THE RESISTANCE OF COPPER WIRES
AT VERY HIGH FREQUENCIES.

By

Wesley M. Roberds,
A.B. University of Kansas, 1925.

A Thesis submitted to the Department of Physics
and the Faculty of the Graduate School of
the University of Kansas in partial
fulfillment of the requirements
for the degree of
Master of Arts

Approved by:
Instructor-in-Charge

August 2, 1926

Head of Department

B I B L I O G R A P H Y

- Lord Rayleigh - Phil. Mag., Vol. 21, p. 381 (1886)
- A. Russell - Phil. Mag., Vol. 17, p. 524 (1909)
- Austin Bailey - Phys. Rev., Vol. 20, p. 154 (1922)
- Jas. D. Stranathan - Phys. Rev. Vol. 26, p. 500 (1925)
- and Master's Thesis
- Wüchel - Ann. der Physic, Vol. 73, p. 422 (1925)
- Bureau of Standards Circular No. 74.
- A. E. Maibauer and T. Smith Taylor of Bakelite Corp. in paper
presented at Kansas City meeting of American
Physical Society, Dec. 1925. (Abstract of
above paper in Vol. 1, No., 1925, of Bulletin
of American Physical Society)
- R. R. Ramsey - Paper presented at Washington meeting of
American Physical Society, April, 1926
(Abstract of above paper in Vol. 1, No. 9,
1926, of the Bulletin of the American
Physical Society.)

TABLE OF CONTENTS

page

I. INTRODUCTION - - - - -	3
Purpose of this Research	
Means of Calculating High Frequency Resistance	
Previous Experimental Investigations	
II. PRELIMINARY TRIALS AND APPARATUS - - - - -	8
III. FINAL APPARATUS AND PROCEDURE - - - - -	11
Apparatus	
Procedure and Method of Calculation	
Calibration of Instruments	
Test on No. 20 Insulated Copper Wire at 35 Meters	
Test on No. 20 Insulated Copper Wire at 30 Meters	
Test on No. 20 Insulated Copper Wire at 20 Meters	
Test on No. 20 Oxide Coated Copper Wire at 20 Meters	
Test on No. 20 Bare Silver Wire at 20 Meters	
Test on No. 16 Oxide Coated Copper Wire at 35 Meters	
IV. DISCUSSION OF RESULTS - - - - -	35
V. SUMMARY AND CONCLUSIONS - - - - -	40

INTRODUCTION

The purpose of this research was to investigate the manner in which the very high frequency resistance obeyed the theoretical laws which have been deduced from theoretical considerations. Frequencies from approximately 8×10^6 cycles to 15×10^6 cycles were used on number 20 B. and S. guage copper wire and number 20 silver wire.

There are a number of formulas from which the high frequency resistance of wires may be calculated but the more convenient ones are those of Lord Rayleigh¹ and A. Russell². Neither of these formulas will hold for all sizes of wire and so in the calculations of this experiment both formulas were used.

Lord Rayleigh's formula, in convenient form, is:-

$$R_{ac} = R_{dc} \left(1 + \frac{h^2}{48} - \frac{h^4}{2880} + \frac{h^6}{58647} \dots \right) \quad \text{where} \quad h = \frac{\mu \cdot f \cdot d^2 \pi^2}{\rho}$$

in which μ is the permeability, f the frequency, d the diameter in centimeters, and ρ the resistivity in electromagnetic units. Then, if R_{dc} is in ohms, R_{ac} will also come out in ohms.

Russell's formula is:-

$$R_{ac} = R_{dc} \left[\frac{q}{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{2q} + \frac{3}{8q^2\sqrt{2}} - \frac{1}{2q^4\sqrt{2}} \dots \right) \right]$$

in which $q = \pi \cdot d \sqrt{\frac{2\mu f}{\rho}}$; where μ , d , f and ρ have the same meaning as before.

1. Phil Mag. Vol. 21, p.-381 (1886)

2. Phil. Mag. Vol. 17, p.-524. (1909)

Now it can be easily seen, as indeed it has been shown³, that if d and f are of such values that h is comparatively small, Lord Rayleigh's formula will rapidly converge while that of Russell will be more cumbersome if not entirely useless. Therefore, for small values of h the Rayleigh form is more applicable. But, on the other hand, if h or q is large Russell's formula is the more rapidly convergent and hence is the better.

Dr. Austin Bailey⁴ has shown that under certain conditions the observed resistance of copper wire at fifteen million cycles is four or five times the calculated resistance and that this discrepancy increased as the frequency was raised. He interpreted at least some of this discrepancy as due to the conditions at the surface of the wire; oxide coated wire showing a higher resistance than bright polished wire.

Wuckel⁵ has measured the resistance of wires of brass, nickel, German silver, and constantin at wave lengths of four and five meters and finds that the observed values fit the calculated values very closely. This work shows that wires of such materials may be used as excellent standards of resistance at these frequencies. However, since all practical work in short wavelength radio must be done with

3. J. D. Stranathan's Master's Thesis. p.-7.

4. Phys. Rev., Vol. 20, p.-154. (1922).

5. Ann. der Physic. Vol. 73, p.-427.

much lower resistance materials and since in most cases copper is used, it is very important to know how copper obeys the theoretical laws of resistance; especially in view of the great discrepancy between observed and calculated values for it, found in previous experiments.

The object of this research was to determine whether or not such a discrepancy did exist and if so, to what it might be due.

METHODS OF MEASUREMENT

Of the many ways of measuring resistance, the method promising the most accurate results and the method most generally used in this sort of work is the substitution method⁶. This is the method whereby a known standard resistance is inserted in a circuit whose resistance is desired. If the impressed e.m.f. is kept the same, of course the introduction of this known resistance will cause a decrease in the current in the circuit. If the current is known both before and after the insertion of the standard resistance, from the ratio of these two currents and the value of the standard resistance, the resistance of the unknown part of the circuit can be easily calculated.

