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A STUDY OF VARIOUS DEVICES FOR CONTROLLING VIBRATING FLOOR SYSTEMS

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By
William C. Lyons

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THE UNIVERSITY OF KANSAS CENTER FOR RESEARCH IN ENGINEERING SCIENCE
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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

During the past decade the search for modern, economical, building construction techniques has led to the development of the steel joist-concrete slab floor system. The steel joist used in the above floor system is made of hot-rolled steel angles and bar stock. The upper chord of the steel joist consists of two lengths of angles; the lower chord, of two lengths of steel bar stock. The upper and lower chords are welded to a webbing of bent steel bar stock. This simple steel joist construction is light in weight, relatively inexpensive to mass produce, and is easy to handle at construction sites. To complete the total steel joist-concrete slab floor system, centering or corrugated steel sheets are welded to the upper chords of the steel joist. The concrete slab is then cast upon the corrugated steel sheet. The use of this type of floor system has eliminated many costly operations, such as the use of concrete forms to cast the concrete slab-joist system and the elimination of work stoppage due to the time required for concrete curing. In this investigation the structural soundness of the steel joist-concrete slab floor system is not in question.

The successful use of this floor system in the modern buildings of today is proof of its structural ability.

The replacement of many of the concrete and wooden parts of the old style floor systems by more advanced steel joist and concrete slab construction has led to a decrease in the dimensions and the massiveness of the new steel joist-concrete slab floor systems. This change results in relatively no loss in strength. In fact, usually a greater reserve strength is realized. The steel joist-concrete slab floor system, having less mass, requires less force to deflect and excite small vibrations. Small vibrations, although causing no structural damage, may cause discomfort to the human occupants on the floor system if within certain ranges of frequency and amplitude.[2] The major exciting force of these vibrations is the movement of humans on the floor systems, where every footfall is an impact force which excites the floor systems. This problem is common to all modern construction, especially that utilizing prestressed concrete and high-strength steels.

The Object

For the past three years a research program has been conducted by Dr. Kenneth H. Lenzen for the Steel Joist Institute. The research program has been conducted at the University of Kansas "Center of Research in Engineering Science". The executive director of the Center being Dr. John S. McNown. The program has been composed of three definite phases. The first phase was to derive an analytical method for computing the natural frequencies of vibrating steel joist-concrete slab floor systems. It was found from this initial phase of investigation that the natural frequency of vibration of the steel joist-concrete slab floor system can be closely approximated by the "T-Beam" analogy for normal construction. More refined methods are desirable for unusual construction. The second phase was to investigate the human sensitivity to small vibrations of floor systems. These first two phases of research were conducted by James A. Wiley[1] and Joseph E. Keller[2] respectively. It was found from the study of the first two phases of research that excellent correlation existed between the theoretical investigation and the practical field investigation.

The third phase was to investigate the possibility of damping or eliminating the vibrations in those floor systems already constructed. Out of this third phase of research, a damped vibration absorber was developed and used with great success in experiments by Gerald W. Barr[3]. This damped vibration absorber (Figure 6) consists of a spring-mass-dash-pot system which can be attached underneath the floor between or on the upper chords of the joists. The system can be designed completely within the joist system. When the floor system is excited by an impact force, the energy is transferred to the mass of the damped vibration absorber and then dissipated as heat by the viscous dash pot before the energy can be transferred back to the floor system.

This paper is a continuation of the third phase of the research program. To be specific, to investigate many possible devices that could damp or eliminate the vibrations of the steel joist-concrete slab floor systems.

The Problem

The problem is to investigate economical methods of damping or eliminating vibrations in those floor systems already constructed. The following are the principal areas of investigation:

1) The action of the damped vibration absorber in altered floor system.

2) The energy absorbing characteristics of

standard ceiling designs which are attached to the floor systems.

3) The damping due to various cable attachments in floor systems.
4) The damping due to various bridging arrangements.
5) The feasibility of increasing the rigidity of the floor systems.

Each of the five vibration controlling devices was tested on an experimental floor system. In some cases, analytical solutions were obtained. These solutions coupled with the experimental data gave a good indication of the practicality of each vibration controlling device.

CHAPTER II

DETAILED CONSTRUCTION AND ALTERATIONS

The test floor is located in the "Center for Research in Engineering Sciences" at the University of Kansas at Lawrence, Kansas. This floor was constructed to evaluate methods of damping steel joist-concrete slab floor systems. It was designed for maximum vibration from excitation and minimum damping.

The test floor was constructed with nine (9) No. SJ-123 joists which have a nominal depth of 12 inches and an approximate weight per foot of 4.94 pounds. The joists have an overall span of 24 feet 8 inches, were simply supported, and placed on 24 inch centers. The large span to depth ratio of the joists gave the test floor system minimum structural damping. The nine (9) joists support a concrete floor 2½ inches thick, cast on corrugated sheet steel. The floor system was supported by end walls 42 inches high built of concrete blocks. In the original construction the two long sides of the test floor system were left unsupported. A more detailed discussion of the original test floor system is given in Reference 3.

To obtain more complete and practical experimental data, the experimental test floor system was altered so that the two long, unsupported sides of the floor could be supported by I-beams (See Figure 2). This would give a test floor system which is often found in actual practice. One I-beam 23 feet long, 8 inches in depth, and weighing 13 pounds per foot was placed under each of the two unsupported sides of the test floor. One end of each I-beam was placed on the edge of a concrete block in such a way that the upper surface of the I-beam was not in contact with the lower surface of the floor system (See Figure 3). The other end of each I-beam was free to be lifted by a scissor jack into contact with the lower surface of the floor system. In this way the total length of each of the two long sides of the floor system can be supported (See Figure 2). Figure 4 shows the I-beam on one side of the floor before it is lifted into contact. Figure 5 shows the I-beam lifted into contact.

<u>Testing Procedure</u>

Vibrations in the test floor system were produced by hanging a weight of 170 pounds from the bottom chord at the center of the floor and releasing it suddenly. This gave the desired impact effect. When the weight was released, vibrations were induced in the floor system which in turn produced a signal in a seismic pickup located at the center of the floor. The signal from the seismic pickup was amplified by a Consolidated Vibration Meter to indicate the time-displacement graph of the center of the floor system. A Brush Oscillograph permanently recorded these displacements. Figures 7 and 8 show, respectively, the seismic pickup and the instrument complex.

The initial tests conducted on the floor were those to obtain the natural frequency and the amount of structural damping of the floor system. First, the I-beams were left down, not in contact with the floor's under surface. The test floor was considered an anisotropic plate, supported on two opposite sides and free on the other two sides. The natural frequency obtained from this test was 7.00 cps. Also for this system, the coefficient of solid damping, or structural damping factor, was equal to 0.00368. The mathematical development and explaination of the structural damping factor is presented in Appendix A of this paper.

The above natural frequency value is higher than the 6.67 cps. which was obtained in experiments conducted earlier on the same floor system.[3] The difference in the two frequency values was due to the additional curing of the floor between the two series of tests. The long curing time would have a tendency to increase the stiffness or modulus of elasticity, E, of the concrete, thereby increasing the frequency of the floor system. In this case the modulus of elasticity of the concrete increased 0.01%. Although the 7.00 cps is higher than the previous frequency obtained, the value is still close to the calculated one of 6.77 cps in which E of concrete was assumed to be 3 x 106 pounds per square inch.

Next, the I-beams were raised by the scissor jacks until the free edges of the floor were completely supported. Then, the natural frequency and the structural damping factor were obtained in the same way as before. The natural frequency for the altered (i.e., I-beams up) system is 7.41 cps; the structural damping factor, 0.00600. The higher values of the structural damping factor indicate more damping in the system.

The same general testing procedure was used to determine the structural damping factor for all the damping devices discussed in this paper.

CHAPTER III

VIBRATION ABSORBER APPLIED TO THE ALTERED FLOOR SYSTEM

The damped vibration absorber was to be tested for its damping ability on a floor system that was to be simply supported on two opposite sides and simply supported by I-beams on the other two sides. This floor system, which is typical of much of the present day construction, was obtained by altering the original floor system which was used in the initial investigations of the vibration absorber. The alteration of the original floor system is described in Chapter II of this paper.

From the initial investigations of the damped vibration absorber it was found that six units, which were damped with SAE #20 motor oil, provided the most effective damping for the test floor system. These units approached the theoretical optimum with a mass ratio of 0.02 and a structural damping factor of 0.150 or a viscous damping factor of 0.075 (See Appendix A). The mass ratio is that ratio between the total mass of the vibration absorber units and the floor system mass.

The same set of effective damping components (i.e. six units with SAE #20 motor oil) were applied to the altered floor system to compare these results to those obtained from the earlier investigation on the original floor system. From Figures 18 and 19 it can be seen that the six vibration absorbers damp vibration effectively on the altered floor system. The structural damping factor of the altered floor system with the above damping components was 0.170.

This test indicates that the damped vibration absorber, which is designed for use on a floor system which is simply supported on two opposite sides and free on the other two sides, is slightly more effective on the altered floor system. This is because the damped vibration absorber units were originally tuned at a frequency slightly below that of the original floor system. The difference in natural frequency between the damping units and the floor system would produce a phenomenon known as beating, or preform vibrations at a beat frequency. When the beat frequency is at a maximum amplitude, all energy is stored in the floor system. At zero amplitude, all the energy is stored in the damped vibration absorber units. If the natural frequency of the floor system is slightly increased, as is the case of the altered floor system, the vibration absorbers may be able to absorb energy in less time, since the zero amplitude position of the beat frequency becomes more frequent. No optimum beat frequency has been computed.

Therefore, to design damped vibration absorbers for either floor system, (i.e. the original or the altered type) the design techniques presented in Reference 3. can be used without altering the effectiveness of the device.

CHAPTER IV

DAMPING DUE TO ACOUSTIC TILE CEILINGS

The absorbing of energy in a vibrating floor system can be accomplished by the addition of a system of coulomb or friction forces to the floor in such a way that each vertical movement of the floor is opposed. A simple acoustic tile ceiling attached to the lower chords of the joists and to the side walls would act as a friction force system. In effect the addition of such a friction force system is the same as increasing the structural damping of the floor system.

The original test floor system had supporting walls only under the two simply supported edges. To obtain complete and accurate experimental data, it was necessary to construct side walls along the free sides of the original floor system. This was done by supporting, with concrete blocks, a 2 inch by 8 inch wood beam 23 feet long, running parallel to each I-beam mentioned earlier (See Figure 1).

There are two methods of attaching acoustic tile ceiling, they are the rigidly attached ceiling and the hanging or loosely attached ceiling. The former, the attached ceiling, is fastened directly below and against the lower chords of the joists

(See Figure 9). The latter, the hanging ceiling, is hung from the lower chords of the joists by wire. In the experiment the wire was approximately 4 inches long as shown in Figure 10. The installation of a ceiling is shown in Figure 11.

The installation of the above ceilings on the test floor system was done by local contractors. This was done to insure typical installation. The tile used in the installation was 12 inches by 12 inches by 3/4 of an inch rigid, dense, multicellular board made of mineral fibers.

Attached Ceiling, I-Beams Down

The attached ceiling was installed on the floor system and tested. This ceiling was tested with I-beams down, or on a floor with two free and two simply supported sides. The structural damping factor for this system was 0.00574. This is an increase in structural damping of the original floor system of 0.00206 or 56%.

Attached Ceiling, I-Beams Up

The floor system was next altered by lifting the I-beams into contact with the lower surface of the floor. Again, with the attached ceiling, the total system was tested. The structural damping factor obtained was 0.00712. This was an increase of 0.00112 or 19% in structural damping of the altered floor system.

<u>Hanging Ceiling, I-Beams Down</u>

The hanging ceiling was tested on the orginal floor system. The structural damping factor obtained was 0.00570. This was an increase of 0.00202 or 54% in the structural damping of the original floor system.

<u>Hanging Ceiling, I-Beams Up</u>

The structural damping factor for the altered floor system with the hanging ceiling was 0.00690, an increase in structural damping of 0.00090 or 15%.

Each of the above tests were rerun with the ceilings detached from the four side walls. It was found that this had no significant effect on the values presented above.

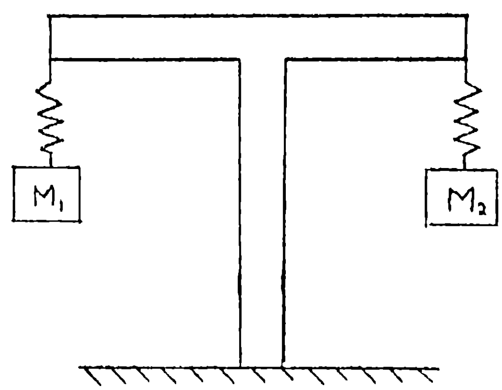
CHAPTER V

DAMPING DUE TO THE ACTION OF CABLES

From early investigations conducted by Maney and Masters and Modjeski on the subject of vibrations of suspension bridges, it was found that by placing sets of prestressed, diagonal hanging, crossed cables between the large suspension cables and the bridge floor, structural damping of the total system was increased. This additional structural damping of the bridge system is due partially to the coulomb or friction damping in each diagonal hanging cable. The coulomb damping results when the individual strands of each cable slide on one another as they are twisted and untwisted. This twisting and untwisting is due to the elongation and relaxation of each cable as the bridge floor tends to move relative to the large suspension cables during the bridge system's vertical displacements.

Also, during vertical oscillations energy is transferred from the large suspension cables and the bridge floor to the diagonal hanging cables. Under live loads or winds the diagonal hanging cables deliver components of force to the large suspension cables and bridge floor which causes these members to act to some extent as chords of a truss, tending to stiffen the bridge. When alternate diagonal cables go out of direct action, as a result of large bridge deflections, or approach that condition, they would cease to act in a purely elastic manner, and quickly tend to damp out any vibration.

A simple example of this type of energy transfer and loss can be illustrated by the simple dynamic system shown below.



M_1, only, is given an initial velocity or displacement, so that it performs vibrations about its equilibrium position. The vibration energy will be transferred through the frame to M_2. As M_1 approaches its equillibrium position, M_2 will approach maximum amplitude of vibration. After M_2 has reached its maximum amplitude, its vibration will begin to die out, while M_1 again starts to vibrate. But, in the frame as the energy is passed from one mass to the other, part of the energy will be absorbed in each transfer by internal damping forces.

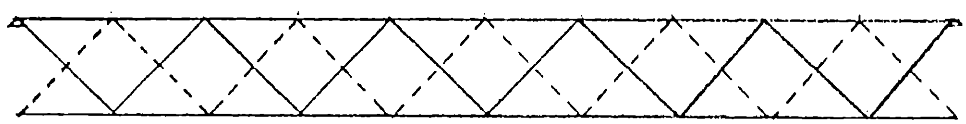
It was thought that the above damping technique could be used to damp vibrations in the steel joist-concrete slab floor system. This technique was applied to the test floor system by using cables attached in the steel joist webbing in the hope that the structural damping of the total system would be increased. The cables were applied only to the middle five joists of the test floor system. Since these were the most active joists while the floor system oscillated, the results obtained would be a good representation of the cables applied to every joist in the floor system.

Below, a typical steel joist is drawn and each configuration of cables used is indicated with dashed lines. A discription of the configuration of the cable systems is also given. All configurations were run on the original (I-beams down) and the altered (I-beams up) floor system. Also, all configurations were tested with varied tension forces.

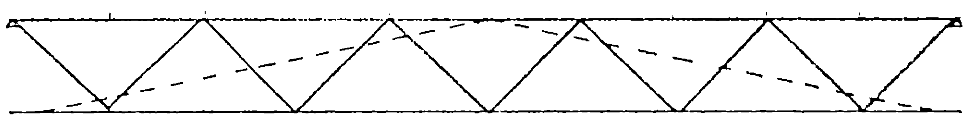
None of the cable systems indicated a measurable increase in the structural damping in either test floor system.

I The cables used in this investigation were

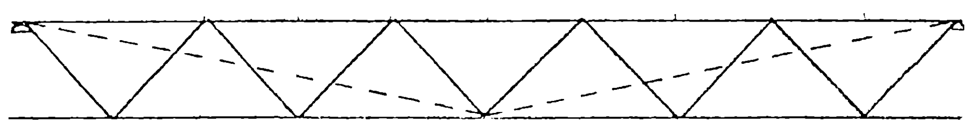
1/8" in diameter. This was the largest diameter cable that could be used in the confining space of the joist. The cables were prestressed and securly attached at all contact points along the upper and lower chords of the joist so that each cable section acted independently. A turn buckle was used to stress each cable.



