strong substitutability ( $\alpha_g = 1.5$ ) between public spending and private consumption.



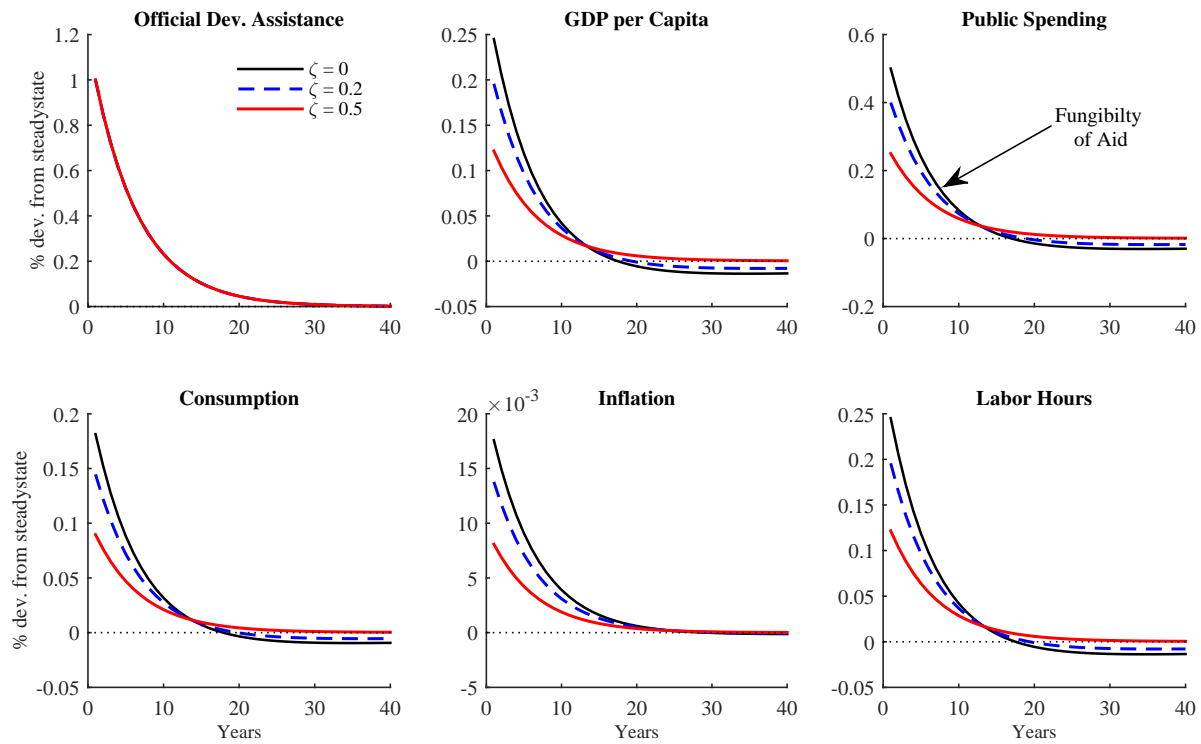


Figure 2.4: Effects of institutional inefficiencies governed by  $\zeta$  following a 1 percent shock to official development assistance when consumption and public spending are complements with ( $\alpha_g = -1.5$ ).  $\zeta = 0.2$  and  $\zeta = 0.5$  represent moderate and is high inefficiencies in the public sector respectively.  $\zeta = 0$  represents no institutional inefficiencies.

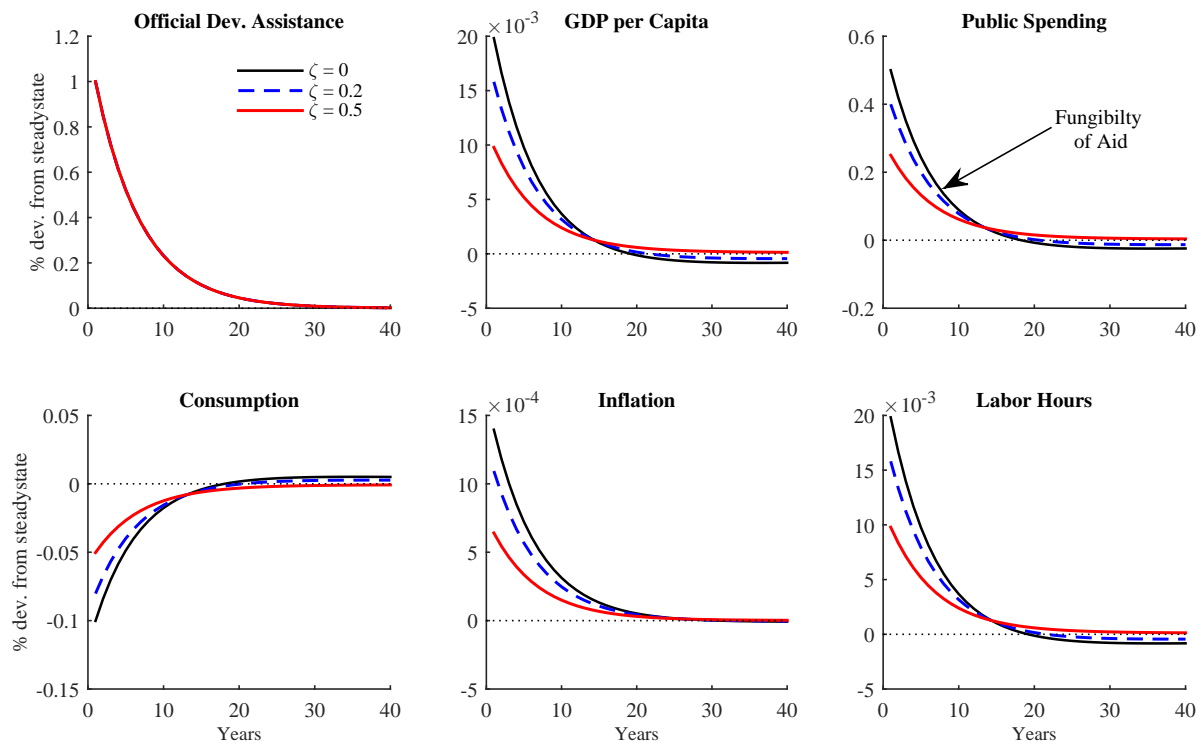


Figure 2.5: Effects of institutional inefficiencies governed by  $\zeta$  following a 1 percent shock to official development assistance when consumption and public spending are weak substitutes with ( $\alpha_g = 0.75$ ).  $\zeta = 0.2$  and  $\zeta = 0.5$  represent moderate and is high inefficiencies in the public sector respectively.  $\zeta = 0$  represents no institutional inefficiencies.

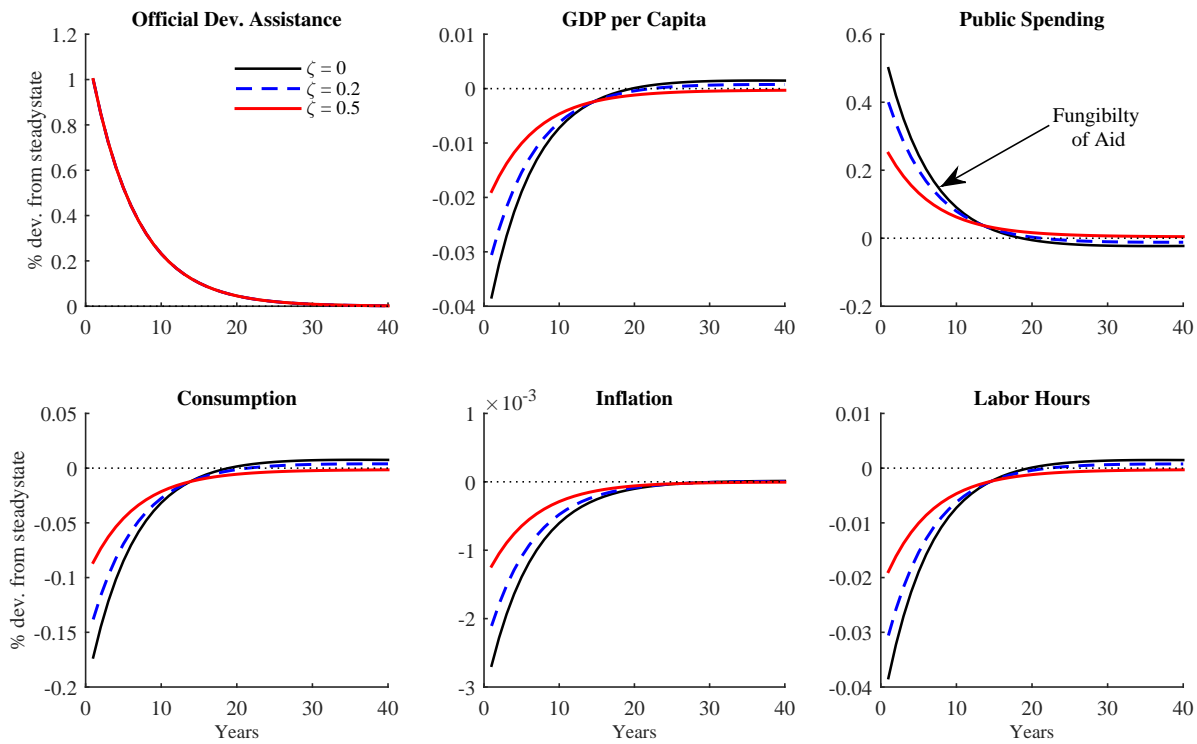


Figure 2.6: Effects of institutional inefficiencies governed by  $\zeta$  following a 1 percent shock to official development assistance when consumption and public spending are strong substitutes with ( $\alpha_g = 1.5$ ).  $\zeta = 0.2$  and  $\zeta = 0.5$  represent moderate and is high inefficiencies in the public sector respectively.  $\zeta = 0$  represents no institutional inefficiencies.

# Appendix C

## Appendices

### C.1 Direct Estimation Results

Table C.1: Cointegration Regressions Results

	FM-OLS		CCR		DOLS	
	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$
Egypt						
FS: 1975 - 2015	0.4230** (0.1691)	-0.5581** (0.2280)	0.4216** (0.1699)	-0.5562** (0.2301)	0.4212** (0.1853)	-0.5677** (0.2464)
SS: 1985 - 2015	-0.1448 (0.2483)	0.1514 (0.3213)	-0.0918 (0.2020)	0.0842 (0.2647)	-0.0603 (0.2266)	0.0375 (0.0375)
Kenya						
FS: 1960 - 2015	0.7201*** (0.1756)	-0.1267 (0.2753)	0.6820*** (0.1395)	-0.1878 (0.2179)	0.7468*** (0.1968)	-0.0961 (0.3111)
SS: 1970 - 2015	1.0432*** (0.1030)	0.4409** (0.1649)	1.0426*** (0.1052)	0.4399** (0.1694)	1.0761*** (0.1151)	0.5004** (0.1865)
Nigeria						
FS:1982 - 2015	0.9680*** (0.0764)	0.8735** (0.0992)	0.9664*** (0.0795)	0.8739*** (0.0985)	0.9426*** (0.0873)	0.9093*** (0.1112)
SS: 1990 - 2015	0.9141*** (0.0636)	1.0008*** (0.0944)	0.9102*** (0.0668)	1.0059*** (0.0979)	0.8773*** (0.0760)	1.0462*** (0.1086)

**Notes:** The dependent variable is  $\ln(C_t/G_t)$ . FS=Full Sample and SS= sub-Sample. Standard errors in parentheses. FM-OLS is Fully modified OLS; CCR is Canonical cointegrating regression; and DOLS is Dynamic OLS. FM-OLS and CCR use Andrew's automatic bandwidth selection method in computing the long-run variance matrix. DOLS includes one lead and one lag of the first difference of the regressors in the augmented regression.

## C.2 Model with Heterogenous Agents

Consider two sets of households in the economy,  $\{R, N\}$  where  $R$  and  $N$  represents *Ricardian Households* and *Non-Ricardian* households respectively, where their masses sum up to one and the proportion of non-Ricardian household is  $\lambda$ . Also for each of the the household the associated variable will be  $x_{R,t}$  and  $x_{N,t}$ . The goal here is to show that the log-linearized intratemporal Eq. (2.12) holds even with heterogenous households. By showing this, it suffices that the key propositions of the model holds. It is important to know that, the associated intertemporal Euler equation associated with the model with heterogenous agent is fundamentally different, see [Gali et. al \(2007\)](#):

$$\text{Ricardian Household :} \quad \chi N_{R,t}^\eta = (C_{R,t} + \alpha_g G_t)^{-1} w_t \quad (1.1B)$$

$$\text{non-Ricardian Household :} \quad \chi N_{N,t}^\eta = (C_{N,t} + \alpha_g G_t)^{-1} w_t \quad (1.2B)$$

log-linearizing equation 1c and 2c while assuming  $\bar{G} = 0$  and normalizing  $w_t = 1$  yields:

$$\eta \hat{n}_{R,t} = -(\hat{c}_{R,t} + \alpha_g \hat{g}_t) \quad (1.3B)$$

$$\eta \hat{n}_{N,t} = -(\hat{c}_{N,t} + \alpha_g \hat{g}_t) \quad (1.4B)$$

consider now the aggregated consumption and labor hours weighted by the portion of each household in the economy:

$$C_t = \lambda C_{R,t} + (1 - \lambda) C_{N,t} \quad (1.5B)$$

$$N_t = \lambda N_{R,t} + (1 - \lambda) N_{N,t} \quad (1.6B)$$

following the simplifying assumption  $\bar{N} = \bar{N}_N = \bar{N}_R = 1$  and  $\bar{C} = \bar{C}_N = \bar{C}_R = 1$  it is easy to show that the log-linear versions of Eq.(1.5B) and (1.6B)

$$\hat{c}_t = \lambda \hat{c}_{R,t} + (1 - \lambda) \hat{c}_{N,t} \quad (1.7B)$$

$$\hat{c}_t = \lambda \hat{n}_{R,t} + (1 - \lambda) \hat{n}_{N,t} \quad (1.8B)$$

Now multiplying Eq (1.3A) and (1.4A) by  $\lambda$  and  $1 - \lambda$  respectively and adding the resulting equations yields:

$$\eta(\lambda \hat{n}_{R,t} + (1 - \lambda) \hat{n}_{N,t}) = -(\lambda \hat{c}_{R,t} + (1 - \lambda) \hat{c}_{N,t}) - \alpha_g(\lambda \hat{g}_t + (1 - \lambda) \hat{g}_t) \quad (1.9B)$$

employing Eq (1.7B) and (1.8B) we obtain an equation equivalent to the log-linearized version of Eq(2.12):

$$\eta \hat{n}_t = -(\hat{c}_t + \alpha_g \hat{g}_t) \quad (1.10B)$$

### C.3 The Steady-State and Implied Parameters

Steady state values of the economic variables in the model are defined such that, for any time period  $t$ ,  $X_t = X_{t+1} = \bar{X}$ . Hence, at steady-state, the variable  $X_t$  is time invariant so the time subscripts are dropped. Below are the equations defining steadystate values of the economic variables that have closed form solutions.

$$\bar{R} = \frac{\bar{\pi}}{\beta} \quad (1.1C)$$

$$\bar{\Psi} = \frac{\varepsilon - 1}{\varepsilon} \quad (1.2C)$$

$$A = \frac{\bar{Y}}{\bar{N}} \quad (1.3C)$$

$$\bar{w} = A \bar{\Psi} \quad (1.4C)$$

$$\chi = (\bar{C} - \alpha_g \bar{G})^{-\gamma} \frac{\bar{w}}{\bar{N}^\varphi} \quad (1.5C)$$

$$Aid = \frac{\bar{R}}{\bar{\pi}} \bar{b} + \bar{G} - \bar{T} - \bar{b} \quad (1.6C)$$

## C.4 Full Log-Linearized Model

The dynamic economic problem presented in the paper takes on a system of non-linear difference equations. Since there are no closed form solutions, I employ a first order Taylor expansion to approximate the nonlinear model around the neighborhood of its steady-state and solve it numerically. Particularly, for a smooth arbitrary function  $h(x_t)$ , the function is approximated linearly as:

$$h(x_t) = h(x) + h'(x)(x_t - x)$$

A variable,  $\hat{x}$  represents the variable's deviation from its log steady-state value.

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{g}\hat{g}_t \quad (1.1D)$$

$$\hat{w}_t - (\sigma\bar{c}/(\bar{c} + \alpha_g\bar{g}))\hat{c}_t - (\sigma\alpha_g\bar{g}/(\bar{c} + \alpha_g\bar{g}))\hat{g}_t = (\eta\bar{n}/(\bar{n} - 1))\hat{n}_t \quad (1.2D)$$

$$\sigma\bar{c}/(\bar{c} + \alpha_g\bar{g})(\hat{c}_{t+1} - \hat{c}_t) + \sigma\alpha_g\bar{g}/(\bar{c} + \alpha_g\bar{g})(\hat{g}_{t+1} - \hat{g}_t) + \hat{\pi}_{t+1} - \hat{r}_t = 0 \quad (1.3D)$$

$$\hat{y}_t = \hat{n}_t \quad (1.4D)$$

$$\hat{\Psi}_t = \hat{w}_t \quad (1.5D)$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \kappa\hat{\Psi}_t \quad (1.6D)$$

$$\bar{T}\hat{t}_t = \bar{r}\bar{b}/\bar{\pi}(\hat{b}_{t-1} + \hat{r}_{t-1}) + \bar{g}\hat{g}_t - \bar{b}\hat{b}_t - \bar{a}\hat{a}_t \quad (1.7D)$$

$$\hat{g}_t = (1 - \zeta)\phi^F\hat{a}_{t-1} - \phi^B\hat{b}_{t-1} + \varepsilon_t^g \quad (1.8D)$$



$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a \quad (1.9D)$$

$$\hat{r}_t = \phi_\pi \hat{\pi}_t \quad (1.10D)$$

$$\hat{t}_t = \alpha \hat{b}_{t-1} \quad (1.11D)$$

## C.5 Calibrating Institutional Inefficiencies, $\zeta$

In the calibration for the numerical example we calibrate  $\zeta$  using the World Banks' Country Policy and Institutional Assessment (CPIA) transparency, accountability, and corruption in the public sector rating (1=low to 6=high). We construct this index so that it lies between zero and one.

*Definition:* Transparency, accountability, and corruption in the public sector assess the extent to which the executive can be held accountable for its use of funds and for the results of its actions by the electorate and by the legislature and judiciary, and the extent to which public employees within the executive are required to account for administrative decisions, use of resources, and results obtained. The three main dimensions assessed here are the accountability of the executive to oversight institutions and of public employees for their performance, access of civil society to information on public affairs, and state capture by narrow vested interests. Accordingly, higher scores can be attained by a country that, given its stage of development, has a policy and institutional framework that more strongly fosters growth and poverty reduction.