It is known that for very fine wires the skin effect is practically negligible. Also, Mr Stranathan has shown⁷ that

6. Bureau of Std's. Circular No. 74. p.-183.

7. Thesis & Phys. Rev. Vol. 26. p.-500.

platinum is very constant in its resistive properties at moderately high frequencies. Therefore, fine platinum wire .0069 cm. in diameter was chosen as the standard to be used. Using Rayleigh's formula it was found that the high frequency of this wire exceeded its d.c. resistance by only about 0.5% for the frequencies used.

As a source of high frequency oscillation a Western Electric 5 watt power tube, self excited, was used.

The wavelength radiated by the oscillator was measured with a General Radio wave filter, especially made and calibrated from 65 meters down to 15 meters. By setting this instrument at the desired wavelength and placing it near the oscillator then tuning the latter until the oscillating current showed a sharp deflection (which indicated resonance with the wave filter) the oscillator could be tuned to any desired frequency to within an error of 2%.

The diagram of connections for the oscillator was as shown in Plate I.

PLATE I
THE OSCILLATOR

PRELIMINARY TRIALS AND APPARATUS

The first problem which had to be considered was the devising of an arrangement whereby the test circuit would be as nearly isolated as possible. Accordingly, three circuits were constructed. The first was a loop in the circuit of the oscillator. This loop radiated the energy and was called the oscillator loop. The second circuit was to be the test circuit whose resistance was to be measured. It consisted of a single large square or rectangular loop coupled inductively to the oscillator loop. The third or measuring circuit consisted of one or more loops in which was inserted a thermojunction connected to a sensitive galvanometer for measuring the amount of current induced in it from the test circuit; and thus as the current in the test circuit varied the current in the measuring circuit would vary proportionally.

The first plan tried was the coupling of the test circuit to the radiating circuit inductively and then coupling the third or measuring circuit to the test circuit in such a position that the radiating circuit would have no direct effect on the third or measuring loop. Theoretically such a position would be found when there was a 45° angle between the test loop and the measuring loop. Such an arrangement was used by Bailey⁸ and fully described by him.

However, this plan did not work satisfactorily in the present experiments owing to the great amount of reflection

8. Austin Bailey: Phys. Rev., Vol. 20, p.-154

from the walls. No position could be found where the measuring circuit could be coupled to the test circuit without receiving a considerable amount of energy directly from the oscillator.

The next plan tried was to make the test circuit a long narrow rectangular loop. The oscillator was coupled at one end and the measuring circuit at the other. Under these conditions a negligible amount of energy was coupled directly from the oscillator to the measuring circuit. The loop was broken at one end so that it could have a resonance frequency and a little capacity was added by wrapping the insulated ends of the wires together at the break. The oscillator was then tuned to the loop by noting the sudden fall of oscillating current in the oscillator when resonance was reached.

A few tests were run on this type of loop and it was found that the observed resistance was sometimes as much as fifteen times the calculated resistance. However, it was noticed that if the size of the loop was changed, the resistance per unit length changed. Sensitive ammeters were placed in various parts of the loop and it was found that the ammeter in the end of the loop opposite the break or condenser might have three or four times the current in it as an ammeter near the break. The obvious explanation was that the two sides of the loop were acting as plates of a small condenser (although they were 50 cm. apart) and at the high frequency used, most of the current was leaking across before it

came to the small condenser at the break.

A large condenser was substituted for the twisted ends of the loop and it was found that as the capacity of this condenser was increased the current became more uniform. However, as the capacity in series with the loop was increased of course the resonant frequency of the loop was decreased so that when the current became almost uniform throughout the loop, the wavelength was several hundred meters. If the loop was decreased in size, more energy would be coupled from the oscillator directly to the receiving circuit.

Therefore, the only solution was to put the current measuring device directly in the test circuit. This necessitated measuring the resistance of the thermojunction for each frequency used. This was done by inserting a second thermojunction and the one to be measured in a small test circuit containing a variable condenser. This second junction was connected to a D'Arsonval galvanometer and a high frequency current induced in the loop until the galvanometer showed a considerable deflection. The test-junction was then removed and fine platinum wire inserted in its place. The size of the wire was chosen so that a length of it approximately equal to the distance between the terminals of the 'junction would have the same resistance of the 'junction. The length of this inserted wire was then carefully adjusted until the galvanometer showed exactly the same deflection as

when the test-junction was in. Thence calculating the resistance of the fine platinum wire, the resistance of the junction was known at the given frequency. It was found that the resistances of a good junction could thus be found with a possible error of only 2%.

Thus knowing the resistance of the thermojunction and assuming the resistance of the condenser to be negligible (an assumption which afterward proved erroneous) the resistance per unit length of the wire in any loop can be measured by the substitution method outlined above.

FINAL APPARATUS AND PROCEDURE

Several shapes of loops for the test circuit were tried and it was found that no difference in resistance per unit length could be detected between round, square, or slightly oblong loops. Therefore, for convenience, a rectangular loop twice as long as wide was adopted as the type to be used throughout the tests. (A diagram of the final test loop apparatus is shown in Plate II) At one of the lower corners of the test loop was mounted a thirteen plate variable condenser. The condenser was mounted rigidly on a corner of a table and the loop suspended by means of cotton strings from the walls of the room so that it was practically isolated from any solid, bulky material. The condenser gave some trouble because of losses in it due to ohmic resistance

PLATE II
THE TEST LOOP

and energy losses in the dielectric insulating material. An extremely low loss condenser was finally built consisting entirely of metal except for six small pieces of hard rubber used as insulators. The frame of the condenser was of brass while the plates were copper, soldered together to reduce resistance. On the shaft of the condenser was mounted a wood pulley about fifteen centimeters in diameter. Around this pulley was wrapped a cotton string so that the condenser might be varied without the observer being closer than a meter from the loop. Such precautions had to be taken because of the extreme sensitiveness of the loop to the presence of an observer.

