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VIBRATION OF STEEL BEAM-CONCRETE SLAB FLOOR SYSTEMS

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A_o = first maximum amplitude

A_s = cross-sectional area of steel beam (including cover plate).

b = effective width of concrete slab.

E = modulus of elasticity.

F = magnitude of impact function.

f_c' = compressive strength of concrete.

f_n = natural nth mode frequency.

g = acceleration of gravity, 386 in/sec^2.

$g_n(t)$ = the normalized response for the equivalent one-degree of freedom system of the nth mode.

I = cross-sectional moment of inertia of the beam section about the x-axis.

I_t = cross-sectional moment of inertia of the composite tee-section transformed to steel.

n = modulus of elasticity ratio; integer counter.

$p(x,t)$ = total force per unit length applied perpendicular to the beam.

$p_1(x)$ = the load distribution along the beam.

t = time; thickness of concrete slab.

t_d = duration of impact function.

t_o = time of first maximum amplitude.

W = total weight of tee-beam section.

w = weight per inch of tee-beam section.

w_c = unit weight of concrete.

x = distance measured along the beam.

$y(x,t)$ = vertical displacement of the neutral axis of the beam from its static equilibrium position.

y_b = distance from top of concrete slab to centroid of steel beam (including cover plate).

y_o = first amplitude.

y_n = nth amplitude.

δ = logarithmic decrement.

ζ = critical damping ratio.

ρ = mass per unit length of beam.

ϕ_n = modal characteristic shapes.

ω_n = nth mode radian frequency

INTRODUCTION

The advent of high strength steels and concretes, lightweight concrete, composite construction, and plastic design has reduced the mass of concrete slab-steel beam floor systems. This reduction, coupled with the use of lightweight hung ceilings, has occasionally created the problem of transient vibrations set up by small impacts. Thus, the efficiently designed modern floor system, having adequate static strength, may be susceptible to vibration which can be annoying to human inhabitants.

Human perceptibility of vibration seems to depend on three factors; frequency, initial amplitude, and duration (damping). The vibration of older, less efficiently designed floors, is usually not within the range of human perceptibility. In addition, older types of floor coverings (slate, marble, etc.) and ceilings (plaster) together with extensive partitioning sufficiently increases the damping of the structural system. Modern building construction tends to change these factors in an adverse way.

Since 1959 a research program dealing with structural vibrations of floor systems has been conducted at the University of Kansas. The first phase of the program dealt with steel joist-concrete slab floor systems,[1] including an investigation of human sensitivity and the development of mechanical dampers. The second phase has been limited to steel beam-concrete slab floor systems.

The objectives of the research can be summarized as follows: 1. To determine what constitutes an annoying vibration. 2. To establish a mathematical model of the floor system which will accurately predict the response of the floor system to a given impact. 3. To develop an analytical expression to be used as a design guide.

To accomplish these objectives each phase of the project was divided into two parts: an experimental portion and a theoretical portion. The experimental portion consisted of obtaining data (amplitude, frequency, and subjective evaluation) on actual floors which had been built using standard modern construction techniques. The theoretical portion consisted of the development of mathmatical models and design criteria.

This report deals with the research to date (April, 1968) conducted on the vibration of steel beam-concrete slab floor systems.

HUMAN SENSITIVITY

Human sensitivity to vibration seems to depend on three factors: frequency, amplitude, and damping. During the steel joist-concrete slab portion of the research project it was determined that a criterion for establishing an annoying floor would be similar to that shown in Fig. 1 or Fig. 2. Figure 1 is from a paper by Reiher and Meister,[2] but the vertical scale has been multiplied by a factor of ten. To obtain the original plot, Reiher and Meister subjected a group of standing people to steady state vibrations (frequencies ranging from 3 to 70 cycles per second with amplitudes ranging from 0.0004 to 0.40 inches) and recorded the subjective reaction of the participants. The results were assessed in varying levels of sensitivity from barely perceptible to intolerable severity. Figure 2 is from a paper by Goldman[3] with the vertical scale multiplied by ten. Goldman took all of the available data on steady state vibration tests and summarized the results on the basis of the three fundamental levels of subjective sensations: perceptible, unpleasant, intolerable. Reference 4 is an excellent summary of the research done in this area.

Forty-six floors were tested during the steel joist phase of the project, and the reaction of individuals in the test area was recorded. When the field measurements were compared to the original plots it was found that a majority of the floors were in the disturbing range. Yet, in only three floors were there vibrations that were perceptible to the occupants. It was then concluded that transient vibrations are much less annoying than steady state vibrations. The plots were changed by a factor of ten. Replotting of the field data resulted in consistent prediction of human perceptibility to vibrations.

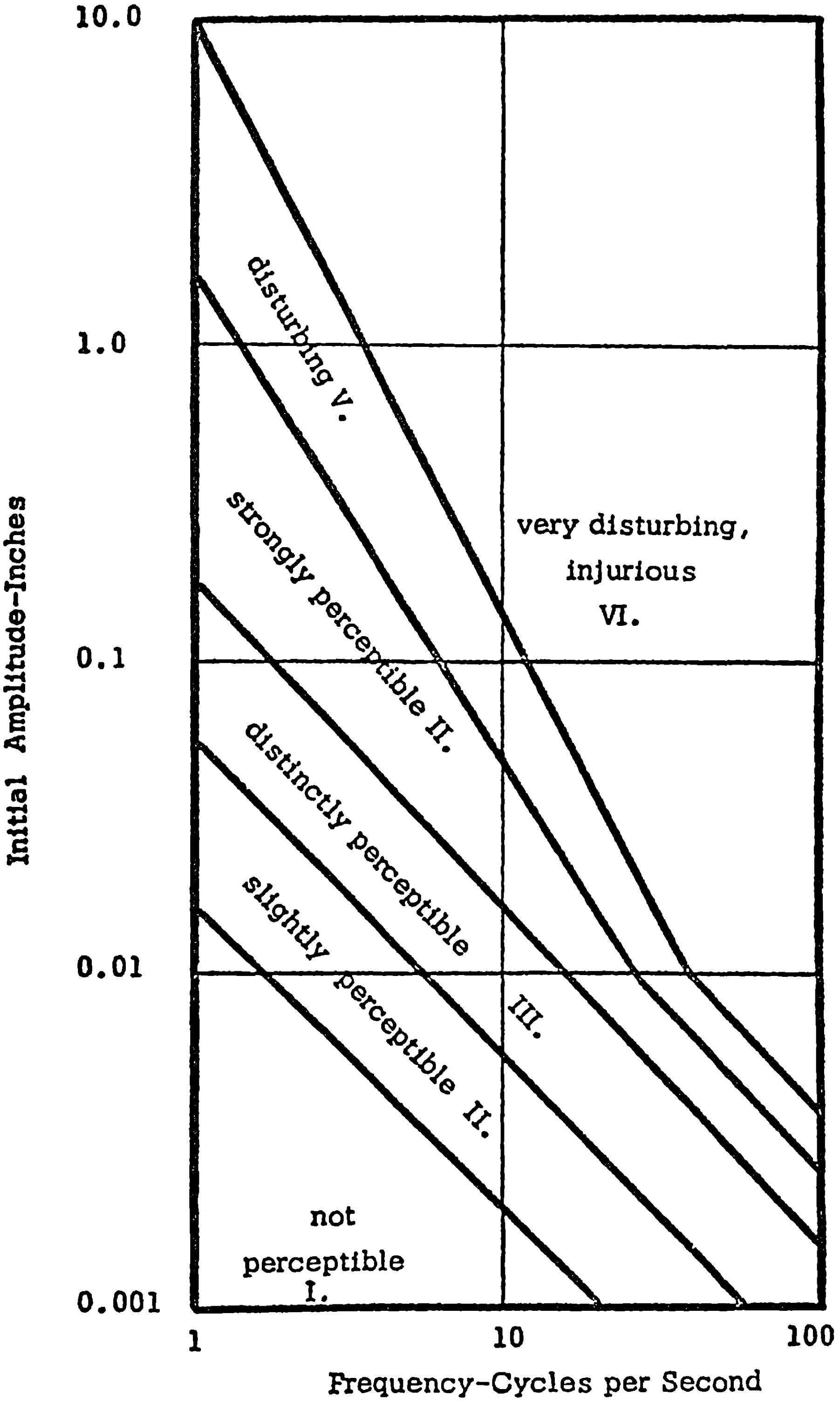


Figure 1. Domains of Various Strengths of Sensations for Standing Persons Subject to Vertical Vibration, Adjusted for Transient Vibrations, After Reiher and Meister.