II $\frac{1}{4}$" diameter cable.



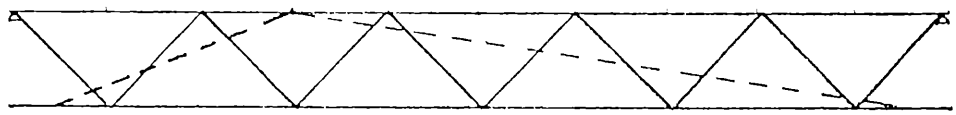
III $\frac{1}{4}$" diameter cable.



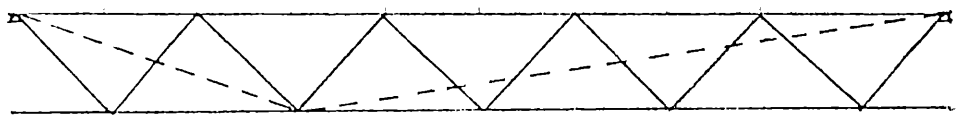
IV A) $\frac{1}{4}$" diameter cable.

B) Configurations II and III used alternatly on the five center joists with II on the center joist.

V $\frac{1}{4}$" diameter cable.



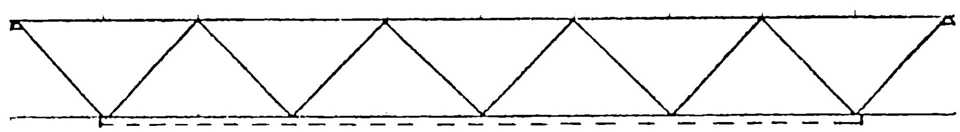
VI $\frac{1}{4}$" diameter cable.



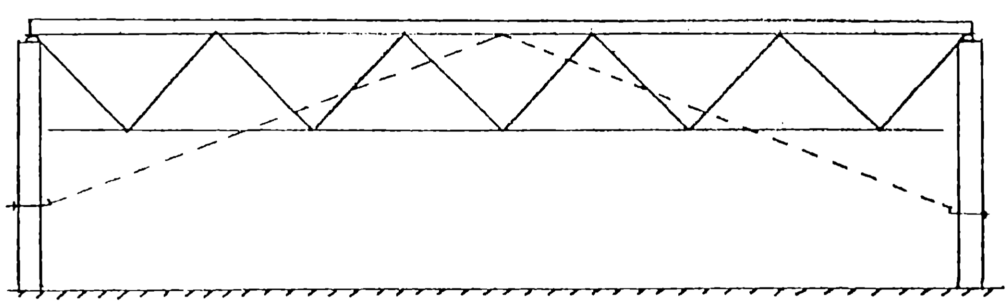
VII A) $\frac{1}{4}$" diameter cable.

B) Configuration V and VI used alternatly on five center joists with V on the center joist.

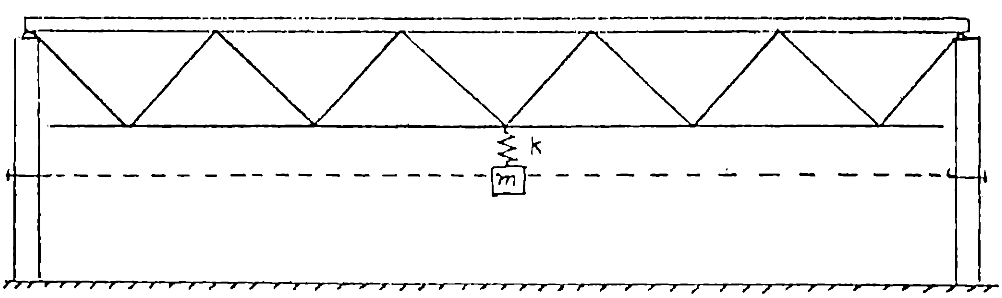
VIII $\frac{1}{4}$" diameter cable.



IX $\frac{1}{4}$" diameter cable.



Since an increase in structural damping was not observed in any of the above systems tested, it was concluded that the displacements of the vibrating floor system were far too small to bring about sufficient twisting and untwisting action of the cables. It was decided that the cables had to be attached to a system with larger displacements in order to obtain damping. The spring-mass combination used to amplify displacements for viscous damping in the vibration absorber units could be used for the same purpose in this investigation. Instead of the viscous dash pot for damping, cables in tension were attached between the mass and the side walls in such a way that any movement of the mass would increase the tension and thereby twist the cables causing coulomb damping.



Six of the spring-mass and cable units described above were applied to the original floor system (I-beams down) and tested. The structural damping factor obtained from this test was 0.00316.

The structural damping factor of the same original floor system with the spring-mass units alone (i.e. without cables attached) was 0.000682. Damping in the cables was indicated when the above two structural damping factors were compared.

The structural damping factor for the cable damped original floor system described above (i.e. 0.00316) was lower than the structural damping factor of the original floor system in its natural state (i.e. 0.00368). This indicates that the cable damped units are inadequate even though the cables did cause damping in the system.

An analytical solution for the above cable damped floor system was obtained by using the electrical analog computer and is shown in Appendix C. Although the analytical solution did not yield any immediate results to the above problem, it did emphasize the difficulty of simulating structural damping on an analog computer.

CHAPTER VI

THE EFFECT OF BRIDGING

The purpose of this investigation was to determine whether or not the structural damping of the test floor system could be increased by the addition of various types of horizontal and cross bridging.

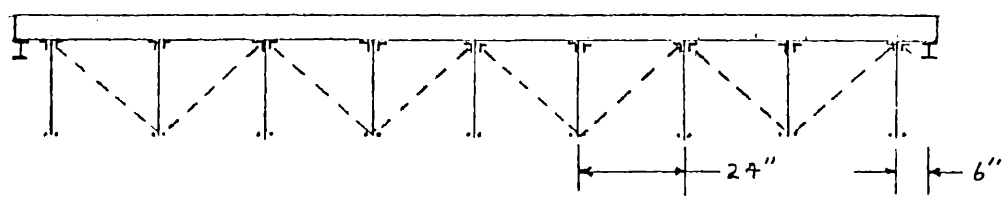
When the floor system was first constructed, horizontal bridging was bolted to the lower chords of the test floor system. This bridging caused the floor to have a natural beat frequency. The beat frequency was caused by transverse waves propagating along the horizontal bridging perpendicular to the axis of the joists. The structural damping factor for this system was 0.00369, which is almost the same as the value for the original floor system (i.e. 0.00368) with no bridging. It can be concluded that with the above type of bridging we have not gained in damping but have a more complicated wave form.

Between the horizontal bridging and the contacts with the lower chords of the test floor system, rubber pads were placed to test whether energy could be absorbed by these pads. The results of this test showed that negligible energy was absorbed. The same test was tried using insulation pads in the place of the rubber pads. This again showed negligible energy absorbtion.

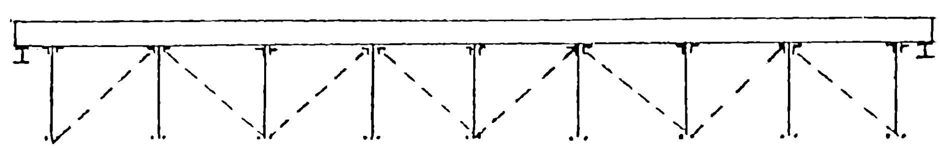
In the next test series, cables were used as cross bridging. The cable bridging configurations were run perpendicular to the axis of the floor joists. Each specific cable bridging configuration was applied at 6 foot intervals along the 24 foot length of the floor. The cables were secured to the joists at every possible point in order to let each cable section act independently. One fourth inch diameter steel cable was used in each of the configurations shown below.

The dashed lines in the sketches below indicate the cables.

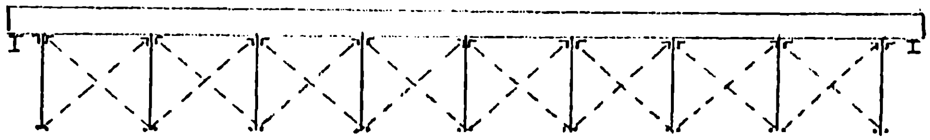
I



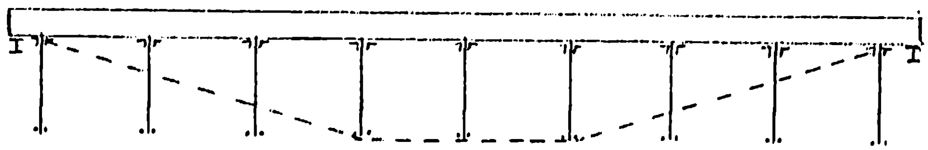
II



III



IV



The tests for the above cable cross bridging were run on the original (I-beams down) and the altered (I-beams up) floor system. None of the configurations tested increased the structural damping factor of either floor system. The reason structural damping was not increased was because the edges of the floor system deflected some even when the I-beams supported the edges. This was proven to be true when the seismic pickup was placed at the center of one of the edges when the I-beams were up in contact with the floor. Because of this fact, there was little twisting or untwisting of the cables or any extra supporting forces.

CHAPTER VII

INCREASED RIGIDITY OF FLOOR SYSTEMS

In some cases it may be desirable to have the rigidity of a floor system increased to eliminate vibrations. From the "T-beam" analogy given in Appendix D of this paper, it is seen that the original floor system can be represented by a simple supported beam under a distributed load. The static deflection of such a beam is:

$$\delta = \frac{5wL^4}{384\ EI}$$

The spring constant k_1 of the beam is represented by the load divided by the static deflection of the beam,

$$k_1 = \frac{wL}{\delta} = \frac{384\ EI}{5L^3}$$

Using the basic equation for the frequency of a mass and spring combination we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1}{M_1}} = \frac{1}{2\pi}\sqrt{\frac{384\ EI}{M_1\ 5L^3}}$$

where M_1 is the mass of the beam.

When the value of the moment of inertia (i.e.I) is increased, the deflection of the beam would decrease proportionately. The frequency increase would be proportional to the square root of the moment of inertia increase, therefore, the frequency increase would be small as compared to the deflection decrease. One can conclude that annoying vibrations would be greatly decreased, if not totally eliminated, depending on the increase in the moment of inertia.

For practical purposes the above vibration elimination could be accomplished by the welding of channels or angle sections along the lower chord of each of the joists of the floor system in question. This could only be done after the effective load was removed from the floor system. If the load was not removed then none of the new sections would carry any of the load and their dead weight would decrease the frequency and the structural damping. The removing of the effective load could be accomplished by jacking up the floor system at or near its center. When the floor system was jacked up to the unloaded state, the new sections were welded to the joists. When the jacks were removed the new sections would take a share of the load.

The above method was used with success by Mr. William Bradbury of Sheffield Steel to decrease vibrations in a floor system in the Church of Perpetual Adoration located at Omaha, Nebraska.

CHAPTER VIII

DISCUSSION

This paper has dealt with the experimental investigations of various damping devices and methods for the steel joist-concrete slab floor system. The results obtained from the investigations clearly show that only a few devices or methods are able to damp or eliminate vibrations. The use of steel cables as a damping device was found to be totally inadequate for floor systems. In addition, bridging can be eliminated as an effective method of damping vibrations. The general cause of the above failures was that the displacements and internal strains in the floor system were extremely small; and, thus the individual cables were unable to act as damping units.

The relative success of the various ceiling arrangements to absorb energy in the vibrating floor system must be taken lightly. When the case of the largest structural damping factor increase is examined (i.e. an increase of 56%), the vibrations still are not damped out in a reasonable period of time. In fact, the time required to damp the transitory vibrations is approximately 15 seconds. Thus, if a floor is highly responsive to vibrations this additional damping may not be sufficient. Floors with mild human annoyance values may be damped sufficiently to eliminate awareness.

The damped vibration absorber units damp transitory vibrations in 5 cycles or 0.7 of a second. In this way the vibration response of each step of a human on the floor system would be damped almost completely before the impact of the next step. The occupant could not sense the transitory motion caused by the preceeding step. For this reason, the damped vibration absorber units are considered the most effective device tested.

The original design procedures, developed by Gerald W. Barr, are to design damped vibration absorber units for a floor system with specific edge conditions. These conditions consist of two opposite edges simply supported and two free edges. Damped vibration absorber units designed originally for a test floor system with the above specific edge conditions were tested on an altered floor system which had a natural frequency higher than that of the original test floor system. It was found that the units were more effective in the altered floor system with a higher natural frequency. This was because the damped vibration absorber units were able to absorb energy in less time since the zero amplitude position of the beat frequency of the floor system became more frequent.

From the above results it can be concluded that an effective set of damped vibration absorber units can be designed from the original design procedures for any floor system with any set of edge conditions. For this particular problem, the natural frequency of the floor system in question is slightly higher than that frequency obtained for the floor system from the "T-beam" analogy. In all cases a floor system with any other set edge conditions other than the designed edge conditions will give a higher natural frequency.

An effective method of eliminating vibrations in floor systems is to increase the rigidity of the floor system. To increase a floor system's rigidity entails increasing the rigidity of the joists of that system. This can be done by welding to the lower chords of the joists additional sections of bar stock or angles, but only after the effective load (including dead load) has been removed from the floor system. Not all the joists need have additional sections welded to them. The number of joists reinforced is dependent upon the severity of the vibrations of the floor system in question. The practicality of this method is dependent upon the accessibility of the lower chords of the joists.

There are situations where the damping characteristics of ceilings can be used advantageously. Take for example, a floor system designed for office spaces or other small separate rooms which exhibits undesirable oscillations. This system can be sufficiently damped by applying a ceiling arrangement to the lower chords of the floor and using other natural constraints on the floor to compliment the ceiling arrangement such as wall partitions. If the same example floor system is only to be used as a meeting hall or room where no partitions can be constructed, then the only alternatives for damping the undesirable oscillations would be either the use of the damped vibration absorbers or by increasing the floor rigidity.

CHAPTER IX

CONCLUSIONS

The results of this study indicates that:

1. The dynamic damped vibration absorber has been proven by experimental means to be an effective damping device for any floor system with any set of edge conditions.
2. Increasing the rigidity of a floor system has been shown to be an effective method of eliminating undesirable transitory vibrations.
3. The increase in damping of vibrations in floor systems due to various ceiling designs is too small to be of any practical use unless coupled with other constraints on the floor system.
4. The damping of floor systems due to various cable arrangements was found to be inadequate.
5. Bridging was also found to be an inadequate damping device.

BIBLIOGRAPHY

1. Wiley, James A., A Study of the Vibration of Rectangular Anisotropic Plates by the Ritz Method., The University of Kansas Center of Research in Engineering Science, June, 1960 (Mimeographed).

2. Keller, J. E., Damping Considerations in Vibration Response of Humans., The University of Kansas Center for Research in Engineering Sciences, May, 1960 (Mimeographed).

3. Barr, Gerald W., Vibration Damping of Anisotropic Plates., The University of Kansas Center for Research in Engineering Science, May, 1961 (Mimeographed).

4. Lenzen, K. H. and Keller, J. E., Vibration of Steel Joist-Concrete Slab Floor System., The University of Kansas Center for Research in Engineering Sciences, September, 1959, (Mimeographed).

5. Modjeski and Masters, "Supension Bridges and Wind Resistance", Engineering News-Record, October 23, 1941, Vol. 127, p. 565.

6. Maney, Geo., "New Type Supension Bridge Processes", Engineering News-Record, April 24, 1941, Vol. 126, p. 609.

7. Timoshenko, S., Vibration Problems in Engineering, Third Edition, D. Van Nostrand Co. Inc., New York, New York, 1955.

8. Scanlan, R. H., and Rosenbaum, R., Aircraft Vibrations and Flutter. The MacMillan Company, New York, New York, 1951.

9. Myklestad, N. O., Fundementals of Vibration Analysis, McGraw-Hill Book Company, Inc., New York, New York, 1956.

10. Thomson, W. T., Mechanical Vibrations, Second Edition, Prestice-Hall, Inc., Englewood Cliffs, N. J., 1956.

11. Courant, R., and Hilbert, D., Methods of Mathematical Physics, Vol. I, Interscience Publishers, Inc., New York, New York, 1953.

12. ------------- Open Web Steel Joists, Sheffield Steel Corporation, Division of Armco Steel Corporation, Member of Steel Joist Institute.