At the opposite corner of the loop was fixed a sensitive thermojunction mounted rigidly on the end of a wooden arm. A microfarad condenser was shorted across the potential terminals of the junction to bypass the high frequency past the galvanometer circuit. Two wires were led from these terminals along the supporting arm and to a wall galvanometer some two meters away. A second one-microfarad condenser was placed across the terminals of the galvanometer. On one of the "heating" terminals of the thermojunction was fastened a copper spring clamp for gripping one end of the inserted resistance. A similar clamp held the other end of the resistance. The clamps were kept constantly spaced by the tension in the wire and a short piece of cotton string holding them together.

The oscillator was set about a meter and a half from the loop and at this distance the test circuit introduced no resistance in the oscillator circuit which could be detected. For the smaller loops the oscillator had to be moved slightly closer but at no time was it nearer than a meter.

The procedure was as follows:-

Starting with a rather large loop, i.e., one that could just be tuned with the smallest condenser setting, a piece of wire similar to that in the loop was placed in the clamps, the loop carefully tuned and the maximum deflection noted. This wire was then replaced with a piece of the standard resistance platinum wire and again the loop tuned and the maximum deflection noted. The length of the inserted wire was adjusted until the initial current was approximately cut in half when the platinum wire was inserted. Because of the slight difference in inductance between the platinum wire and the piece of test wire which it replaced the tuning position of the condenser was very slightly different for the two. But the difference was so slight as to be negligible. The piece of test wire was then replaced and the readings repeated. This was done three or four times. Most of the time the oscillator current was very constant and so an average of the readings for each of the two insertion pieces could be taken, or what was usually done was to take the highest value obtained for each and then use the ratio of the currents corresponding to these two deflections. But where the oscillator

current varied, each set of results was figured separately and an average of the ratios taken.

From a calibration curve of the thermojunction and galvanometer the current corresponding to any deflection could be found. Let the ratio of "the current when the resistance was in" to the "current when the test wire was in the circuit" be M . If R_j is the resistance of the thermojunction; r_p the resistance of the inserted Pt wire; r_c the resistance of the inserted piece of test wire (which was usually copper); and R_c be the resistance of the test wire in the circuit which was not changed, then it can be seen that

$$\frac{I_{Pt}}{I_c} = M = \frac{\frac{E}{R_j + r_p + R_c}}{\frac{E}{R_j + r_c + R_c}} = \frac{R_j + r_c + R_c}{R_j + r_p + R_c} \quad \text{since } E \text{ is constant.}$$

If L is the length of R_c and l is the length of r_c then

$$r_c = \frac{l}{L} R_c, \quad \text{since } r_c \text{ and } R_c \text{ are the same kind of wire.}$$

$$\text{Therefore, } M = \frac{R_j + \left(\frac{l}{L} + 1\right) R_c}{R_j + r_p + R_c}$$

$$\text{or, } M(R_j + r_p + R_c) = R_j + \left(\frac{l}{L} + 1\right) R_c$$

$$R_c = \frac{M R_j + M r_p - R_j}{\frac{l}{L} + 1 - M} = \frac{(M - 1) R_j + M r_p}{1 + \frac{l}{L} - M}$$

For each loop L and l were carefully measured and since the resistance of the Pt per unit length was known $r_p = p \cdot l$ was easily computed, where p equals resistance per unit length. R_j and M are known so R_c is easily calculated from the above formula. This, of course, is the observed resistance. The calculated resistance is calculated from Russell's formula

and the length L . The ratio of the observed resistance to the calculated resistance was then tabulated.

This ratio is in error since the observed resistance is made abnormally high by the non-uniform current distribution. The loop was then reduced in size, keeping the same relative dimensions. The parallel capacity was increased sufficiently to tune this smaller loop to the same frequency as before. Under these conditions the resistance was again determined. A similar procedure was carried out for still smaller loops. It is evident that as the size of the loop is decreased and the parallel capacity correspondingly increased, the current distribution will become more and more nearly uniform. It follows, therefore, that if the resistance ratios are plotted against the corresponding condenser settings, the resulting curve will approach the ratio holding for uniform current distribution as the capacity becomes very large. Such a series of readings was taken for number 20 copper wire triple cotton covered; for the same wire after it had been exposed to the weather for ten weeks and had received a thorough coating of oxide; for number 20 silver wire, (a metal which does not easily form an oxide); and for number 16 copper wire which had been exposed for years and which was very brown with oxide.

CALIBRATIONS.

The thermo-couple was connected to the galvanometer just as it was to be used and a known direct current sent through it and the corresponding galvanometer deflection noted. The same current was then sent through the 'couple in the opposite direction and the deflection noted. These two deflections were then averaged and plotted on a calibration curve according to the current which caused them. The thermo-junction and galvanometer were thus calibrated for current before the experiment and again just before the experiment was finished, the points falling very closely on the same curve. The calibration data was as shown in Table I which follows.

TABLE I

FIRST CALIBRATION		(0 point at -.3)	
I_H^*	D_f^{**}	D^{***}	Ave.D.
			(Corrected)
15	2.7	2.46	2.88
25	7.42	6.88	7.38
30	10.29	9.84	10.37
35	12.96	12.51	13.04
40	15.54	15.12	15.63
43	17.18	16.77	17.28
46	18.71	18.29	18.80
50	20.81	20.33	20.87
53	22.16	21.74	22.25
56	23.28	22.98	23.48
59	24.75	24.32	24.84

SECOND CALIBRATION			
23.00	6.10	5.93	6.01
25.15	7.35	7.13	7.24
28.2	9.20	8.92	9.06
45.0	18.15	17.80	17.98
48.0	19.73	19.40	19.57
51.0	21.35	21.10	21.23
55.0	23.15	22.85	23.00
57.2	24.20	23.85	24.02

* Heating current through the thermojunction in milliamperes.