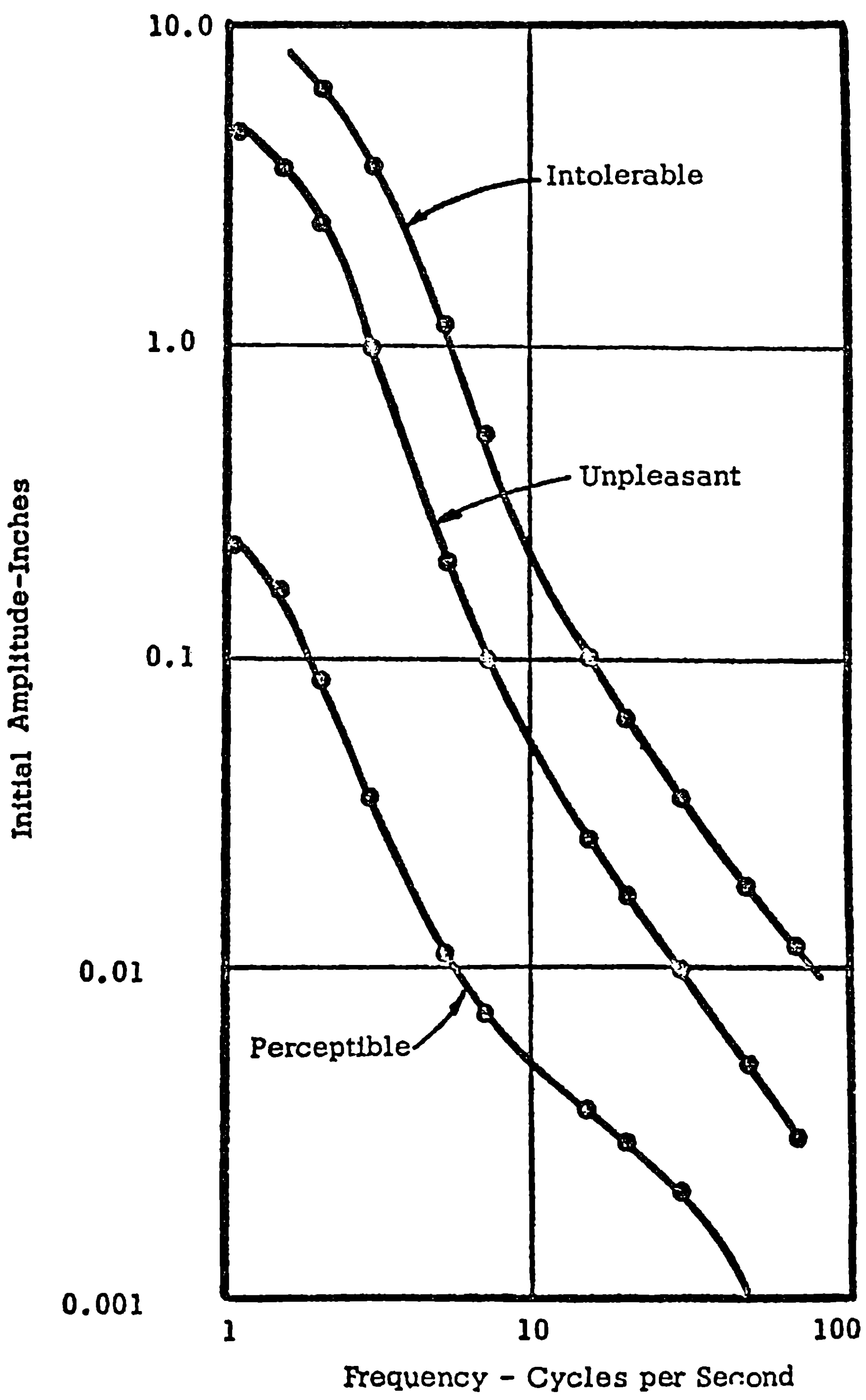


Figure 2. Subjective Response of the Human Body to Vibratory Motion, Adjusted for Transient Vibrations, After Goldman.

It was also concluded that the main factor influencing the effect of vibrations on the human was the damping in the system. A series of tests was conducted on a steel joist laboratory floor system where frequency, amplitude and damping could be varied. If the floor was damped to a small amplitude prior to five cycles of vibration, the participant felt only the initial impact, no vibration. If the vibration persisted after 12 cycles, the participant responded to the vibration just as to steady state vibration. The response to vibration between these ranges was a function of the number of cycles before the amplitude became negligible. Negligible vibration being defined as the amplitude of that cycle being less than one-fifth the initial amplitude.

Figure 3 is a plot of the number of cycles for 80% reduction in amplitude versus percent of critical damping for a single degree of freedom system with viscous damping. From this plot, it can be concluded that for a vibration to be perceptible to a human the damping in the floor system must be less than 5% of critical. Thus the modified Reiher and Meister curves are valid only when the damping in the system meets this criterion.

Further, the effect of room partitions and dividers on the damping of the floor system was observed. Partitions or dividers which were attached securely to the floor system at a minimum of three points significantly increased the damping of the system to a degree that annoying vibrations are non-existent.

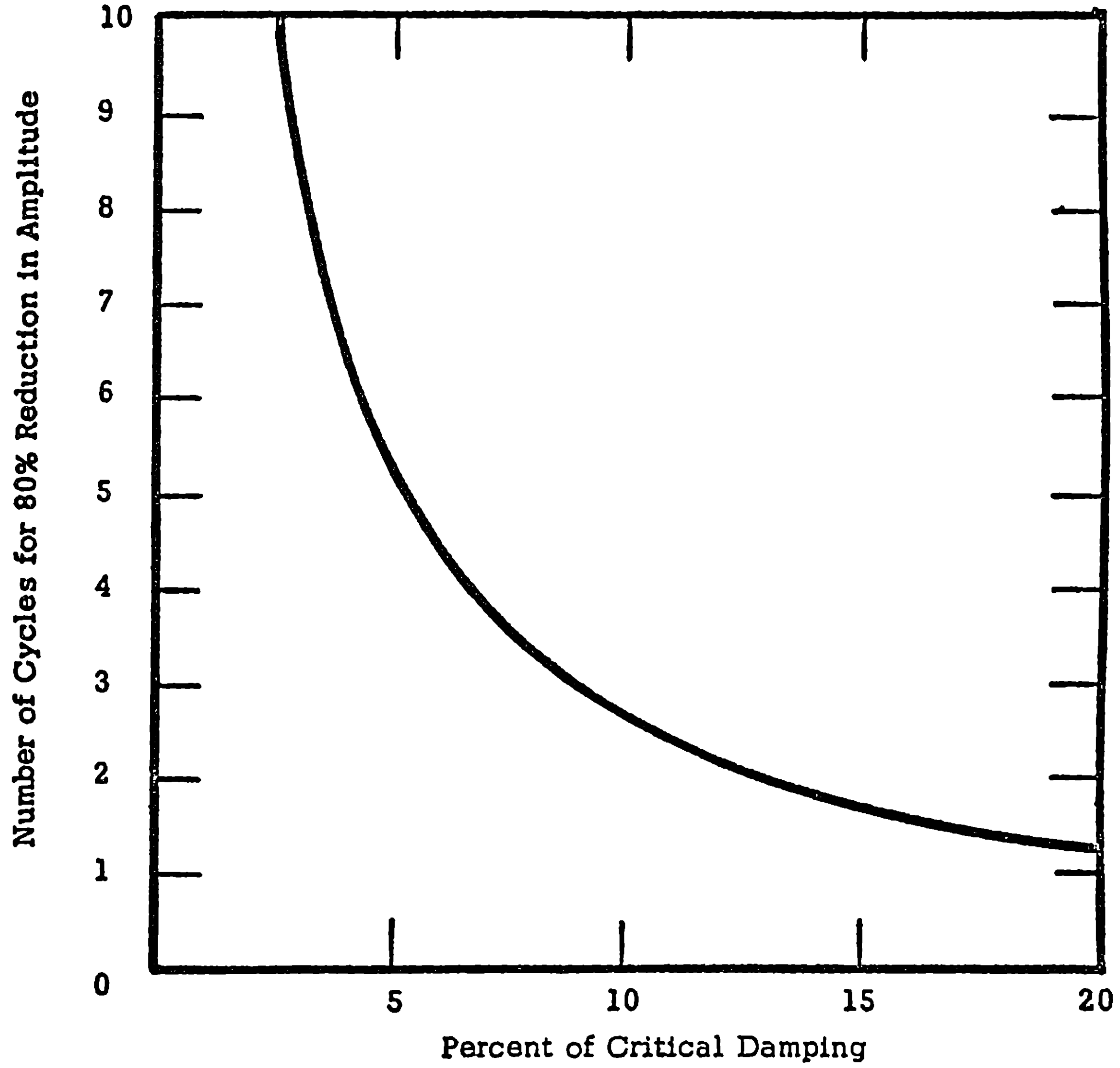


Figure 3. Number of Cycles for 80% Reduction in Amplitude versus Percent of Critical Damping.

EXPERIMENTAL APPROACH

Twenty buildings located throughout the northeastern quarter of the United States were chosen to study the effect of as many variables as possible. Included in this series of buildings were floor systems of

A. Composite construction
   1. Solid Slab
   2. Slab on steel deck
B. Non-composite construction
   1. Solid slab
   2. Slab on steel deck.

In addition two small floor systems were constructed in the laboratory.

The data was obtained by impacting the floors and permanently recording the resulting oscillations on film. Each floor was impacted by two different methods.

The first method utilized a mechanical impactor specifically designed for this purpose (Fig. 4). The impactor consists of a cylindrical steel weight of 31.5 pounds and a vertical steel frame. The weight may be dropped at any set height from 1.5 to 5.5 inches. A device included on the impactor prevents the weight from striking the floor more than once. This method of impacting was used since it was quite easy to repeat without variation.