13. ------------- Sound Control Ceilings, Johns-Manville, (Litho).

APPENDIX A

THE DAMPING FACTOR

The decay of amplitude in free vibrations is due to dissipation of energy due to damping force. It is of interest to determine the nature of the damping force and the energy dissipated by it. In the general case of a harmonic force lagging the displacement by a phase angle ϕ , the displacement and force may be represented as:[10]

$$x = X_o \sin \omega t \qquad (A.\ 1)$$

$$F = F_o \sin(\omega t - \phi) \qquad (A.\ 2)$$

The work done by this force per cycle of motion becomes

$$\Delta W = \int F\ dx = \int_0^{\frac{2\pi}{\omega}} F_{\phi} \frac{dx}{dt} \cdot dt$$

$$= \omega F_o X_o \int_0^{\frac{2\pi}{\omega}} \cos \omega t \sin (\omega t - \phi) dt$$

$$= - \pi F_o X_o \sin \phi \qquad (A.\ 3)$$

The maximum energy dissipated is when the phase angle is 90^o, or when the force is lagging the displacement by 90^o.

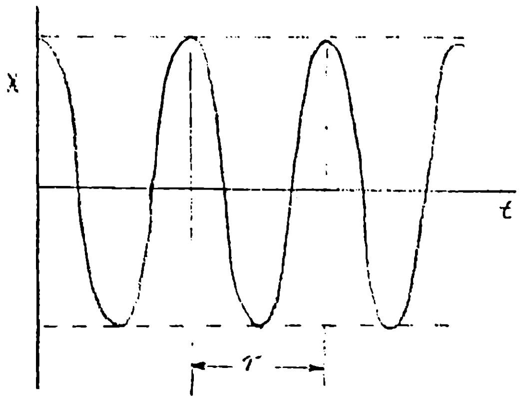


Figure A-1
Undamped oscillations.

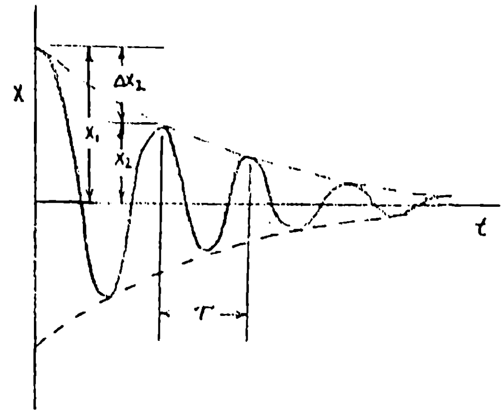


Figure A-2
Damped oscillations.

The amount of damping is often specified by giving the logarithmic decrement δ, where:

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} \cdots\cdots = \frac{x_{n-1}}{x_n} = e^{\delta}$$

which can be written

$$\frac{x_1}{x_n} = \left(\frac{x_1}{x_2}\right)\left(\frac{x_2}{x_3}\right)\left(\frac{x_3}{x_4}\right)\cdots\cdots\left(\frac{x_{n-1}}{x_n}\right) = \left(e^{\delta}\right)^n = e^{n\delta}$$

In general,

$$\delta = \frac{1}{n} \ln \frac{x_1}{x_n} \tag{A. 4}$$

For systems having small damping,

$$\delta = \ln \frac{x + \Delta x}{x} = \frac{\Delta x}{x} - \frac{1}{2}\left(\frac{\Delta x}{x}\right)^2 + \frac{1}{3}\left(\frac{\Delta x}{x}\right)^3 + \cdots$$

and, therefore,

$$\delta \approx \frac{\Delta x}{x} \tag{A. 5}$$

Assuming Hooke's law to be valid, the total vibrational energy becomes proportional to the square of the amplitude, and can be expressed as

$$W = Kx^2 \qquad (A.\ 6)$$

in which K is a constant of the system.

To determine the energy dissipated per cycle, the vibrational energy is considered a cycle later, and is

$$K\,(x - \Delta x)^2 = W - \Delta W$$

From this and equation (A. 5),

$$\delta = \frac{\Delta x}{x} = \frac{\Delta W}{2W} \qquad (A.\ 7)$$

This enables one to determine the energy dissipated per cycle when δ is known.

The three types of damping forces discussed in this paper are structural damping, viscous damping and coulomb damping.

Structural Damping

Structural damping[10] is due to internal friction within the material itself. Experiments indicate that this type of damping is independent of frequency and proportional to the maximum stress of the vibration cycle. Since stress and strain are proportional to one another in the elastic range, it can also be stated that structural damping force is proportional to displacement. Such a damping force proportional to displacement, but independent of frequency can be expressed by the equation

$$F = \gamma kx \qquad \text{(A. 8)}$$

in which k is the spring constant of the material and γ is the constant of proportionality written as a nondimensional damping factor.

For structural damping where γ is assumed small, the energy dissipated during the cycle is given by equation (A. 3).

$$\Delta W = \pi F_o X_o \sin \phi = \pi \gamma k X_o^2$$

in which $F_o = \gamma$ kx, and is independent of the frequency.

Since

$$W = \tfrac{1}{2} k X_o^2$$

then

$$\delta = \frac{\Delta W}{2W} = \frac{\pi \gamma k X_o^2}{k X_o^2} = \pi \gamma \qquad \text{(A. 9)}$$

Viscous Damping

Viscous damping is encountered by bodies moving at moderate speeds through a fluid. This type of damping leads to a resisting force proportional to the velocity,

$$F = -cv$$

in which c is a constant of proportionality and v the velocity.

The viscous damping factor ζ is defined in terms of a nondimensional ratio,

$$\zeta = \frac{c}{c_c} \tag{A. 10}$$

in which c_c is the critical damping coefficient.

The relation between the viscous damping factor and the logarthmic decrement in which $\zeta < 1$ can be expressed mathematically as

$$\delta \cong 2\pi\zeta \tag{A. 11}$$

The derivation of this value can be found in any vibration book.

<u>Coulomb Damping</u>

Coulomb damping arises from the sliding of dry surfaces. This type of friction force is nearly constant and depends on the nature of the sliding surfaces and the normal pressure between them. This is expressed by the equation of kinetic friction

$$F = \mu N \tag{A. 12}$$

in which μ is the coefficient of kinetic friction and N, the normal force.

In this paper various damping devices were applied to the test floor system in an attempt to increase the damping in that structural system. Since this is a structural system in which damping is being investigated, then the structural damping factor will be used to describe this damping.

The structural damping factor γ is used in this paper to describe damping in the structural floor system regardless of the type (coulomb or viscous) of damping device applied to the test floor.

Given experimental data, such as that given below in Figure A-1 and in Appendices D and E, one can obtain the damping factor from the formulas obtained earlier in this appendix.

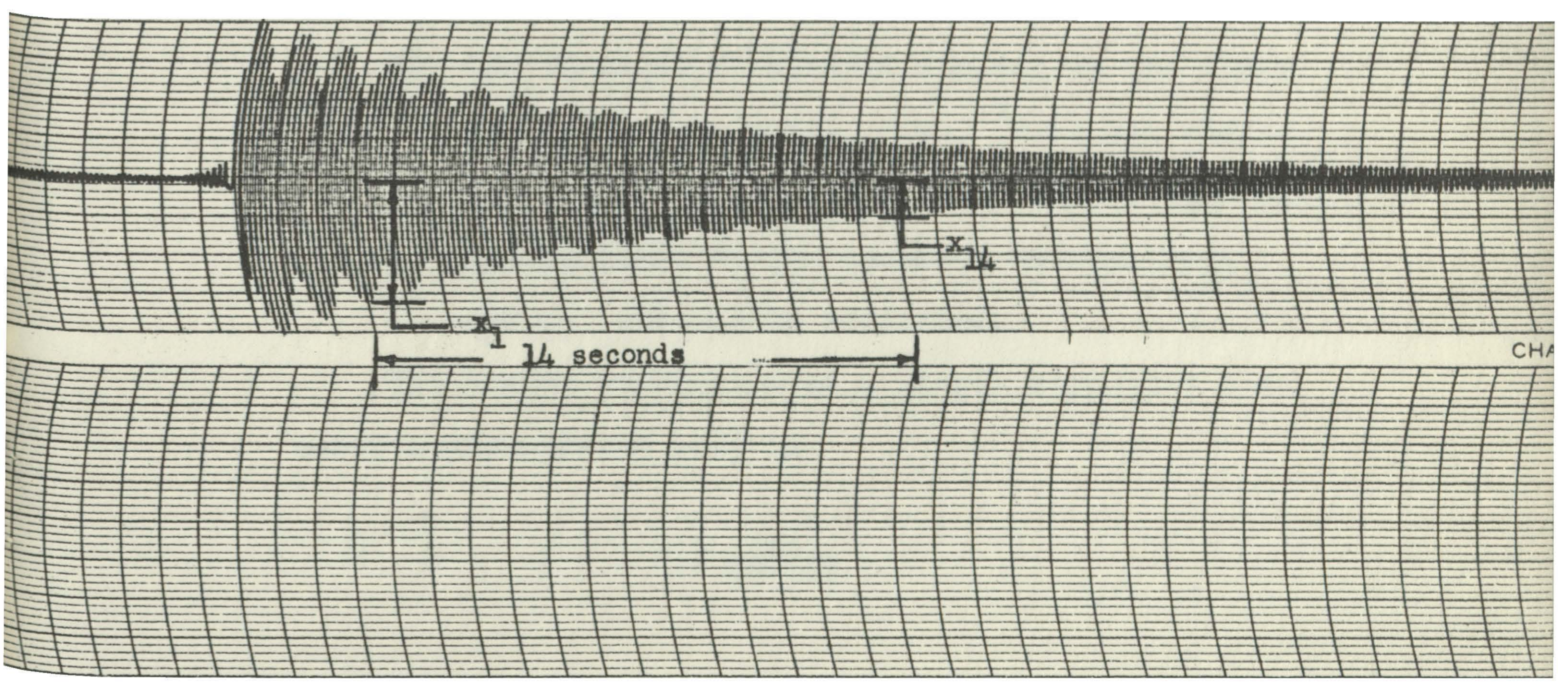


Figure A-1

From equation (A. 4) the logarithmic decrement

$$\delta = \frac{1}{n} \ln \frac{x_1}{x_n}$$

$$\delta = \frac{1}{14(7.00)} \ln \frac{3.10}{1} = 0.0116$$

in which the system tested is the original floor system described in Chapter I. The original floor system has a natural frequency of 7.00 cps. Multiplying the natural frequency of the system by the number of seconds between x_1 and x_n we obtain the number of cycles in that interval (i.e. n).

Since

$$\delta = \gamma \pi$$

then, γ for this system is

$$\gamma = \frac{0.0116}{\pi} = 0.00368$$

APPENDIX B

NATURAL FREQUENCY OF THE TEST FLOOR

In determining the natural frequency of the test floor it was assumed that;[2]

1) The floor may be represented as a series of simple "T-Beams". Each beam consists of a web of the steel joist and a flange of the concrete floor of width equal to the distance between joists,
2) Full interaction occurs between the joist and the slab,
3) The ends are simply supported,
4) The centering and steel mesh are neglected,
5) The thickness of the concrete was measured to the bottom of the corrugated centering, and,
6) The internal structural damping of the floor system is very small, i.e. $\gamma^2 << 1$ and therefore the damped natural frequency is approximately equal to the undamped natural frequency, and may be neglected.

The static deflection of a simply supported beam under a distributed load is

$$\delta = \frac{5wL^4}{384\ EI}$$

and the spring constant, k_1, of a beam is equal to the impressed load divided by the static deflection of the beam,

$$k_1 = \frac{wL}{\delta} = \frac{384\ EI}{5L^3}$$

Substituting in the values

$E = 30 \times 10^6$ psi

$I = 726.00\ in^4$ (I from Appendix D of reference 3).

$L = 295$ in

the spring constant of the floor system is obtained

$$k_1 = 65{,}200 \text{ lbs./in.}$$

Using the basic equation, from any book on vibrations,

$$\omega_1^2 = k_1/m_1$$

where $m_1 = 13960/386 = 36.15$ lbs.-sec.^{2} in.

the undamped natural frequency of the floor is

$$\omega_1 = 42.5 \text{ Rad/sec.}$$

or since

$$f_1 = \omega_1/2\pi$$

the natural frequency of the floor is found to be

$$f_1 = 6.77 \text{ cps}$$

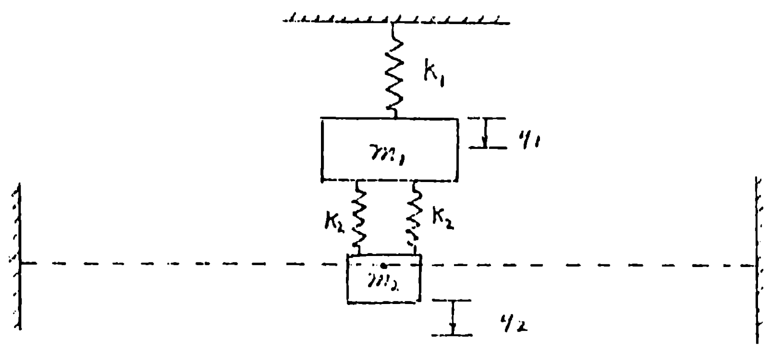
APPENDIX C

ANALOG SOLUTION OF THE CABLE DAMPED VIBRATION ABSORBERS

The floor system will be considered as a single degree of freedom spring-mass system capable of vertical motion only. In reality a floor has an infinite number of degrees of freedom with an infinite number of modes and frequencies of vibration, but because of the inability of the human occupant to detect these higher frequencies of vibration and smaller amplitudes, the above assumption is valid. The following is also assumed:

1) The time duration of impact is great enough that the floor and the cable damped vibration absorbers are merely given an initial displacement and released at time equal to zero,
2) After the initial impact, the floor will vibrate freely in the vertical plane at its resonate frequency,
3) With the addition of the cable damped vibration absorbers, the system will be a linear two degree of freedom system.

The coupled system is represented in the figure below, in which m_1 and k_1 are mass and spring constant of the floor as determined before, and m_2 and k_2 are the mass and spring constant of the vibration absorber units. The cables have some initial tension S. The datum planes $y_1 = 0$ and $y_2 = 0$, are the static equilibrium positions of the system; therefore, the force of gravity acting on the system may be neglected. The downward direction is considered positive.



By initially deflecting the entire system in the positive direction from its equilibrium position the equations for the kinetic and potential energies of the system can be determined;

$$T = Tm_1 + Tm_2 + T_c$$

$$V = Vk_1 + Vk_2 + V_c$$

in which

T =total kinetic energy of system

Tm_1=kinetic energy of $m_1 = \frac{1}{2}m_1\dot{y}_1^2$

Tm_2=kinetic energy of $m_2 = \frac{1}{2}m_2\dot{y}_2^2$

T_c =kinetic energy of cable

V = total potential energy of system

Vk_1 = potential energy of $k_1 = \frac{1}{2}k_1 y_1^2$

Vk_2 = potential energy of $k_2 = k_2(y_2 - y_1)^2$

V_c = potential energy of cable.

Assuming only the first mode of vibration for the cable (i.e. $y = y_2 \sin \frac{\pi x}{2L}$).

$$T_c = \frac{w}{g} \int_0^L \left(\frac{dy}{dx}\right)^2 dx = \frac{wL}{2g} \dot{y}_2^2$$

and

$$V_c = \frac{S}{2} \int_0^L \left(\frac{dy}{dx}\right)^2 dx = \frac{S y_2^2 \pi^2}{8L}$$

in which w is weight per unit length of cable.

Then,

$$T = \frac{wL\dot{y}_2^2}{2g} + \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2$$

$$V = \frac{S y_2^2 \pi^2}{8L} + \frac{1}{2} k_1 y_1^2 + k_2(y_2 - y_1)^2$$

The equations of motion for the system are obtained by the Lagrangian equation.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_S}\right) - \frac{\partial T}{\partial q_S} + \frac{\partial V}{\partial q_S} = 0$$

which gives

$$\ddot{y}_1 = \frac{2k_2}{m_1} y_2 - \left(\frac{k_1}{m_1} \quad \frac{2k_2}{m_1}\right) y_1$$

and

$$\ddot{y}_2 = \frac{2k_2 g}{(wL \ m_2 g)} y_1 - \left[\frac{S g \pi^2}{4(wL \ m_2 g)L} \quad \frac{2k_2 g}{(wl \ m_2 g)}\right] y_2$$

The above system of differential equations were run on the analog computer.