Data Source:

<http://databank.worldbank.org/data/reports.aspx?source=world-development-indicators>

The reconstruction is as follows of  $\zeta$

$$\zeta = 1 - \frac{\text{CPIA}_{corr}}{\text{Max. Value}}$$

Hence, when  $\text{CPIA}_{corr} = 6 \equiv 1 = \zeta^c$  and  $\zeta^c = 0 \equiv 1 = \text{CPIA}_{corr}$ .

# Bibliography

- [1] Addison, T., and Tarp, F. (2015). Aid Policy and the Macroeconomic Management of Aid. *World Development*, (69), 1-5.
- [2] Alesina, A., and Dollar, D. (2000). Who gives foreign aid to whom and why?. *Journal of Economic Growth*, 5(1), 33-63.
- [3] Alesina, A., Campante, F. R., and Tabellini, G. (2008). Why is fiscal policy often procyclical?. *Journal of the European Economic Association*, 6(5), 1006-1036.
- [4] Arndt, C., Jones, S., and Tarp, F. (2016). What is the aggregate economic rate of return to foreign aid?. *The World Bank Economic Review*, 30(3), 446-474.
- [5] Arndt, C., Jones, S., and Tarp, F. (2015). Assessing foreign aids long-run contribution to growth and development. *World Development*, 69, 6-18.
- [6] Aschauer, D. A. (1985). Fiscal Policy and Aggregate Demand. *American Economic Review*, 75, 117-127.
- [7] Asongu, S. A. (2013). On the effectiveness of foreign aid in institutional quality. *European Economic Letters*, 2(1), 12-19.
- [8] Bailey, M.J. *National Income and the Price Level: A Study in Macroeconomic Theory. 2d ed.* New York: McGraw-Hill
- [9] Banerjee, A. V., and Duflo, E. (2005). Growth theory through the lens of development economics. In P. Aghion and S. N. Durlauf, editors, *Handbook of economic growth*, volume 1A, chapter 7, 473-552. North-Holland, Amsterdam
- [10] Barro, R. J. (1981). Output effects of government purchases. *Journal of Political Economy*, 89(6), 1086-1121.

- [11] Berg, E. (2013). Are poor people credit-constrained or myopic? Evidence from a South African panel. *Journal of Development Economics*, 101, 195-205.
- [12] Bräutigam, D. A., and Knack, S. (2004). Foreign aid, institutions, and governance in sub-Saharan Africa. *Economic development and cultural change*, 52(2), 255-285.
- [13] Burnside, C. and D. Dollar, (2000), Aid, Policies and Growth. *The American Economic Review*. 90 (4), September, 847-868.
- [14] Campbell, J. Y., and Mankiw, N. G. (1989). Consumption, income and interest rates: Reinterpreting the time series evidence. In *NBER Macroeconomics Annual 1989, Volume 4* (pp. 185-246). MIT Press.
- [15] Chatterjee, S., Giuliano, P., and Kaya, I. (2012). Where has all the money gone? Foreign aid and the composition of government spending. *The BE Journal of Macroeconomics*, 12(1), 1-31.
- [16] Dalgaard, C. J., and Hansen, H. (2001). On aid, growth and good policies. *Journal of Development Studies*, 37(6), 17-41.
- [17] Davig, T., and Leeper, E. M. (2008). Endogenous monetary policy regime change. In *NBER International Seminar on Macroeconomics 2006* (pp. 345-391). University of Chicago Press.
- [18] d'Agostino, G., Dunne, J. P., and Pieroni, L. (2016). Government spending, corruption and economic growth. *World Development*, 84, 190-205.
- [19] Deaton, A. (1992). Household saving in LDCs: Credit markets, insurance and welfare. *The Scandinavian Journal of Economics*, 253-273.
- [20] DeJong, D. N., and Dave, C. (2011). *Structural Macroeconometrics*. Princeton University Press.
- [21] Dixit, A. K., and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67(3), 297-308.
- [22] Duflo, E. (2003). Grandmothers and granddaughters: oldage pensions and intrahousehold allocation in South Africa. *The World Bank Economic Review*, 17(1), 1-25.
- [23] Easterly, W. (2003). Can foreign aid buy growth?. *The Journal of Economic Perspectives*, 17(3), 23-48.
- [24] Edmonds, E. V. (2006). Child labor and schooling responses to anticipated income in South

- Africa. *Journal of development Economics*, 81(2), 386-414.
- [25] Féve, P., Matheron, J., and Sahuc, J. G. (2013). A pitfall with estimated DSGE-based government spending multipliers. *American Economic Journal: Macroeconomics*, 5(4), 141-178.
- [26] Feyzioglu, T., Swaroop, V., and Zhu, M. (1998). A panel data analysis of the fungibility of foreign aid. *The World Bank Economic Review*, 12(1), 29-58.
- [27] Galí, J., ópez-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1), 227-270.
- [28] Ganelli, G., and Tervala, J. (2009). Can government spending increase private consumption? The role of complementarity. *Economics Letters*, 103(1), 5-7.
- [29] Hauck, V., Hasse, O., and Koppensteiner, M. (2005). EC budget support: thumbs up or down?. Maastricht: European Centre for Development Policy Management (Discussion Paper 63)
- [30] Heckelman, J. C., and Powell, B. (2010). Corruption and the institutional environment for growth. *Comparative Economic Studies*, 52(3), 351-378.
- [31] Hjertholm, P., Laursen, J. and White, H. (2000). Macroeconomic Issues in Foreign Aid. Institute of Economics discussion paper, University of Copenhagen.
- [32] Jones, S., and Tarp, F. (2016). Does foreign aid harm political institutions?. *Journal of Development Economics*, 118, 266-281.
- [33] Karras, G. (1994). Government spending and private consumption: some international evidence. *Journal of Money, credit and Banking*, 26(1), 9-22.
- [34] Knack, S. (2001). Aid dependence and the quality of governance: cross-country empirical tests. *Southern Economic Journal*, 310-329.
- [35] Koeberle, S., Stavreski, Z., and Walliser, J. (Eds.). (2006). *Budget support as more effective aid?: recent experiences and emerging lessons*. World Bank Publications.
- [36] Kolstad, I., Fritz, V., and O'Neil, T. (2008). Corruption, anti-corruption efforts and aid: Do donors have the right approach. *Good Governance, Aid Modalities and Poverty Reduction: Linkages to the Millennium development goals and implications for Irish aid research project*

(RP-05-GG) of the Advisory Board for Irish Aid.

- [37] Kwan, Y. K. (2007). The Direct Substitution between Government and Private Consumption in East Asia. In *Fiscal Policy and Management in East Asia, NBER-EASE, Volume 16* (pp. 45-58). University of Chicago Press.
- [38] Leiderer, S. (2012). Fungibility and the Choice of Aid Modalities The Red Herring Revisited. *UNU-WIDER Working Paper*, (2012/68).
- [39] Marć, Ł. (2016). The Impact of Aid on Total Government Expenditures: New Evidence on Fungibility. *Review of Development Economics*.
- [40] McGillivray, M., and Morrissey, O. (2000). Aid fungibility in assessing aid: Red herring or true concern?. *Journal of International Development*, 12(3), 413.
- [41] McGillivray, M. and O. Morrissey (2004) Fiscal Effects of Aid, in T. Addison and A. Roe (eds), *Fiscal Policy for Development: Poverty, Reconstruction and Growth*, Basingstoke: Palgrave Macmillan for UNU-WIDER.
- [42] Mitchell, D. J. (2005). The impact of government spending on economic growth. *The Heritage Foundation*, 1831, 1-18.
- [43] Morrissey, O. (2006). Fungibility, prior actions, and eligibility for budget support. *Budget support as more effective aid*, Recent Experiences and Emerging Lessons, Washington D.C.: World Bank, 333-343.
- [44] Morrissey, O. (2015). Aid and government fiscal behavior: assessing recent evidence. *World Development*, 69, 98-105.
- [45] Ng, S. and Perron, P. (1997). Estimation and inference in nearly unbalanced nearly cointegrated systems. *Journal of Econometrics* 79(1), 53-81
- [46] Njeru, J. (2003). The impact of foreign aid on public expenditure: The case of Kenya (Vol. 135). Nairobi, Kenya: African Economic Research Consortium.
- [47] Ogaki, M. (1992). Engle's law and cointegration. *Journal of Political Economy*, 100(5), 1027-
- [48] <sup>46</sup>Ogaki, M. and Joon, Y.P (1997). A cointegration approach to estimating preference parameters. *Journal of Econometrics*, 82(1), 107-34.

- [49] Ogaki, M. and Reinhart, C.M (1998). Measuring intertemporal substitution: The role of durable goods. *Journal of Political Economy*, 106(5), 1078-98.
- [50] Okada, K., and Samreth, S. (2012). The effect of foreign aid on corruption: A quantile regression approach. *Economics Letters*, 115(2), 240-243.
- [51] Ouattara, B., (2006). Foreign aid and government fiscal behaviour in developing countries: Panel data evidence. *Economic Modelling*, 23(3), 506-514.
- [52] Park, Joon Y. (1992). Canonical cointegrating regression. *Econometrica*, 60, 119-43.
- [53] Pettersson, J. (2007). Foreign sectoral aid fungibility, growth and poverty reduction. *Journal of International Development*, 19(8), 1074-1098.
- [54] Phillips, Peter C. B., and B. Hansen. (1990). Statistical inference in instrumental variables regression with I(1) processes. *Review of Economic Studies*, 57:99125.
- [55] Richter, A. W. (2015). Finite lifetimes, long-term debt and the fiscal limit. *Journal of Economic Dynamics and Control*, 51, 180-203.
- [56] Rotemberg, J. J. (1996). Prices, output, and hours: an empirical analysis based on a sticky price model. *Journal of Monetary Economics*, 37(3), 505-533.
- [57] Rothmann, I. and A. ten Have (2004). Tracking Financial Flows to Education for All (EFA), Draft Synthesis Report, Rotterdam: ECORYS-NEI
- [58] Singer, H. W. (1965). External aid: for plans or projects?. *The Economic Journal*, 539-545.
- [59] Stock, J., and Watson, M. (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica*, 61 (4): 783820.
- [60] Swaroop, V., and Devarajan, S. (1998). The implications of foreign aid fungibility for development assistance. *World Bank Policy Research Working Paper*, (2022).
- [61] Tavares, J. (2003). Does foreign aid corrupt?. *Economics Letters*, 79(1), 99-106.
- [62] Urama, N. E., Oduh, M. O., Nwosu, E. O., and Odo, A. C. (2013). Price Rigidity and Monetary Non-Neutrality in Developing Countries: Evidence from Nigeria. *International Journal of Economics and Financial Issues*, 3(2), 525-536.
- [63] Van de Sijpe, N. (2012). Is foreign aid fungible? Evidence from the education and health sectors. *The World Bank Economic Review*, lhs023.

- [64] Wagstaff, A. (2011). Fungibility and the impact of development assistance: Evidence from Vietnam's health sector. *Journal of Development Economics*, 94(1), 62-73.
- [65] Zampelli, E. M. (1986). Resource fungibility, the flypaper effect, and the expenditure impact of grants-in-aid. *The Review of Economics and Statistics*, 33-40.



## Chapter 3

Is There A Role for Discretionary  
Government Spending In A Fiscally  
Stressed Economy?: Extending Davig,  
Leeper and Walker (2010)

## Abstract

Evaluating fiscal policy is difficult and usually not exhaustive. In this chapter, I extend Davig, Leeper and Walker(2010) and study the implication of discretionary spending as an additional policy instrument in an economy that faces a dual economic problem of severe economic downturn and fiscal stress– explosive transfers, tax limit and high debt. Two key findings emerge: (1) although a particular policy stance– either contractionary or expansionary– on discretionary government spending cannot eliminate the fiscal stress and severe recession caused by the explosive transfers, their use as additional policies can mitigate/worsen these effects. The mitigation or worsening effects of a specific policy stance on discretionary spending vary in both the short run and long run; (2) The short-run/long-run effectiveness of a particular policy stance on discretionary government spending depends critically on how households internalize government spending in their utility– either complements or substitute to private consumption.

**JEL Classification:** E61, E62, H60

**Keywords:** Discretionary government spending policy, Policy Uncertainty, Fiscal Limit, Useful government spending

### 3.1 Introduction

In the U.S, the Congressional Budget Office (CBO) projections show that demographic shifts due to aging population coupled with rising health care costs, and the expansion of federal subsidies for health insurance will result in exploding government transfers. This will put an upward pressure on the federal budget. In the absence of any aggressive policy changes and strategies, the debt to GDP ratio will deviate from its long-run average of 38 percent.

Of course, projections of any economy are events that are yet to happen and hence several uncertainty surrounds them. Moreover, to telescope their implications into the present requires succinct modelling of economic and policy uncertainty. The case of the CBO's projections and their corresponding policy options for resolution of the projections are no different.

In a growing literature, Davig, Leeper and Walker (2010); Davig and Leeper (2011); Richter (2015) for instance show that these projections by the CBO will undoubtedly have huge macroeconomic implications on the future health of the economy if they do materialize. In fact, these studies find that the “mere” probability of them occurring can have effects on economic agents expectations which can impact current decisions. It is also clear that any dire macroeconomic implications of these projections will occur in the presence of high debt levels—creating a dual problem of recession and fiscal stress.

In light of the above, this paper focuses on discretionary government spending by studying its role in an economy that faces the possibility of fiscal stress and severe economic downturn due to the realization of an exploding transfer payment regime. In particular, to understand the role and the implications of discretionary government spending policy in such an economy, this paper extends Davig, Leeper and Walker (2010) (DLW henceforth) and study how different discretionary spending policy options can impact the consequences of exploding transfer regime in the presence of a fiscal limit.

DLW show that in the presence of a fiscal limit, how the exponential transfers are financed in the future either through inflation or default on promised payments can cause severe economic downturn and fiscal stress well before the fiscal limit is reached. However, they assume discre-

tionary government spending as constant in their framework. This limits its role as a policy. Hence this paper introduces discretionary as additional policy to policymakers in addressing the dual economic problem in their model.

How important is discretionary spending in an economy that faces severe economic downturn and fiscal stress? First, discretionary spending forms a significant component of the government budget, assuming it to be constant in any analysis limits its role in the dynamics of the budget.<sup>1</sup> In particular, to address the fiscal stress part of the dual problem, the fiscal authority can adapt contractionary policy stance on discretionary spending, by cutting spending, to supplement any increase in taxes. It is important to note that although cuts in discretionary spending cannot eliminate all the risk factors that create uncertainty about budgetary outcomes, it can mitigate the budgetary implications of those factors and in some cases “buy time” to create room for more sustainable policies.

Moreover, because discretionary spending is “discretionary” it is more flexible and politically feasible to cut in early stages of fiscal stress than its counterpart, mandatory spending.<sup>2</sup> In fact in the CBO (2013, 2014, 2015) choices of deficit reduction report suggest that, one option for reducing the projected debt is by reducing discretionary spending. They outline two options. For short term savings, discretionary spending can be cut through regular appropriation process and for longer-term savings, lawmakers can set limits on the amount of appropriations that may be provided in future years. This strategy is not readily available to policymakers in the case of mandatory spending.

Second, if on the other hand government wants to address the recession component of the dual problem, then discretionary spending can be used as an expansionary policy tool in response to the economic downturn. Specifically, when tax policy and monetary policy are constrained due to the presence high debt, fiscal limit and rising inflation respectively, both policies automatically relinquish their role as instruments to offset any additional macroeconomic implication that may

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<sup>1</sup>For instance, in the 2014 and 2015 fiscal year all discretionary government spending( (defense plus non-defense)) was about 34 percent of total spending

<sup>2</sup>In public finance, discretionary spending is government spending implemented through an appropriations bill. This spending is an optional part of fiscal policy, in contrast to entitlement programs for which funding is mandatory (Mandal (2007)).

arise due to the realization of the exploding transfers. For instance, due to the high rising debt and inflation, taxes can not fall to stimulate consumption neither can the monetary authority cut policy rates to affect aggregate demand. However, since increases in government spending contributes a dollar to aggregate demand an expansionary policy stance on discretionary spending policy can serve as a plausible policy option that can mitigate some of the contractionary effects of the exponentially rising transfer payments.

Finally, an often discounted factor in fiscal policy evaluation is the long-run relationship between private consumption and government spending, two important components of aggregate demand determination. Indeed, the CBO argues that the implications of any policy and thus discretionary government spending policy can vary according to the economic agents preferences and priorities.<sup>3</sup> On aggregate, discretionary government spending is not totally wasteful, it is can be utility enhancing (Karass (1994); Kwan (2007); Gannelli and Tervala (2009)). This means the degree of substitution (complementarity) between consumption and government spending matter in fiscal policy effectiveness.<sup>4</sup> Thus if designed properly, the long-run relationship between government spending and private consumption can serve as an additional channel to improve the effectiveness government spending policy. Other fiscal policy instruments do not have this direct characteristics with private agent behavior.

The points discussed above inherently raise two tradeoff issues: if policy objective on discretionary spending is focused on addressig the economic downturn by increasing discretionary spending then this will put additional pressure on the budget. Moreover, depending on the component of spending that is increased, the classic issue of crowding out of private consumption and investment by government spending can deepen the downturn. if instead government cuts discretionary spending to serve as buffers to budgetary pressures then that will have direct negative impact on aggregate demand. Both of these tradeoffs can have an amplifying or mitigating effect on an economy that faces a dual problem of fiscal stress and economic downturn.

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<sup>3</sup>See the Congressional Budget Office 2013 Choices for Deficit Reduction: An Update.

<sup>4</sup>For instance, as argued by Bailey (1971) the degree of substitutability between private and government consumption plays a crucial role of determining the extent of the expansionary effects of government spending. See also Barro (1981) and Bailey(1971) for a deeper discussion.

### 3.1.1 DLW Model and Contribution to the Literature

Projections of any economy are events that are yet to happen and hence several uncertainty surrounds them. Moreover, to telescope their implications into the present requires succinct modelling of economic and policy uncertainty. The case of the CBO's projections and their corresponding policy options for resolution of the projections are no different.

DLW recently develop an optimizing dynamic model capable of thoroughly capturing these policy uncertainty and projections in a tractable way. To generate an economy that face both fiscal stress and severe recession, I augment different endogenous policy options on discretionary spending to their policy block in the model developed. Specifically, they model in a New Keynesian framework, an economy that faces exponential transfer payments, fiscal limit and several layers of policy monetary and fiscal uncertainty that seeks to resolve the growing debt after the fiscal limit is reached. Transfer payments are modelled such that it switches from a stable regime to an exponentially increasing regime at some unknown time. When in the exponentially rising regime, transfers are financed through debt which in turn puts an upward pressure on taxes.

As taxes rising, policymakers face increasing political pressure and a rising probability of reaching the fiscal limit. At the fiscal limit, either the fiscal authority defaults on its promised transfer payments or monetary authority must adjust its policy to stabilize debt. There is a possibility that the fiscal authority will not default on its promised transfer since defaulting is politically costly, in such an instance, monetary policy must adjust the nominal interest rate less than one-for-one with inflation. Since taxes are constant at the fiscal limit, growing transfers obligations increase the price level until the real value of debt stabilizes.