For the second method, the same member of the research team executed a heel drop on each floor. This was accomplished by the researcher assuming a natural stance with knees straight, then shifting his weight to the balls of his feet and lifting his heels approximately two and one-half inches off of the floor. A sudden relaxation allowed his body weight to

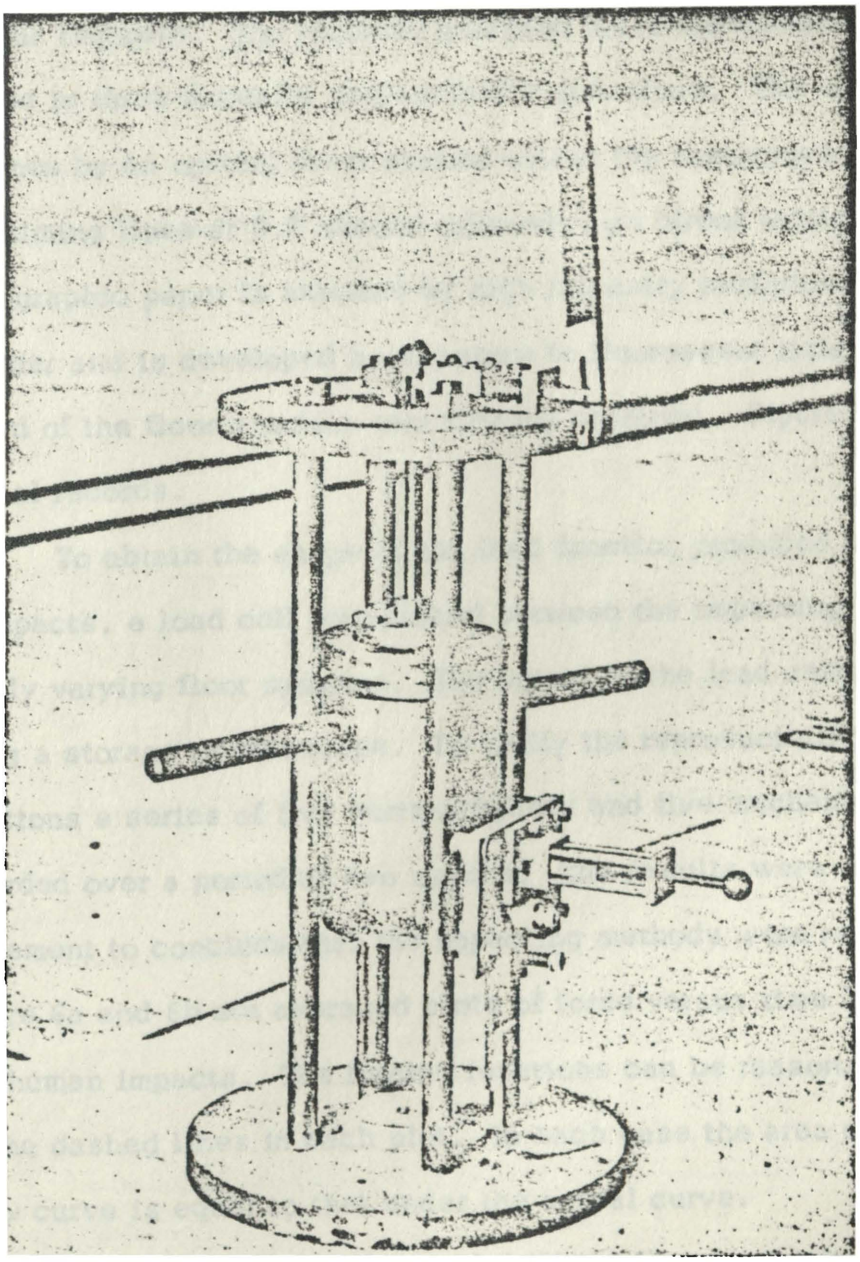


Figure 4. The Mechanical Impactor

essentially free fall to the floor, terminating in an impact. This method of impacting was used to approximate the type of impact normally found due to human occupancy.

The response of the floor to the impact was recorded on a portable seismic recorder. The recorder produces three independent records of response in three mutually perpendicular directions. The vibration is magnified 50 times by an optical lever system within the instrument and recorded, along with timing lines at 0.2 second intervals, on direct writing paper. This photographic paper is exposed by high intensity incandescent light in the recorder and is developed by exposure to fluorescent light. A permanent record of the floor's motion was thereby obtained. Figure 5 shows several typical records.

To obtain the shape of the load function produced by the two types of impacts, a load cell was placed between the impacting system and two widely varying floor systems. The output of the load cells was recorded using a storage oscilloscope. To verify the reproducibility of the load functions a series of five human impacts and five mechanical impacts was recorded over a period of two months. The results were within sufficient agreement to conclúde that the impacting methods were reproducible. Figure 6a and 6b are averaged plots of force versus time for mechanical and human impacts. The forcing functions can be reasonably approximated by the dashed lines in each plot. In each case the area under the appromimate curve is equal to that under the actual curve.

As a check of the shape of the load function a simply supported single beam (8WF31, length=12 feet) was subjected to the impacts and the resulting response recorded. The solution of the differential equation governing the motion of simply supported elastic beams, excited by a unit