The solution, Figure C-1, shows that the system will vibrate at the same heat frequency regardless of the tension in the cables. The first test was run with negligable cable tension and the second with 1000 pounds cable tension.

These results emphasize that the increased damping obtained by experimental investigations of this same system was an internal damping in the cables themselves. The nature of this damping is such that it could not be represented on the analog computer.

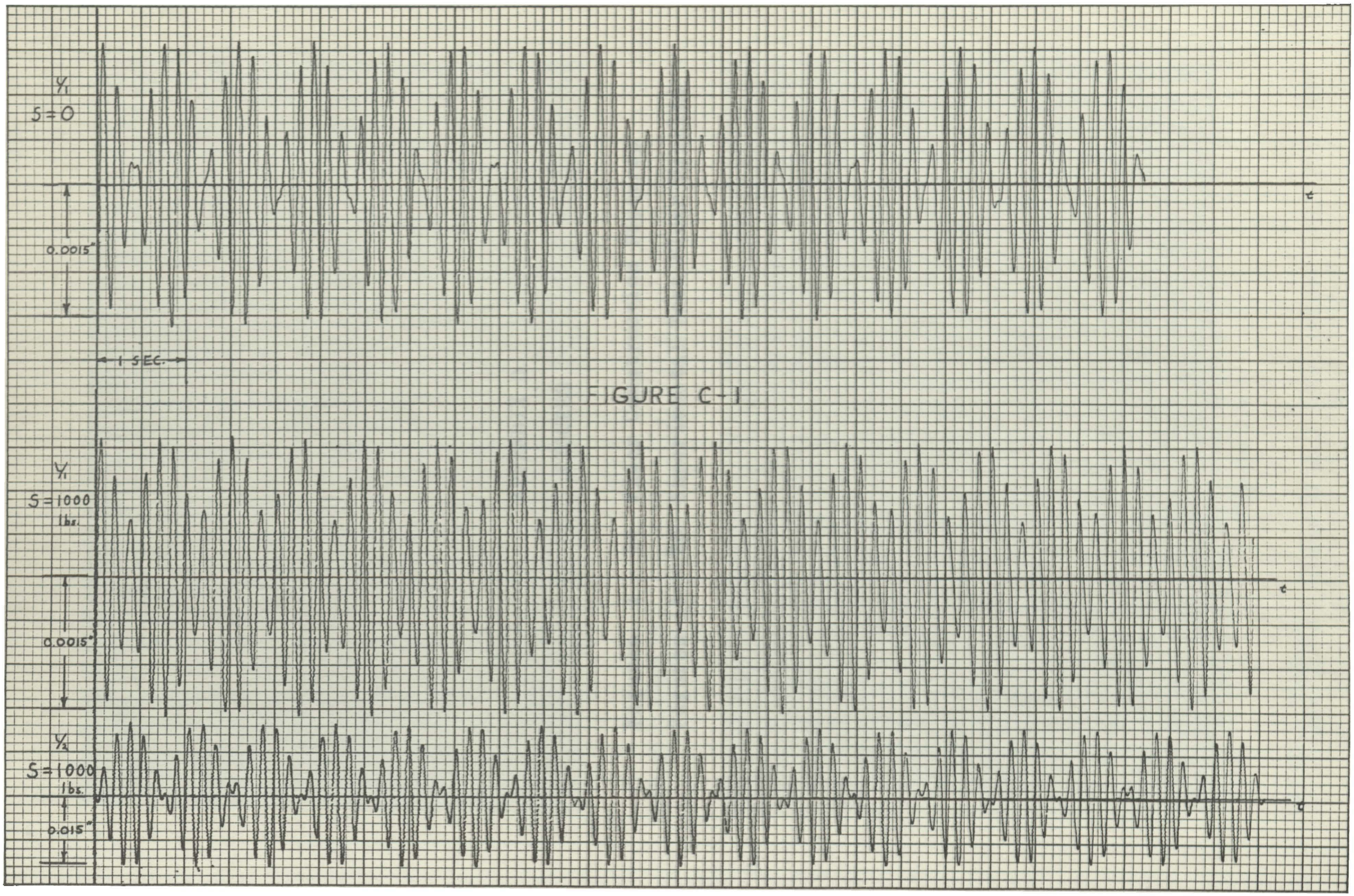


FIGURE C-1

APPENDIX D

FIGURES

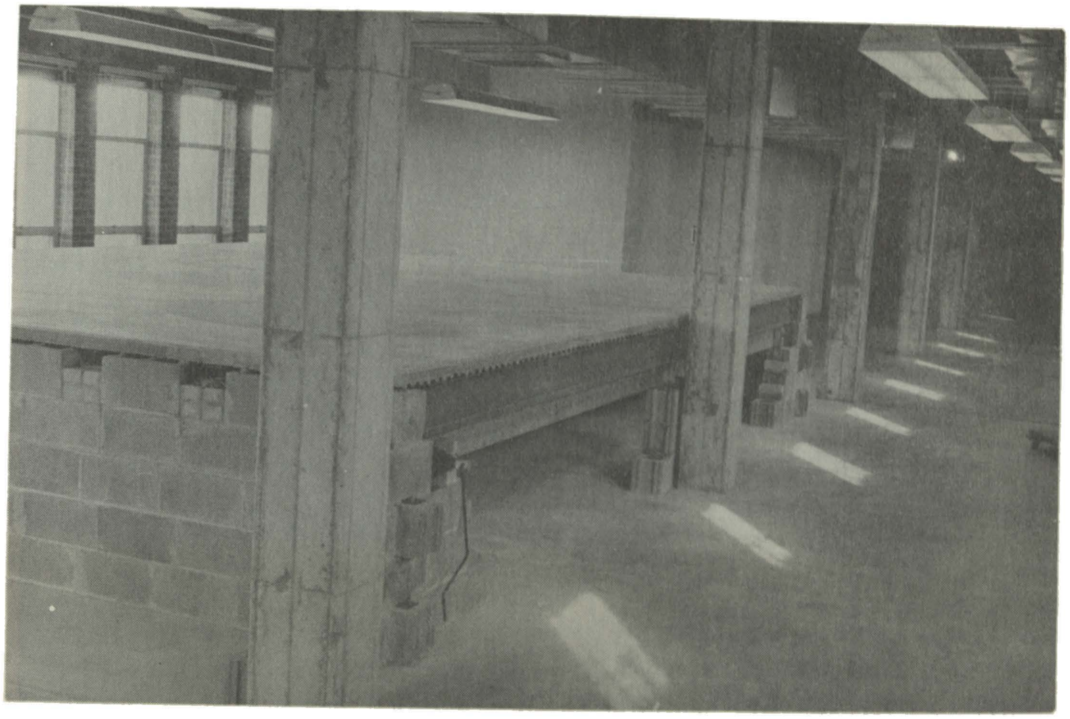


Figure 1

General view of
test floor system with I-beam supporting free edge.

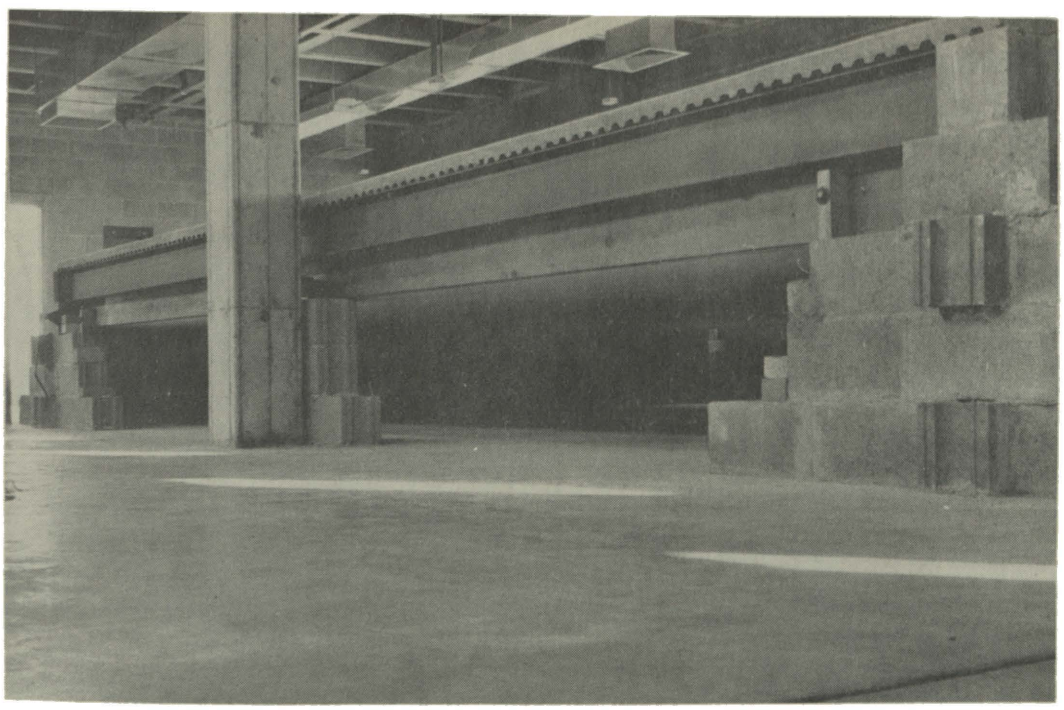


Figure 2

General view of
test floor system with I-beam supporting free edge
and side walls with ceiling attached.

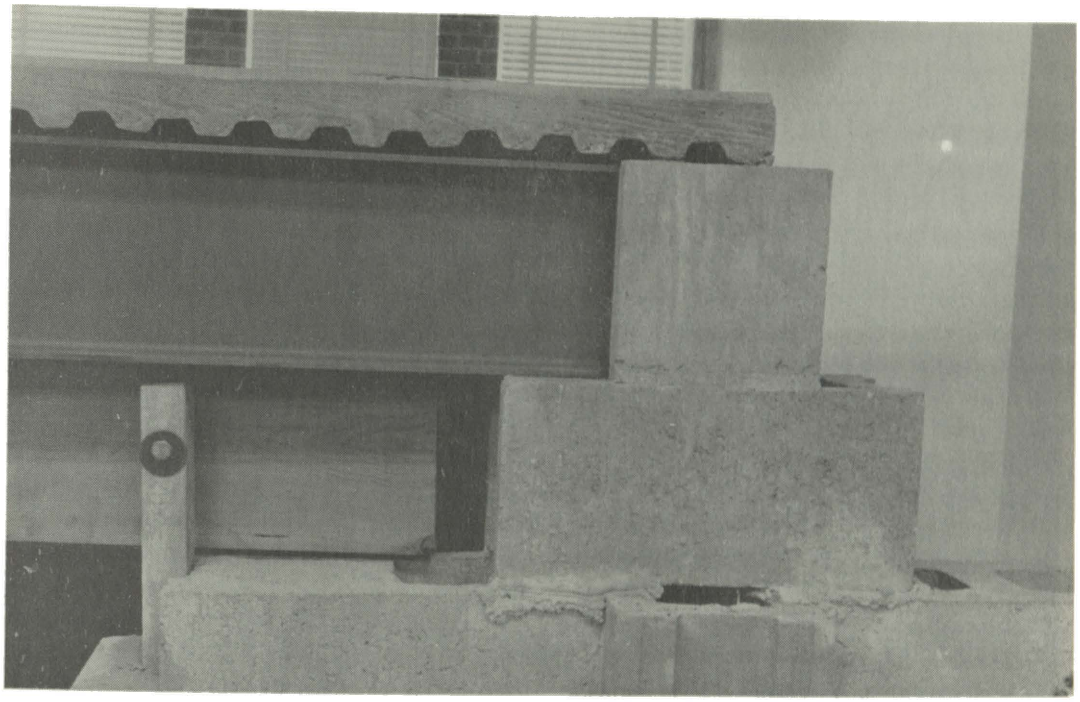


Figure 3

I-beam supported on one end by concrete blocks.

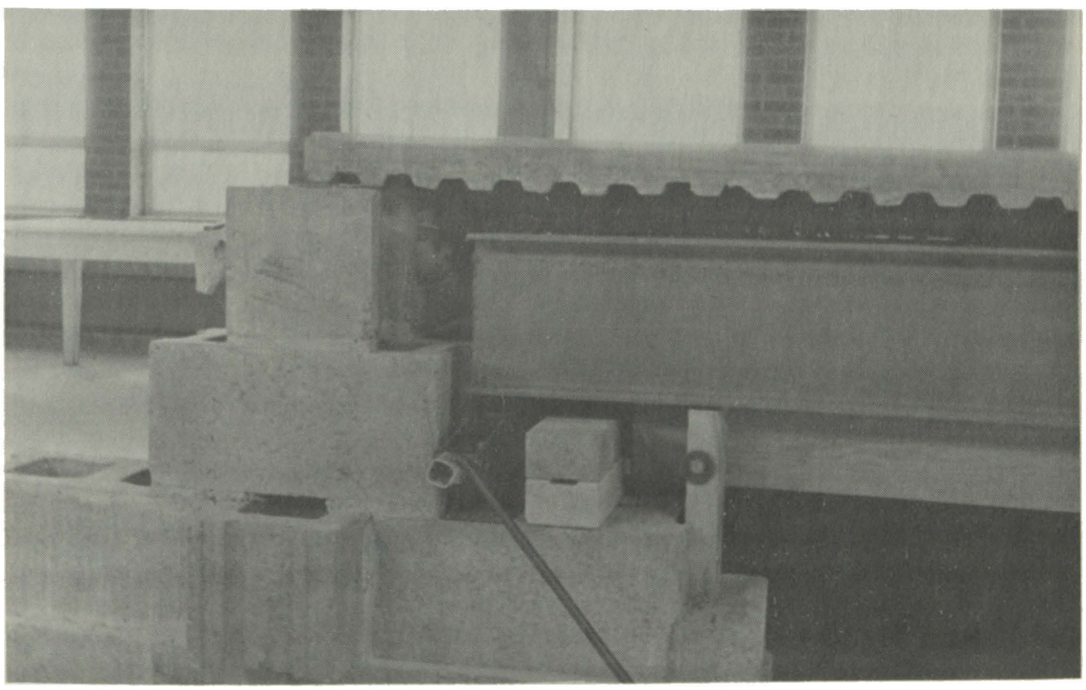


Figure 4

I-beam down, not in contact with concrete slab.

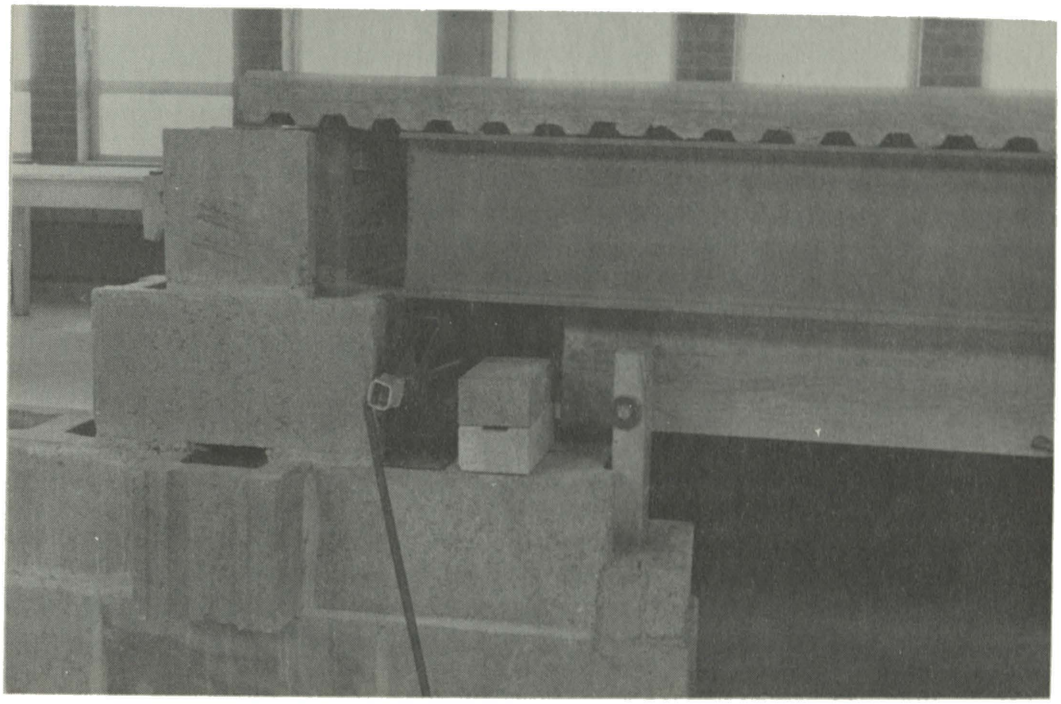


Figure 5

I-beam up, in contact with concrete slab.