** Deflection of galvanometer when the current was in
/ direction.

*** Deflection of galvanometer when the current was reversed.

The average of the galvanometer deflections for direct and reversed currents gives the true deflection for alternating current as shown by Stranathan⁹.

This data when plotted is shown in the curve of Plate III.

For 14.5 meters of number 20 triple cotton covered copper wire the resistance as measured by two different Wheatstone bridges was found to be .499, .499 and .500, .501 ohms, respectively. During these measurements the current flowing through the wire was adjusted to be approximately that which would flow when the wire was being used in the loop. Thus the D. C. resistance was $.5003 / .450$ or .0003450 ohms per cm.

Some number 16 copper wire was also used and so its resistance per cm. was also determined as follows:- With a Kelvin bridge the resistance of 38.1 cm of this wire was found to be .0051 ohms (accurate to 1%) Hence the resistance per cm. is $\frac{.0051}{38.1}$ or .000134 ohms.

The resistance of a specimen of Pt wire .0069 cm. in diameter and 14.3 cm. in length which we shall designate as type number 1 Pt wire, was measured with an accurate voltmeter and ammeter using currents of 25 milliamperes to 35 milliamperes which is the range of current values actually used in the experiment. The results are given in the following table. _ _ _

TABLE II

Current in Amperes	Voltage Drop	Resistance in ohms	Resistance per Unit Length
.025	.125	5.00	.351 ohms
.030	.155	5.16	.362
.035	.180	5.13	.360
			--- --
			.357 Avg.

Another type of Pt wire which shall be called number 2 was used somewhat. It was .012 cm. in diameter. The resistance of 62.8 cm. of this wire was measured on a good Wheatstone bridge and found to be 6.64 and 6.65- ohms, respectively, which is a resistance per cm. of .106 ohms. Here, also, the same precautions, as to the value of current flowing during the measurement, were taken as in the case of the copper wire.

A third type or number 3 Pt wire was .0069 cm. in diameter and 96 cm. of it had a resistance, as found on a box bridge, of 27.20, 27.20, 27.2, 27.30 ohms, which is a resistance per cm. of .284 ohms.

The resistance of number 20 silver wire was also determined as follows:- The resistance of 1865 cms. was found by means of a Wheatstone bridge to be .615, .615, .615 ohms. Hence the resistance per cm. was $\frac{.615}{1865}$ or 3.30×10^{-4} ohms.

PLATE III

The high frequency resistance of the thermo-junction was determined at various frequencies as described above. At 35 meters the resistance of the junction was just equal to 6.6 cm. of number 2 Pt wire. (accurate to 1mm. or 1.5%) At this frequency the a.c. resistance of number 2 Pt wire is 1.015 times its d.c. resistance. At 20 meters wavelength the resistance of the thermo-junction was found to be just equal to 6.6 cm of number 2 Pt wire (accurate to 1 mm. or 1.5%)

TESTS ON NO. 20 COTTON INSULATED COPPER WIRE.

Test at 35 Meters Wavelength or $f = 8.58 \times 10^6$ Cycles.

In these tests number 1 Pt wire was used i.e., .069 mm. in diameter. The a.c. resistance for this wire must be calculated with the Rayleigh formula rather than the Russell formula. The resistivity of the metal in this wire is $\rho = \frac{R\pi d^2}{4}$ where R is the resistance per cm. of the wire and d is its diameter.

Thus $\rho = \frac{.357 \times (.0069)^2 \times \pi}{4}$ or to convert to abohms we multiply by 10^9 . Hence, $\rho = \frac{.357 \times (.0069)^2 \times 3.14}{4} \times 10^9 = 13,400 \text{ abohms/cm}^2$.

Then $h = \frac{\pi^2 d^2 f}{\rho} = \frac{9.86 \times (.0069)^2 \times 8.58 \times 10^6}{13400} = .301$

and $R_{ac} = .357 \times \left(1 + \frac{(.301)^2}{48} - \frac{(.301)^4}{2880} \dots\right)$

Using only the first two figures in the parenthesis

$R_{a.c.} = .357 \times 1.0063 = .359$ ohms per cm. The resistance of the thermojunction is in terms of the length of number 2 Pt wire.

For this wire $\rho = \frac{3.14 \times (0.012)^2 \times 10^6}{4} \times 10^9 = 12,000 \text{ abohms/cm}^3$
 and $h = \frac{(3.14)^2 \times (0.012)^2 \times 8.58 \times 10^6}{12,000} = 1.015$ hence
 $R_{ac} = .106 \left(1 + \frac{(1.02)^2}{48} - \frac{(1.02)^4}{2880} \dots \right) = .108$

Since the thermo-junction resistance is equal to 6.6 cm. of this wire then its ohmic resistance is $6.6 \times .108 = .712 \text{ ohms}$.

The calculated high frequency of the number 20 copper wire is obtained as follows: $\rho_c = \frac{\pi d^2 R}{4} \times 10^9$ where the symbols refer to the same quantities as for the platinum.