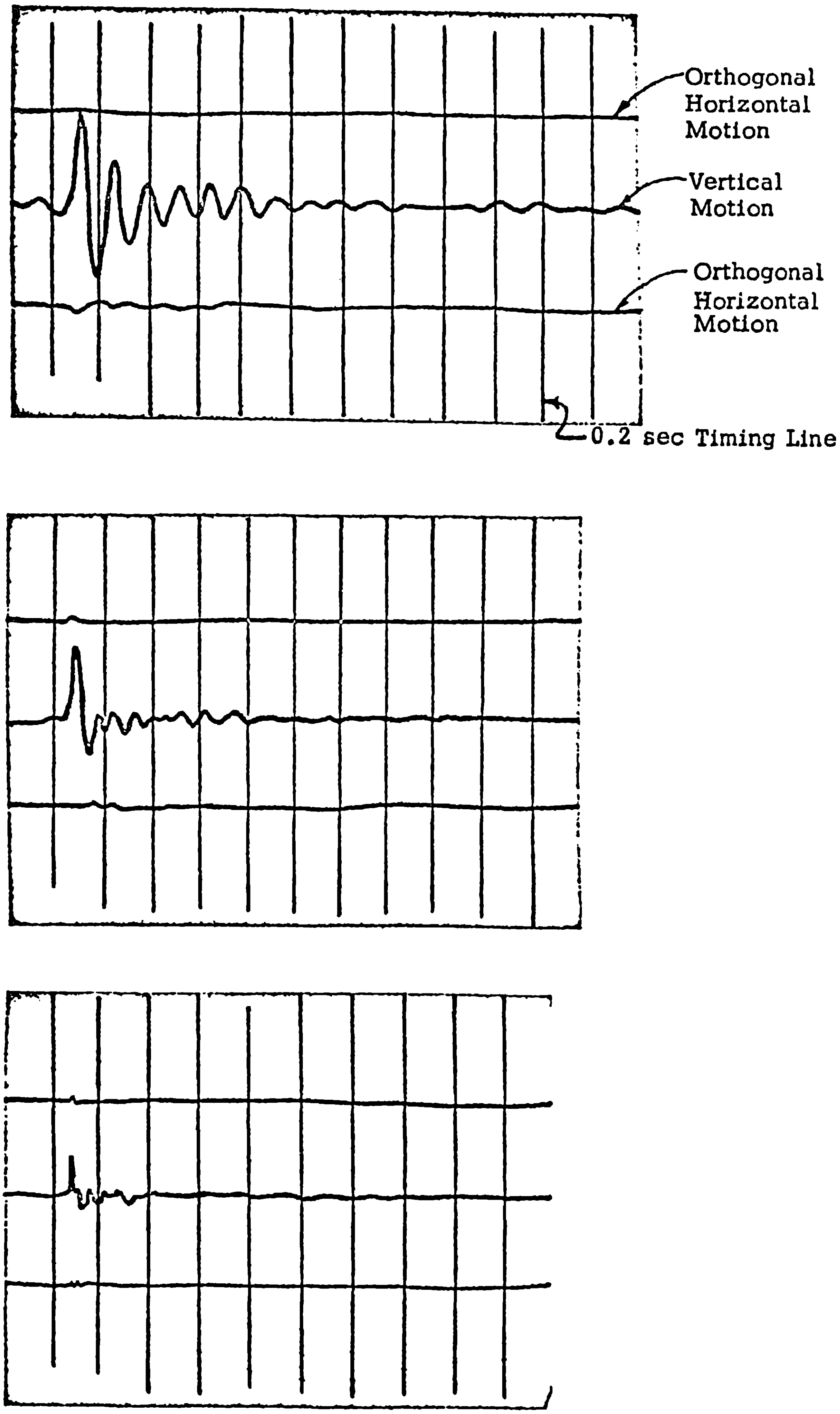


Figure 5. Typical Floor Response Records

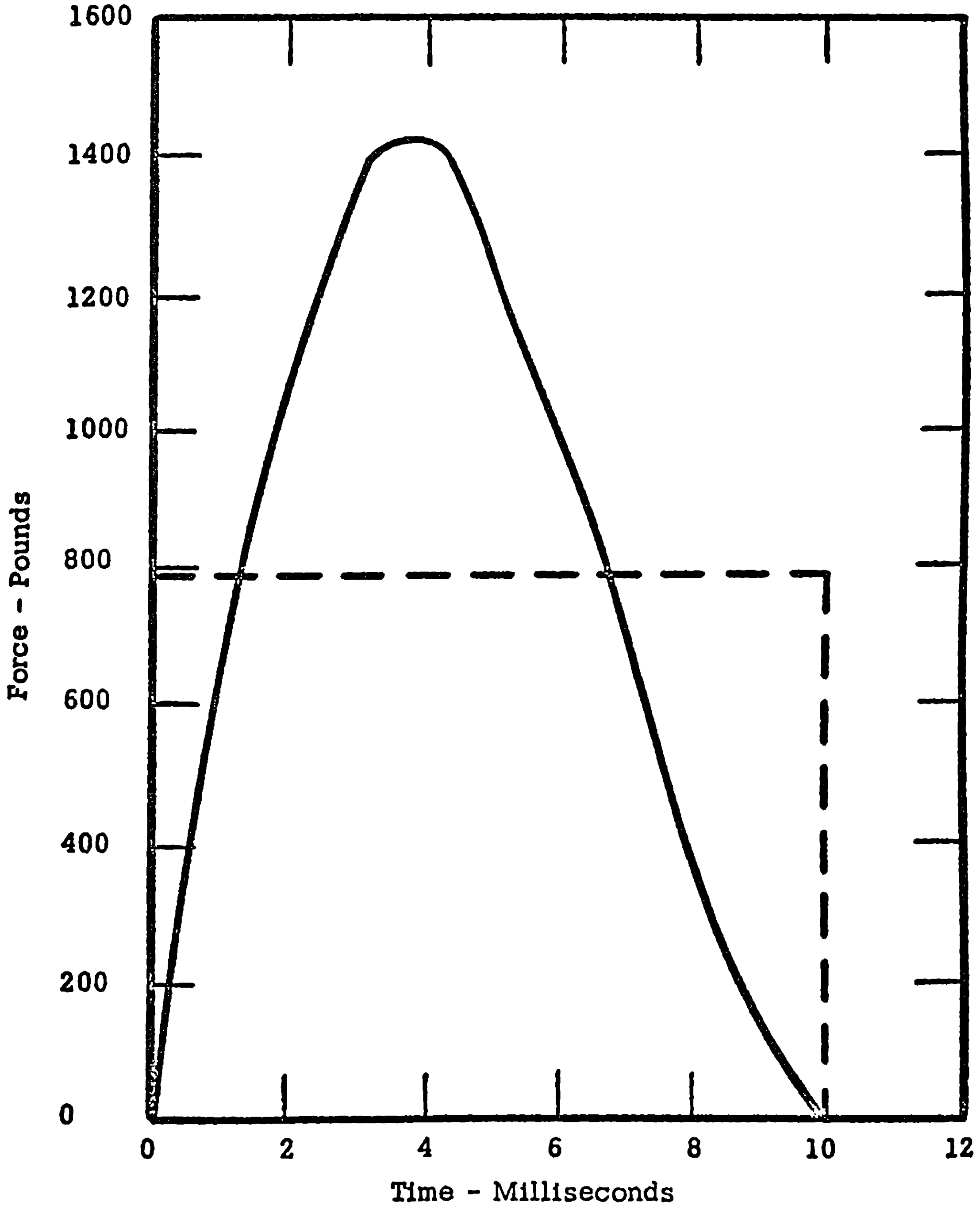


Figure 6a. Averaged Plot of Force Versus Time for Mechanical Impact.

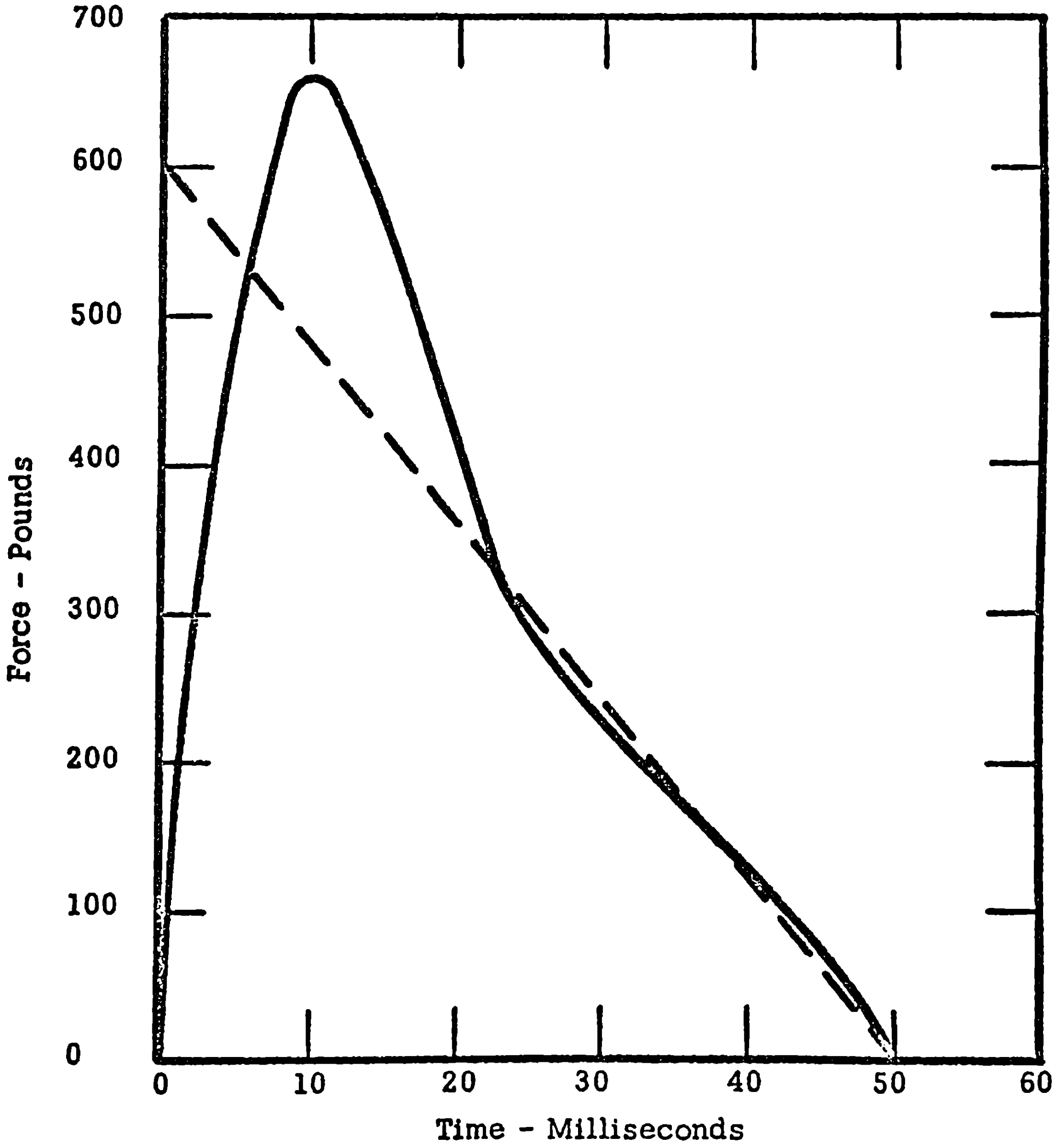


Figure 6b. Averaged Plot of Force Versus Time for Human Impact.

impulse at midspan, was integrated numerically using the actual shape of the load function and the results compared. For the human impact, the first maximum measured amplitude was 0.0079 inches and theoretical 0.0071 inches. For the mechanical impact the first measured amplitude was 0.0193 inches and the theoretical was 0.0198 inches. The reproducibility of the impacts was again checked here and found to be satisfactory.

THEORETICAL APPROACH

Two theoretical models of the floor system were considered. First, a single tee-beam, consisting of a portion of the concrete slab and the steel beam, was treated as an elastic beam with simple end-supports and free sides (Fig. 7). This model was chosen since it was hoped that the form of the analytical solution could be conveniently used in a design office. Second, the floor system was treated as an isotropic stiffened thin plate. It was hoped that this more precise theoretical model would justify the use of the tee-beam model.

Tee-Beam Model

The differential equation governing the motion of an elastic beam is

$$EI \frac{\partial^4 y}{\partial x^4} + \rho \frac{\partial^2 y}{\partial t^2} = p(x,t) \tag{1}$$

in which

x = the distance measured along the beam

$y = y(x,t)$ = the vertical displacement of the neutral axis of the beam from its static equilibrium position

t = time

ρ = mass per unit length of the beam

$p(x,t)$ = total force per unit length applied perpendicular to the beam

E = modulus of elasticity

I = cross-sectional moment of inertia of the beam.