A key finding in DLW is that how rising debt due to exponential rising transfer is resolved in the future can impact current economic outcomes. In particular, the current and expected rise in tax rate on capital and labor reduces consumption and output while the possibility of financing the debt with higher inflation causes current inflation to increase well before the fiscal limit is hit. Clearly, this outcome produces a dual problem of fiscal stress and economic downturn.

Despite the fact that the model in DLW capture and highlight very salient points about the

policy implications of the rising transfer payments, it assumes discretionary government spending to be constant.<sup>5</sup> It is important to note that, although the assumption of constant discretionary spending may be an innocuous one, it unambiguously limits the role of discretionary spending in the policy analysis. Furthermore, DLW acknowledge that their work comprises only a small set of possible policy scenarios which further underscores the importance of what this paper investigates and its contribution to the literature.

The paper extends DLW by studying the role of an additional policy option, discretionary spending policy, available to policymakers in addressing the consequences of explosive transfers. Specifically, to understand the role and the implications of discretionary government spending policy in the economy described above, I contrasts three discretionary spending policy scenarios in the context DLW-like economy. The policy options considered include<sup>6</sup>:

1. A case where discretionary spending is constant so that it plays no policy role in the economy<sup>7</sup>;
2. A scenario where discretionary spending policy increases in response to recessions but government promises to cut it in the future when the fiscal limit is hit—run fiscal deficits today to stimulate activity, but ”promise” to reduce deficits in the future; and
3. A case where in addition to raising taxes in response to increases in debt, discretionary spending is used as an contractionary policy tool by cutting it in response to the increases in debt— a case of complete contractionary policy.

As mentioned earlier, the effectiveness of a particular policy depends on how consumers internalize(absorb) government spending in their preferences matter. Indeed, the CBO argues that the implications of any policy and thus discretionary government spending policy can vary according to the economic agents preferences and priorities. Hence, in addition, I examine these policy

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<sup>5</sup>Moreover, in the DLW economy if instead discretionary spending was introduced into the model as an exogenous process (simple AR(1) process) and hence it does not respond to the state of the economy it will be a redundant equation.

<sup>6</sup>Fiscal policy is difficult to model, study and quantify its impact on the economy for several reasons (See Leeper (2015)). Hence it is important to note that, these options/scenarios are by no means exhaustive of all the possible scenarios, nonetheless, they highlight some important aspects of policy options available to policymaker in a broader context.

<sup>7</sup>This is the case of DLW

choices when discretionary spending is purely wasteful and when it is useful and utility enhancing.<sup>8</sup>

Two key findings emerge from the model: (i) although a particular policy stance— either contractionary or expansionary— on discretionary government spending cannot eliminate the effects and risks of the rising transfer payments, their use as additional policies can mitigate/worsen these effects both in the short-run and the long-run; (ii) The effectiveness of a particular policy stance on discretionary government spending depends critically on how households internalize government spending in their utility— useful or wasteful. These results are central to understanding the tradeoffs of choosing a particular discretionary spending policy over the other in an economy faced with fiscal stress and severe recession.

### 3.2 Evidence of Long-run Relationship between C and G in the U.S

The goal of this section is to provide evidence of the usefulness of government spending at the aggregate level in the U.S. Specifically, I focus on showing the degree of substitutability between private consumption (C) and government spending (G). I then employ annual data for 1970 to 2015 from the World Development Indicators (World Bank (2015)) to estimate Eq. (3.1) or Eq. (3.2):

$$\ln(C_t/G_t) = \beta_0 + \beta_1 \ln(P^g/P^c) + e_t \quad (3.1)$$

or

$$\ln(P^g/P^c) = \alpha_0 + \alpha_1 \ln(C_t/G_t) + u_t \quad (3.2)$$

Since the discussion focuses on the long-term relationship between private and government consumption, Eq. (3.1) and (3.2) are cointegration regressions that relate the logarithm of private and government consumption ratio,  $C_t/G_t$ , to their corresponding logarithm price ratio  $P^g/P^c$ .<sup>9</sup> The gradient parameter,  $\beta$  which is approximately equal to  $\frac{1}{\alpha_1}$ , governs the elasticity of substitu-

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<sup>8</sup>See for example Ganelli and Tervela(2009); Kwan(2007); Karass(1994)

<sup>9</sup>The choice of equation specification for estimation depends on which series is more integrated Ng and Perron (1997). I discuss this more under the series properties.



tion between household and government consumption.<sup>10</sup> Negative values of  $\beta$  mean that private and government consumption are complements while a positive value means the two goods are substitutes. I start by examining the time series properties of the log transformed data of the consumption and price ratio, that is  $\ln(C_t/G_t)$  and  $\ln(P_t^g/P_t^c)$  respectively.

It is important to note that the specifications in Eq. (3.1) and (3.2) are reduced form specifications and do not have any structural interpretations. To lend structural interpretation to Eq. (3.1) I follow Kwan(2007), Ogaki and Park (1997), Ogaki and Reinhart (1992), and Ogaki (1992) and assume a representative consumer who gains utility from two goods, private and public. The agents expected lifetime utility function is governed by Eq. (3.2) and it is subject to stationary preference shocks:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^e) \quad (3.3)$$

where  $C^e$  is given by

$$C^e = [\theta \epsilon_t C_t^{1-(1/\eta)} + (1-\theta) \epsilon_t G_t^{1-(1/\eta)}]^{1/(1-(1/\eta))} \quad (3.4)$$

The optimal consumption bundle  $C^e$  satisfies the equality condition between marginal rate of substitution and relative price given the assumption of time separability of the consumer's utility function. Hence the condition:

$$\frac{\partial U_t / \partial G_t}{\partial U_t / \partial C_t} \equiv \frac{\epsilon_t \theta C_t^{1/\eta}}{\epsilon_t (1-\theta) G_t^{1/\eta}} = \frac{P_t^g}{P_t^c} \quad (3.5)$$

To allow for easy interpretation and estimation, I take the logarithm of Eq. (3.3) to obtain,

$$\ln \left( \frac{C_t}{G_t} \right) = -\eta \ln \left( \frac{1-\theta}{\theta} \right) + \eta \ln \left( \frac{P_t^g}{P_t^c} \right) - \eta \ln \left( \frac{\epsilon_t}{\epsilon_t} \right) \quad (3.6)$$

The corresponding equation for the empirical estimation is then given as:

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<sup>10</sup>It is important to note that in finite sample the estimates from the direct and reverse regressions may be far from being reciprocal to each other and they can also have drastically different statistical properties (Kwan(2007))

$$\ln\left(\frac{C_t}{G_t}\right) = \alpha_0 + \alpha_1 \ln\left(\frac{P_t^g}{P_t^c}\right) + \nu_t \quad (3.7)$$

Note that the parameter of interest here is  $\alpha_1$ .

### 3.2.1 Data and Estimation

I employ annual data for 1960 to 2015 from the World Development Indicators (World Bank 2015) for the United States. To obtain the consumption ratio,  $C_t/G_t$  I divide, household final consumption expenditure by general government final consumption expenditure, both in 2010 constant Dollars. In light of Kwan (2007), the corresponding prices  $P_t^c$  and  $P_t^g$  are computed as the implicit price deflators which are constructed by dividing the nominal private and government consumption series by their respective constant price series.

As standard in the literature, I begin by studying the time series properties of the constructed series  $\ln(C_t/G_t)$  and  $\ln(P_t^g/P_t^c)$ . Figure 3.1 shows the time path of log consumption and price ratio in the U.S. Individually, both series show strong persistence which suggest that they may be cointegrated and possibly I(1).<sup>11</sup>

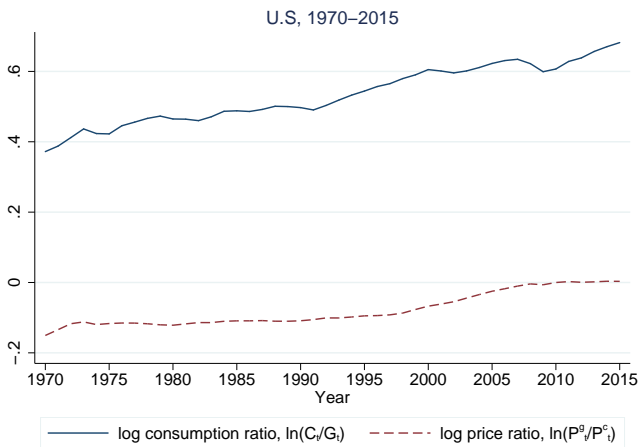


Figure 3.1: Policy Options: No Useful Government spending  $\alpha_g = 0$

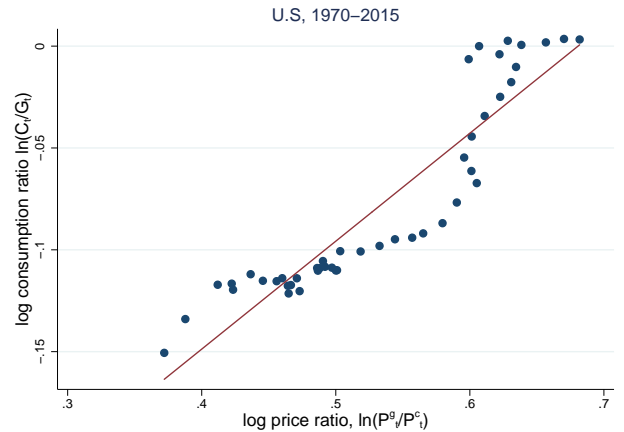


Figure 3.2: Correlation between consumption ratio,  $\ln(C_t/G_t)$  and price ratio,  $\ln(P_t^g/P_t^c)$

Figure 3.2 provides preliminary correlation analysis of the two series for the full sample

<sup>11</sup>Formal unit root tests are provided in the appendix. Results from the unit root suggests that Eq. (3.2) is adopted.

period and sub-period. As shown in the figure, there is a consistent positive correlation between the price and consumption ratios. This shows tendency of government and private consumption, on aggregate, to be substitutes.

## Estimation Results

Table 3.1 reports the cointegrating regression results from estimating Eq. (3.2). The table also reports cointegrating results for sub-samples. It is important to check for robustness, hence three different estimation methodologies—the fully modified ordinary least square (FM-OLS) by Phillips and Hansen (1990); the canonical cointegrating regression (CCR) by Park (1992); and Stock and Watson’s(1993) dynamic ordinary least square (DOLS)— are employed. These estimation methods are all asymptotically efficient procedures for estimating cointegration regressions.

Table 3.1: Cointegration Regressions: United States

	FM-OLS		CCR		DOLS	
	$\alpha_0$	$\alpha_1$	$\alpha_0$	$\alpha_1$	$\alpha_0$	$\alpha_1$
1970 - 2015	-0.3651*** (0.0401)	0.5418*** (0.0740)	-0.3670*** (0.0352)	0.5461*** (0.0678)	-0.3712*** (0.0397)	0.5524*** (0.0728)
1980 - 2015	-0.4466*** (0.0396)	0.6811*** (0.0700)	-0.4503*** (0.0385)	0.6894*** (0.0701)	-0.4469*** (0.0415)	0.6902*** (0.0752)
1990 - 2015	-0.5378*** (0.0646)	0.8290*** (0.1088)	-0.5334*** (0.0581)	0.8241** (0.1005)	-0.5026*** (0.0673)	0.7732*** (0.1134)

Notes: Standard errors in parentheses. FM-OLS is Fully modified OLS; CCR is Canonical cointegrating regression; and DOLS is Dynamic OLS. FM-OLS and CCR use Andrew’s automatic bandwidth selection method in computing the long-run variance matrix. DOLS includes one lead and one lag of the first difference of the regressors in the augmented regression.

The parameter of interest here is the reciprocal of  $\hat{\alpha}_1$  given as  $\frac{1}{\hat{\alpha}_1}$ . This value governs the degree of substitution/complementarity between private and government consumption. Generally, the estimated parameter values for  $\hat{\alpha}_0$  and  $\hat{\alpha}_1$  are stable across the estimation methods. The positive sign and statistical significance of  $\hat{\alpha}_1$  across estimation methods for the full sample and sub-sample period show that government spending on aggregate is a substitute to private consumption.

Particularly, the value governing the degree of substitutability,  $\frac{1}{\alpha_1}$  ranges between 1.2 to 1.85. Since  $\frac{1}{\alpha_1}$  is great than 1, this implies the government spending on aggregate is a strong substitute in the U.S in the long-run. This results is consistent with estimates from Kwan (2009)

Although, this section focuses on the long-run relationship between government spending and consumption, our choices of estimation assumes that government spending is exogenous and hence precludes cross equation restrictions that arise due to endogenous government policy. For instance Fève, Matheronn, and Sahuc (2013), in a full information estimation model find that omitting the countercyclical component of government spending policy when estimating the relationship between private consumption and government spending can lead to lower estimates of complementarity. Their estimates however, do not explicitly consider the possible long-run relationship between consumption and government spending.

Moreover, several empirical studies find that an increase in government spending crowds in private consumption. The values of the degree of substitutability obtained in this section do not favor this empirical finding. Instead, theoretical work have found that the possibility of complementarity between private consumption and government spending favors the empirical finding that increases in government spending crowds in private consumption.

In the model that follow, government spending is not exogenous, it is modelled as endogenous thus it is important not to discount the aforementioned issues. To expand the analyse, the counterfactual exercises conducted in the model section of the paper considers cases where government spending is: (i) wasteful (ii) substitute to private consumption and (iii) complement to private consumption.

### **3.3 The Economic Model**

The analysis is conducted in an otherwise standard new Keynesian model. Nominal rigidities are introduced using Rotemberg (1982) pricing by monopolistically competitive intermediate good firms. The policy block is governed by Davig, Leeper and Walker (2010) with a key introduction of an additional fiscal policy choice in the form of different policy options of discretionary government

spending. I also model government spending as utility enhancing.

### 3.3.1 Households

The representative household chooses consumption ( $C_t$ ), Labor hours ( $N_t$ ), money ( $M_t$ ), money ( $K_t$ ) and risk free one period nominal bonds ( $B_t$ ) to maximize:

$$E_t \sum_{j=0}^{\infty} \beta^j U_{t+j} \quad (3.8)$$

where  $\beta \in (0, 1)$  is the subjective discount factor and the instantaneous utility is given as:

$$U_t = \frac{(C_t + \alpha_g G_t)^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + \vartheta \frac{(M_{t+1}/P_t)^{1-\kappa}}{1-\kappa} + V(G_t)$$

where  $E_t$  is the expectation operator, conditioned on information available at time  $t$ ,  $\sigma > 0$ ,  $\eta > 0$ ,  $\chi > 0$ ,  $\kappa >$  and  $\vartheta > 0$  and  $C_t$  is a composite private consumption good consisting of differentiated goods,  $c_{it}$ , which is governed by the Dixit and Stiglitz (1977) aggregator. Lastly the parameter  $\alpha_g$  allows government spending to be useful and governs the complementarity/substitutability between private consumption  $C_t$  and public spending  $G_t$ .<sup>12</sup> Specifically if  $\alpha_g < 0$ ,  $G_t$  is complementary to  $C_t$  while  $\alpha_g \geq 0$  means government spending  $G_t$  is a substitute to  $C_t$ . The case where  $\alpha_g = 1$  characterizes the perfect substitution explained in Christiano and Eichenbaum (1992). If  $\alpha_g = 0$ , the model reduces to the standard real business cycle model.<sup>13</sup> The function  $V(G_t)$  is to ensure that the marginal utility of government spending do not attain negative values when  $\alpha_g < 0$ .

The household's budget constraint is

$$C_t + K_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} \leq \frac{R_{t-1}B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \lambda_t z_t + (1 - \tau_t)(w_t N_t + R_t^k K_{t-1}) + \frac{D_t}{P_t}$$

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<sup>12</sup>See Fève, Matheronn, and Sahuc (2013), Ganelli and Tervela (2009), Christiano and Eichenbaum (1992), Finn (1998), McGrattan (1994). In the empirical model, I adopted the CES specification of utility which have been considered by Bouakes and Rebei (2007), McGrattan, Rogerson and Wright (1997). However, as mentioned by Fève, Matheronn, and Sahuc (2013), both specifications yield the same log-linearized equilibrium conditions which in this paper I use for the initial guess for the numerical solution. To allow for tractability I adopt the former specification.

<sup>13</sup>See for instance Aiyarigari, Christiano and Eichenbaum (1992) and Baxter and King (1993)

where  $K_{t-1}$  is the capital stock available to use in production at time  $t$ ,  $w_t$  is real wages paid to labor,  $\tau_t$  is the distorting tax rate common to both labor and capital,  $R_t^k$  is the real rental rate of capital,  $R_{t-1}$  is the nominal return to bonds,  $z_t$  are lump-sum transfers promised by the government,  $\lambda_t$  is the fraction of promised transfers actually received by the household and  $D_t$  is nominal profits.

### 3.3.2 Optimality conditions

$$\text{Consumption :} \quad (C_t + \alpha_g G_t)^{-\sigma} = \lambda_t \quad (3.9)$$

$$\text{Labor :} \quad \chi N_t^\eta = \lambda_t (1 - \tau_t) w_t \quad (3.10)$$

$$\text{Real balance :} \quad \nu \frac{M_t^{-\kappa}}{P_t^{1-\kappa}} = \frac{\lambda_t}{P_t} - \beta \frac{\lambda_{t+1}}{P_{t+1}} \quad (3.11)$$

$$\text{Capital :} \quad \frac{\lambda_{t+1}}{\lambda_t} \{(1 - \tau_{t+1}) R_{t+1}^k + (1 - \delta)\} = 1 \quad (3.12)$$

$$\text{Bonds :} \quad \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} = \frac{1}{R_t} \quad (3.13)$$

where the stochastic discount factor is given as  $\frac{\lambda_{t+1}}{\lambda_t} = \left( \frac{C_{t+1} + \alpha_g G_{t+1}}{C_t + \alpha_g G_t} \right)^{-\sigma}$

### 3.3.3 Firms

The production sector is made up two types of firms, monopolistically competitive intermediate goods producers who produce a continuum of differentiated goods and competitive final goods producers.

## Intermediate goods firm

Intermediate good firm  $i$  has access to a Cobb-Douglas production function:

$$y_t(i) = k_{t-}(i)^\alpha n_t(i)^{(1-\alpha)}$$

and hence minimizes its total cost,  $w_t n_t(i) + R_t^k k_t(i)$  subject to the Cobb-Douglas production technology.

