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ON THE ASYMMETRY OF MARKET RETURNS

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I. Introduction

During the late 1960s and throughout the 1970s, a myriad of tests of the two-parameter capital asset pricing model (hereafter CAPM) have been executed and reported in the literature. Relatively recently, much attention has been focused on the asymmetry--skewness--of realized asset, portfolio, and market return distributions. The intent of the present effort is to report the results of an investigation of the asymmetry of one of these variables--the returns of the market portfolio.

To accomplish this task, Section II of this paper contains a brief literature review designed to emphasize the importance of the issue. Section III contains a discussion of the adopted measure of asymmetry and its estimate's possible behavior under various return generating regimes. Section IV contains summaries of the results of the empirical investigation of the topic. This essay is concluded with a discussion of some of the implications of the results.

II. Measured Asymmetry and Tests of Equilibrium Pricing

One of the most intriguing empirical investigations (and a work initially motivating the present effort) of measured market skewness was executed by Fogler and Radcliffe (hereafter F&R) who concluded that [14, p. 485] ". . . skewness measures . . . (are) highly sensitive to both the size of the differencing interval and initialization point." More particularly, F&R found that the measured asymmetry of the simple returns of the Standard and Poor's Composite Index depended upon whether quarterly, semi-annual, or annual data were used, and upon which quarter of 1950 was used as the beginning date (that is, initialization point) of the time series.

The importance of this observation is most striking when note is taken of the three-moment equilibrium pricing framework of Simonson [24] and Kraus and Litzenberger [16]. These efforts forwarded that equilibrium asset returns depend not only upon systematic risk but also upon systematic skewness. The relevant point here is that, with fixed systematic risk, investors should be rewarded

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with additional expected returns for taking on an asset or portfolio that has relatively large systematic skewness *if* the market is (*ex ante*) negatively skewed. Conversely, if the market is best characterized as being positively asymmetric, then systematic skewness is deemed to be *desirable* so that assets with large values should be priced in order to return *less*. Hence, the behavioral guides of the equilibrium framework depend upon the sign of the market's (*ex ante*) skewness.

That this point is crucial is seen most vividly by noting the empirics of the two works. Simonson found the market (logs of the quarterly Standard and Poor's Composite for 1961 to 1970) to be negatively skewed. Kraus and Litzenberger, on the other hand, found *positive* asymmetry in the return series they used for the market (the sum across their sample of stocks of the monthly excess deflated returns for January 1936 to June 1970).

III. Asymmetry: Measurement and Possible Behavior

The most obvious measure of distributional skewness is the third central moment about the expectation, estimated with the average cubed deviation about the mean. Since such a statistic is scale sensitive and is not amenable to significance testing, relative skewness, defined as the third central moment divided by the cube of the standard deviation, is used. The estimate of relative skewness is defined as:

$$r.s. = \frac{\sum_{t=1}^{T} (x_t - \bar{x})^3/T}{\left[\sum_{t=1}^{T} (x_t - \bar{x})^2/T\right]^{1.5}} \qquad (1)$$

where x_t is the t^{th} observation of the T periods (for a time series study) for a variable with a mean $\bar{x}$.

Since the denominator of equation (1), the cube of the estimated standard deviation, is a scaling factor, the asymmetry of return series of different magnitudes may be compared. Further, enough knowledge about the sampling properties of equation (1) exists to permit at least limited hypothesis testing. In particular, with an estimated value of r.s. of adequate magnitude, a Gaussian null hypothesis may be rejected for a particular confidence level and sample size (= T). (See Pearson and Hartley [21]).

Inspection of equation (1) immediately brings to the fore one important empirical issue--the time series unbiasedness (or at least consistency) of the measure. If the process generating x_t is stable and stationary and if the third central population moment is finite, then increasing the number of observations used to compute r.s. should result in the statistic's estimated values asymptotically approaching the population value. Moreover, if the process is asymmetric stationary but the third moment is *not* finite, r.s. should expand without bound with increasing time series. This should obtain regardless of whether or not the second moment is defined, although the expansion will likely be most pronounced when the second moment is finite and the third is not. Finally, and importantly in light of the empirics of Section IV below, if the process is unstable,[1] the behavior of r.s. should be erratic and extremely sample sensitive whether the moments are finite or not.

In any event, regardless of the true generating process that underlies a market return series, the financial empiricist must cope with at least two other onerous basic problems: the particular index to use and the form of the returns.[2] On the former, researchers typically use equity returns, but they still must choose from among several value weighted and several equally weighted indices. On the second point, the literature abounds with studies using simple periodic returns, while other works use the logs of (unity plus the) returns. As Fama notes, the decision to use one form or another can have a substantial impact on measured asyymetry.[3] Thus, although the log versus simple return decision should

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[1]Observed instability can have a number of causes, including nonstationary sequences of distributions (see Brennar [9]) or mixtures (see [5; 20; 26] and their citations). Moreover, the empiricist will measure instability even if the individual component distributions are themselves stable.

An important note is that the exhaustively studied stable stationary framework provides in its most general form a measure of skewness (see Levy [17]) although virtually all past finance works have used symmetry as a working hypothesis (see Fama [10, Ch. 1]). The stable distribution's measure of asymmetry is an order statistic and hence its estimate might be better behaved in an infinite moment context than r.s. Unfortunately, the estimating equation for the measure requires that individual observations be arbitrarily classified as either having been "extreme" or "not extreme." (For example, one researcher may characterize an observation as having been "extremely large" if it falls more than two standard deviations above the mean, while another may employ a criterion of five mean absolute deviations from the median.) Further, depending upon the definition of extremeness, null sets and hence uncomputable statistics could obtain.