Solution of the associated homogeneous equation

$$EI \frac{\partial^4 y}{\partial x^4} + \rho \frac{\partial^2 y}{\partial t^2} = 0 \tag{2}$$

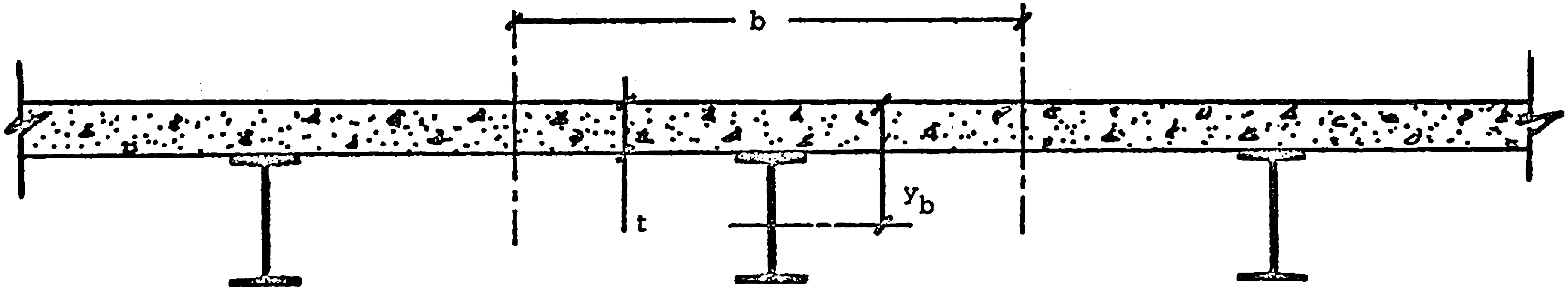


Figure 7. The Tee-Beam Model

assuming simple supports at both ends of a beam of length, L, yields the natural radian frequencies for the normal modes of vibration

$$\omega_n^2 = \frac{EI}{\rho}\;\frac{n^4\pi^4}{L^4} \qquad n = 1,2,3\;.\;.\;. \tag{3}$$

and the modal characteristic shapes

$$\phi_n(x) = \sin\frac{n\pi x}{L} \tag{4}$$

The first natural frequency is then the familiar

$$f_1 = 1.57\left[\frac{gEI}{WL^3}\right]^{1/2} \tag{5}$$

in which

W = the total weight of the beam in pounds.

Using Lagrange's equation the general solution for Eq.(5)is obtained [5]

$$y(x,t) = \sum^{n}\left[\frac{\int_0^L p_1(x)\;\phi_n(x)dx}{\omega_n^2\int_0^L \phi_n^2\,(x)dx}\right] g_n(t)\;\phi_n(x) \tag{6}$$

in which

$p_1(x)$ = the load distribution along the beam

$g_n(t)$ = the normalized response for the equivalent one degree of freedom system of the nth mode.

For a simply supported beam which is subjected to a concentrated dynamic load of magnitude F at midspan

$$\int_0^L p_1(x)\quad \phi_n(x)\;dx = F\;\phi_n(L/2)$$

$$= F\sin\frac{n\pi}{2}$$

and

$$\int_0^L \phi_n^2(x)dx = \int_0^L \sin^2\frac{n\pi x}{L} = \frac{L}{2}\;.$$

Equation (6) can then be written as

$$y(L/2,t) = \sum^{n}\frac{2FL^3}{n^4\pi^4EI}\left(\sin^2\frac{n\pi L}{2}\right)g_n(t). \tag{7}$$

For a suddenly applied constant load F of limited duration t_d (Fig. 8a) and no damping, the normalized response for the equivalent one-degree of freedom system of the nth mode is

$$g_n(t) = 1 - \cos\omega_n t \qquad t \leq td \qquad (8a)$$

$$g_n(t) = \cos \omega_n(t-t_d) - \cos \omega_n t \qquad t \geq td \qquad (8b)$$

Since the maximum value of the normalized response is 2.0 and the value of the sine terms in Eq. (7) are unity for odd modes, the modal contributions are in proportion to $1/n^4$. The maximum modal deflections are then in proportion to 1, 1/81, and 1/625 for the first, third and fifth modes respectively. Thus, the higher modes contribute very little to the midspan deflection and only the first mode will be considered. Equation (7) then becomes

$$y(L/2,t) \cong \frac{2FL^3}{\pi^4 EI}\, g_1(t) \qquad (9)$$

The first maximum amplitude A_o is found by differentiating with respect to "t", equating the result to zero and solving for the time t_o, then substituting this value into Eq. (9).

$$A_o = y(L/2,t_o) = \frac{4FL^3}{\pi^4 EI} \quad \text{at } t_o = \frac{\pi}{\omega_1} \leq t_d \qquad (10a)$$

$$A_o = y(L/2,\, t_o) = \frac{4FL^3}{\pi^4 EI} \sin \frac{\omega_1 t_d}{2} \quad \text{at } t_o = \frac{\pi}{2\omega_1} + \frac{t_d}{2} \geq t_d \qquad (10b)$$

For a suddenly applied load F which decreases linearly to zero at time t_d (Fig. 8b) and no damping, the normalized response for the equivalent one-degree of freedom system of the nth mode is

$$g_n(t) = 1 - \cos\omega_n t + \frac{\sin \omega_n t}{\omega_n td} - \frac{t}{t_d} \qquad t \leq t_d \qquad (11a)$$

$$g_n(t) = \frac{1}{\omega_n t_d}\left[\sin \omega_n t - \sin \omega_n(t-t_d)\right] - \cos\omega_n t \qquad t \geq t_d \qquad (11b)$$

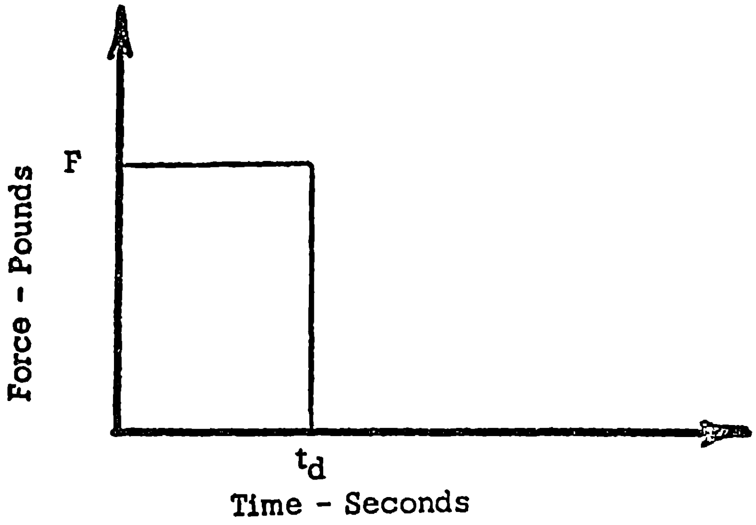


Figure 8a. Rectangular Load Pulse

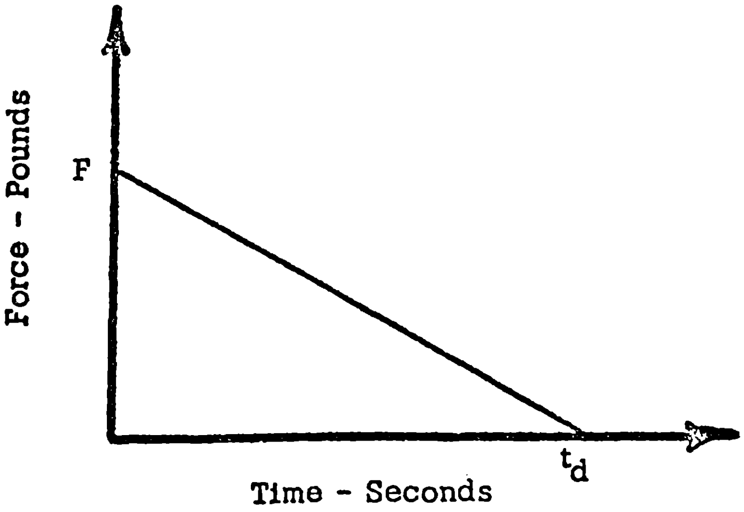


Figure 8b. Triangular Load Pulse

If $\omega_n t_d \geq 1$, the maximum modal deflections are, at most, in proportion to $1/n^4$ in Eq. (7). Therefore, as in the case of a rectangular pulse load, only the first mode will be considered.

Proceeding in a similar manner the first maximum amplitude is found to be

$$A_o = y(L/2,\, t_o) = \frac{2FL^3}{\pi^4 EI}\,(2 - t_o/t_d) \tag{12a}$$

$$\text{at } t_o = \frac{2}{\omega_1}\tan^{-1} t_d\omega_1 \leq t_d$$

$$A_o = \frac{2FL^3}{\pi^4 EI}\left[\frac{1}{\omega_1 t_d}\sqrt{2(1-\omega_1 t_d \sin \omega_1 t_d - \cos \omega_1 t_d) + (\omega_1 t_d)^2}\right] \tag{12b}$$

$$\text{at } t_o = \frac{1}{\omega_1}\tan^{-1}\left(\frac{1-\cos \omega_1 t_d}{\sin \omega_1 t_d - \omega_1 t_d}\right) \geq t_d$$

The first results apply to the approximated mechanical forcing function and the second to the approximated human forcing function.