$\rho_c = \frac{3.14 \times .081^2 \times .000345}{4} \times 10^9 = 1.777 \frac{\text{abohms}}{\text{cm}^3}$ The R.a.c. for the copper must be calculated from Russell's formula:-

$$q = \pi d \sqrt{\frac{2f}{\rho}} = 3.14 \times .081 \sqrt{\frac{2 \times 8.58 \times 10^6}{1.777}} = 25.0 \text{ and } R_{ac} = R_{dc} \left[\frac{q}{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{2q} + \frac{3}{8\sqrt{2}q^2} \dots \right) \right]$$

Here again only the first two figures inside the inner parenthesis need be used, hence,

$$R_{ac} = .000345 \left[\frac{25}{2} \left(.707 + \frac{1}{50} \right) \right] = 3.22 \times 10^{-3} \text{ ohms/cm.}$$

Loops of the wire were constructed and the tests run as described above. The results in tabular form are shown in Table III.

In this and the following tables the following symbols will have these meanings:-

Size of loop	- Dimensions of loop in centimeters.
C. S.	- Setting of condenser in degrees (180° being maximum capacity)
D _o	- Galvanometer deflection when the inserted wire is the same as the test wire
D _p	- Galvanometer deflection when the inserted wire is the platinum resistance wire

M - The ratio of the currents corresponding to D_p and D_c , respectively, i.e.

L - The length in centimeters of the test wire in the loop which remains unchanged for each loop

l - The length in cm. of the insertion wire.

R_0 - The resistance in ohms of the length L of the test wire.

Calc. R_c - The calculated resistance for this same length of test wire.

Ratio - The ratio of
$$\frac{\text{Observed Resistance}}{\text{Calculated Resistance}}$$

T A B L E III

Size of Loop	C.S.	D _c	D _p	M	L	l	R _c	Calc R _c	Ratio
110 x 220	34-	18.77	9.97	.641	645	8.6	4.54	2.08	2.18
		18.70	10.00						
		18.74	9.97						
		18.78	9.98						
95 x 190	47-	22.05	10.43	.576	555	8.5	3.32	1.78	1.86
		21.97	10.40						
		22.05	10.42						
		22.06							
80 x 160	64	22.15	10.85	.589	468	6.4	2.50	1.51	1.65
		22.20	10.84						
		22.17	10.84						
		22.22							
70 x 140	80/	22.45	9.20	.529	408	6.4	1.83	1.31	1.40
		22.40	9.25						
		22.40	9.25						
65 x 130	90-	22.40	8.50	.505	378	6.35	1.57	1.22	1.29
		22.45	8.55						
		22.43	8.53						
60 x 120	100-	22.55	10.45	.567	350	4.45	1.35	1.13	1.19
		22.60	10.48						
		22.54	10.45						
55 x 110	114	22.25	9.35	.535	320	4.35	1.06	1.03	1.03
		22.30	9.34						
		22.35	9.35						
50 x 100	130	19.80	7.67	.534	290	4.25	1.005	.93	1.08
		19.82	7.70						
		19.75	7.78						
45 x 90	150	18.23	6.20	.515	260	4.25	.880	.838	1.05
		18.20	6.23						
		18.21	6.20						
41 x 82	171	20.20	8.65	.555	236	3.43	.796	.760	1.05
		20.19	8.63						
		20.22	8.62						

The data above is plotted in ^{Black} curve on Plate IV.

PLATE IV

Wavelength - 35 Meters

o Cotton Covered No. 20 Copper Wire

o Oxide Coated No. 16 Copper Wire

Ratio
 $\frac{\text{Obs. Res.}}{\text{Calc. Res.}}$

2.50
2.25
2.00
1.75
1.50
1.25
1.00

20 40 60 80 100 120 140 160 180

Condenser Setting in Degrees

Test on the Same Wire at 30 Meters or

$$f = 10^7 \text{ Cycles}$$

In the following tests No. 3 Pt wire was used as the inserted resistance. The resistivity of the metal in this wire was, making calculations as above:

$$\rho = \frac{R \pi d^2}{4} \times 10^9 = \frac{.284 \times 3.14 \times .0069^2}{4} \times 10^9 = 10,700 \text{ abohms/cm}^2$$

And at a frequency of 10^7 cycles per second, for this wire

$$h = \frac{10^7 \times 3.14^2 \times .0069^2}{10700} = .439. \text{ Hence } R_{ac} = .284 \left(1 + \frac{.44}{.48} \right) = 1.009 \times .284 = .286 \text{ ohms per cm.}$$

For the copper test wire at this frequency the resistance is calculated as follows:-

$$q = \pi d \sqrt{\frac{2f}{\rho}} = 3.14 \times .081 \times \sqrt{\frac{2 \times 10^7}{1777}} = 27.0$$

$$\text{And } R_0 \text{ per centimeter is } .000345 \times \left[\frac{27}{2} \left(.707 + \frac{1}{54} \right) \right] = 3.39 \times 10^{-3} \text{ ohms}$$

(The resistance of the thermojunction at this frequency was assumed to be equal to 6.6 cm. of No. 2 Pt wire or .715 ohms)

The following data was then taken and the calculations made as in Table IV..