## Price setting

The final good firm then purchases  $y_t(i)$  units from each intermediate goods firm to produce the final good,

$$Y_t \equiv \left( \int_0^1 y_t(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}$$

where  $\theta > 1$  is the elasticity of substitution between goods. Profit maximization by the final goods producing firm then yields a demand for each intermediate good which is given by:

$$y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t$$

Monopolistically competitive intermediate goods producing firm  $i$  chooses prices  $P_t(i)$  to maximize the expected present value of profits:

$$E_t \sum_{k=0}^{\infty} Q_{t,k} \tilde{D}_k(i)$$

where  $\tilde{D}_k(i)$  is real profit and given as:

$$\tilde{D}_k(i) = \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} Y_t - MC_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t - \frac{\varphi}{2} \left( \frac{P_t(i)}{\pi^* P_{t-1}(i)} - 1 \right)^2 Y_t$$

where  $\varphi$  governs adjustment cost,  $MC_t(i)$  is real marginal cost. Price adjustment follows Rotemberg (1982) quadratic costs of adjustment. The functional form of adjustment cost  $\left( \frac{P_t(i)}{\pi^* P_{t-1}(i)} - 1 \right)^2 Y_t$

is that of Ireland (1997).

With costly price adjustment, the aggregate resource constraint is given as

$$Y_t = C_t + K_{t-1} - (1 - \delta)K_{t-1} + G_t - \frac{\varphi}{2} \left( \frac{P_t(i)}{\pi^* P_{t-1}(i)} - 1 \right)^2 Y_t \quad (3.14)$$

### 3.3.4 Davig, Leeper, Walker Policy Block with Discretionary Spending Policy

The government finances purchases  $G_t$  and actual transfer delivered.  $\lambda_t z_t$  with income tax revenues, money creation, and the sale of one-period nominal bonds. The government's flow budget constraint is given as

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} + \tau_t(w_t N_t + R_t^k K_{t-1}) = \frac{R_{t-1} B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \lambda_t z_t + G_t$$

The rest of the policy blocks follows closely, DWL and Richter (2015). The policy block encompasses several layers of uncertainty around tax, transfer, and monetary policy. In particular, this policy formulation captures the non-stationary process of transfer payments as projected by the CBO in their 2013.

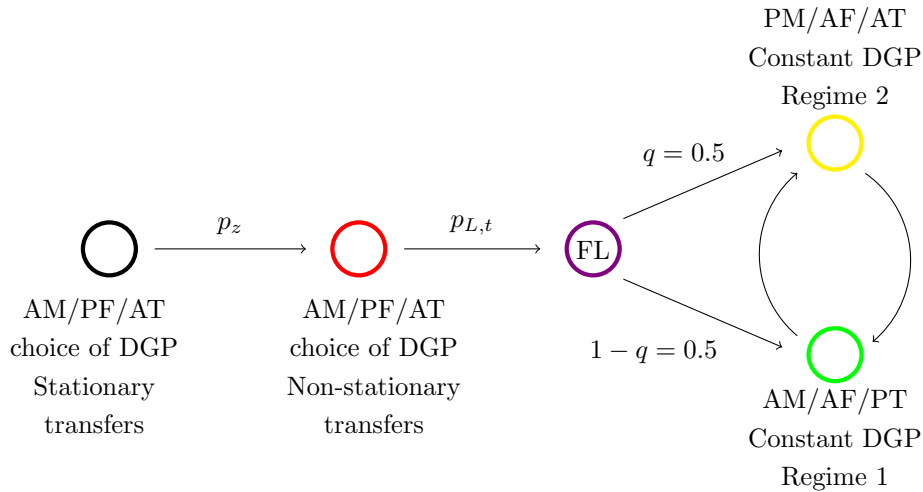


Figure 3.3: A graphical representation of policy evolution of the DLW economy augmented with government stance on discretionary government spending policy (DGP).

Figure 3 describes graphically how the uncertainty unfolds. The fiscal authority sets transfer payments exogenously according to a Markov chain; the economy starts at the black node with a



stationary process of transfers, a choice of discretionary spending policy, active inflation targeting (AM) by the central bank, and a tax policy that raises tax rates passively in response to an increasing debt (PF). Governed with some positive probability  $p_z$ , the transfer process switches from its stationary regime into its non-stationary regime by moving from the black node to the red node. It is assumed that non-stationary regime is an absorbing state, so that when the non-stationary regime is entered it does not switch back to its stationary path. The transfer process is therefore given as:

$$z_t = \begin{cases} (1 - \rho_z)z^* + \rho_z z_{t-1} + \varepsilon_t & \text{for } S_{z,t} = 1 \\ \mu z_{t-1} + \varepsilon_t & \text{for } S_{z,t} = 2 \end{cases} \quad (3.15)$$

$0 < \rho_z < 1$  and  $\mu > 0$  however  $\beta\mu < 0$  ensuring square summability. This formulation captures the potential upward trend in transfers. The regimes,  $S_{z,t}$  follow a two state Markov chain that evolve according to

$$M_z = \begin{bmatrix} 1 - p_z & p_z \\ 0 & 1 \end{bmatrix}$$

The exponential growth in transfer is initially financed by new debt issuance, which is backed by increasing tax rates. However, as emphasized by DLW, there is a fiscal limit to the amount of debt that can be financed through tax increases. This is due to either reaching the peak of the Laffer curve or political resistance to tax hikes. To capture the fiscal limit, the fiscal limit is modelled as setting  $\tau_t = \tau_{FL}$  for  $t \leq T$ , where  $T$  is the date at which the economy hits the fiscal limit and it is stochastic. Tax policy is then set according to

$$\tau_t = \begin{cases} \bar{\tau}(b_{t-1}/b^*)^\gamma, & \text{for } S_{p,t} = 1, \text{ Fiscal Limit doesn't bind} \\ \tau_{max}, & \text{for } S_{p,t} \in \{2, 3\}, \text{ Fiscal Limit bind} \end{cases} \quad (3.16)$$

where  $b^*$  is the target debt-output ratio and  $\bar{\tau}$  is the steady state tax rate. In the light of DLW, Davig, Leeper, Walker (2011), and Richter (2015) the probability of hitting the fiscal limit,

$p_{Lt}$ , is endogenously determined through the logistic function

$$p_{FL,t} = 1 - \frac{\exp(\eta_0 - \eta_1(\tau_{t-1} - \bar{\tau}))}{1 + \exp(\eta_0 - \eta_1(\tau_{t-1} - \bar{\tau}))}$$

where  $\eta_0$  and  $\eta_1 > 0$  pin down the intercept and slope of the logistic function. Thus the probability of hitting the fiscal limit is increasing in taxes. Since taxes increase when debt increases (passive response), the probability of hitting the fiscal limit increases with debt.

Monetary policy evolves between active/passive monetary regime (AM/PM)

$$R_t = \begin{cases} \bar{R}(\pi_t/\pi^*)^{\rho^A} & \text{for } S_{p,t} \in \{1, 3\} \\ \bar{R}(\pi_t/\pi^*)^{\rho^P} & \text{for } S_{p,t} = 2 \end{cases} \quad (3.17)$$

where active monetary policy is in regimes  $S_{m,t} \in \{1, 3\}$  and passive monetary policy is in regime  $S_{m,t} = 2$

Depending on the short-run goals of fiscal authority, discretionary spending policy can be set to case 1 for contractionary policy or case 2 for expansionary policy goals:<sup>14</sup>

**Case 1:** Systematic cuts to discretionary spending in response to debt (Contractionary Type Policy)

$$G_t = \begin{cases} \bar{G}(b_{t-1}/\bar{b})^{\phi^B} & \text{for } S_{p,t} \in \{1, 3\} \\ \bar{G}(b_{t-1}/\bar{b})^{\phi^F} & \text{for } S_{p,t} = 2 \end{cases} \quad (3.18)$$

where  $\phi^B < 0$  and  $\phi^F = 0$  so that discretionary spending is contractionary prior to the fiscal limit and constant at the fiscal limit.<sup>15</sup>

**Case 2:** Systematic increase in government spending in response to economic downturn (Expan-

<sup>14</sup>There are more general formulations to the policy rules below (See for instance, McGrattan (1994) and Jones (2002)) while Fève, Matheron, and Sahue (2013); Moura (2015) and Leeper, Plante and Traum (2010) ) For computational speed purposes I do not include the lag term ( $g_{t-1}$ ) in the policy rule for discretionary spending as this will add to the state space and will significantly increase the computational time. Numerical analysis with  $g_{t-1}$  for the fixed regime yielded similar qualitative results.

<sup>15</sup>In the CBO (2013) choices of deficit reduction report for instance, one option for reducing the projected debt is reducing discretionary spending. They outline two options. For short term savings discretionary spending can be cut through regular appropriation process and for longer-term savings, lawmakers can set limits on the amount of appropriations that be provided in future years.

sionary Type Policy)

$$G_t = \begin{cases} \bar{G}(Y_t/\bar{Y})^{\phi^C} & \text{for } S_t(g) \in \{1, 3\} \\ \bar{G}(Y_t/\bar{Y})^{\phi^F} & \text{for } S_t(g) = 2 \end{cases} \quad (3.19)$$

where  $\phi^C < 0$  and  $\phi^F = 0$ . Notice that when  $\phi^C = \phi^F = 0$  and the parameter governing the usefulness of discretionary government spending in consumer utility,  $\alpha_g$  is zero then the model collapses to the model in DLW.

At the fiscal limit (FL), tax policy becomes active (AF) so that it can not respond to debt any longer. Policy combinations must adjust to stabilize debt. Specifically, if the fiscal authority honors its promised transfers (AT), the monetary authority stabilizes debt by abandoning its active targeting of inflation to a passive policy (PM). The economy then moves from the red node to the yellow node in figure 3. Since the non-stationary transfer regime is absorbing, at the yellow node, transfers continue to follow an unsustainable path, this leads to continued increases in debt and, without a central bank response, higher inflation. The high inflation reduces the real value of debt and allows the fiscal authority to avoid default on promised transfer (i.e.  $\lambda = 1$ ).

However, if monetary authority decides to actively target inflation (AM), the fiscal authority is forced to renege on its promised transfer (i.e.  $\lambda < 1$ ) and the economy moves from the red node to the green node. At the green node, reductions in the promised transfer should be sufficient enough to stabilize real debt, so that the post-fiscal limit regimes produce paths that are consistent with a long-run equilibrium. In both cases (green and yellow node), government concern is on the long-run stabilization of debt hence, fiscal authority puts a cap on discretionary spending by simply keeping it constant.

At fiscal limit, the monetary authority stabilizes debt and the fiscal authority reneges on its transfer commitments with probability  $q$  and  $1 - q$  respectively. This initial policy adjustment is not invariant. After the fiscal limit is hit, policy evolves according to a first-order two-state Markov chain with transition matrix  $P[s_t = j | s_{t-1}] = p_{ij}, i, j \in 2, 3$  so that in each period either the monetary or fiscal authority stabilizes debt when the fiscal limit binds. These realizations are shown by movements between yellow and green node.

### 3.3.5 Calibration

The study focuses on long-run policy uncertainty as in Davig, Leeper and Walker (2010) thus the model is calibrated in annual frequency. The household preference parameters are calibrated as follows. In the baseline model, the relative risk aversion,  $\sigma$  and the inverse Frisch elasticity of labor  $\eta$  are set to 1 while the parameter governing the complementarity between government spending and private consumption is set to -1.5 [Ganelli and Tervala (2009) and Fève, Matheron, and Sahuc(2013)]. For substitutability between government spending and consumption is set to 1.83 in the baseline case. This value is obtained from the empirical study in section 2. The parameter that determines the interest elasticity of real balances,  $\kappa$  is set to 2.6 [Chari, Kohoe and McGrattan (2000)]. The annual real interest rate is set 2 percent so that the discount factor,  $\beta = 0.9804$ . Meanwhile for real balances,  $\nu$  is set such that the velocity in the deterministic steady state, defined as  $cP/M$ , matches average U.S monetary base velocity at 2.4. This value is consistent with Davig and Leeper (2006).

The price elasticity of demand and the Rotemberg adjustment cost coefficient are set to 7.666 and 10 respectively. The parameter value of price elasticity of demand means firms markup the prices of their goods over marginal cost by 15 percent

Table 3.2: Baseline Calibration

Parameter Name	Symbol	Value	Parameter Name	Symbol	Value
Substitutability b/n $C_t$ & $G_t$	$\alpha_g$	$[-2, 2]$	Inflation coefficient: active MP rule	$T/Y$	1.5
Frisch elasticity of labor supply	$1/\eta$	1	Inflation coefficient: passive MP rule	$G/Y$	0
Elasticity of Intertemporal substitution	$1/\sigma$	1	Debt coefficient: passive fiscal rule	$b/Y$	0.40
Price elasticity of demand	$\theta$	7.666	Prob. of moving to PM/AF/AT regime after FL	$q$	0.5
Rotemberg Adjustment Cost Coef.	$\psi$	10	Initial prob. of moving to PM/AF/AT regime after FL	$p_{22}$	0.5
Capital depreciation rate	$\delta$	0.10	Prob. of staying in AM/AF/PT regime after FL	$\pi_{33}$	0.99
Cost share of capital	$\alpha$	0.33	Prob. of non-stationary transfer occurring	$p_S^z$	0.2
Steady-state gross inflation rate	$\bar{\pi}$	0.02	AR coefficient: stationary transfer process	$\rho_S^Z$	0.90
Steady-state gross nominal interest rate	$\bar{R}$	0.04	Growth rate: non-stationary transfer process	$\rho_S^Z$	1.01
Steady-state labor	$\bar{N}$	0.04	Tax rate after FL	$\tau^{FL}$	0.244
Steady-state government spending share	$\bar{G}/\bar{Y}$	0.08	Output coefficient: government spending rule	$\phi^Y$	-1.2
Steady-sytate debt-to-GDP ratio	$\bar{b}/\bar{Y}$	0.385	Debt coefficient: government spending rule	$\phi^B$	-0.05

For the policy block, the steady state government spending per out is set to 8 percent while transfer per out is set to 9 percent. While in the stationary transfer regime, the coefficient governing

the transfer process is set to  $\rho_z = 0.9$ . The steady state tax rate is set such that the debt to output ratio is consistent to its long term average of 0.385. Meanwhile the steady-state inflation rate,  $\bar{p}_i$  is 2 percent. Prior to the reaching the fiscal limit, policies are “normal” hence tax policy is passive and monetary policy is active ( $S_p = 1$ ) with reaction coefficient  $\gamma = 0.2$  and  $\phi_A = 1.5$ . Discretionary spending policy is endogenous and hence in the case where government spending responds to deviation of output from its steady state,  $\phi^Y$  is set to -1.2 while  $\phi^B = -0.1$  in the case where it responds to debt. The values for  $\phi^Y$  and  $\phi^B$  are set to generate sizeable changes in government spending. The expected duration of the stationary regime is five year ( $p_z = 0.8$ ) after which transfers grow at 1 percent per year once the switch from the stationary to the non-stationary regime is realized, ( $\mu = 1.01$ ).

Even after transfers switches to a regime where it is rising exponentially, monetary and fiscal policies remain the same until the economy reaches its fiscal limit. The probability of hitting,  $p_{FL}$  rises according to the logistic function. The parameters governing the logistic function,  $\eta_0$  and  $\eta_1$  are set so that the initial probability of hitting the fiscal limit is 2 percent when  $\tau_t = \bar{\tau}$  and a 5 percent probability when  $\tau_t = \tau^{FL}$ .

As explained, at the fiscal limit taxes can not rise anymore due to political pressures or economic reasons so taxes remain constant such that  $\tau^{max} = 0.24$ . This tax rate in the stationary regime of transfer can support a steady-state debt to GDP ratio of 2.1, a debt to output level unusually high and unseen in the U.S history. Post fiscal limit, taxes relinquishes its policy role and government cannot use it as a policy option to stabilize debt, hence two potential resolutions are allowed, both of which occur with a 50 percent chance. The first possibility is a switch to passive monetary policy, where the monetary authority simply pegs the nominal interest rate so that  $\rho_A = 0$  and fiscal authority continues to deliver fully promised transfers. The second resolution possibility requires, fiscal authority to renege on promised transfer payments since monetary policy here remains active.

In both scenarios, if government decides to pursue an expansionary discretionary spending policy in the stationary regime then at the fiscal limit it promises to put a cap on expenditure by keeping  $G_t$  constant and hence  $\phi^B = \phi^Y = \phi^F = 0$ , as a supplement to any of resolution that is realized at the fiscal limit. On the other, if government takes a contractionary stand on

government expenditure in the stationary transfer regime in addition to increasing taxes then at the fiscal limit it simply put a cap of expenditures by keeping it constant.

Clearly at the fiscal limit, a standalone passive monetary policy can not completely stabilize debt as transfers continue on their explosive path. In order to stabilize debt, a recurring regime change between the passive monetary regime that provides full amount of promised transfers and the active monetary regime with reneging is allowed. Each regime is calibrated such that, the regime where monetary authority adjusts policy by adopting a passive monetary policy has an expected duration of 10 years and the regime where fiscal authority adjust policy by reneging on promised transfer payments has an expected duration of 100 years.<sup>16</sup>

The full nonlinear model is solve numerically by employing the monotone map method (MMM) described in Davig and Leeper (2006). See the Appendix

### 3.3.6 Simple Illustrative Example

Recall that government spending is introduced into the model so that it is useful and a substitute/complement to private consumption based on the empirical results in section 3.2. More importantly, policy stance on discretionary government spending is endogenous, and thus fiscal authority has the option of either increasing government spending in response to a recession or cut spending in response to increasing debt.

Before presenting the result from the full model, I analyze the fixed regime version of the full nonlinear model. First, a simple analytical intuition is provided using the fixed regime with lump-sum taxes and show how the degree of substitutability/complementarity between private and government consumption interact with endogenous discretionary spending to affect policy effectiveness. Second, I study impulse response functions following a government transfer shock for the fixed regime.

For the simple analytical illustration, I assume a cashless simple productive economy without capital. The model is as follows: a representative agent receives wages from work effort, lump-sum

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<sup>16</sup>These values reflect the political reality that in the long-run some modifications to entitlement benefits will occur, but because of their politically toxic nature, debt revaluation always remains a possible financing outcome, Richter (2015).

transfers, pays lump-sum taxes ( $Z_t$ ) and chooses sequences of labor hours ( $N_t$ ), consumption ( $C_t$ ) and a risk-free nominal bonds ( $B_t$ ) which pays a gross interest rate  $R_t$ , to maximize  $\sum_{t=0}^{\infty} U(C_t, N_t)$  subject to  $C_t + B_t/P_t + T_t = N_t + Z_t + R_{t-1}B_{t-1}/P_t$  so that the first order conditions are as follows:

$$(C_t + \alpha_g G_t)^{-1} = \lambda_t \quad (3.20)$$

$$\chi N_t^\eta = \lambda_t \quad (3.21)$$

where  $\lambda_t$  is the marginal utility of consumption.

The production sector of the economy is such that  $Y_t = N_t$  and the market clearing condition is given as  $C_t + G_t = Y_t$ .