Figures 9a and 9b are plots of normalized amplitude versus frequency for mechanical and human impacts respectively. The curves marked "actual" were obtained by numerically integrating over the actual load functions (Fig. 6) the solution of Eq.(1)for a unit impulse. The curves marked "rectangular" and "triangular" were obtained by using the first eleven terms of Eq.(7)and the approximated load function of Fig. 6. The curves marked "one term" were obtained using Eq.(10)and(12). In addition a rectangular approximation was made for the human load function of Fig. 6b and the resulting curve using Eq.(7)is shown in Fig. 9b.

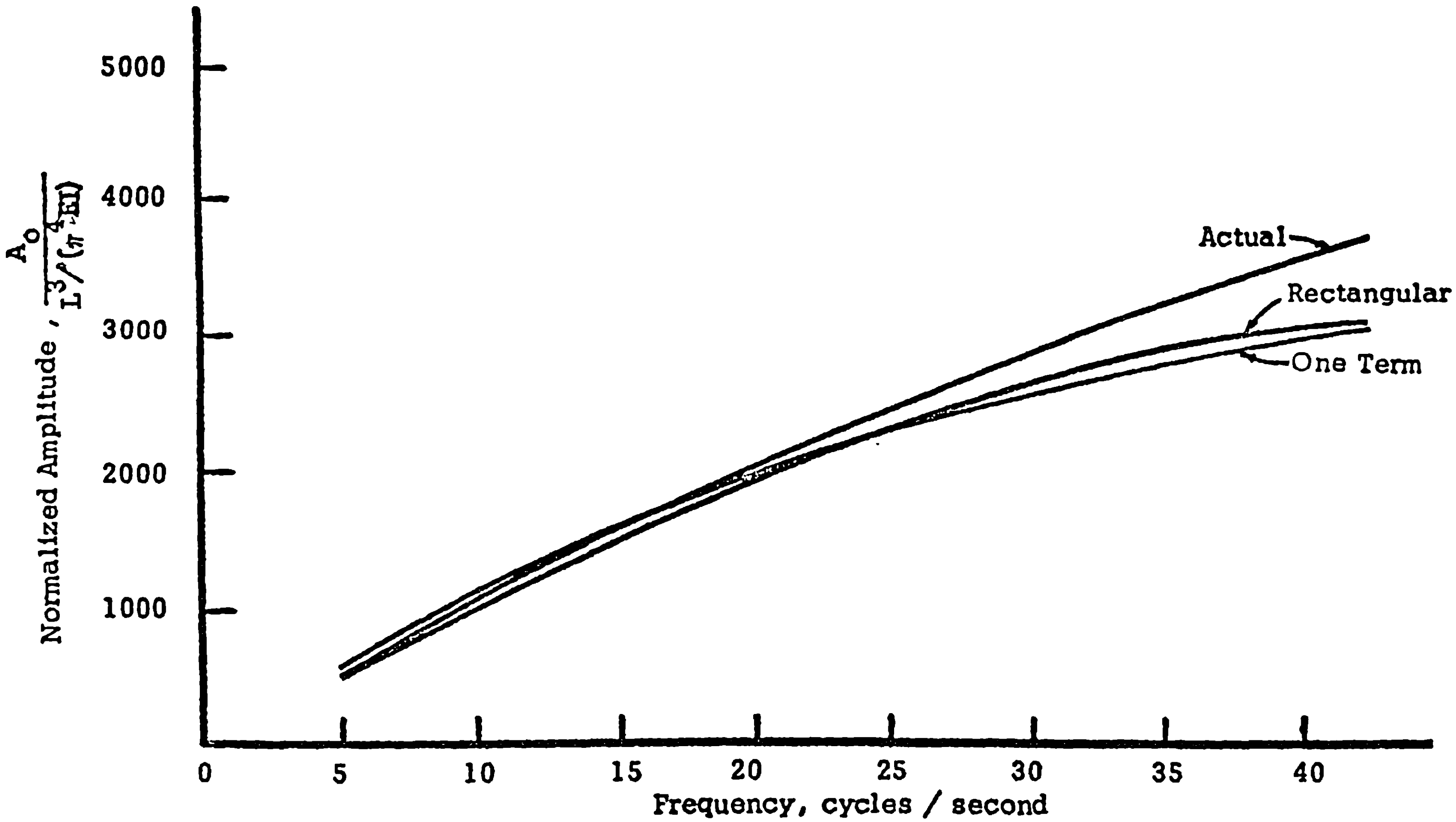


Figure 9a. Normalized Initial Amplitude Versus Frequency for Mechanical Impact

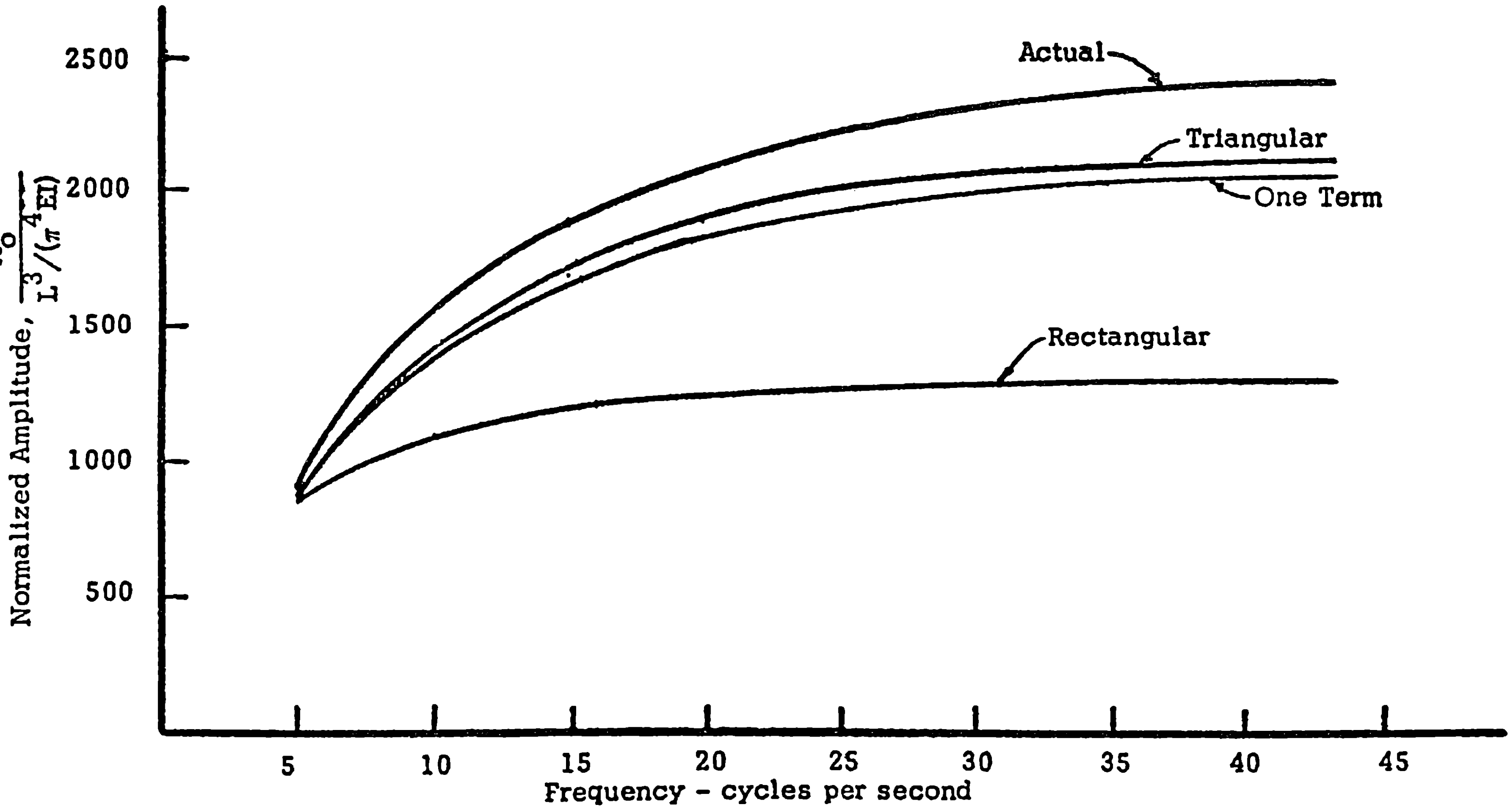


Figure 9b. Normalized Initial Amplitude Versus Frequency for Human Impact

Plate Model

A study of a stiffened thin plate subjected to impacts can be found in Ref. 6. This study determined the response of a beam-slab floor to a periodic excitation by considering one bay of the floor system as a stiffened, simply supported, rectangular plate. The study includes a discussion of a series of test floors subjected to both mechanical and human impact.

The results of these analytical and experimental investigations indicate that a single tee-beam may not be an accurate model.

RESULTS OF EXPERIMENTAL AND THEORETICAL STUDIES

Appendix A contains partial floor plans of all of the buildings tested.* The locations of all impacts where the impactor and recorder were at the center of the span and over a beam are noted. The tape records for each of these impacts are also included. The vertical lines on these records are timing lines at 0.2 second intervals. Since the tape drum runs at a constant speed, the distance between timing lines varies depending on the amount of paper tape on the drum. Because of the optical lever system in the recorder, the actual deflection is 1/50th of that shown on the tape records.