T A B L E IV

Size	C.S.	D _c	D _p	M	L	l	R _c	Calc. R _c	Ratio
70 x 140	63	23.7	10.8	.558	402	8.7	2.32	1.36	1.77
		23.7	10.00	.531		10.2	2.46		
		23.7	10.0	.531		10.2	2.41		
60 x 120	84	22.6	8.7	.510	343	8.8	1.80	1.17	1.55
		22.5	8.6						
		22.6	8.7						
55 x 110	98	22.2	8.5	.514	312	7.7	1.53	1.06	1.45
		22.1	8.5						
		22.2	8.5						
50 x 100	111	23.3	10.2	.548	283	6.4	1.42	.96	1.48
		23.2	10.25						
		23.2	10.10						
(Condenser was tightened and the above repeated)									
50 x 100	111	22.85	9.3	.526	283	6.9	1.40	.96	1.46
		22.85	9.3						
45 x 90	132	22.0	8.7	.523	252	6.65	1.31	.855	1.53
		21.6	8.5	.525					
		23.2	9.55	.528					
(Condenser capt. changed by removing one stationary plate)									
45 x 90	132	22.4	8.80	.518	252	6.65	1.25	.855	1.46
		22.5	8.9						
(Condenser capt. changed. Space between plates decreased)									
45 x 90	132	21.8	8.3	.514	252	6.65	1.23	.855	1.44
		21.5	8.1						
		21.6	8.25						
(Another plate added & condenser tightened all over)									
45 x 90	132	23.0	8.65	.495	256	6.65	1.09	.855	1.27
		23.2	8.75						
(Another copper plate condenser and loop on a different day gave the following)									
45 x 90	132	24.5	9.00	.475	254	7.3	1.12	.858	1.30
		24.0	8.80						
(The above results repeat the preceeding ones to less than 3%)									
37½ x 75	198	21.9	7.90	.503	211	5.65	.855	.715	1.22
		21.7	8.05						
(Again with the copper plate condenser)									
32½ x 65	145	23.2	8.85	.494	179	5.15	.683	.608	1.12
	on 250	23.8	8.9						

The above results are plotted in Plate V.

Ratio
 $\frac{\text{Obs. Res.}}{\text{Calc. Res.}}$

1.80

1.60

1.40

1.20

1.00

50

75

100

125

150

175

200

225

250

Condenser Setting in Degrees

PLATE V

Wavelength - 30 Meters

Cotton Covered No. 20 Copper Wire

Test on No. 20 Insulated Wire at 20 Meters.

A test was run on the cotton insulated No. 20 wire at 15×10^6 cycles or a wavelength of 20 meters. No. 3 Pt wire was used as the substitution resistance and at the above frequency h for this wire is

$$h = \frac{3.14^2 \times .0069^2 \times 15 \times 10^6}{10700} = .656$$

and $R_{a.c.} = .284 \times \left(1 + \frac{.656^2}{48}\right) = .288 \text{ ohms per cm.}$

For the copper wire $q = 3.14 \times .081 \sqrt{\frac{30 \times 10^6}{1777}} = 33.1$
and hence for the copper wire $R_{a.c.} =$

$$= .000345 \times \left[\frac{33.1}{2} \left(.707 + \frac{1}{66.2} \right) \right] = 4.11 \times 10^{-3} \text{ ohms/cm.}$$

At this frequency h for the No. 2 Pt wire was

$$h = \frac{3.14^2 \times .072^2 \times 15 \times 10^6}{12000} = 1.75 \text{ and } R_{ac} = .106 \left(1 + \frac{1.75^2}{48} \right) = .110 \text{ ohms/cm.}$$

The resistance of the thermojunction at this frequency was equal to 6.6 cm. of No. 2 Pt wire. Hence its resistance was $6.6 \times .110 = .725 \text{ ohms.}$

The results of the test are given in Table V

Test on Oxide Coated Wire at 20 Meters.

Some of the same wire as was used in the previous test was carefully stripped of the insulation and laid out in the weather for ten weeks. At the end of that time it was well coated with oxide.

A test was then run on this wire at 15×10^6 cycles. No. 3 Pt was again used as the substitution resistance and the results obtained are given in Table VI.

T A B L E V

Frequency - 15×10^6 cycles per second.

Test wire - Cotton insulated No. 20 copper.

Size	C.S.	D _o	D _p	M	L	l	R _c	Calc. R _c	Ratio
45 x 90	27	24.9	8.6	.466	249	(#1pt) 12.7	3.08	1.02	3.02
		24.0	8.2	.479					
		23.8	7.8	.473					
40 x 80	35	22.0	8.4	.526	222	8.2	2.36	.922	2.56
		22.1	8.4						
		22.1	8.4						
(Repeat on different day with a different loop and thus repeats to 4.5%)									
40 x 80	35	22.9	9.20	.524	220	7.8	2.21	.904	2.45
35 x 70	43	23.6	10.7	.550	195	5.8	1.65	.804	2.06
		23.6	10.6	.550					
		19.6	7.9	.551					
30 x 60	59	24.2	9.35	.493	163	(#3pt) 7.6	1.285	.672	1.91
		24.1	9.35						
		24.2	9.40						
25 x 50	78	21.3	7.35	.485	133	6.45	.934	.548	1.70
		21.35	7.40						
		21.3	7.35						
20 x 40	108	21.7	8.10	.501	107	5.3	.700	.440	1.59
		21.75	8.10						
		22.8	8.6	.495					
		22.8	8.6	(used)					

The above results were plotted in the black curve on Plate VI.

PLATE VI

Wavelength - 20 Meters

● Cotton Covered Copper Wire No. 20

● Oxide Coated Copper Wire No. 20

● Bare Silver Wire No. 20

Ratio
 $\frac{\text{Obs. Res.}}{\text{Calc. Res.}}$

3.00

2.50

2.00

1.50

1.00

20

40

60

80

100

120

140

Condenser Setting in Degrees

T A B L E VI.

Frequency - 15 x 10⁶ cycles

Test Wire - Oxide coated No. 20*

Size	C.S.	D _o	D _p	M	L	l	R _o	Calc. R _o	Ratio
40 x 80	30	24.3	11.6	.560	223	8.65	2.24	.917	2.45
		24.3	11.6						
		24.0	11.4						
35 x 70	40	21.6	8.2	.506	193	8.65	1.68	.794	2.11
		21.6	8.2						
30 x 60	55	22.6	8.35	.497	164	7.5	1.29	.675	1.91
		22.4	8.20						
		22.7	8.40						
25 x 50	74	22.3	8.10	.494	135	6.25	.944	.555	1.70
		22.5	8.35						
		22.2	8.10						
20 x 40	106	22.4	10.2	.534	106	4.5	.696	.436	1.60
		23.6	10.1						
		23.9	8.9	.482	107	5.35	.648	.440	1.47
		23.9	8.9						
17 x 34	112	21.8	9.8	.558	90	3.6	.537	.370	1.45
		21.8	9.8						
		22.1	8.1	.498	89	4.5	.511	.366	1.40
		21.9	8.1						

(Ave 1.52)

(ave 1.41)

The above data was plotted in the red curve of Plate VI.