To allow for interpretive ease, the log-linearized versions of the model is Eq. (3.20), Eq (3.21), the production function and the resource constraint are employed. I further make the simplifying assumption of setting the steadystate level of discretionary government spending to zero.<sup>17</sup> Combining the log-linearized version of Eq. (3.20) and Eq. (3.21) yields:

$$\eta \hat{n}_t = -(\hat{c}_t + \alpha_g \hat{g}_t) \quad (3.22)$$

meanwhile, the log-linearized version of the production function yields:

$$\hat{n}_t = \hat{y}_t = \hat{c}_t + \hat{g}_t$$

combining the log-linearized resource constraint and equation in Eq. (3.22), I obtain:

$$\eta(\hat{c}_t + \hat{g}_t) = -(\hat{c}_t + \alpha_g \hat{g}_t) \Rightarrow (1 + \eta)\hat{c}_t = -(\alpha + \eta)\hat{g}_t \quad (3.23)$$

solving out for  $\hat{c}_t$  yields:

$$\hat{c}_t = -\left(\frac{\eta + \alpha_g}{1 + \eta}\right) \hat{g}_t \quad (3.24)$$

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<sup>17</sup>This is a special case of  $\bar{G} > 0$ , see Ganelli and Tervala (2009) for similar modelling assumptions

furthermore, employing the production function  $\hat{n}_t = \hat{y}_t$  and the resource constraint  $\hat{y}_t = \hat{c}_t + \hat{g}_t$ , Eq. (3.22) can be written as,

$$\hat{n}_t = \left( \frac{1 - \alpha_g}{1 + \eta} \right) \hat{g}_t \quad (3.25)$$

The critical part of Eq. (3.24) and (3.25) is the design of endogenous discretionary government spending policy in responds to the state of the economy. In particular, whether it is designed to respond in an expansionary fashion to the economic downtown component of the dual economic problem or in a contractionary way in response to an increasing debt. To investigate the effects of each policy option, I substitute the log-linearized version for each discretionary spending policy rule in Eq. (3.18) and Eq. (3.19) into Eqs. (3.24) and (3.25) to yield,

$$\hat{c}_t = \begin{cases} \hat{c}_t^Y = -\phi^Y \left( \frac{\eta + \alpha_g}{1 + \eta} \right) \hat{y}_t^R & \text{if } G_t \text{ is expansionary} \\ \hat{c}_t^B = -\phi^B \left( \frac{\eta + \alpha_g}{1 + \eta} \right) \hat{b}_t^S & \text{if } G_t \text{ is contractionary} \end{cases} \quad (3.26)$$

and

$$\hat{n}_t = \begin{cases} \hat{n}_t^Y = -\phi^Y \left( \frac{1 - \alpha_g}{1 + \eta} \right) \hat{y}_t^R & \text{if } G_t \text{ is expansionary} \\ \hat{n}_t^B = -\phi^B \left( \frac{1 - \alpha_g}{1 + \eta} \right) \hat{b}_t^S & \text{if } G_t \text{ is contractionary} \end{cases} \quad (3.27)$$

where  $\phi^Y < 0$  while  $\phi^B < 0$ . The variable  $\hat{y}_t^R < 0$  represents an economic downturn while the variable  $\hat{b}_t^S >$  represents increasing debt.

The transmission mechanism at work when public and private consumption are substitutes (complement) implies that an increase in government spending lowers (raises) the marginal utility of household consumption. These results depend on the value of  $\eta$  (see Ganelli and Tervala (2009)). Setting  $\eta$  to such  $\eta < |\alpha_g|$  yields the following:

- When public and private consumption are substitutes an increase (decrease) in government spending aggravates (mitigates) the marginal disutility of increasig labor supply. It is therefore straightforward to see that, an expansionary (contractionary) policy on discretionary



spending will cause contractionary (expansionary) macroeconomic effects— causing falls in consumption, labor hours, and output.

- When public and private consumption are complements an increase (decrease) in government spending mitigates (worsens) the marginal disutility of increasing labor supply. Hence, an expansionary (contractionary) policy on discretionary spending will lead to (contractionary) expansionary macroeconomic effects— causing increase in consumption, labor hours, and output.

The budgetary implication of each policy stance of discretionary spending is unambiguous. Irrespective of the path of debt, at each time  $t$  an expansionary policy choice on government expenditure will put debt at a higher point relative to a policy choice that aims to cut government spending (contractionary) or keep spending constant. It is however, important to note that these spending policies can only either mitigate or exacerbate budgetary risks but can not fully eradicate these risks.

### **3.3.7 Numerical results: No fiscal Limit**

This section provides numerical results for the fixed regime version of the model described in section 3.3 without fiscal limit. Specifically, conditioning on the active monetary policy and passive fiscal policy with a fixed policy choice on discretionary spending, I conduct two exercises. First, I compare the implications of each policy choice on discretionary spending policy by studying impulse response functions of the macro-variables in the model following a government transfer shock. Second, for each of the policy stance on discretionary government spending, the role of the degree of the intra-temporal substitution between government and private consumption is examined to evaluate the efficacy of each policy choice.

### **3.3.8 Transmission Mechanism**

Figures 3.4, 3.5, and 3.6 compare the implications of discretionary spending policy choices in the economy following a 10% government transfer shock for the case where government spending is nonuseful ( $\alpha_g = 0$ ), useful but a substitute to private consumption with  $\alpha_g = 1.83$  and useful but

complementary to consumption with  $\alpha_g = -1.5$  consumption respectively. Additionally, figures 3.7 to 3.10 considers one discretionary policy at a time and show how the degree of substitutability/complementarity between consumption and government spending affects each policy.

To better understand the role of discretionary spending and its transmission channels, I compare all the results to the scenario where discretionary spending policy has a limited role. That is, discretionary government spending is assumed to be constant so that it can neither be used as an additional tool to finance debt through contractionary measures nor can it be employed in an expansionary fashion to stimulate the economy in response to an economic downturn ( $\phi^Y = 0$  and  $\phi^B = 0$ ). In addition, government spending in this case is assumed to be non-useful/wasteful so that  $\alpha_g = 0$ .

For the case where government spending is constant and does not respond endogenously to the state of the economy, a positive shock on transfer payments financed by higher distortionary taxes on the incomes of factors of production, labor and capital. This generates a disincentive for agents to work and invest directly leading to gradual fall in labor hours and investment which in turn causes output and consumption to fall steadily. The steady fall in output over time will raise marginal cost and hence inflation. This scenario is given by the black solid line in all the figures.

### **Discretionary spending policy under Non-useful government spending**

Figure 3.4 compares the efficacy of each endogenous discretionary spending policy given that government spending is non-useful. The blue dash line represents contractionary discretionary spending – fiscal authority cuts spending in response to increasing debt. The red line shows the effect of transfer shock when policy choice on government spending is expansionary.

Assuming fiscal authority does not only finance the higher transfer payment by nominal debt and distortionary taxes but also cuts discretionary spending in response to rising debt so that discretionary spending serves as a lump-sum tax, two effects will be generated. First, because government spending contributes an immediate dollar to aggregate demand, reduction in spending will lead to a fall in output in the short and medium run relative to when government spending is constant. Second, in the longer run, a combination of crowding in of private consumption, increase

in labor hours worked and less pressure on distortionary taxes due to the cuts in government spending in response to higher transfer payment causes a stronger rebound of consumption, labor hours worked, capital and consequently output compared to the case where spending is constant.

On the other hand, if fiscal authority instead decides to address the economic downturn component of the consequence of the transfer shock and conducts expansionary discretionary spending policy by responding to output loss then the opposite effect will occur. Specifically, if in response to the economic downturn caused by the shock to transfer payment described above government increases discretionary government spending, then this will lead to an even higher distortionary taxation labor and capital as well as severe crowding out of private consumption. Both effects will further exacerbate the economic downturn in the long-run.

Thus when government is non-useful/wasteful, contractionary discretionary government spending policy mitigates the contractionary effects of the transfer shock. Expansionary discretionary government policy on the other hand worsens the economic effects of the transfer shock.

### **Discretionary Spending Policy under Useful Government Spending**

I now consider the efficacy of discretionary spending policy when government spending is useful and a substitute/complement to private consumption. In this case, changes in discretionary spending will not only have the standard wealth effect on households since other transmission channels now come into play. As explained earlier in the analytical example in section 3.6, given the standard calibration of the household preference parameters, changes in government spending is capable of affect the marginal utility of household labor effort and consumption.

*Discretionary Government policy when  $G_t$  and  $C_t$  are Substitutes:* Figure 3.5 compares the impulse responses for policy options on discretionary government spending following a transfer shock when government and private consumption are substitutes. When the transfer shock hits, a discretionary government spending policy that responds to output will exacerbates the contractionary effects of the transfer shock. This is because when  $\alpha_g = 1.83$  an increase in government spending lowers the marginal utility of private consumption leading to further falls in consumption. Moreover, the increase in government spending causes additional pressure taxes which further

affect labor hours worked and investment. The negative effect on consumption, labor, and capital leads to a fall in output.

In contrast, if fiscal authority cuts government spending as an additional financing source to the increasing debt, the marginal utility of private consumption increases and thus there is crowding in consumption. Additionally, the cuts in government spending reduces the upward pressure on distortionary taxes as it serves as an additional financing source. This means lower distortionary taxes on productive capital and labor hence less fall in capital and labor hours worked. Consequently, the crowding in of consumption alongside the positive effects on labor hours and capital mitigates the negative impacts of the transfer shock compared to the case of expansionary or constant government spending.

*Discretionary Government Policy when  $G_t$  and  $C_t$  are Complementarity:* Figure 3.6 compares marginal effects of the three discretionary policy options when government spending is complementary to private consumption. When government spending policy is expansionary, standard wealth effects makes the household agent poorer due to higher taxes. However, because government spending is a complement to private consumption, the marginal utility gains from increasing consumption following an increase government spending can mitigate some of the wealth effects. There is therefore gains in consumption, labor, capital, and hence output relative to the case where government spending is constant following the transfer shock. A contractionary policy stance on government spending generates the opposite effects.

*Role of the degree of substitutability:* Figure 3.7 and 3.8 show the role of the degree of substitutability between government spending and consumption given that discretionary government spending contractionary and expansionary respectively. From figure 3.7, it is clear that contractionary spending policies improve when substitutability between consumption and government spending increases. On the other hand, as shown in figure 8 the lower degrees of substitutability between government spending and consumption improves the efficacy of expansionary government spending policy.

*Role of the degree of complementarity:* Figure 6 and 7 show the role of the parameter that governs the degree of complementarity of discretionary government spending in the utility function.

The qualitative analysis above for the fixed regime emphasizes two important points. First, different categories of spending government chooses to cut or increase in response to a debt-induced recession can have mitigating or worsening effect on the recession. Second, for a particular policy stance government takes on discretionary spending the degree of complementarity of spending in the household utility can either a mitigating or worsening effect on the economic downturn.

### 3.4 Numerical result: full non-linear model

I now turn to the full nonlinear model described in section 3. In this section, I explore the role of discretionary spending policy as additional policy and its impact on the consequences of explosive transfers by employing counterfactual exercises that condition on a particular monetary, tax, transfer and a choice of discretionary spending policy regime as well as Monte Carlo simulations. Moreover, the study is conducted while accounting for the explicit role of the transmission mechanism (utility enhancing channel,  $\alpha_g \neq 0$ ) of each discretionary spending policy.

#### 3.4.1 Equilibrium transition paths

I start the analysis from normal times, in this context defined such that monetary authority targets inflation actively while fiscal authority passively adjusts tax rate to stabilize debt and deliver in full stationary transfer payments. The counterfactual analysis thus conditions on normal times. However, as described in figure 3.3, in period 5 although monetary policy and tax policy remain active and passive respectively, transfers switch to its non-stationary path ( $S_z = 2$ ) as in Eq. (3.14). To clearly understand the transmission mechanism and marginal effects of discretionary spending policy, all simulations results are compared to the results from the baseline model where discretionary spending is non-useful ( $\alpha_g = 0$ ) and constant (*i.e.*  $\phi^Y = \phi^B = 0$ )—this is the case describe in DLW.

Figure 3.11 shows the economic implications of exploding transfer conditional on the initial policy mix for the baseline case. Note that real debt and hence tax rates rise due to steady increase in transfer payment when it switches to its nonstationary path. The rising distortionary taxes levied against the incomes of factors of production discourages labor hours worked and the level

of investment. This causes reductions in labor and capital. The fall in labor and capital leads to a decline in both consumption and output. Declines in consumption usually leads to reductions in inflation, however, the possibility of moving to a regime where debt is revalued leads to a steady increase in expected and hence realized inflation.

Essentially, in the baseline case with constant government spending, rising transfer payments generates a dual economic problem of systematic economic downturn and fiscal stress.

### Discretionary spending as an additional policy

Here discretionary spending is now introduced as an additional endogenous policy tool. Specifically, depending on the policy objective, government may choose to increase discretionary spending in response to a recession irrespective of debt pressures described in Eq. (3.26) or in addition to taxation, cut current spending in response to increasing debt pressures as in Eq.(3.27). These policy choices on discretionary spending are studied in the context where: (i) government spending is wasteful/non-useful; (ii) useful but a substitute to consumption; and (ii) useful and complementary to private consumption.

Figures 3.12, 3.13, and 3.14 describes the cases where government spending is wasteful, a substitute, and a complement to private consumption respectively. The blue dashed lines represents a contractionary discretionary spending policy choice while the red line represents a discretionary spending policy that is expansionary in nature. To appreciate the marginal contribution of each policy choice of discretionary spending, the results are compare to the case where government spending is constant.

**Non-useful government spending and endogenous spending policy:** Figure 3.13 characterizes consequences of the exploding transfer for the case where government spending is wasteful ( $\alpha_g \neq 0$ ) but responds endogenously either to output or debt. It is clear that regardless of the policy stance on discretionary government spending, the economic consequences of the exploding transfers are present. However, when government spending is wasteful and responds endogenously to the economic down by increasing government spending (red line), there are additional standard crowding out effects on private consumption and investment. This deepens the fall in consump-

tion and capital. Although it is expected that labor will be substituted for consumption when government spending is increased, the effect of the persistent rise in tax rates on labor offsets this substitution effect. Thus there is no change in labor hours and hence output.

In contrast, if fiscal authority decides to additionally finance the increase debt by cutting discretionary spending ( $\phi^B \neq 0$ ), this will lead to crowding in of private consumption since the competition of resources by government with household is mitigated—this is given as the blue dashed lines. However, these cuts in public spending virtually have no impact on labor or capital as the impact of high taxes dominates. Additionally, since government spending forms an essential component of output in the economy, a contractionary policy choice on it will directly impact aggregate demand negatively. Thus, even with the effect of crowding in of consumption the direct negative impact due to cuts discretionary spending causes a deeper trough for output relative to the case of constant and expansionary spending.

**Useful Government Spending:** The empirical estimates in section 3.2 shows that public spending and private consumption are strong substitutes and hence an increase in government consumption crowds out private consumption. Empirical studies however, find that private consumption rises after an increase in government spending. Edgeworth complementarity between private and government consumption has been cited as a factor contributing the positive response of private consumption. Moreover, Fève, Matheron and Sahuc (2013) show that due to cross-equation restrictions an omission endogenous government spending at the estimation leads to an underestimation of the degree of complementarity and the long-run government multiplier. The empirical estimation in section 3.2 omits the endogenous feedback channel of government policy in the estimation procedure.

To expand the analyses on the role of discretionary spending policy, this section assume that private and government spending are complements and set  $\alpha_g < 0$ . Policy choices on discretionary spending can now have additional impact on household consumption and labor supply through the utility (disutility) it generates directly for households. That is, the standard wealth effects that causes crowding in or crowding of private consumption in the case of wasteful government spending are not the only channels at work. Concretely, alongside the baseline calibration of the

household preference parameters,  $\alpha_g$  is set such that  $\eta + \alpha_g < 0$ , this ensures that an increase in government spending leads to an increase in private consumption.<sup>18</sup>

In order to achieve this, I conduct the same numerical exercises as in the case where private and public spending are substitutes for the case where both goods are complements. For the case complementarity,  $\alpha_g$  is set to a value of -1.5.<sup>19</sup> For the ease of discussion, I present results from the nonlinear impulse response when private and government consumption are complements. Specifically, I compare the three policy options on discretionary spending policy while assuming complementarity. Second, for each policy I examine the role of substitutability/complementarity.

*(i) Substitutability and endogenous government spending policy:* Figure 3.13 shows the role and implications of fiscal policy choices on discretionary spending when government spending is useful and a substitute to private consumption,  $\alpha_g = 1.83$ . Here, since government spending is a substitute to private consumption, an expansionary government spending policy reduces the marginal utility of consumption and increases disutility of labor. Thus in addition to the consequences of the transfer payments, expansionary discretionary spending policy causes both consumption and labor to fall compared to the case of a constant government spending. Moreover, the increase in government spending further crowds out capital leading to a deeper fall in capital. The fall in capital, consumption, and output means a deeper reduction in output relative to the case where government spending is constant.

A contractionary government spending on the other hand has the opposite effect. Although this policy mitigates the fall in consumption, the wealth effects caused by rising taxes are very strong and offsets the positive impacts of the policy. Thus there is seemingly no effect on labor and capital. The increase in consumption is not enough to mitigate any falls in output.

If government instead chooses to supplement taxation in financing the increase debt by cutting discretionary government spending, then this will lead to crowding in of private consumption since the competition of resources by government with household is mitigated. However, these reductions in government spending virtually have no impact on labor or capital as the impact of high taxes

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<sup>18</sup>It is easy to inspect this from from Eq. (3.24) and 3.(25). That is from these equations the restriction  $\eta + \alpha_g < 0$  means increasing government spending generates a simultaneous increase in private consumption and labor hours.

<sup>19</sup>This value is in the neighborhood of estimates from Fève et al. (2013)



dominates. Moreover, government spending forms an essential part of final consumption in the economy hence cuts in it will directly impact aggregate demand negatively. Thus, even with the effect of crowding in of consumption the direct negative impact cuts in government spending have on aggregate demand worsens the fall in output compared to the case where spending is kept constant. This fall is however marginal as compared to the case where government is increased in spending in response to the recession.

*(ii) Complementarity and endogenous government spending:* Figure 14 characterizes the role and implications of fiscal policy choices on discretionary spending when public spending is useful and complementary to private consumption. Specifically, despite the extra crowding out effect on capital, persistent tax pressures, the endogenous increase in useful government spending increases labor hours worked and consumption—albeit small through the marginal utility channel. Hence, although the devastating effects of the transfer shock are present, an expansionary government spending policy mitigates the severity of the economic downturn relative the case where government spending is simply kept constant.

In contrast, if fiscal authority instead chooses to supplement taxation in financing the increase debt by cutting government spending, then this will lead to crowding in of private consumption since the competition of resources by government with household is mitigated. Moreover, since government spending is complementary, contractionary government spending policy increases the disutility of work thus there is a marginal fall in labor hours worked. Additionally, cuts in government spending directly impacts aggregate demand negatively through the resource constraint. Thus, a contractionary policy choice on discretionary government spending leads to a more severe economic downturn if government spending is complementary.