Table 1 is a summary of the structural components of the floor systems tested. For a measure of the damping in the system, the equivalent critical viscous damping ratio, ζ , was determined using the relationships[7]

$$\zeta = \frac{\delta}{(4\pi^2 + \delta^2)^{1/2}} \qquad (13)$$

$$\delta = \frac{1}{n} \ln {}^{y}o/y_n$$

---

* The floor system of Building 9 consists of deep trusses and is not included in this report.

in which

δ = logarithmic decrement

y_o = first amplitude

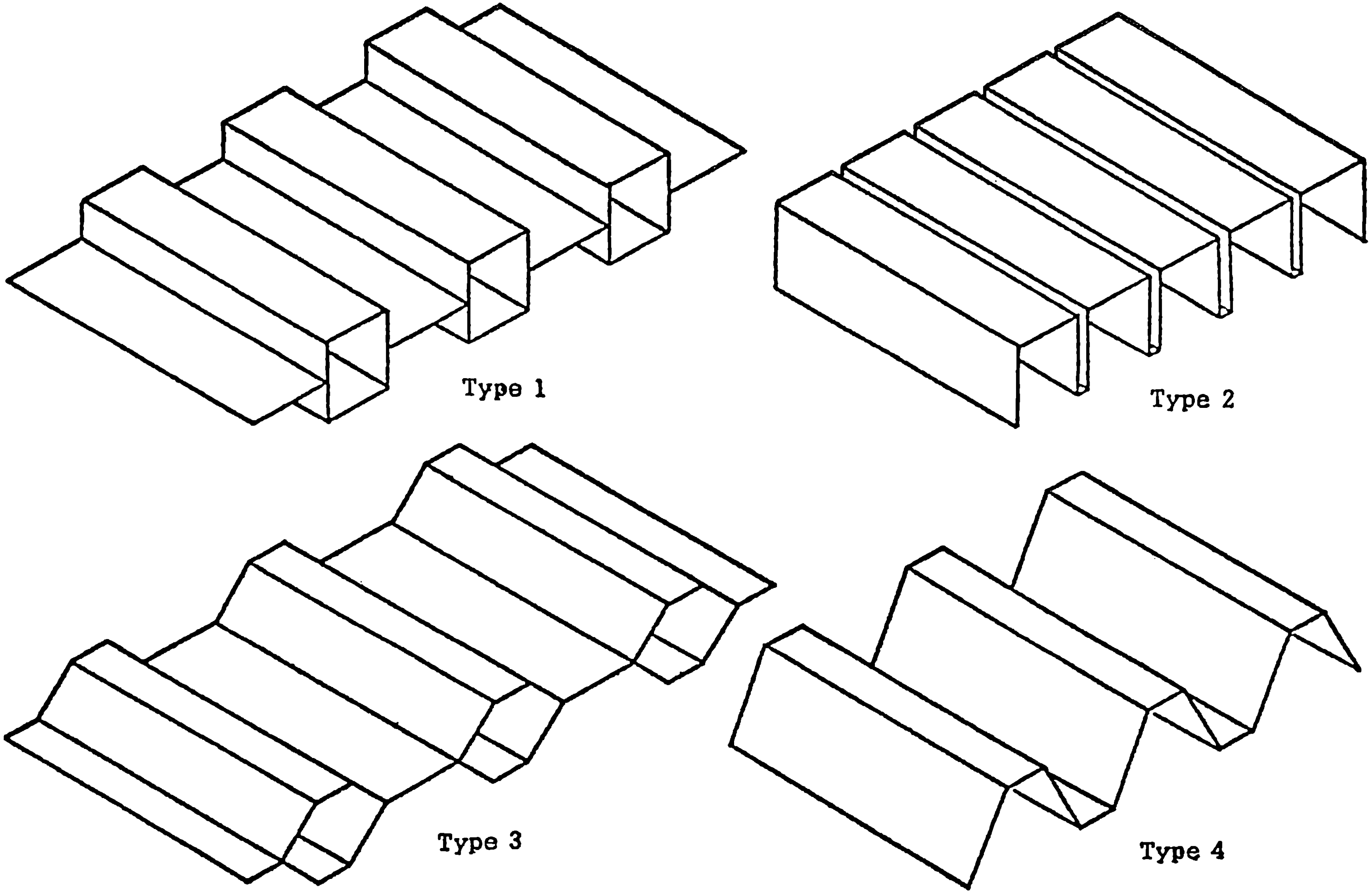
y_n = nth amplitude

ζ = critical damping ratio

and is included in the table. The number of cycles to one-fifth first amplitude is also included.

A response noted "damped" in Table 1 is one where the response was not of sufficient length to accurately determine the damping ratio. The response of some of the floors was definitely a random vibration and is noted as "noise". Figure 10 shows the types of decking indicated in the table.

Table 2 is a summary of the beam and slab properties. Also included is the weight per linear inch and the moment of inertia of the composite tee-beam. Preliminary calculations indicated that, regardless of the actual type of construction, close agreement between the measured frequency and that given by Eq. (5) was obtained only if the slab was considered to act composite with the steel beam. It is thought that the small deflections encountered are not sufficient to produce movement between the two materials, and, therefore, acted as a composite section. The effective width of the slab was considered: first, as the sum of one-half the clear distances to the adjacent beams; second, as the width allowed in the AISC Specification for the Design, Fabrication, and Erection of Structural Steel for Buildings, 1963 revision.[8] Finally non-structural concrete (topping) in the floor system was considered to act compositely with the structural floor slab and the steel beam, and is included when computing the effective width using the AISC Specifications. An explanation of the code in Table 2 follows:



Type 1

Type 2

Type 3

Type 4

Figure 10. Types of Decking.

A - Topping included; effective slab width taken as the sum of one-half the distances to adjacent beams

B - Topping included; effective slab width as limited by AISC Specification

C - Same as A except no topping

D - Same as B except no topping

1 - Cover plates on beam

2 - Castellated beam.

Table 3 is a summary of the theoretical and measured frequencies, and of the theoretical and measured first amplitudes for both mechanical and human impacts. The theoretical frequencies were obtained by using the data from Table 2 and Eq. (5). The theoretical amplitudes were obtained by using the data from Table 2 and Eqs. (10) and (12). Only one tee-beam was considered to act for these calculations. Also included in Table 3 is the ratio of the theoretical amplitude to the measured amplitude. Since the theoretical amplitude is inversely proportional to the moment of inertia of the section, this ratio represents the number of tee-beams effective in resisting the impacts.

A sample record and the procedure used to determine the measured frequency, first amplitude, and damping ratio is shown in Appendix B. Sample calculations for the theoretical frequency and amplitude for a tee-beam are shown in Appendix C.

					Concrete					Damping		Cycles to 0.2 Ao		
Fig.	Floor	Loc.	Beam	Fire-proofing	Slab	Topp'g	C&NC	Centering Type	Partitions	Mech.	Hum.	Mech.	Hum.	Remarks
1	2	1	16B26	spray-on	5" L.W.		comp.	Type 4	Above	D.		1		
	2	2	16B31	" "	" " "		"	" "	"	"		1		
	2	3	"	" "	" " "		"	" "	"	"	9.4%	2	5	
2	4	1	24WF84	spray-on	5" L.W.		comp.	Type 4		N.R.				
3	R	1	31WF68	none	$4\frac{1}{2}$"Reg.		comp.	none			8.0%		10	castellated
	R	2	16B26	"	" "		"	"			11.7%		3	
	2	3	31WF76	"	5"Reg.		"	"		N.	7.7%	1	5	castellated
	2	3	"	"	" "		"	"		"		1		
	2	3	"	"	" "		"	"		"		1		
	2	3	"	"	" "		"	"		"		1		
	2	4	16WF40	"	" "		"	"		D.	7.9%	4	7	
	2	4	"	"	" "		"	"			8.0%		9	
	2	5	18WF45	"	" "		"	"		9.6%		3		cover plate
	2	6	"	"	" "		"	"			8.2%		5	
	2	7	31WF76	"	" "		"	"		D.	8.9	1	3	castellated
	2	8	18WF45	"	" "		"	"		D.	D.	1	1	cover plate
4	2	1	16WF36	spray-on	$4\frac{1}{2}$"L.W.		comp.	Type 4		15.9%	D.	1	1	large live load
	2	2	"	" "	" " "		"	"		D.	11.1%	4	4	
	2	2	"	" "	" " "		"	"		10.4%		1		
5	10	1	16WF36	spray-on	$4\frac{1}{2}$"L.W.		comp.	Type 4		5.7%	4.1%	5	6	
	10	2	"	" "	" " "		"	"		D.	4.5%	1	3	
	10	3	21WF55	" "	" " "		"	"		D.	D.	1	2	
6	7	1	12B19	ceiling	4" L.W.		comp.	Type 4		6.3%	11.5%	5	7	under construction
		2	16WF45	" "	" " "		"	"		D.				