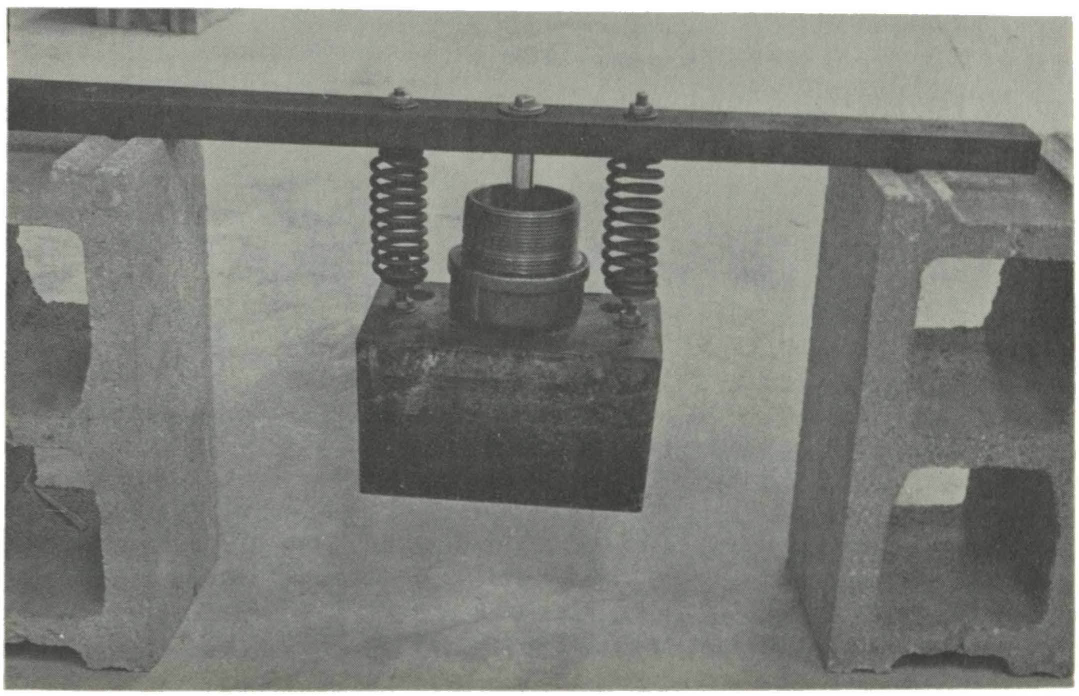


Figure 6

Detail view of Damped Vibration Absorber.

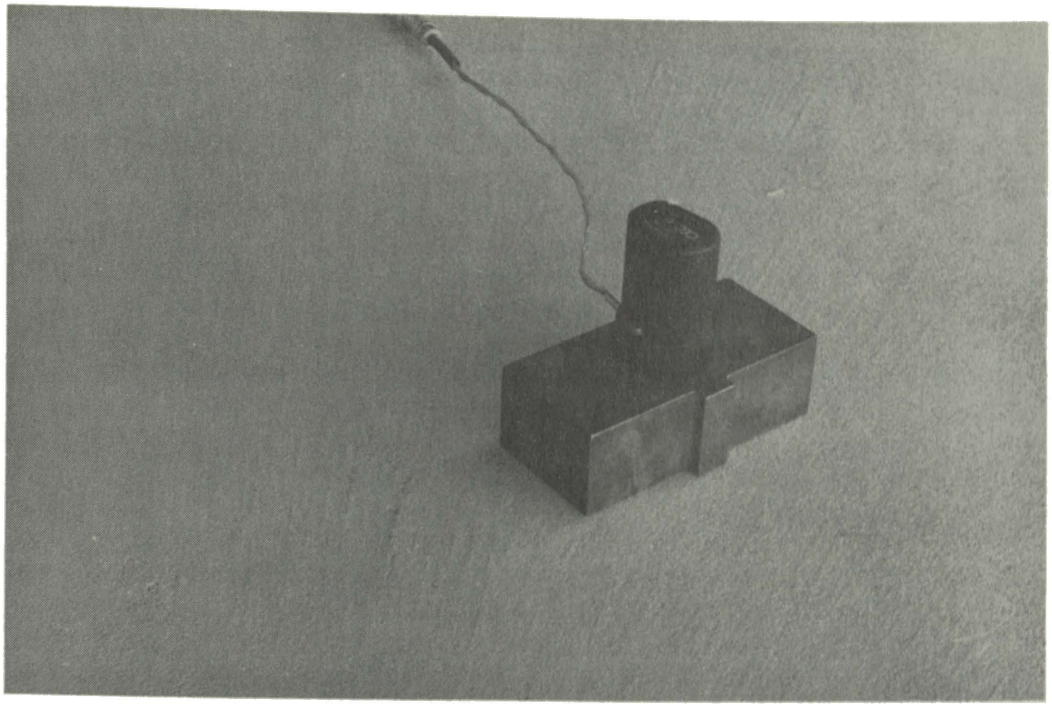


Figure 7

Seismic instrument used to pick up amplitude and frequency of vibration.

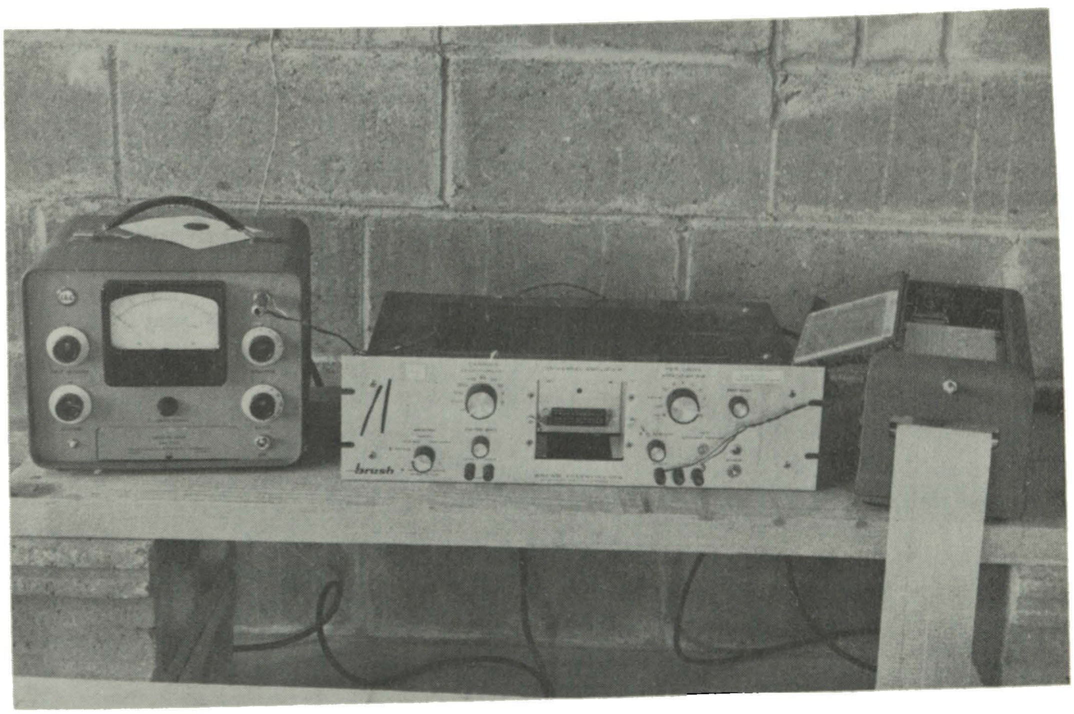


Figure 8

Amplifying and recording instruments used in experimental work.
Consolided Vibration Meter; Brush Universial Amplifier and Recorder.

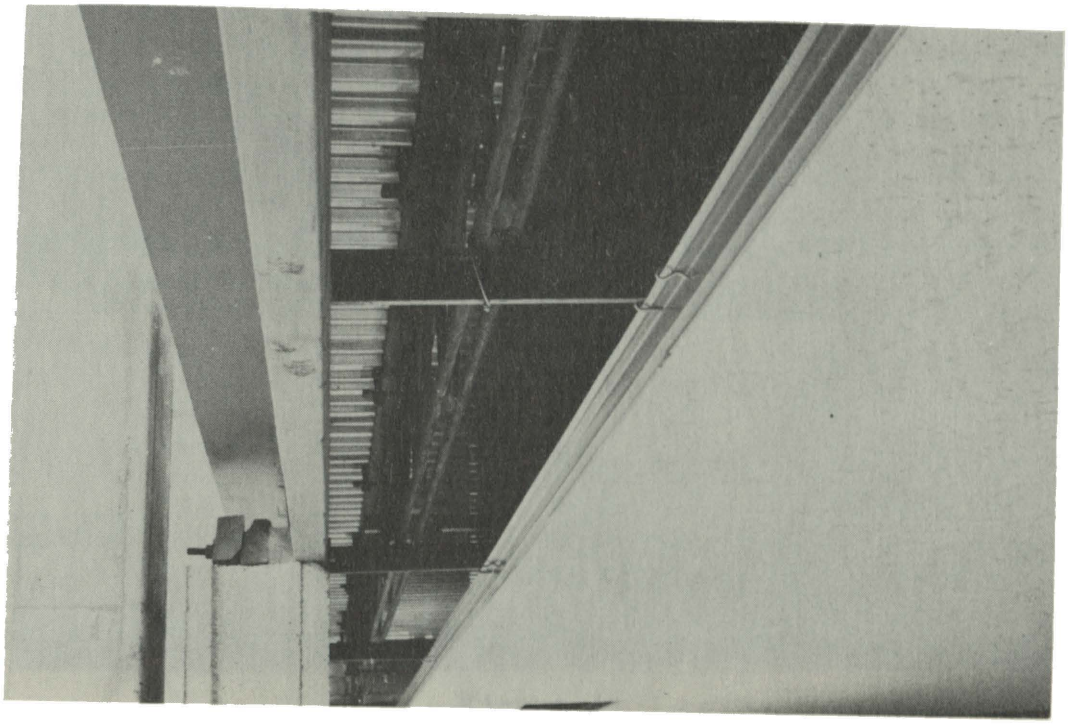


Figure 9

Acoustic tile ceiling rigidly attached to lower chord of joists.

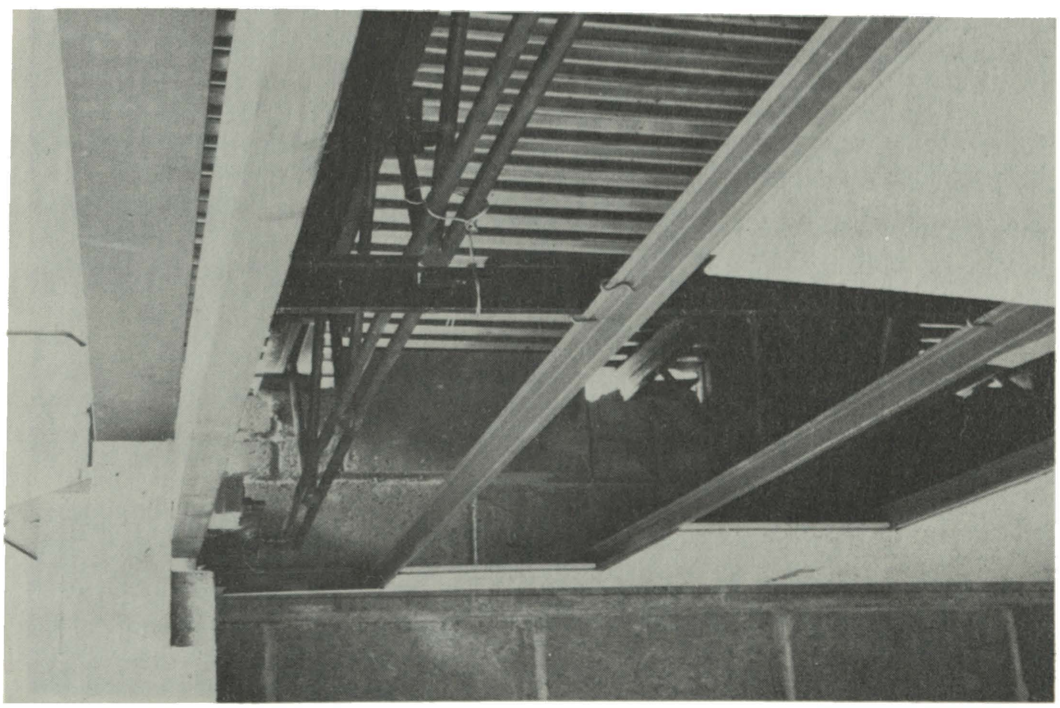


Figure 10

Acoustic tile ceiling loosely hanging from lower chord of joists.

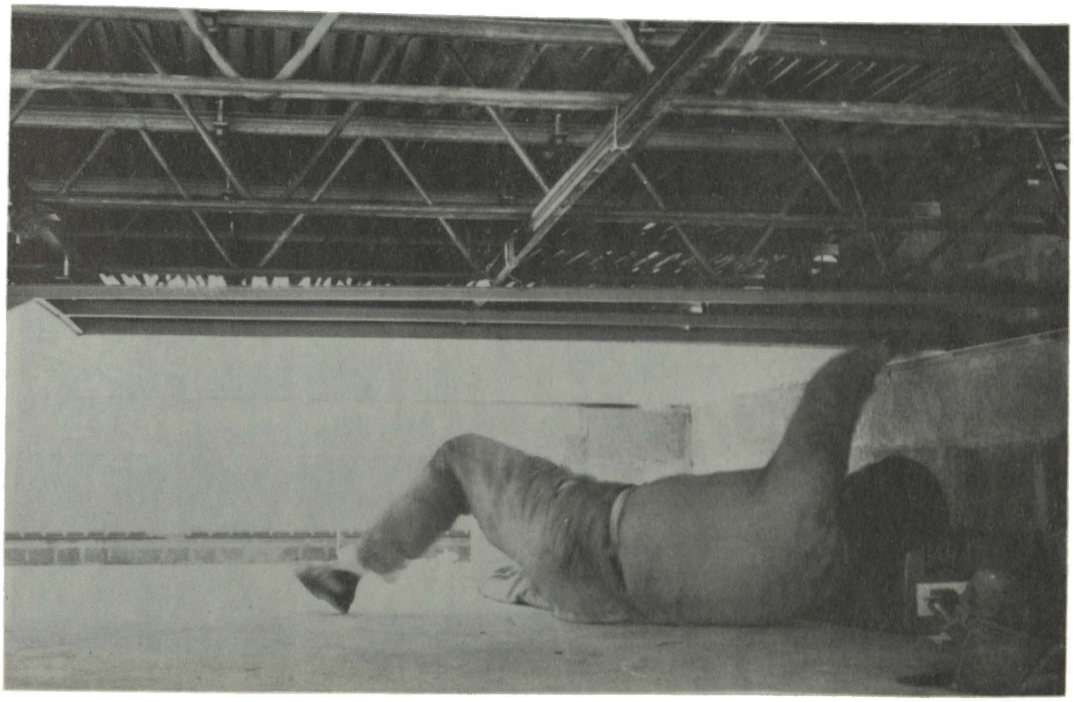


Figure 11

Installation of Acoustic Tile Ceiling.