### **3.4.2 Quantitative Analysis**

The numerical results in Section 4.1 illustrates the expectational effects of the possibility of reaching the fiscal limit. The result are however, based on a specific sequences of policy regimes and thus excludes alternative policy scenarios. In order to demonstrate the spectrum of outcomes I follow DLW and Richter (2015) and conduct 50000 Monte Carlo simulations of the model by drawing

sequences of regimes and transfers shocks. The simulations starts from the initial policy regime of active monetary policy, passive fiscal policy, stationary transfer and a choice on discretionary spending policy.

Figure 15 shows 10th and 90th percentile bands of time paths for selected variables for the baseline case where discretionary spending has a limited row. It clear that over the short run (i.e. in the first 10 years) when the probability of hitting the fiscal limit is low, irrespective of the policy option on discretionary spending, there are moderate deviations from the stationary distribution— with low debt and inflation rates. This is because although there is a chance of hitting the fiscal limit, the initial probability of reaching the fiscal limit is low and economic agents therefore expect the drastic policy adjustments to take place far into the future and consequently heavily discount these outcomes.

However, once past the short run effects and the probability of reaching fiscal limit rises, outcomes can range from very moderate to extreme fiscal stress coupled with high inflation and deep economic downturn. The large spectrum of outcomes shows the uncertainty surrounding the fiscal limit and the realization of nonstationary transfers. Specifically, the fiscal limit can be hit and transfers can switch to a non-stationary process at any point or not occur at all.

Figures 15, 16 and 17 plots the 10th and 90th percentile bands of time paths for selected variables with discretionary spending as an additional policy tool for the case where  $\alpha = 0$ ,  $\alpha = 1.83$ , and  $\alpha_g = -1.5$  respectively. In all the figures both the short run and long run effects are present irrespective of the policy choice on discretionary spending. Generally, expansionary policy choice on discretionary spending extends the moderate deviations from steady levels and dispersion in the distribution and dispersion in the distribution regardless of government spending relationship with private consumption. The most notable effect is shown in figure 17 for the case where government spending is a substitute to consumption. In the figure, when government spending is a substitute to private consumption, an expansionary government spending policy extends the moderate contractionary effects for roughly another 5 years.

This short run effect of expansionary policy on discretionary spending when  $\alpha_g = 1.83$  is rather puzzling. This puzzling effect can however be explained through the expectational channel

as agents expect that a cap will be placed on government spending that is substitutable to private consumption will be kept constant. Economic agents therefore discount this optimism to the present which in extends the moderate contractionary period.

However, over time when the probability of reaching the fiscal limit increases and fiscal authority continues to conduct expansionary discretionary spending policy, this will lead to a more severe contractionary effect for the case where  $\alpha_g = 0$  in figure 16 and  $\alpha_g = 1.83$  in figure 17. Specifically, as shown in figure 17 the long run effect of introducing expansionary discretionary spending to mitigate the economic downturn component of the dual economic problem can lead to devastating outcomes— causing output to fall to double digit while raising the debt to output ratio to almost 125 percent. Although, the scenarios studied in this paper is not exhaustive, even with expansionary discretionary policy, when agents condition on policy adjustments that stabilize the debt to output ratio, the debt/output ratio never reaches the levels projected by the CBO.

Contractionary policy choice on discretionary spending on the other hand have very small impact on the dispersion of the distributions relative to its expansionary counterpart, but it keep the debt to output ratio at decent rates below the 100 percent mark for all values of  $\alpha_g$ . This long-run outcome of tightening of discretionary spending policy as an additional choice to reducing the debt/output ratio is consistent with projections by the CBO.

To give a panoptic view of the short and long run effects of a particular stance on discretionary government spending, table 3 below depicts the computed averages of output loss from the 10th and 90th percentile band. The short-run average is computed over a period of 10 years while the long-run average is computed using a 25 year period after the first 10 years.<sup>20</sup>

Table 3 highlights two points. First, the short run efficacy of expansionary spending policy compared to the more conservative counterpart of cutting government spending. Second, contractionary spending policies such as automatic cuts in  $g$  as shown can in the long run mitigate the economic downturn while keeping debt below 100%. On the other hand, increases in spending even with the promise of holding it constant in the long-run can have the opposite effect.

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<sup>20</sup>These horizons are chosen to closely match CBO projections horizons.

Table 3.3: Short and Long Run Effect of Policy Options on Discretionary Government Spending

Discretionary Spending Type	SHORT/MEDIUM RUN EFFECT					
	Avg. output loss outcomes under Constant G		Avg. output loss outcomes under Contractionary G		Avg. output loss outcomes under Expansionary G	
	90th	10th	90th	10th	90th	10th
	—		—		—	
Wasteful, $\alpha_g = 0$	[-2.90	-0.04]	[-2.90	-0.04]	[-1.82	-0.04]
Useful, $\alpha_g = 1.83$	—		[-3.04	-0.03]	[-0.96	-0.05]
Useful, $\alpha_g = -1.5$	—		[-2.77	-0.05]	[-2.14	-0.03]
	LONG RUN EFFECT					
Wasteful, $\alpha_g = 0$	[-7.66	-1.65]	[-7.26	-1.67]	[-8.04	-1.70]
Useful, $\alpha_g = 1.83$	—		[-8.00	-1.49]	[-9.13	-2.37]
Useful, $\alpha_g = -1.5$	—		[-6.58	-1.85]	[-7.09	-1.36]

**Notes:** 10 year average output loss under the two discretionary government spending policy actions when varying the degree of complementarity of government spending in the consumer utility function. The computations are based on the fixed policy regimes and they are percentage points deviation from the steadystate output following a 10% government transfer shock. The average output loss when there is no discretionary spending policy action is -0.2594

### 3.5 Conclusion

In this chapter, I have extended Davig, Leeper and Walker (2010) by introducing discretionary spending as an additional policy instrument available to policymakers for addressing the dual problem of fiscal stress and recession in their model. First, I provide evidence of useful government spending in U.S. Second, incorporating useful government spending in the household utility, I study how expansionary/contractionary policy choices on discretionary government spending impact the consequences of an explosive transfer regime. Two broad results emerge:

1. although a particular policy stance— either contractionary or expansionary— on discretionary government spending can not eliminate the effects and risks of the rising transfer payments, their use as additional policies can mitigate/worsen these effects both in the short-run and the long-run;
2. The effectiveness of a particular policy stance on discretionary government spending depends critically on how households internalize government spending in their utility— useful or wasteful.

These results are central to understanding the tradeoffs of choosing a particular policy on discretionary spending over the other in an economy faced with a dual problem of fiscal stress and severe recession. Additionally, the results serves as robustness check to the results found in DLW since it expands the policy options available to government.

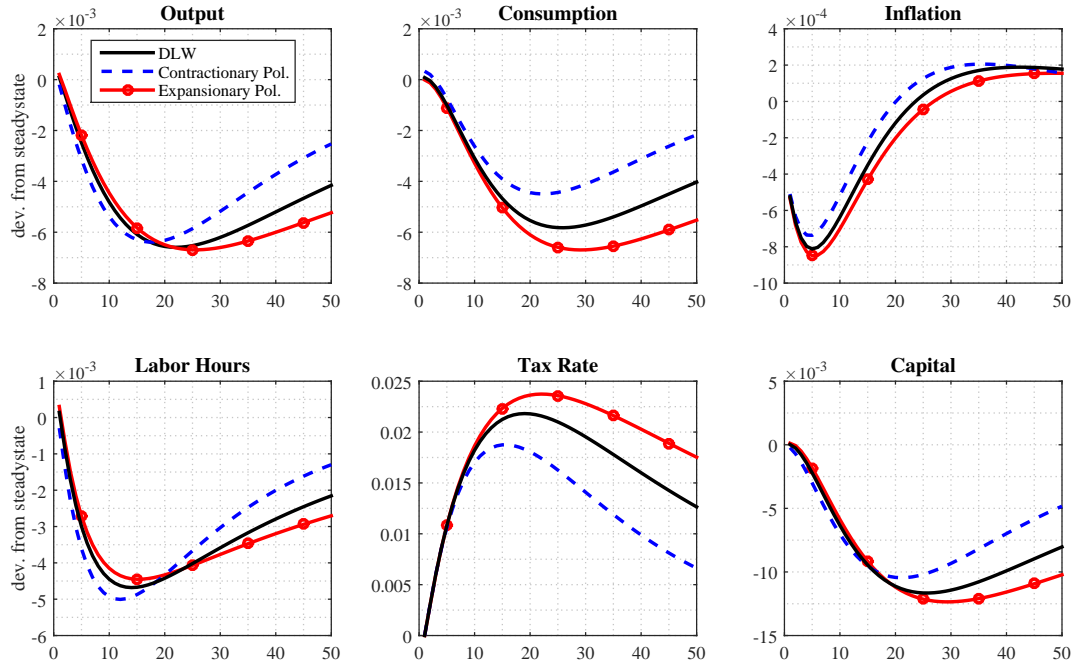


Figure 3.4: Effects of different discretionary spending policy choices on the consequences of transfer payment shock when government spending is non-useful ( $\alpha_g = 0$ )

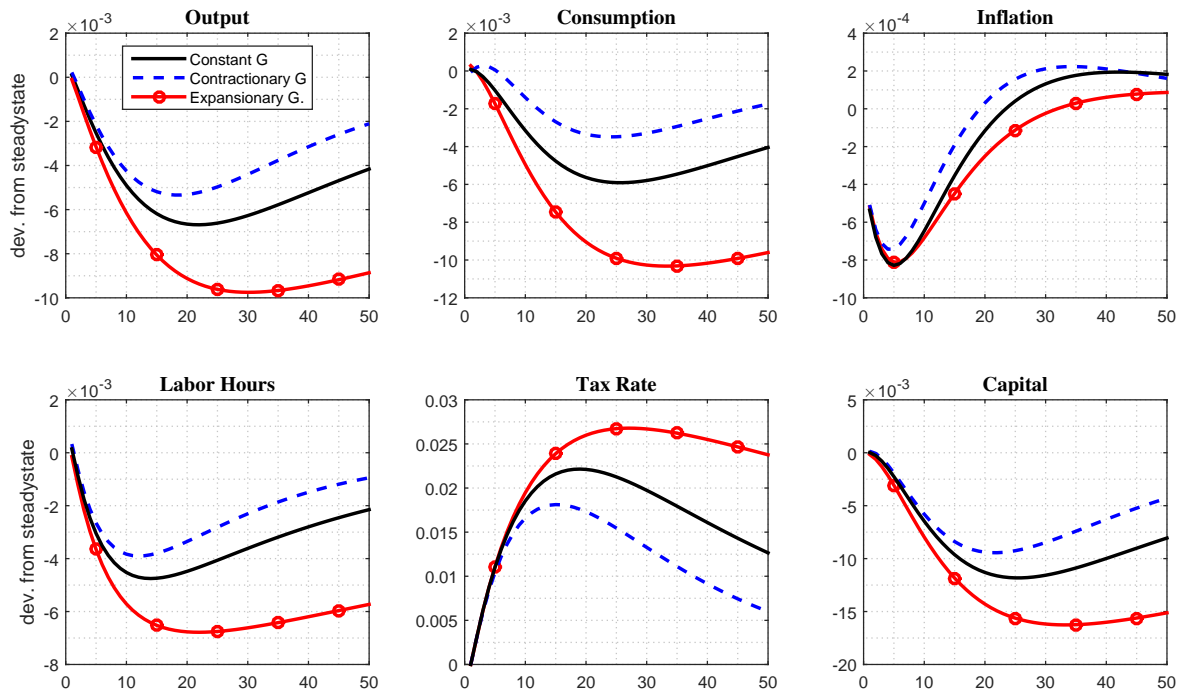


Figure 3.5: Effects of different discretionary spending policy choices on the consequences of transfer payment shock when government spending is useful but a substitute to private consumption ( $\alpha_g = 1.83$ )

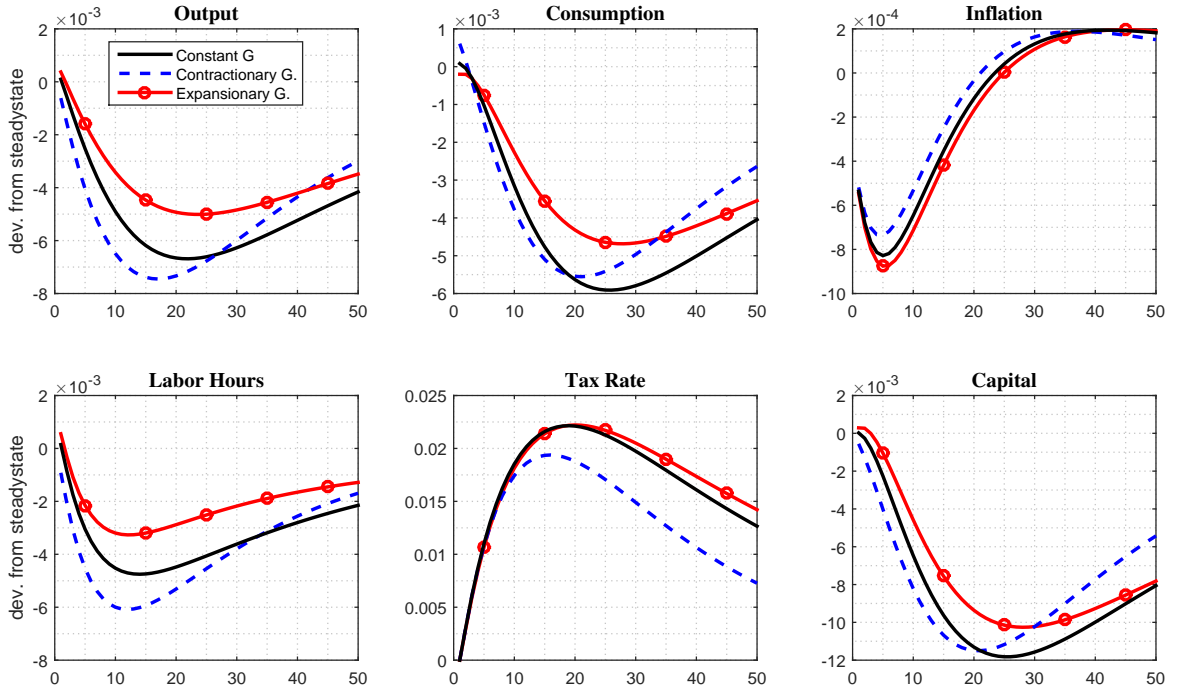


Figure 3.6: Effects of different discretionary spending policy choices on the consequences of transfer payment shock when government spending is useful but a complement to private consumption ( $\alpha_g = -1.5$ )

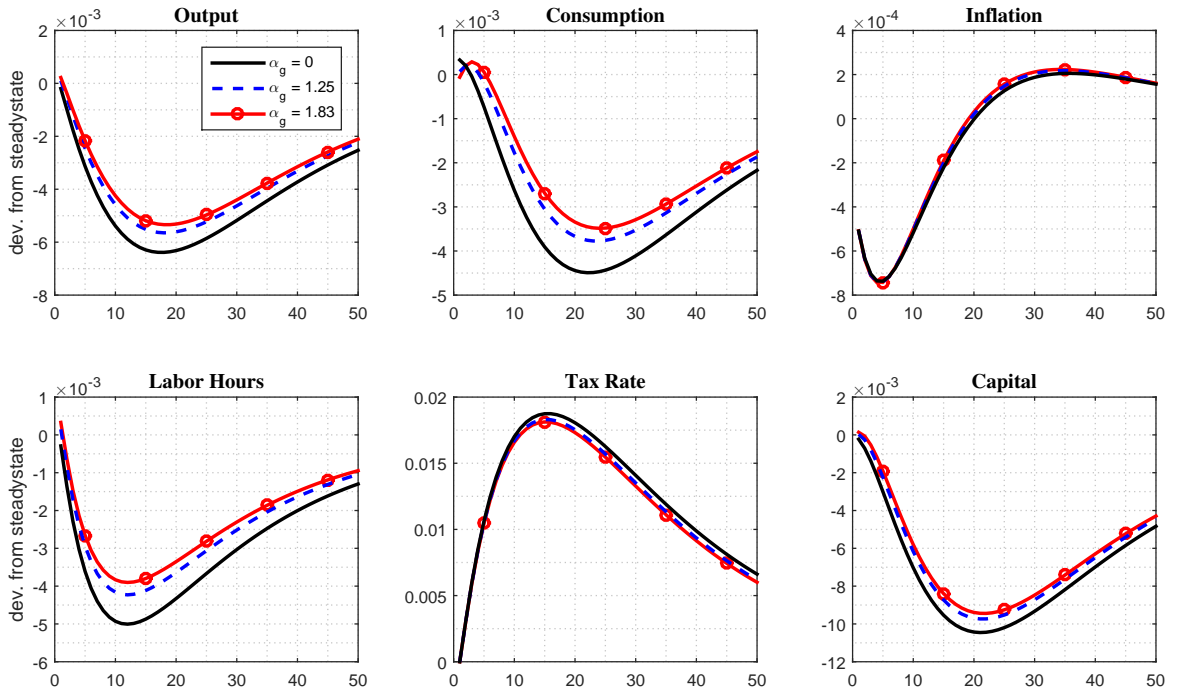


Figure 3.7: The efficacy of contractionary discretionary government policy under different degrees of substitutability between private and government consumption

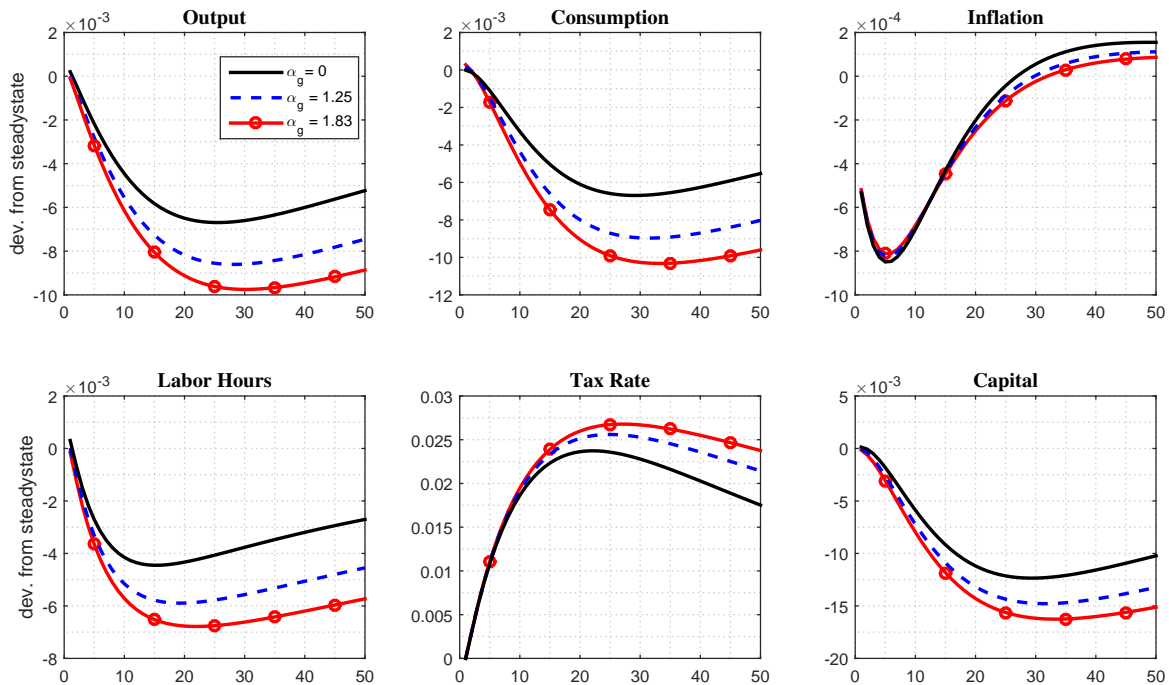


Figure 3.8: The efficacy of expansionary discretionary government policy under different degrees of substitutability between private and government consumption