* Prepared by stripping some of the wire used in the previous experiment and exposing to weather for ten weeks.

It was known that silver does not easily form an oxide. Therefore, a test was run on No. 20 silver wire. For the metal in this wire the resistivity is

$$\rho = \frac{3.14 \times .081^2 \times 3.30 \times 10^{-4}}{4} \times 10^9 = 1700 \text{ abohms/cm}^3$$

$$\text{And, } q = 3.14 \times .081 \sqrt{\frac{30 \times 10^6}{1700}} = 33.8 \quad \therefore R_{ac} = 3.3 \times 10^{-4} \left[\frac{33.8}{2} \left(.707 + \frac{1}{676} \right) \right] = 4.02 \times 10^{-3} \frac{\text{ohms}}{\text{cm.}}$$

The results obtained from this test are shown in

Table VII. No. 3 Pt wire was used as the insertion resistance.

T A B L E VII

Frequency - 15×10^6 cycles per second
Test wire - Bright, bare, No. 20 silver.

Size	C.S.	D_0	D_p	M	L	l	R_c	Calc. R_c	Ratio
50 x 100	24	22.3	10.8	.558	276	13.5	3.67	1.11	3.31
		22.3	10.8						
40 x 80	40	23.2	8.85	.495	220	11.3	2.24	.88	2.53
		23.1	8.80						
30 x 60	64	22.6	8.1	.489	162	7.7	1.27	.65	1.95
		22.5	8.0						
25 x 50	85	23.1	8.20	.482	134	6.7	.975	.54	1.80
		23.1	8.25						
20 x 40	116	23.8	8.35	.474	105	5.4	.614	.42	1.45
		23.5	8.50						

The above data is plotted in the green curve of Plate VI.

Test on No. 16 Oxide Coated Wire at 35 Meters.

To determine how a different size of wire behaved at this frequency a loop of heavily oxidized No. 16 copper wire was made and the above test repeated with it. Because of the low resistance of this size wire contact resistances become more important and the accuracy of the results is necessarily much lower.

For this wire $\rho = \frac{\pi d^2 R}{4} \times 10^9$ where R is the resistance per centimeter or $\rho = \frac{.000134 \times 3/4 \times 129^2 \times 10^9}{4} = 1750 \text{ abohms/cm}^3$

Also, $q = \pi d \sqrt{\frac{2f}{\rho}} = 3/4 \times 129 \sqrt{\frac{2 \times 858 \times 10^6}{1750}} = 40.2$

Then, $R_{a.c.} = .000134 \left[\frac{40.2}{2} \left(707 + \frac{1}{804} \right) \right] = 1.93 \times 10^{-3} \text{ ohms/cm.}$

No. 3 Pt wire was used as the insertion resistance.

The results of the test are shown in Table VIII.

T A B L E VIII

Frequency - 8.5×10^6 cycles per second
(35 meters wavelength)

Test wire - No. 16 heavily coated with copper oxide

Size	C.S.	D _c	D _p	M	L	l	R _c	Calc R _c	Ratio
90 x 180	54	22.9	9.10	.513	521	11.4	2.60	1.00	2.50
		22.9	9.15						
70 x 140	85	22.8	7.60	.468	401	8.9	1.46	.775	1.88
		22.9	7.55						
55 x 110	118	23.1	9.3	.510	315	5.7	.952	.608	1.57
		23.2	9.2						
50 x 100	134	22.5	7.6	.473	286	5.8	.748	.552	1.35
		22.5	7.65						
45 x 90	153	22.5	7.35	.466	252	5.6	.661	.486	1.36
		22.4	7.30						

These results are plotted in the red curve on Plate IV.

Since the same condenser was not used in both cases it was hard to get the exact values of the abscissae. However, the results here given are accurate enough to show how both curves act as the capacity of the condenser is increased.

DISCUSSION OF RESULTS

In plotting the foregoing curves anything could be plotted as abscissa which varied with the uniformity of the current. That is, size of the loop, or perimeter of loop, or condenser capacity might have been used, but the condenser setting seemed to be the most accurately measured and hence it was used. However, the condenser capacity was not proportional to the setting of the condenser and, therefore, bore no simple relation to the uniformity of the current and hence the actual shapes of the curves mean nothing.

The curves in Plate IV show how copper wire behaves at 35 meters wavelength as the test loop is decreased in size. The insulated No. 20 wire quickly drops to a minimum and remains at this value while the oxide coated No. 16 wire shows no definite minimum although it seems to be flattening out at the largest values of condenser setting. No smaller loops were used, for even at the sizes which were used the resistance of the wire was smaller than the known thermo-couple resistance. Then the length of the insertion resistance wire was so small that its length could not be determined accurately to less

than six percent. Therefore, the greatest degree of accuracy claimed for the latter points plotted would be to ten percent.

The fact that the curve for the No. 20 wire finally reaches a minimum about five or six percent above the calculated value indicates that there might be a resistance in the circuit which was not taken into consideration. Suspicion immediately falls on the condenser. Although such a condenser as used in this experiment has a negligible resistance at lower frequencies it has been recently shown¹⁰ that at the frequencies here used the resistance of a condenser becomes very appreciable. If we assume that such a resistance exists and from "observed resistance" formula calculate approximately what it is we find it amounts to about .17 ohm. Now if this value is considered in getting the observed resistance of the No. 16 wire at this same wavelength it is found that the curve for the latter approaches the calculated value within the limits of possible error. R. R. Ramsey¹¹ of Indiana University has recently measured the resistance of a condenser of a very similar type and size to that used in this experiment and finds that its resistance varied from .05 ohm to .15 ohms at 10^6 cycles. Hence, the resistance of the condenser to 10^7 cycles as used here might be expected to be considerably larger for a given capacity.