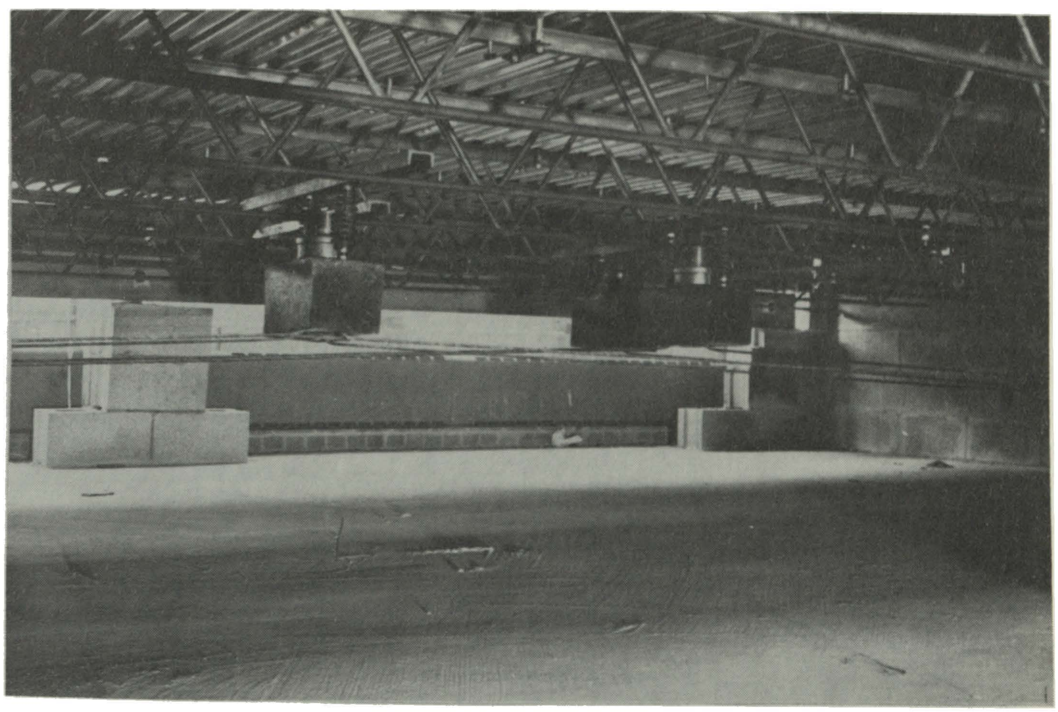


Figure 12

General view of underside of floor with cable damped vibration absorbers.

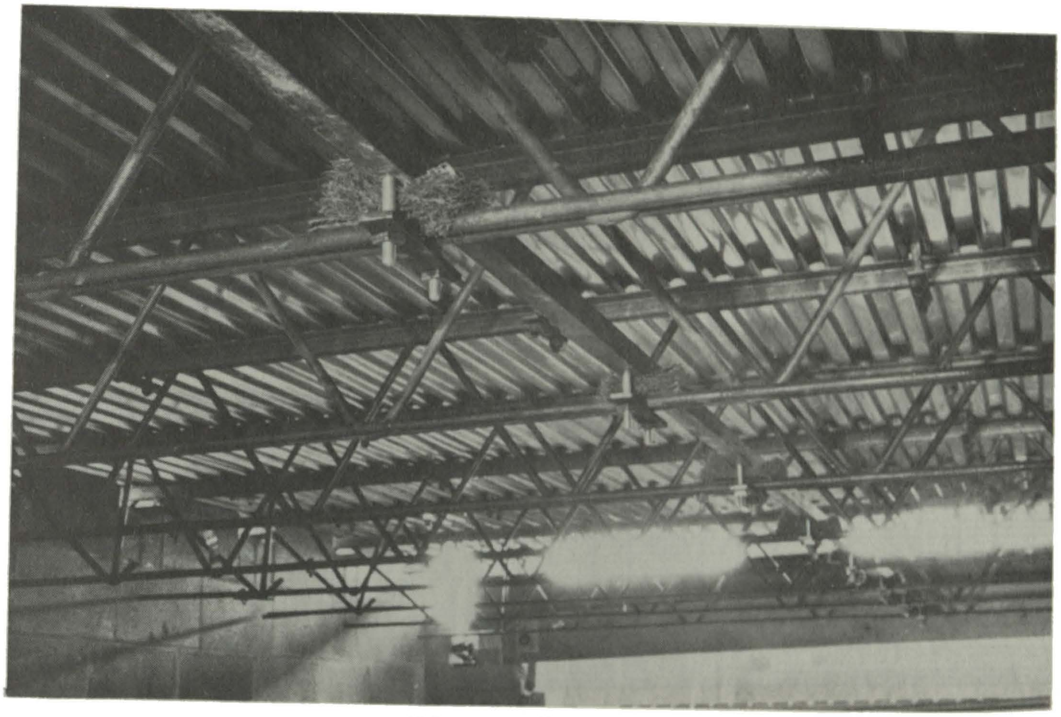


Figure 13

General view of underside of floor with insulation pads between the horizontal bridging and the contacts with the lower chords of the joists.

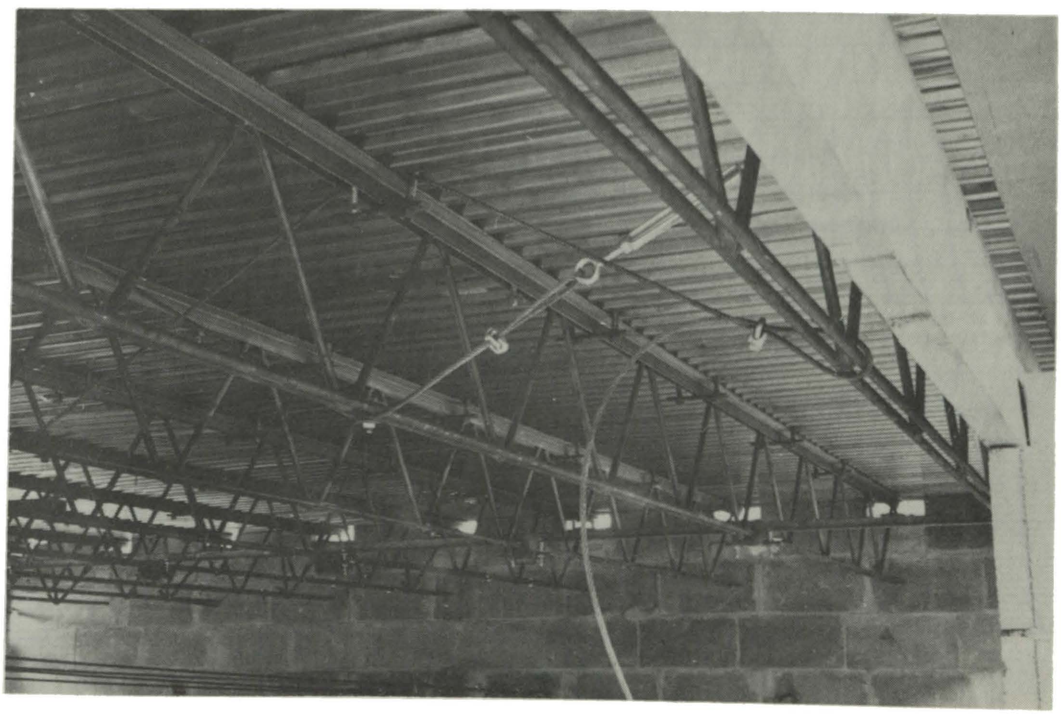


Figure 14

General view of underside of floor with cable cross bridging.

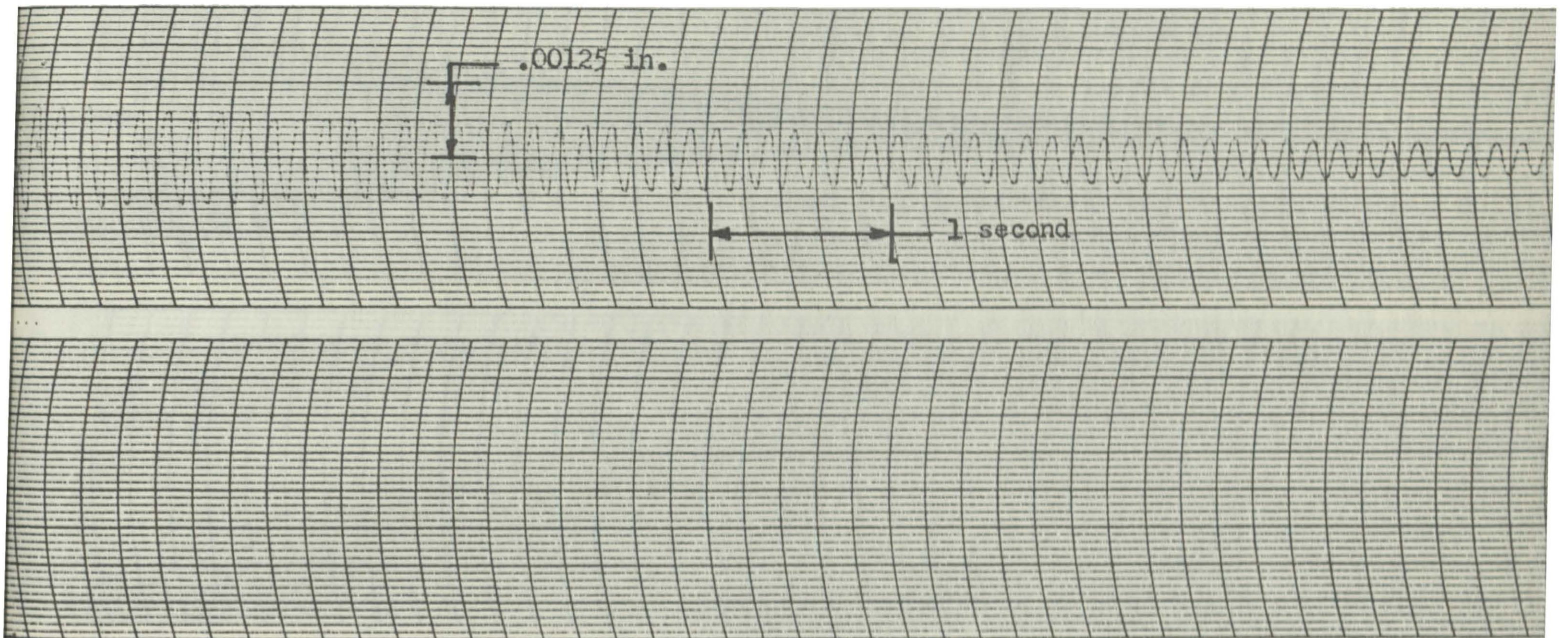


Figure 15

Original Experimental Test Floor system (I-beams down)
Natural Frequency = 7.00 cps

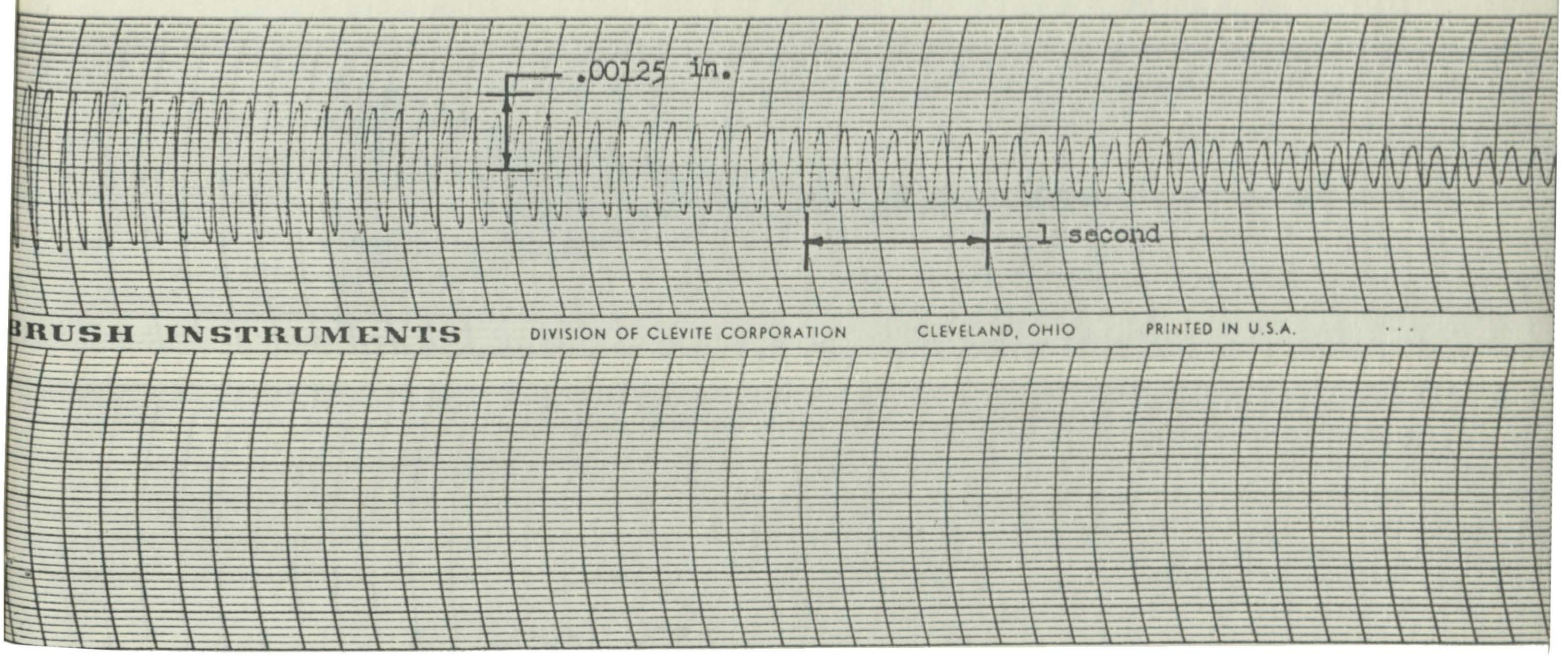


Figure 16

Altered Experimental Test Floor System (I-beams up)
Natural Frequency ≃ 7.41 cps

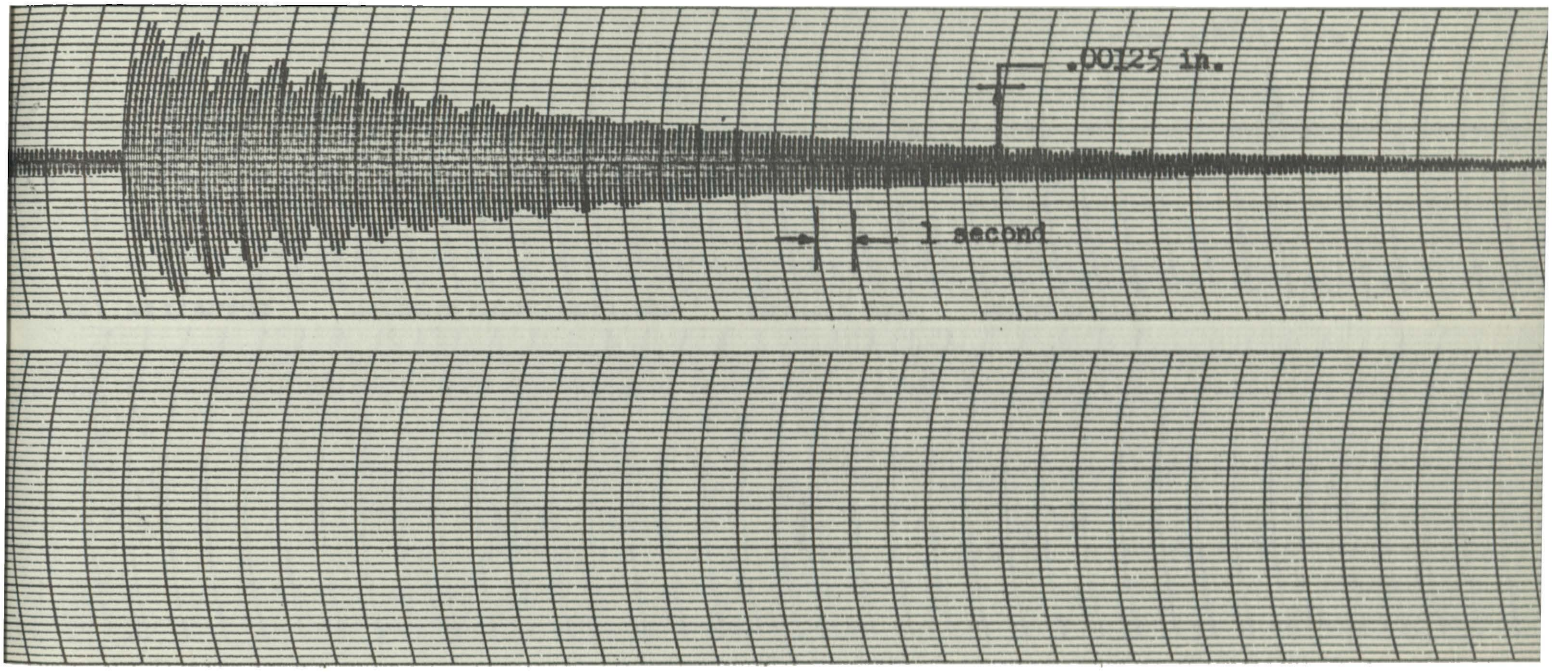


Figure 17

Original Experimental Test Floor System (I-beams down)
Structural Damping Factor = 0.00368