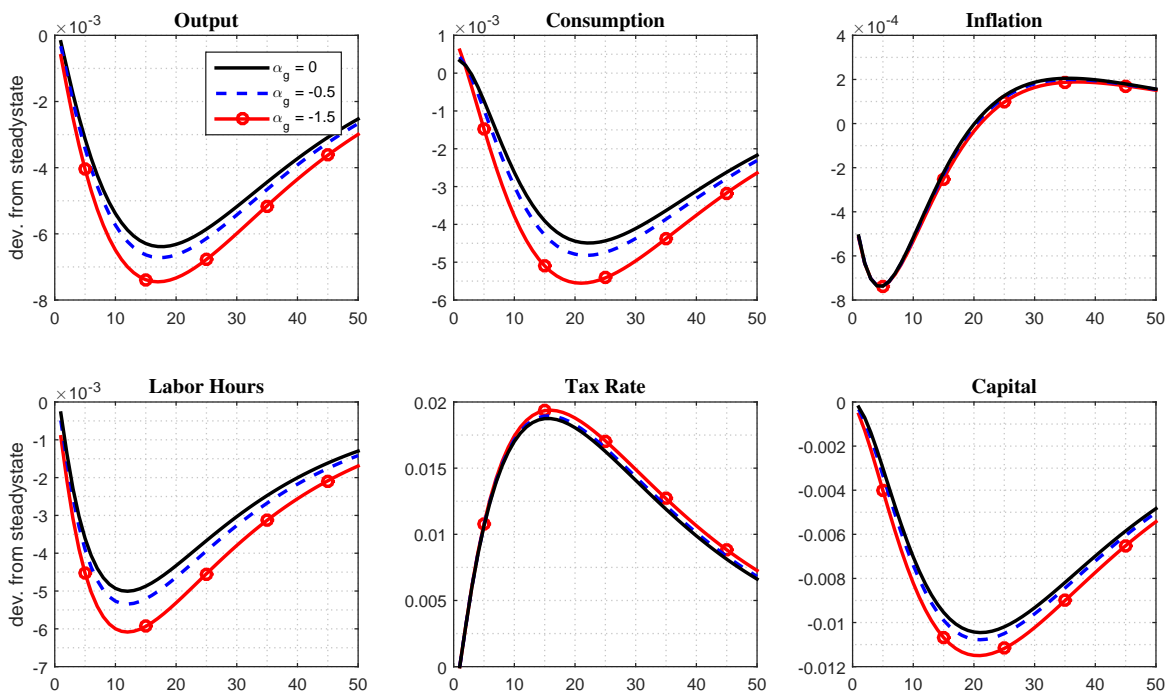


Figure 3.9: The efficacy of contractionary discretionary spending policy under different degrees of complementarity between private and government consumption



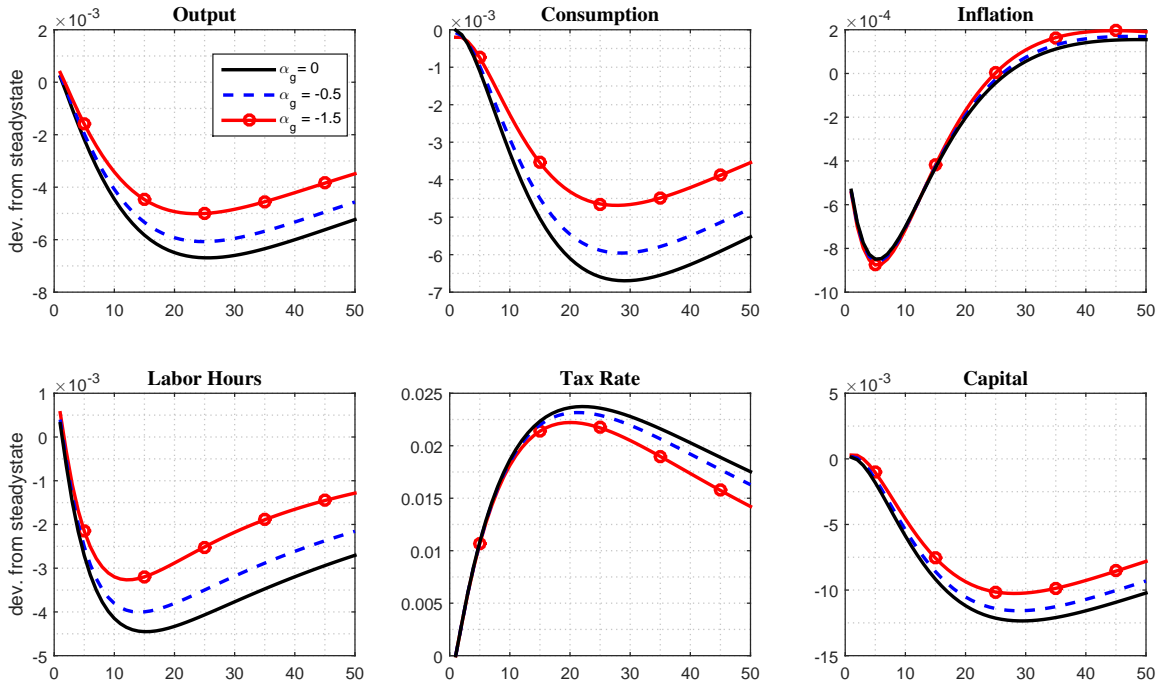


Figure 3.10: The efficacy of expansionary discretionary spending policy under different degrees of complementarity between private and government consumption

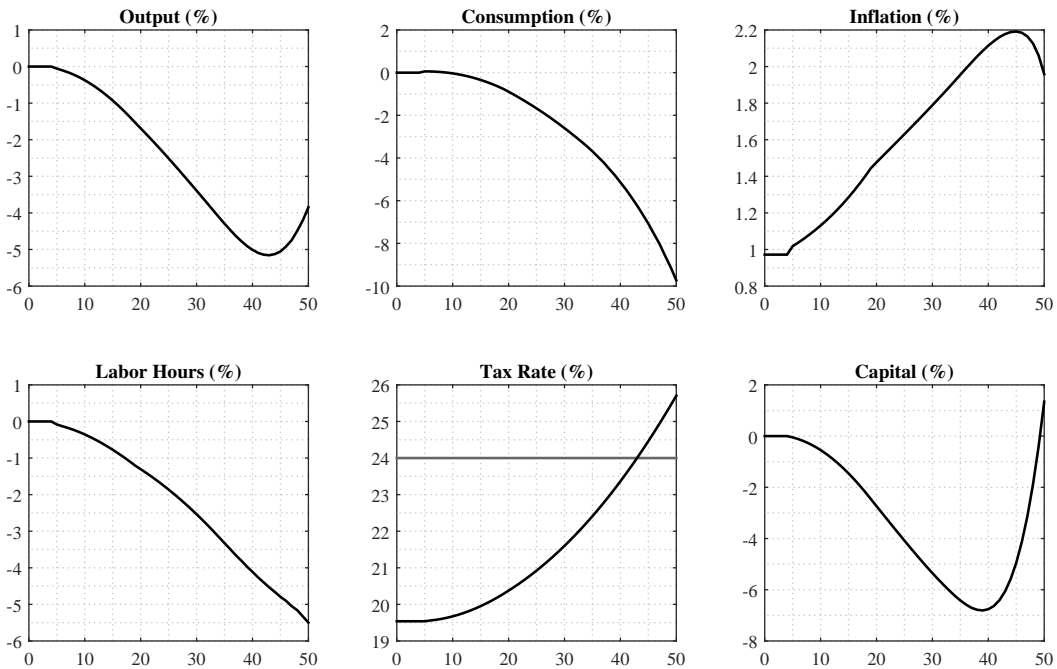


Figure 3.11: Nonlinear impulse response functions of selected variable in response to government transfer payments switching from its stationary regime ( $S_z = 1$ ) to a non-stationary regime ( $S_z = 2$ ) while assuming discretionary spending policy to be useful and a complements to private consumption ( $\alpha_g = 1.83$ ). The analysis is carried out conditioning on active monetary, passive tax, active transfers and a choice of discretionary government spending policy (contractionary or expansionary). Values represent deviations from each simulation's stochastic steady state

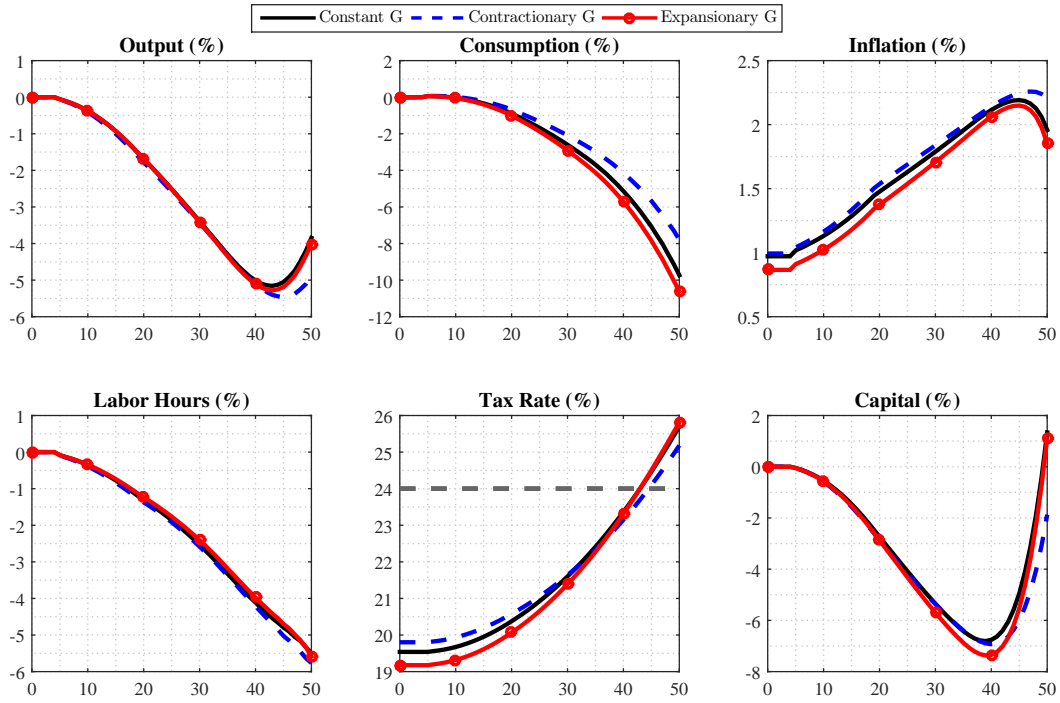


Figure 3.12: Nonlinear impulse response functions of selected variable in response to government transfer payments switching from its stationary regime ( $S_z = 1$ ) to a non-stationary regime ( $S_z = 2$ ) while discretionary spending policy is assumed to be non-useful ( $\alpha_g = 0$ ). The analysis is carried out conditioning on active monetary, passive tax, active transfers and a choice of discretionary government spending policy (contractionary or expansionary). Values represent deviations from each simulation's stochastic steady state

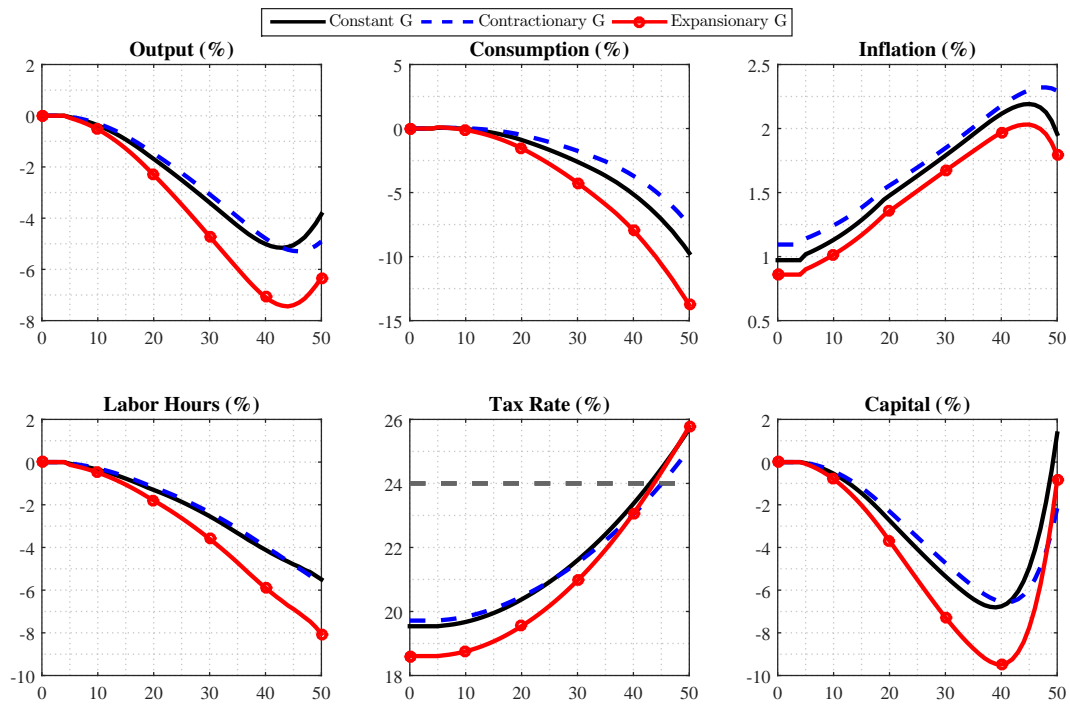


Figure 3.13: Nonlinear impulse response functions of selected variable in response to government transfer payments switching from its stationary regime ( $S_z = 1$ ) to a non-stationary regime ( $S_z = 2$ ) while assuming discretionary spending policy to be useful and a substitute to private consumption ( $\alpha_g = 1.83$ ). The analysis is carried out conditioning on active monetary, passive tax, active transfers and a choice of discretionary government spending policy (contractionary or expansionary). Values represent deviations from each simulation's stochastic steady state

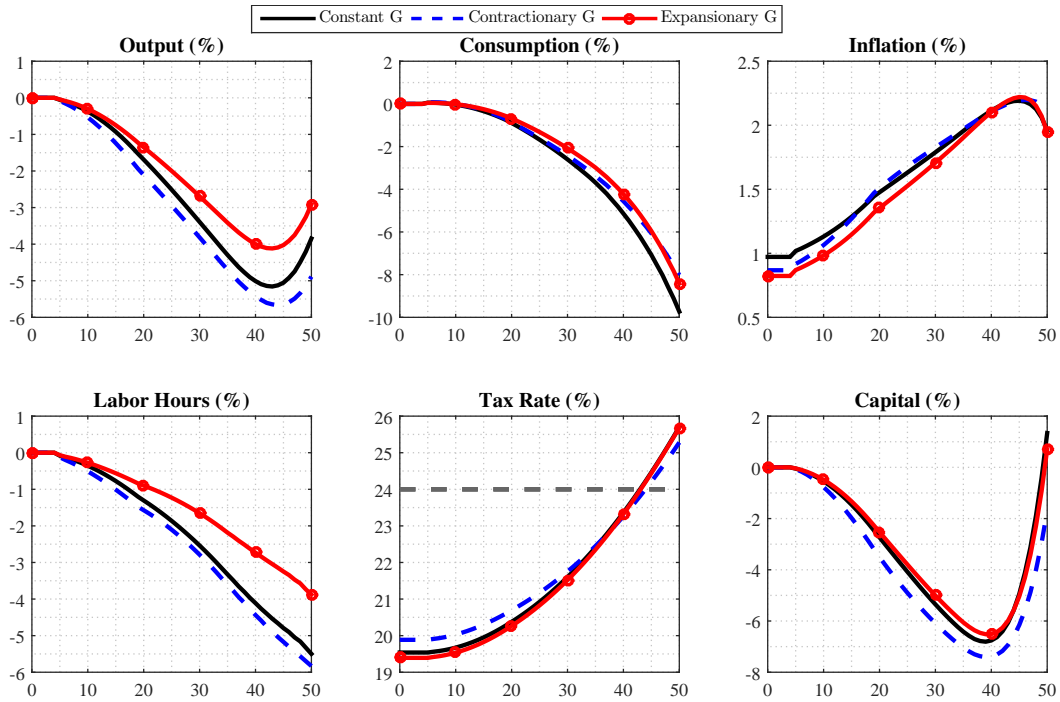


Figure 3.14: Nonlinear impulse response functions of selected variable in response to government transfer payments switching from its stationary regime ( $S_z = 1$ ) to a non-stationary regime ( $S_z = 2$ ) while assuming discretionary spending policy to be useful and a complements to private consumption ( $\alpha_g = -1.5$ ). The analysis is carried out conditioning on active monetary, passive tax, active transfers and a choice of discretionary government spending policy (contractionary or expansionary). Values represent deviations from each simulation's stochastic steady state