There is nothing at all accurate in the above mentioned calculation but it serves to indicate that the

- - - - -

10. R.R. Ramsey: Bul Am. Phys. Soc. Vol. 1. No. 9, 1926.

11. R.R. Ramsey: Bul. Am. Phys. Soc. Vol. 1, No. 9, 1926.

discrepancy here/^{found}~~and~~ between the calculated and observed values for copper wire is more likely due to the condenser resistance than to anything else.

The curve in Plate V. represents the results of the test on No. 20 cotton covered copper wire at 30 meters wavelength. This curve shows the same type of decrease with greater values of condenser capacity as did the former curves. However, in this test experiments were performed on the condenser to see just how the "ratio" might be affected by changes in that instrument. The condenser first used was the best type of low loss air condenser with aluminum plates.

It is seen that at a setting of 111 the ratio started to increase and that a general tightening of contacts of the condenser caused a slight decrease.

Then with the next loop whose tuned position corresponded to a condenser setting of 132, the ratio had increased from 1.46 to 1.53. Then one of the stationary plates of the condenser was removed and the same loop returned. In this case less plate area was idle. That is, a larger proportion of the total area of the plates was opposing another plate. Another run showed the ratio reduced to 1.46.

Still another condenser change was made by decreasing the spacing between the plates. Since this increased the maximum capacity of the condenser, the tuned position or setting in this case was less than in the preceeding case and hence a less proportion of the total area of the plates was

opposed and the test under these conditions gave a ratio of 1.27. (This latter result was checked by using a condenser with copper plates. This condenser had the advantage that its plates could be soldered together on the shaft while the aluminum plates could not)

Thus it appears that the condenser is very largely responsible for the discrepancy between the observed and calculated values of resistance of the wire in the loop, and the above results seem to indicate that a closer spacing of the plates contributes more to reducing its resistance than the elimination of "idle" area of the plates.

These facts are in excellent agreement with the more quantitative results of A. E. Maibauer and T. Smith Taylor of the Bakelite Corporation whose results were presented at the Annual Kansas City Meeting of the American Physical Society, December, 1926.

Plate VI shows very clearly how the same size wire under identical experimental conditions but of different materials and with different surface conditions compared in resistance as the current became more uniform.

The black curve is for the cotton covered copper wire. When stripped of its insulation this wire was bright and shiny showing very little traces of copper oxide. The curve on this wire could be run with a fair degree of accuracy until the ratio of observed to calculated values of resistance was well below 1.5. Even at this point the curve showed a steep decrease.

The red curve is for the same kind of wire after it has gained a heavy coating of oxide. It can be seen that if there is a difference between this curve and the curve of the proceeding paragraph, the curve of the oxide coated wire is the lower. This difference is so slight that nothing definite can be said concerning it. But these results tend to show, if anything, that the oxide coated wire is less in resistance than the brighter wire.

To make sure that the oxide had practically nothing to do with the results here obtained a curve was run with No. 20 bare, silver wire. Since silver does not easily form an oxide the effects of any oxide would not be present in this curve and for the most part its shape is just like that of the copper curves.

This is in disagreement with the interpretation given by Dr. Bailey⁸. It is believed by the author that the discrepancy between experimental and calculated resistances which the above mentioned author was attempting to explain may well be explained by non-uniform current distribution in the test wire along with possibly neglected condenser resistance.

8. Bailey. Phys. Rev. Vol. 20. p.-154 (1922)

SUMMARY AND CONCLUSIONS

The results of this experiment show:-

1. The apparent resistance of the wire in a tuned loop at very high frequencies decreases rapidly as conditions are adjusted to make the current more uniform in intensity throughout the loop.
2. Under favorable conditions the observed resistance of No. 20 B. & S. copper wire approaches to within a few percent of the calculated resistance even when the condenser resistance is neglected.
3. At the frequencies here used the condenser resistance is not negligible for the resistance of the loop changes considerably with changes in the condenser.
4. The effect of oxide on the surface of the wire, if it exists at all, is entirely negligible in comparison to contact resistance, falling off of energy as you go along the loop from the middle toward the ends, condenser resistance, etc.

The errors in the results of this investigation are all on one side, i.e., it is more probable that the actual values of resistance are less than the values here given rather than greater than these values for the following reasons:-

All contact resistances as well as the condenser resistance were neglected in the computation.

The loops tuned extremely sharply but, of course, the tuning was even more sharp when the insertion resistance was taken out than when it was in. This made it much more probable that the loop was accurately tuned when the added resistance was in than when it was out.

Since the observed resistance depended greatly on an accurate value of $\frac{I}{L}$ it is seen that if I_0 is small (which it would be if the loop was not accurately tuned) the ratio is increased and the observed resistance is greatly increased.

Also, the presence of any iron object or of an observer tended to increase the resistance of the loop greatly.

Thus it may be said that no great discrepancy exists between the observed and calculated values of the resistance of copper wire of sizes used here at frequencies up to 15×10^6 cycles per second.

A more accurate verification of the resistance laws awaits accurate means of measuring the condenser resistances at these frequencies.

And in closing, the author wishes to express his appreciation and thanks to Professor J. D. Stranathan for his many helpful suggestions and his continued interest throughout the investigation and to Mr. Lawrence Lynn for his skill in the building of apparatus.

Blake Laboratory,
University of Kansas.
August 2, 1926.