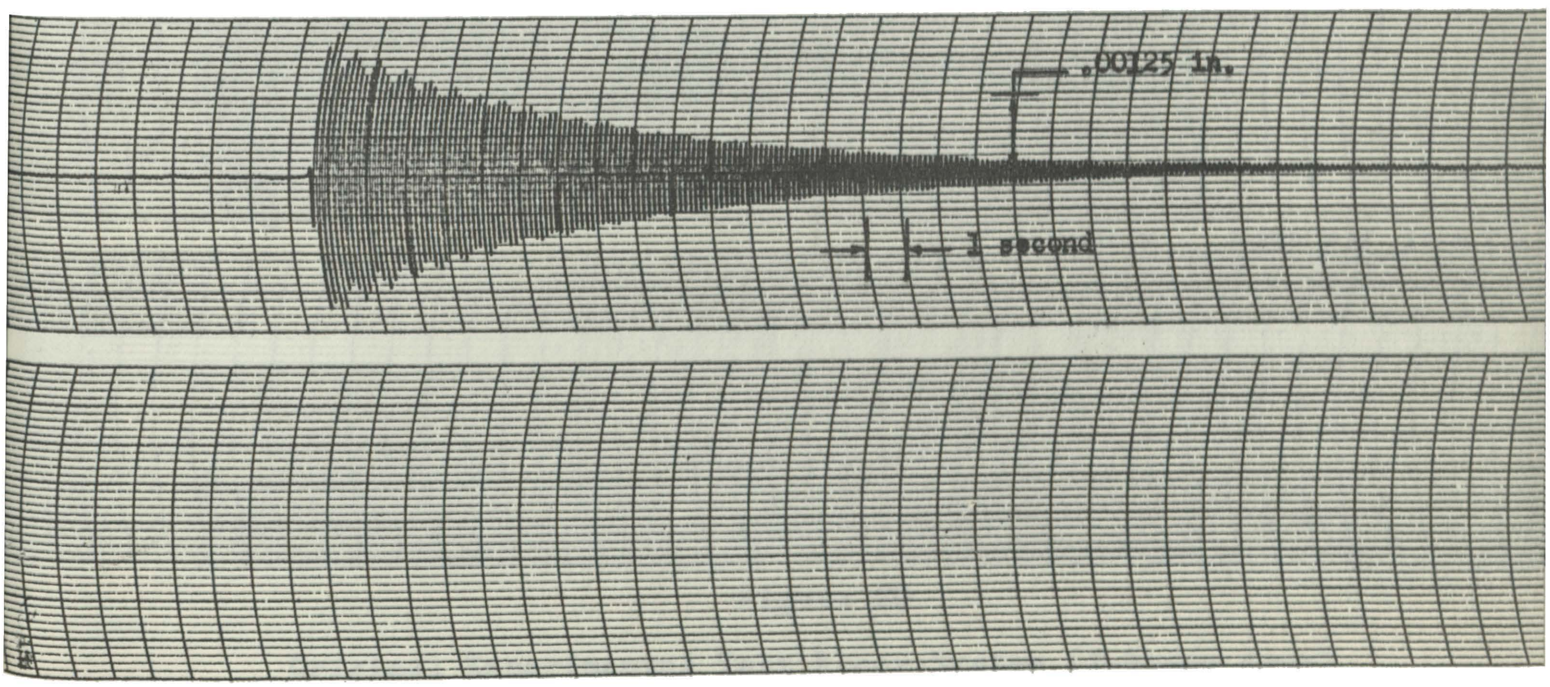


Figure 18

Altered Experimental Test Floor System (I-beams up)
Structural Damping Factor = 0.00600

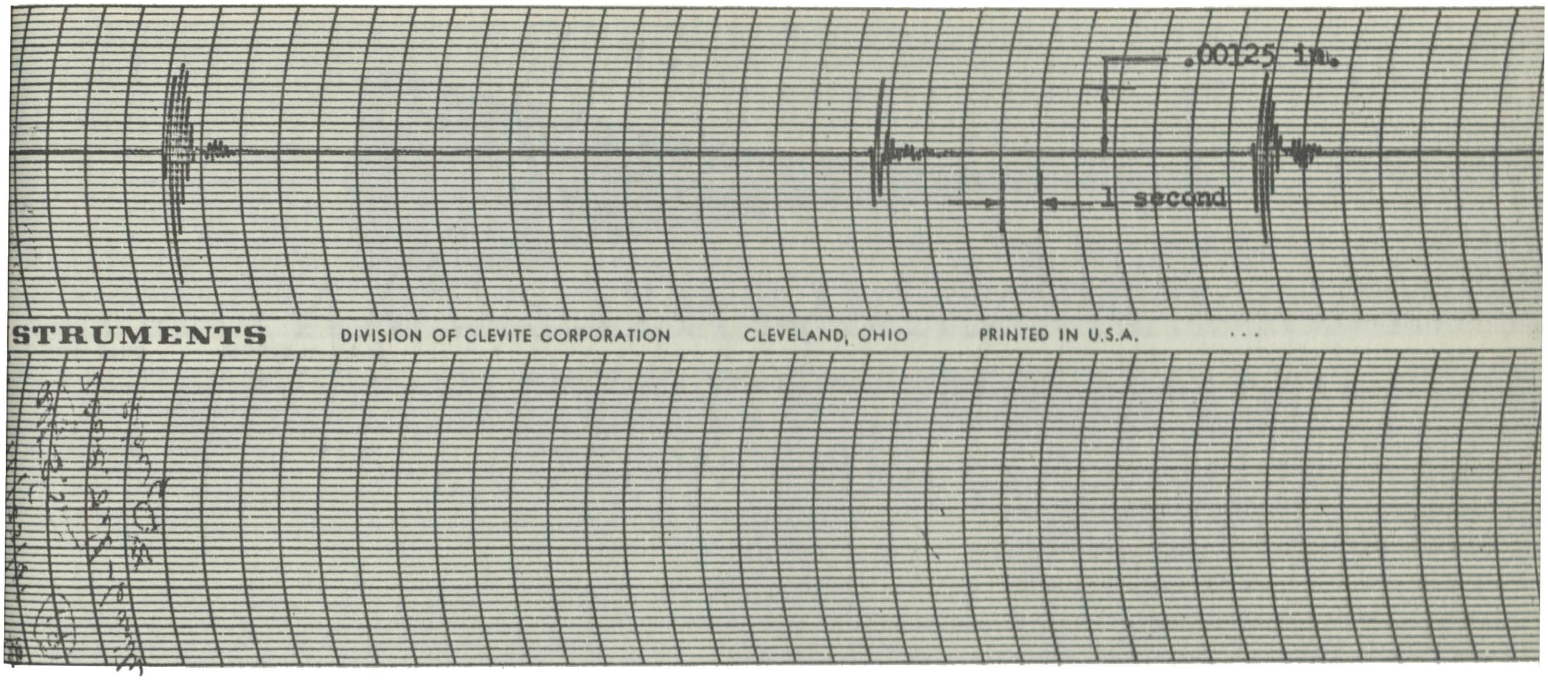


Figure 19

Altered Experimental Test Floor System (I-beams up)
with 6 Vibration Absorber units (i.e. Mass Ratio of .0236)
Damping fluid SAE #20 Motor Oil
Structural Damping Factor = 0.170

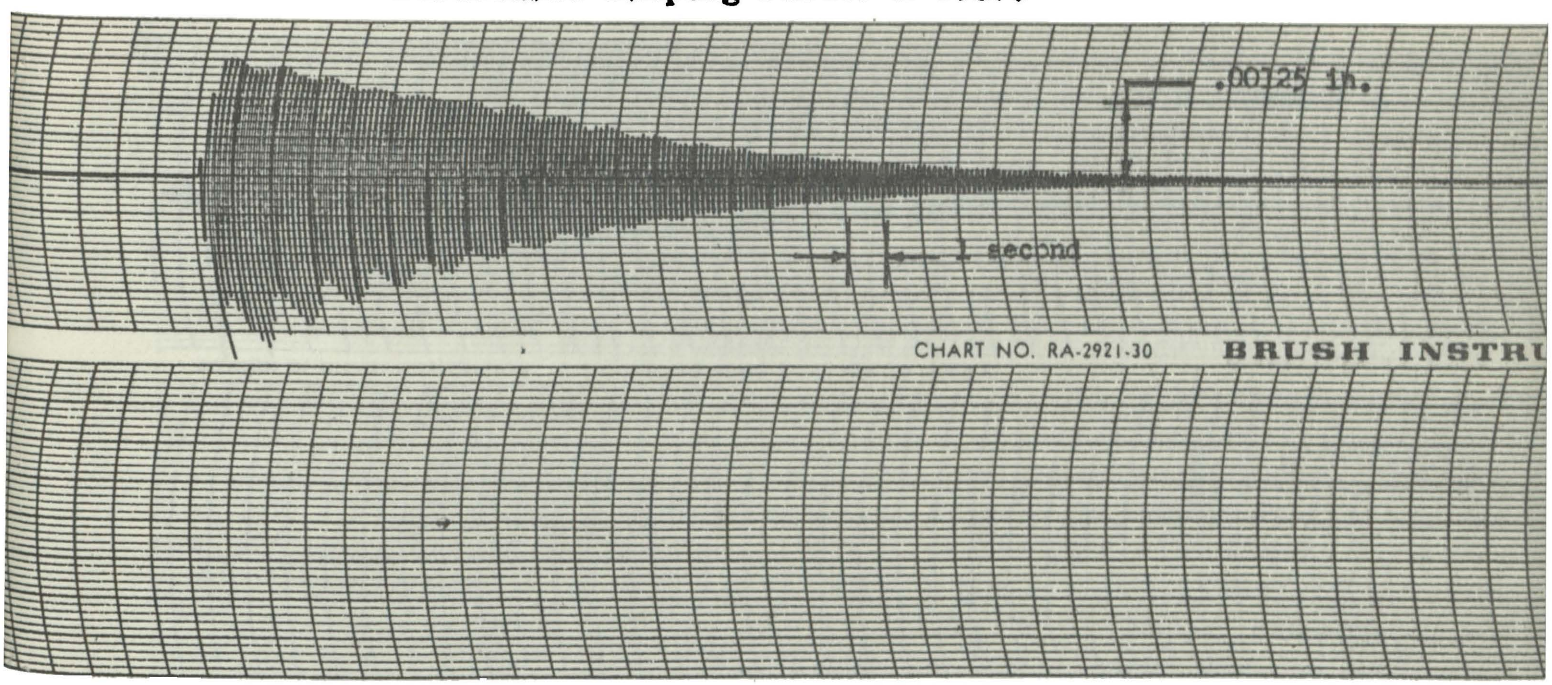


Figure 20

Original Experimental Test Floor System (I-beams down)
with Attached Acoustic Tile Ceiling
Attached to side walls.
Structural Damping Factor = 0.00574

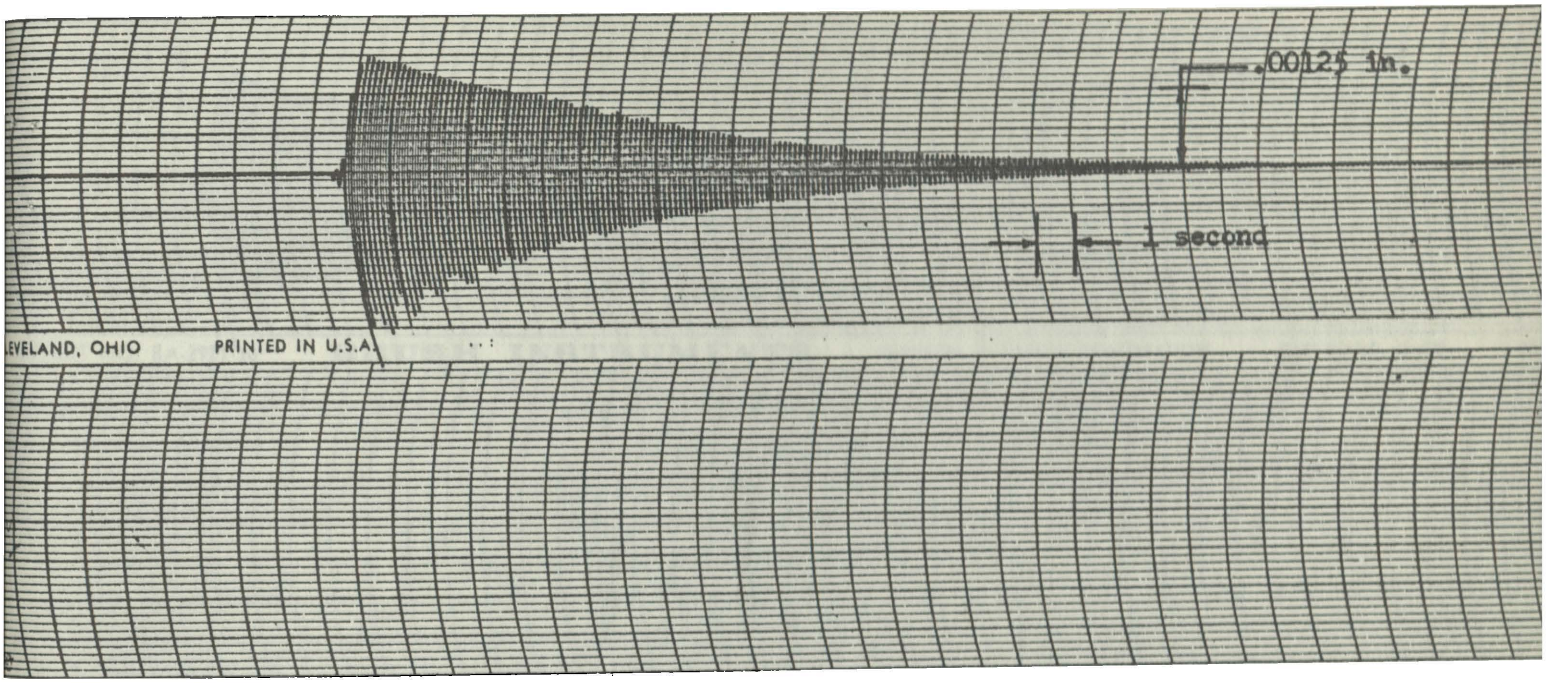


Figure 21

Altered Experimental Test Floor System (I-beams up)
with Attached Acoustic Tile Ceiling.
Attached to side walls.
Structural Damping Factor = 0.00712