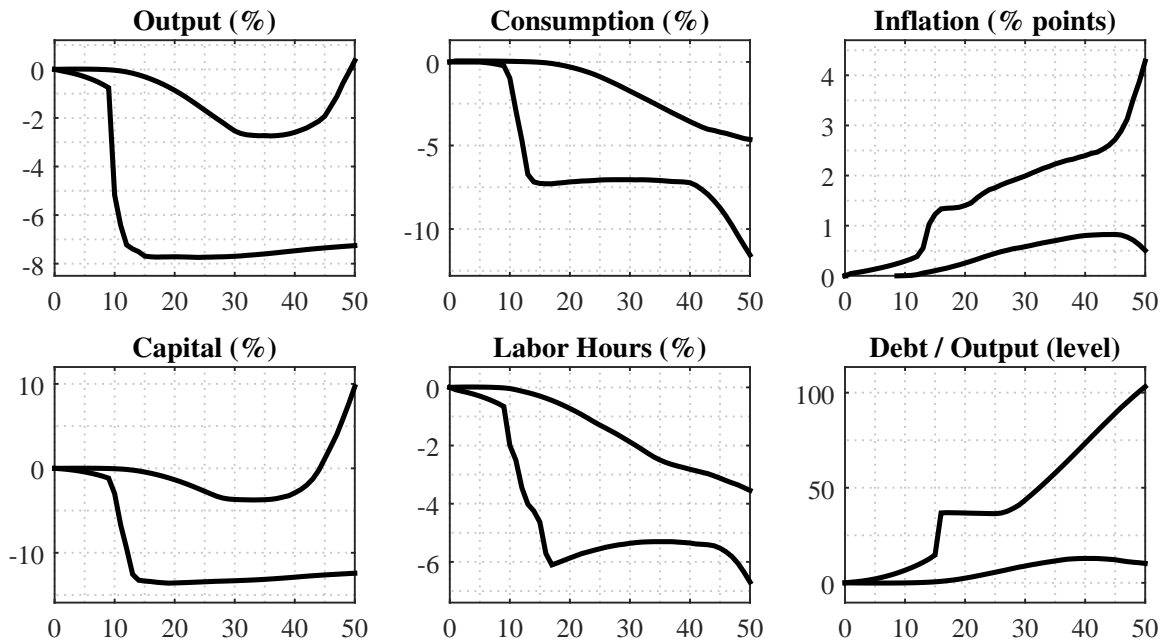


Figure 3.15: Monte Carlo simulation Baseline Model with Constant Government Spending and  $\alpha_g = 0$

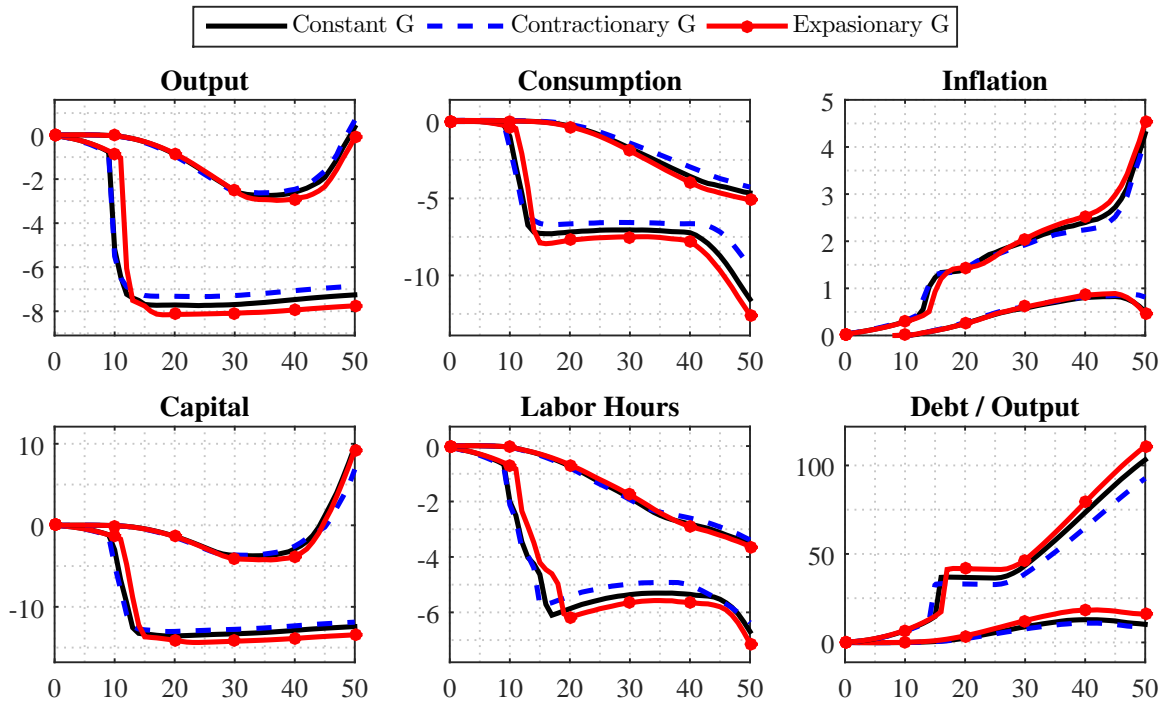


Figure 3.16: Monte Carlo simulation with wasteful Government Spending,  $\alpha_g = 0$

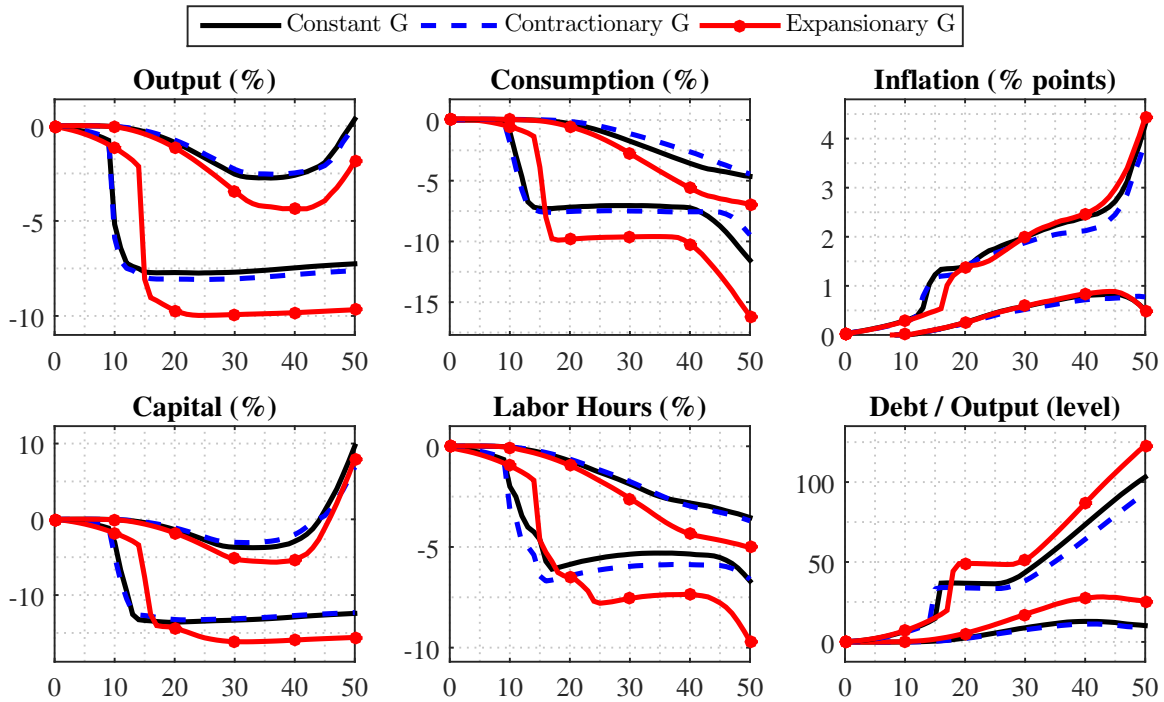


Figure 3.17: Monte Carlo simulation with substitute government spending,  $\alpha_g = 1.83$

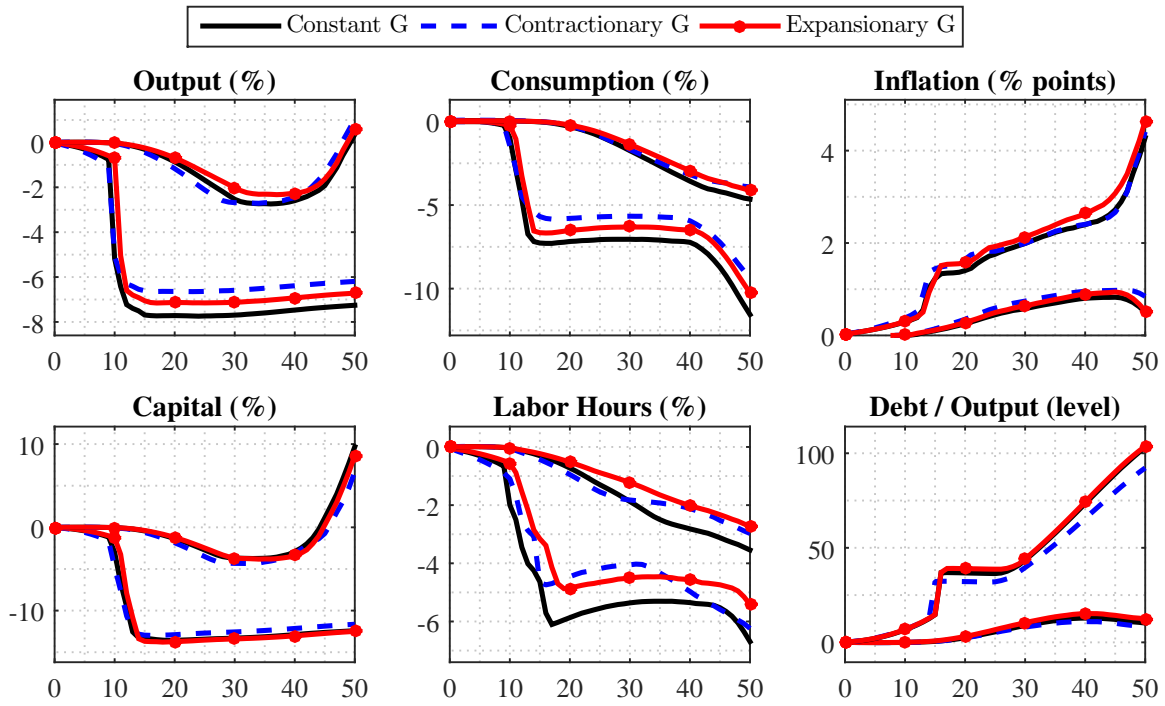


Figure 3.18: Monte Carlo simulation with complementary government spending,  $\alpha_g = -1.5$

# Bibliography

- [1] Aiyagari, S. Rao, Lawrence J. Christiano, and Martin Eichenbaum. (1992). The Output, Employment, and Interest Rate Effects of Government Consumption. *Journal of Monetary Economics*, 30(1), 73-86.
- [2] Bailey, Martin J. (1971). *National Income and the Price Level: A Study in Macroeconomic Theory*. . 2nd ed., New York:McGraw-Hill.
- [3] Barro, R. J. (1981). Output Effects of Government Purchases. *Journal of Political Economy*, 89(6), 1086-1121.
- [4] Baxter, M., and King, R.G. (1993). Fiscal Policy in General Equilibrium. *American Economic Review*, 83(3), 315-34.
- [5] Bouakez, H., and Rebei, N. (2007). Why Does Private Consumption Rise After a Government Spending Shock? *Canadian Journal of Economics*, 40(3): 954-979.
- [6] Christiano, Lawrence J., and Martin Eichenbaum. (1992). Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations. *American Economic Review*, 82(3): 430-50.
- [7] Coleman II., W.J, 1991, Equilibrium in a Production Economy with an Income Tax, *Econometrica* 59, 1091-1104.
- [8] Congressional Budget Office, 2013. Choices for Deficit Reduction: An update, CBO, Washington, D.C.
- [9] Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2000). Sticky price models of the business cycle: can the contract multiplier solve the persistence problem?. *Econometrica*, 68(5), 1151-1179.
- [10] Davig, T., and Leeper, E. M. (2006). “Fluctuating Macro Policies and the Fiscal Theory.

In NBER *Macroeconomics Annual 2006*, ed. by D. Acemoglu, K. Rogoff and Woodford, pp. 247-316. MIT Press, Cambridge.

- [11] Cúrdia, V., and Reis, R. (2010). Correlated Disturbances and U.S. Business Cycles. National Bureau of Economic Research Working Papers 15774.
- [12] Davig, T., Leeper, E.M and Walker, T.B (2010). "Unfunded Liabilities and Uncertain Fiscal Financing." *Journal of Monetary Economics*, 57, 600-619.
- [13] Davig, T., Leeper, E.M and Walker, T.B (2011). "Inflation and Fiscal Limit." *European Economic Review Special Issue Monetary and Fiscal Policy Interaction In Times of Economic Stress* 55, 31-47
- [14] Dixit, A.K. and Stiglitz, J.E. 1977, Monopolistic Competition and Optimum Product Diversity. *American Economic Review*. 67, 297-308.
- [15] Fève, P., Matheron, J., and Sahuc, J. G. (2013). A pitfall with estimated DSGE-based government spending multipliers. *American Economic Journal: Macroeconomics*, 5(4), 141-178.
- [16] Finn, M. G. (1998). Cyclical Effects of Governments Employment and Goods Purchases. *International Economic Review*, 39(3): 635-657.
- [17] Ganelli, G., and Tervala, J. (2009). Can government spending increase private consumption? The role of complementarity. *Economics Letters*, 103(1), 5-7.
- [18] Jones, J. B. (2002). Has fiscal policy helped stabilize the postwar US economy?. *Journal of Monetary Economics*, 49(4), 709-746.
- [19] Karras, G. (1994). Government spending and private consumption: some international evidence. *Journal of Money, credit and Banking*, 26(1), 9-22.
- [20] Kwan, Y. K. (2007). The Direct Substitution between Government and Private Consumption in East Asia. In *Fiscal Policy and Management in East Asia, NBER-EASE, Volume 16* (pp. 45-58). University of Chicago Press.
- [21] Leeper, E.M. (1991), "Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*. 54, 247-266.
- [22] Leeper, E. M. (2015). Fiscal analysis is darned hard (No. w21822). National Bureau of Economic Research.



- [23] Leeper, E. M., Plante, M., and Traum, N. (2010). Dynamics of fiscal financing in the United States. *Journal of Econometrics*, 156(2), 304-321.
- [24] McGrattan, E. R. (1994). The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics*, 33(3), 573-601.
- [25] McGrattan, E. R., Richard Rogerson, and Randall Wright. (1997). An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy. *International Economic Review*, 38(2): 267-90.
- [26] Ogaki, M. (1992). Engle's law and cointegration. *Journal of Political Economy*, 100 (5): 1027-46.
- [27] Ogaki, M. and Joon, Y.P. A cointegration approach to estimating preference parameters. *Journal of Econometrics* 82(1), 107-34.
- [28] Ogaki, M. and Reinhart, C.M (1998). Measuring intertemporal substitution: The role of durable goods. *Journal of Political Economy*, 106(5), 1078-98.
- [29] Okada, K., and Samreth, S. (2012). The effect of foreign aid on corruption: A quantile regression approach. *Economics Letters*, 115(2), 240-243.
- [30] Okubo, M. (2003). Intratemporal substitution between private and government consumption: The case of Japan. *Economic Letters*, 79, 75-81.
- [31] Park, Joon Y. (1992). Canonical cointegrating regression. *Econometrica* 60:1194-3.
- [32] Phillips, Peter C. B., and B. Hansen. (1990). Statistical inference in instrumental variables regression with I(1) processes. *Review of Economic Studies*, 57:991-25.
- [33] Richter, A. W. (2015). Finite Lifetimes, Long-Term Debt and Fiscal limit. *Journal of Economic Dynamics and Control*, 51, 180-203
- [34] Richter, A. W. Throckmorton, N. A and Walker, T.B (2014). "Accuracy, Speed and Robustness of Policy Function Iteration," *Computational Economics*, 44(4) 445-476
- [35] Rotemberg, J.J (1992), "Sticky Prices in the United State", *Journal of Political Economy*, 90, 1187-1211
- [36] Social Security Administration, (2013). The 2013 Annual Report of the Board of Trustees of the Federal OASDI Trust Funds

- [37] Stock, J., and Watson, M. (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica*, 61 (4): 783820.

# Appendix D

## Appendices

### D.1 Steady State Values

Equations characterizing the steadystate equation of the system. Variables without time subscripts represent the steadystate values.

FOC Capital:

$$r_k = \frac{(1/\beta + \delta - 1)}{(1 - \tau)} \quad (\text{D.1})$$

FOC Bond:

$$r = \pi/\beta \quad (\text{D.2})$$

Firm Pricing:

$$\psi = \frac{(\theta - 1)}{\theta} \quad (\text{D.3})$$

Marginal Cost:

$$w = (\psi(1 - \alpha)^{(1-\alpha)}\alpha^\alpha/r_k^\alpha)^{(1/(1-\alpha))} \quad (\text{D.4})$$

Firm FOC:

$$k = \frac{wn\alpha}{(1 - \alpha)r_k} \quad (\text{D.5})$$

Investment:

$$i = \delta k \quad (\text{D.6})$$

Production Function:

$$y = k^\alpha n^{1-\alpha} \quad (\text{D.7})$$

Aggregate Resource Constraint:

$$c = y - i - g \quad (\text{D.8})$$

FOC Labor:

$$\chi = \frac{w(1-\tau)}{((1-n)^\eta (c + \alpha_g g)^\sigma)} \quad (\text{D.9})$$

Velocity of Money Definition:

$$m = c/vel \quad (\text{D.10})$$

FOC Money:

$$\nu = m^\kappa (c + \alpha_g g)^{-\sigma} (r - 1)/r \quad (\text{D.11})$$

Government Budget Constraints:

$$b = \frac{g + z - \tau(wn + r_k k) - m(1 - 1/\pi)}{(1 - 1/\beta)} \quad (\text{D.12})$$

Consolidated Government Liabilities:

$$a = m + rb \quad (\text{D.13})$$

## D.2 Nonlinear Solution Algorithm

The model is solved using the monotone map method from Coleman (1991) which is explained in Davig and Leeper (2006). Richter, Throckmorton and Walker (2013) provides a very flexible routine (Matlab, Mex and Fortran codes) that explains and implements the monotone map method.

- 1 Discretize the state space around the non-stochastic steady state for each state variable (i.e.,  $\Delta_t = b_{t-1}, R_{t-1}K_{t-1}, z_t, S_{z,t}, S_{p,t}$ )
- 2 Conjecture an initial set of decision for capital, labor and inflation ( $\hat{h}_j^k(\Delta_t) = K_t, \hat{h}_j^N(\Delta_t) = N_t, \hat{h}_j^\pi(\Delta_t) = \pi_t$ ) for  $j = 0$ . Decision rules for other endogenous variables can be obtained using the resource constraint.
- 3 At each point in the state space, substitute these decisions rules into the household's FOC.  $t + 1$  endogenous variables depend on  $\Delta_{t+1}$ . Numerical integration is used to integrate over exogenous variables  $g_{t+1}, S_{z,t+1}$  and,  $S_{t+1}$ . This procedure yields updated values for the D.R (e.g,  $\hat{h}_{j+1}^\pi(\Delta_t) = \pi_t$ )
- 4 Repeat step (3) until the decision rules converge at every point in the state space (e.g.,  $|\hat{h}_j^\pi(\Delta_t) - \hat{h}_{j+1}^\pi(\Delta_t)| < \epsilon$ )