Thus, even though estimated relative skewness may be ill-behaved, that statistic has been used for the empirics of this effort since it (i) has a precedent in the finance literature [19;23], (ii) is unambiguously defined, (iii) can be computed for any nonconstant time series, and (iv) is amenable to limited hypothesis testing.

[2]The investigator must also select a differencing interval. See Fogler and Radcliffe [14] and Smith [25] for empirical investigations, and Arditti and Levy [4] for comments on the theory of this point.

[3]See [10], especially the vivid illustration on page 31.

be motivated by theoretical considerations, the comparative distributional characteristics are of interest until the field reaches a consensus regarding the correct form.

IV. Empirical Results

In order to investigate the impacts of the three decision variables itemized above, monthly data covering January 1927 to December 1976 have been used to estimate the relative skewness of both the simple and log return forms of both the equally and value weighted market[4] indices from Chicago's Center for Research in Securities Prices (CRSP) for various time periods. Specifically, the entire 50-year period has been divided into five 120-month subperiods, so that the five beginning dates are January 1927, January 1937, and so forth, to January 1967. The statistic has been computed for periods that increase in length by 120 months, so the five ending dates are December 1936, December 1946, and so forth.[5] The significance of each computed value has also been checked against a null hypothesis of a normal distribution. All of the results are reported in Table 1.

The Table 1 data strongly support the notion of a nonstationary, and hence overall unstable, time series. Although a beginning date of January 1927 results in measured statistics that are not especially sensitive to the ending date, this is not true for the other initialization points. For example, a researcher with 10 years of data beginning in January 1957 using logs of the equally weighted index would find it to be so negatively skewed (-.51) that the null hypothesis of a Gaussian underlying distribution could be rejected beyond the one percent level. The investigator could do the same if 20, rather than 10, years of data were in hand, but would do so based on the magnitude of *positive*, rather than negative, skewness (+.25).

Comparing Table 1's panel a to panel c and b to d provides insights into impacts of index selection. Although the comparative magnitude, significance and even sign of the measured asymmetry are often sensitive to starting and ending dates, without fail the measured asymmetry of the value weighted index is smaller than for the equally weighted. Since (definitionally) less is invested

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[4] According to Roll [22], referring to these return series as "market" indices is incorrect. They are in point of fact only NYSE equity returns. Thus, the corruption of using the term "market returns" is hereby noted, although such does not negate interest in the topic, given the volume of past, ongoing, and in all likelihood future research using the measures.

[5] The selection of 120 months was arbitrary. However, different subperiods do not result in changes in the generalizations offered below.

TABLE 1

MEASURED MARKET SKEWNESS

Equally Weighted Starting Date January 19--	Ending Date: December 19-- 36	46	56	66	76
a. Simple Returns					
27	1.75*	1.54*	1.77*	1.92*	1.85*
37		.27#	.28#	.28*	.42*
47			-.24#	-.29*	.48*
57				-.35*	.63*
67					.78*
b. Log Returns					
27	.80*	.50*	.54*	.57*	.57*
37		-.43*	-.55*	-.57*	-.30*
47			-.36*	-.43*	.13
57				-.51*	.25*
67					.43*
c. Value Weighted Simple Returns					
27	.60*	.41*	.37*	.37*	.37*
37		-.51*	-.63*	-.62*	-.40*
47			-.22#	-.35*	-.09
57				-.53*	-.01
67					.20#
d. Log Returns					
27	-.07	-.30#	-.42*	-.46*	-.41*
37		-1.01*	-1.14*	-1.11*	-.81*
47			-.30*	-.46*	-.26*
57				-.64*	-.20#
67					.01

and * denote significance beyond the .05 and .01 levels, respectively.

in small firms with the value weighted index, these small firms being more asymmetric than large firms could explain the observed comparative skewness.

Insights into the impacts of selection of return form are provided by comparison of panels a and b and of c and d. Although (again by definition) the log returns will be less asymmetric than the simple returns, the changes in relative magnitude are often striking. Indeed, in several instances a significantly positively asymmetric series is changed to significantly *negative* when logs are taken.

V. Summary and Implications

This effort has been nearly exclusively empirical in nature, a tack essentially dictated by the results of the various tests. In other words, the results of the computations are so mixed that little extant theory cannot be rejoined (or supported, for that matter) with some of the data. The only expected pattern that seems always to be observed is less measured asymmetry when log returns are used rather than simple returns. However, this result is not a confirmation of any "theory," but is instead a "confirmation" of a mathematical definition.[6]

This may justify broader inquiries into the phenomena of interest. First, development of the theoretical constructs and empirical tests of the "causes" of asymmetry (or lack of it, as the case may be) are needed. Moreover, if asymmetry does exist, studies of its implications would be appropriate. Examples might include behavioral guides for specialists (in terms of inventory holdings), appropriate margin requirements, and the execution of block trades. Perhaps more valuable would be development of pricing characterizations that are [27] "distribution free." All of these prospects would seem to be worthy of the field's attention.

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[6]At least this result adds some confidence as to the correctness of the computations. Some further confirmation is had when note is made that the computed skews that underlie the tables are consistent in sign with those in previous works. In particular, with monthly data, Kraus and Litzenberger [16, p. 1097] found positive market asymmetry over 1936-1970, while Simkowitz and Beedles [23, Table 2] found negative for 1945-1965. With quarterly data, Fogler and Radcliffe found [14, p. 488] negative skewness for 1949-1969, while Simonson (who used quarterly log returns) found [24, p. 387] negative values for 1961-1965, 1966-1970, and 1961-1970. Fogler and Radcliffe used a semiannual differencing interval and found negative skewness for 1949-1969, positive for 1950-1969, and positive skewness for 1950-1969 using annual data. Finally Arditti [2, p. 911] found positive skewness for 1954-1963 using annual data.

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