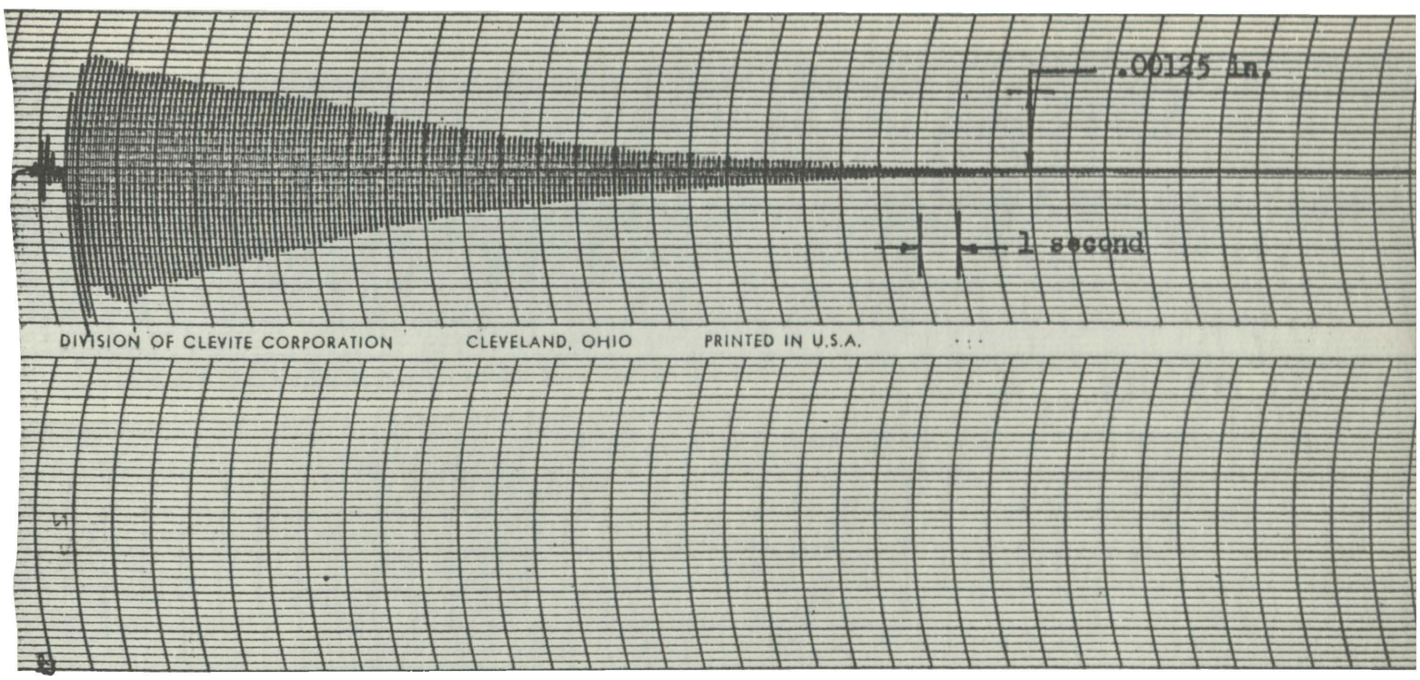


Figure 22

Original Experimental Test Floor System (I-beams down)
with Hanging Acoustic Tile Ceiling.
Attached to side walls.
Structural Damping Factor = 0.00570

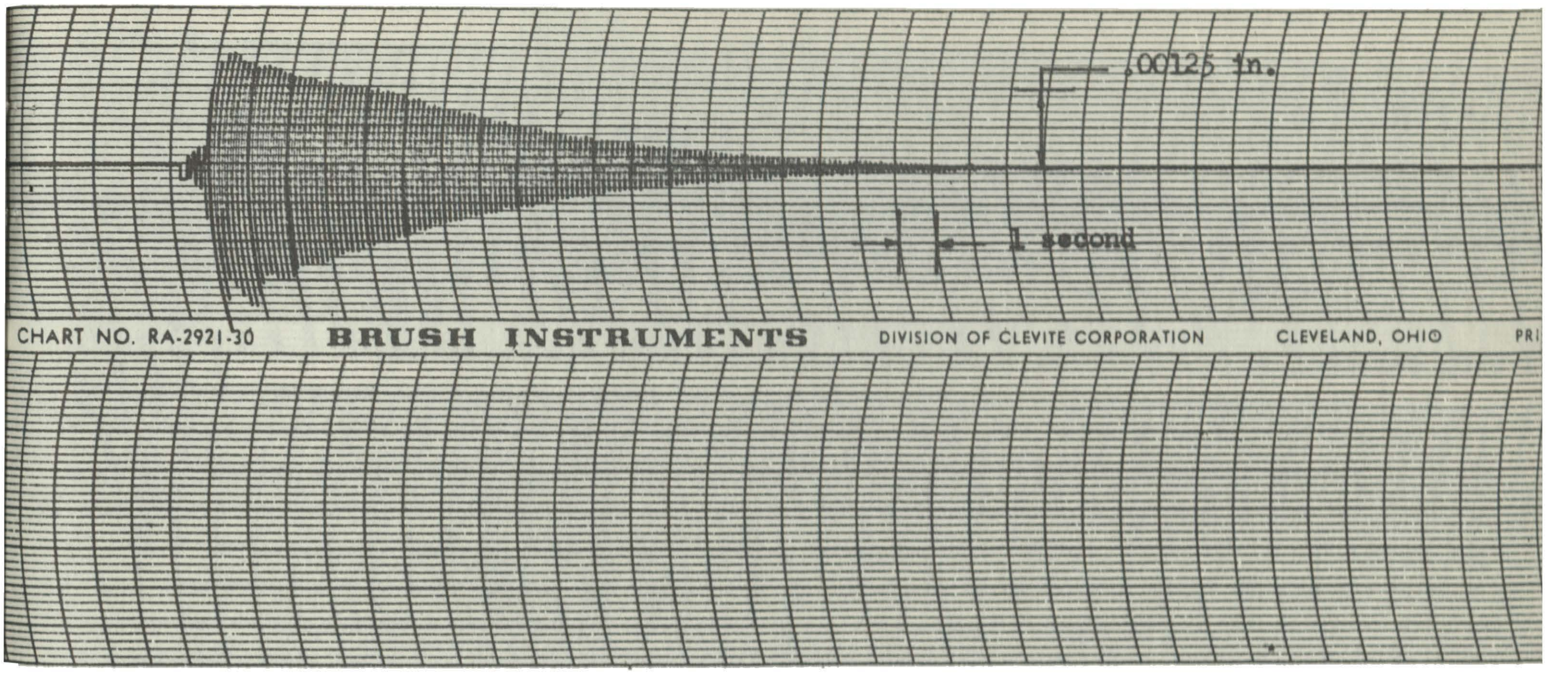


Figure 23

Altered Experimental Test Floor System (I-beams up)
with Hanging Acoustic Tile Ceiling.
Attached to side walls.
Structural Damping Factor = 0.00690

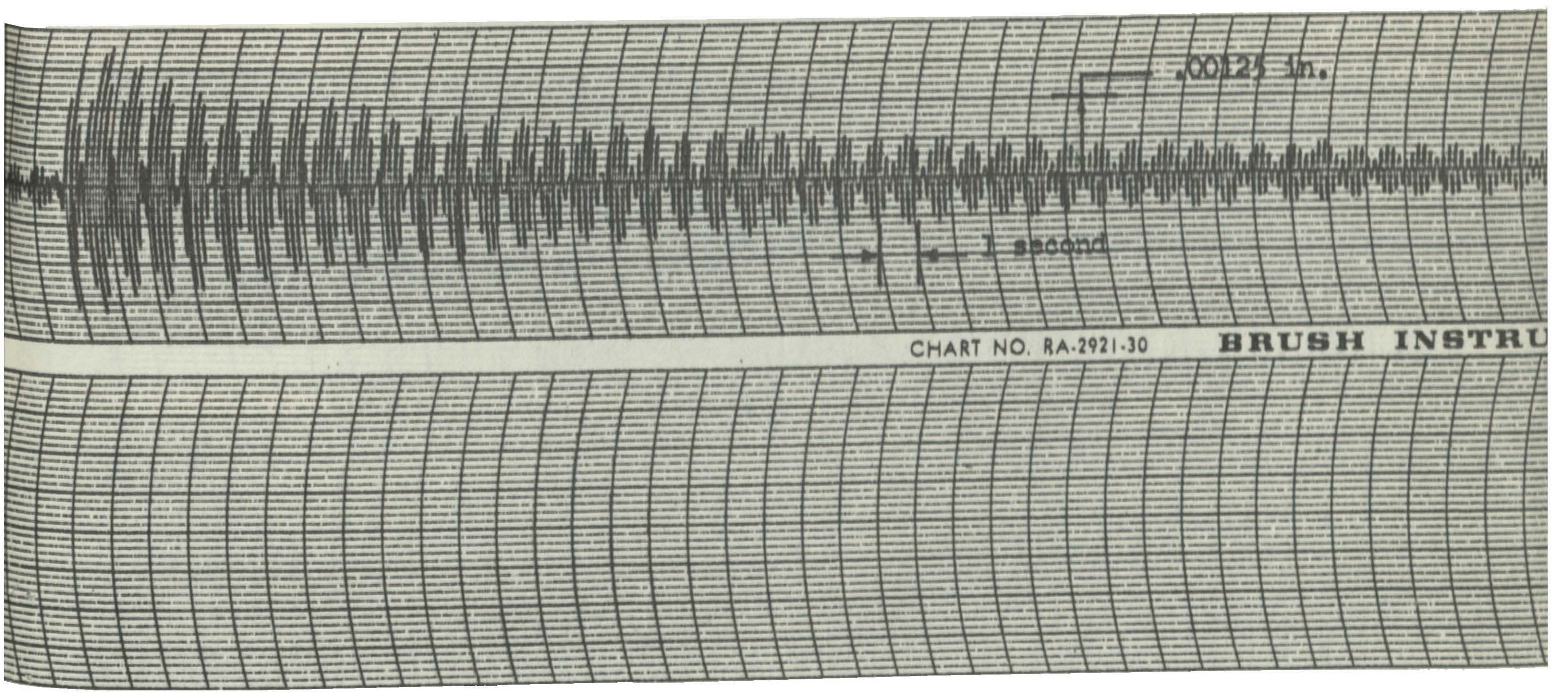


Figure 24

Original Experimental Test Floor System (I-beams down)
with 6 cable Damped Vibration Absorber units.
Structural Damping Factor = 0.00316

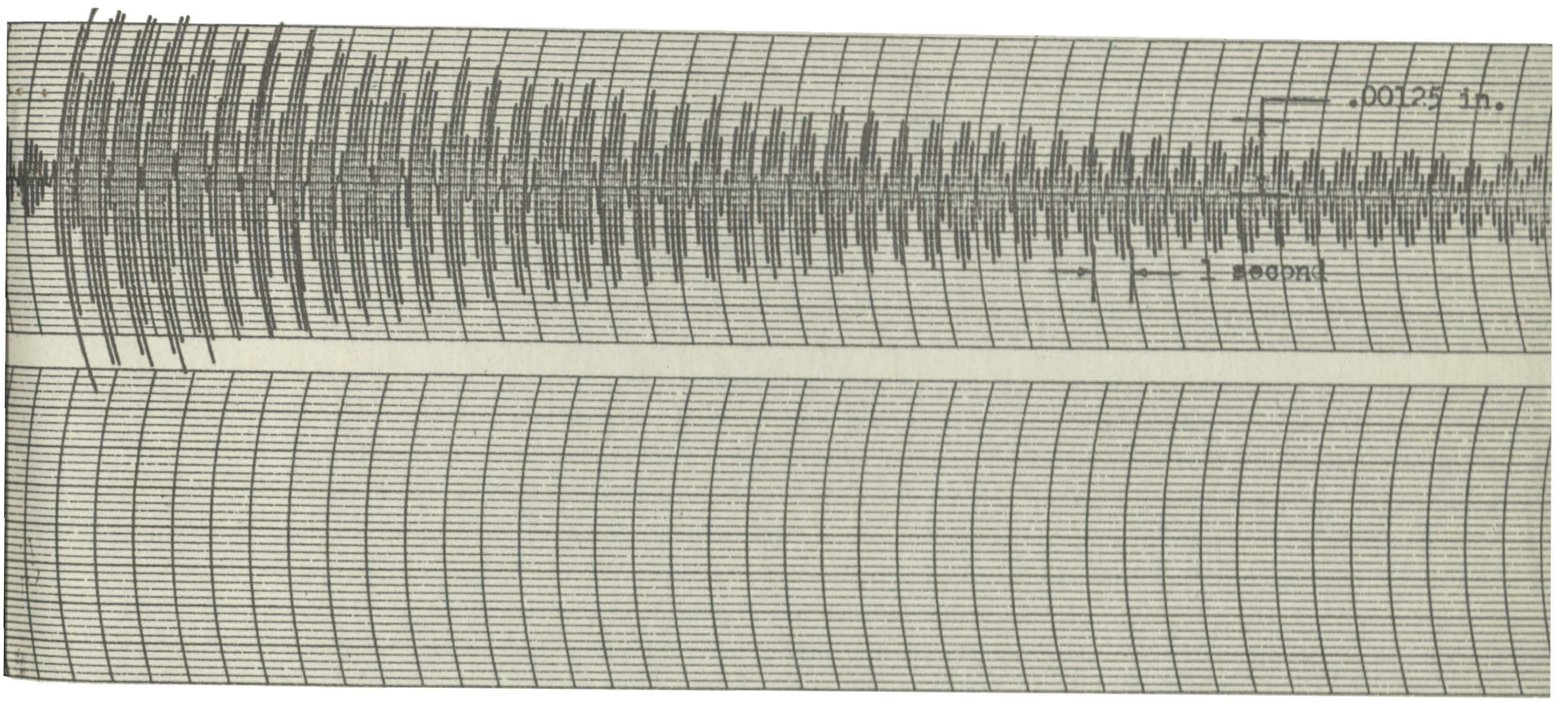


Figure 25

Original Experimental Test Floor System (I-beams down)
with 6 Spring-mass units.
No Damping
Beat Frequency
Structural Damping Factor = 0.000682

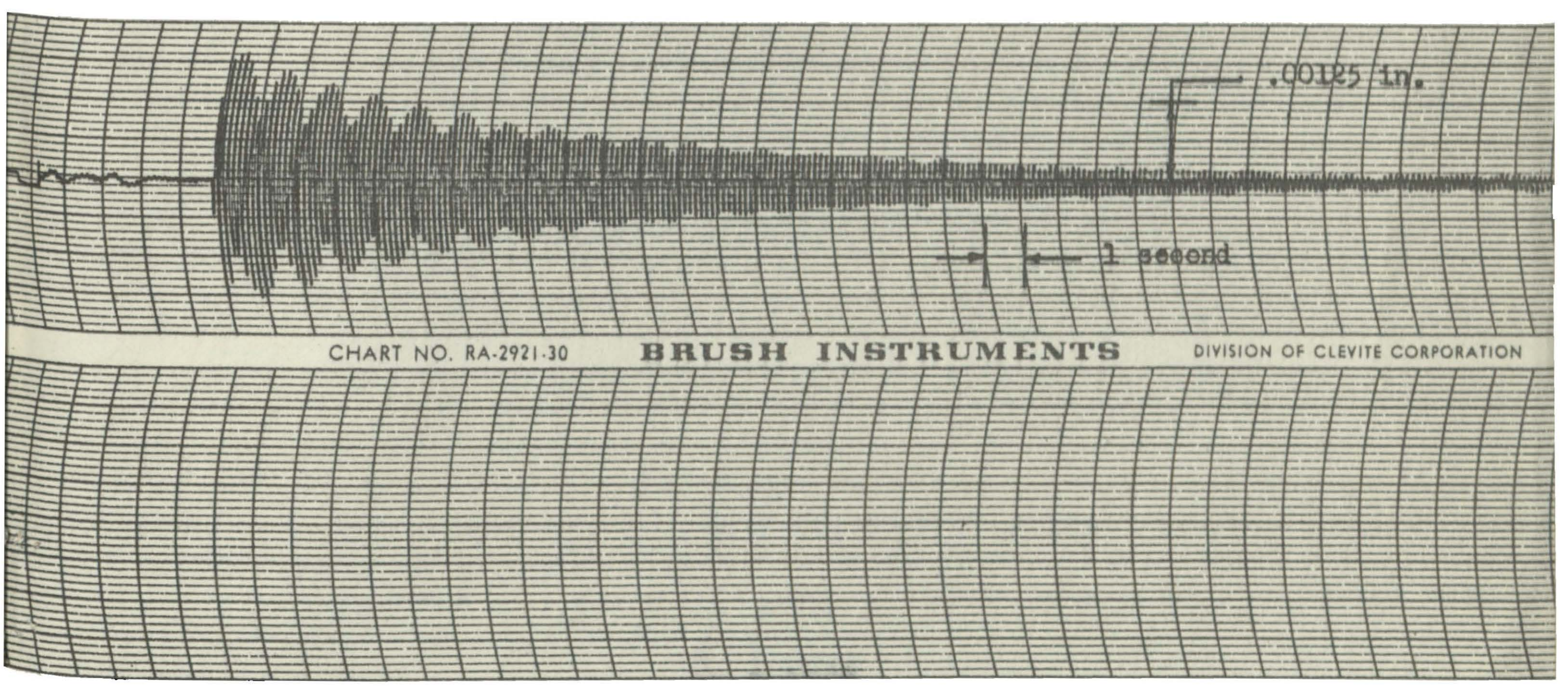


Figure 26

Original Experimental Test Floor System (I-beams down)
with horizontal bridging.
Structural Damping Factor = 0.00369