TABLE 1. Floor Properties and Damping Characteristics

					Concrete					Damping		Cycles to 0.2 Ao		
Blg.	Floor	Loc.	Beam	Fire-proofing	Slab	Topp'g	CorNC	Centering Type	Partitions	Mech.	Hum.	Mech.	Hum.	Remarks
7	1	1	24WF68	N.K.	4"Reg.		N.C.	Type 1						walked on bm.
8	4	1	16B26	spray-on	4½"L.W.		comp.	Type 4		D.	N.	5	1	
	4	2	"	" "	" " "		"	" "		"	4.2	3	3	
	4	3	18WF45	" "	" " "		N.C.	" "		"	D.	N.	1	
	11	4	16B26	" "	" " "		"	" "		N.	N.	1	1	
10	10	1	14B22	spray-on	3"L.W.		N.C.	Type 1		D.	D.	2	1	
	10	2	18WF50	" "	" " "		"	" "		"	"	1	3	
	10	4	"	" "	" " "		"	" "		6.4	7.1	5	7	
11	11	3	16WF45	" "	2½"L.W.		"	Type 2		D.	D.	2	1	
	11	5	16WF45	" "	" " "		"	" "		"	"	1	D.	
12	6	1	16WF50	" "	" " "		"	Type 1		"	*	N.		*walked on bm.
13	4	1	21WF96	" "	" " "		"	" "		"	D.	1	1	
	4	3	14WF30	" "	" " "		"	" "		7.4	7.6	5	4	
14	31	1	21WF55	" "	4½"L.W.	2½"reg	comp.	none		N.	D.	1	2	
	31	5	21WF73	" "	" " "	" "	"	"		D.	N.	2	1	
15	7	1	16WF36	" "	2½"L.W.		"	Type 1		"	D.	N.	1	
	7	3	36WF300	" "	" " "		"	" "		"	"	1	1 1	
	7	4	16WF36	" "	" " "		"	" "		N.	D.	N.	1	
16	3	1	21WF62	lath & plaster	5"Reg.		"	none		4.2	4.1	6	10	cover plate
	3	3	"		" "		"	"		7.4	3.8	4	9	
17	2	1	18WF50	spray-on	5"L.W.		"	Type 4		5.5	6.3	4	4	
	2	3	"	" "	" " "		"	" "		D.	D.	2	2	
	2	4	16B31	" "	" " "		"	" "		10.4	12.8	3	5	

TABLE 1. Floor Properties and Damping Characteristics
(continued)

.Blg.	Floor	Loc.	Beam	Fire-proofing	Concrete			Centering Type	Partitions	Damping		Cycles to 0,2 Ao		Remarks
					Slab	Topp'g	CorNC			Mech.	Hum.	Mech.	Hum.	
17	2	6	16B31	spray-on	5"L.W.		comp.	Type 4		D.	D.	1	1	
	2	7	24WF68	" "	" " "		"	" "		"	"	1	1	
18		1	18WF50	" "	4"Reg.	2.5"	comp.	none		"	"	1	1	
		3	"	" "	" "	"	"	"		N.R.	N.R.	N.R.	N.R.	
		4	12WF27	" "	" "	"	"	"		N.	6.8	N.	5	
		10	18WF50	" "	" "	"	"	"		"	10.1	N.	9	
		12	"	" "	" "	"	"	"		D.	6.4	1	7	
19	1	4	16B31	none	$3\frac{1}{2}$Reg.		"	"						
20	3	2	"	"	5"L.W.		"	Type 4			7.8		8	
	3	2	"	"	" " "		"	" "			8.3		4	
	3	4	16WF36	"	" " "		"	" "			D.		1	
	3	4	"	"	" " "		"	" "			N.T.			

Comp. - Composite Construction
N.C. - Non-composite Construction
N.K. - Not Known
D. - Damped
W. - Noise
N.R. - No Record
L.W. - Light Weight Concrete
Reg. - Regular Weight Concrete

TABLE 1. Floor Properties and Damping Characteristics (continued)

Blg.	Floor	Loc.	Code	Beam	Span ft.	A_s in^2	I in^4	y_b in.	b in.	t in.	w_c pcf	f_c' psi	w lb/in.	I_t in^4
1	2	1	C	16 B26	30.00	7.65	298.1	12.83	95.50	5.0	110.	3000.	32.57	1037.2
1	2	1	D	16 B26	30.00	7.65	298.1	12.83	85.50	5.0	110.	3000.	32.57	1015.8
1	2	2	C	16 B31	30.00	9.12	372.5	12.92	126.50	5.0	110.	3000.	42.85	1252.1
1	2	2	D	16 B31	30.00	9.12	372.5	12.92	85.53	5.0	110.	3000.	42.85	1200.3
2	4	1	C	24WF84	38.08	24.71	2364.3	17.05	312.00	5.0	110.	3000.	106.32	6884.9
2	4	1	D	24WF84	38.08	24.71	2364.3	17.05	89.00	5.0	110.	3000.	106.32	5382.8
3	R	2	C1	16 B26	40.00	11.65	480.6	14.69	120.00	4.0	150.	3000.	44.98	2101.9
3	R	2	D1	16 B26	40.00	11.65	480.6	14.69	69.50	4.0	150.	3000.	44.98	1896.5
3	R	1	C2	31WF68	54.00	16.52	3368.0	19.80	120.00	4.0	150.	3000.	46.36	7465.9
3	R	1	D2	31WF68	54.00	16.52	3368.0	19.80	72.96	4.0	150.	3000.	46.36	6915.3
3	2	3	C2	31WF76	54.00	20.47	4266.3	21.77	120.00	4.5	150.	3000.	52.69	10230.5
3	2	3	D2	31WF76	54.00	20.47	4266.3	21.77	80.99	4.5	150.	3000.	52.69	9569.8
3	2	4	C	16WF40	30.00	11.77	515.5	12.50	120.00	4.5	150.	3000.	50.22	1658.7
3	2	4	D	16WF40	30.00	11.77	515.5	12.50	87.00	4.5	150.	3000.	50.22	1570.6
3	2	5	C1	18WF45	40.00	16.24	910.6	15.13	120.00	4.5	150.	3000.	51.49	3148.7
3	2	5	D1	18WF45	40.00	16.24	910.6	15.13	79.48	4.5	150.	3000.	51.49	2908.9
3	1	7	C2	31WF76	54.00	20.47	4266.3	21.77	120.00	4.5	150.	3000.	52.69	10230.5
3	1	7	D2	31WF76	54.00	20.47	4266.3	21.77	80.99	4.5	150.	3000.	52.69	9569.8
3	1	8	C1	18WF45	40.00	16.24	910.6	15.13	120.00	4.5	150.	3000.	51.49	3148.7
3	1	8	D1	18WF45	40.00	16.24	910.6	15.13	79.48	4.5	150.	3000.	51.49	2908.9
4	2	2	C	16WF36	30.00	10.59	446.3	12.43	120.00	4.5	110.	3000.	37.38	1374.1
4	2	2	D	16WF36	30.00	10.59	446.3	12.43	78.99	4.5	110.	3000.	37.38	1265.4
5	10	1	C	16WF36	30.00	10.59	446.3	12.43	120.00	4.5	110.	3000.	37.38	1374.1
5	10	1	D	16WF36	30.00	10.59	446.3	12.43	78.99	4.5	110.	3000.	37.38	1265.4
5	10	3	C	21WF55	30.00	16.18	1140.7	14.90	360.00	4.5	110.	3000.	107.72	3610.7
5	10	3	D	21WF55	30.00	16.18	1140.7	14.90	80.22	4.5	110.	3000.	107.72	2779.5

Table 2. Tee-Beam Properties

Blg.	Floor	Loc.	Code	Beam	Span ft.	A_s* in^2	I* in^4	y_b in.	b in.	t in.	w_c pcf	f_c' psi	w lb/in.	I_t in^4
6	7	1	C	12 B19	25.00	5.62	130.1	10.80	102.00	4.0	110.	3000.	27.57	534.5
6	7	1	D	12 B19	25.00	5.62	130.1	10.80	68.00	4.0	110.	3000.	27.57	494.3
6	7	2	C	16WF45	25.42	13.24	583.3	12.06	300.00	4.0	110.	3000.	80.15	1860.1
6	7	2	D	16WF45	25.42	13.24	583.3	12.06	71.04	4.0	110.	3000.	80.15	1423.7
7	1	1	C	24WF68	25.00	20.00	1814.5	15.86	300.00	4.0	145.	3000.	106.37	5320.4
7	1	1	D	24WF68	25.00	20.00	1814.5	15.86	72.96	4.0	145.	3000.	106.37	4214.4
8	4	1	C	16 B26	28.50	7.65	298.1	12.33	120.00	4.5	110.	3000.	36.55	1013.0
8	4	1	D	16 B26	28.50	7.65	298.1	12.33	77.50	4.5	110.	3000.	36.55	936.0
8	4	3	C	18WF45	20.00	13.24	704.5	13.43	342.00	4.5	110.	3000.	101.73	2369.3
8	4	3	D	18WF45	20.00	13.24	704.5	13.43	79.48	4.5	110.	3000.	101.73	1840.4
8	11	4	C	16 B26	20.00	7.65	298.1	12.33	114.00	4.5	110.	3000.	34.83	1004.1
8	11	4	D	16 B26	20.00	7.65	298.1	12.33	77.50	4.5	110.	3000.	34.83	936.0
10	10	1	C	14 B22	7.50	6.47	197.4	12.99	45.00	3.0	110.	3000.	10.43	717.3
10	10	1	D	14 B22	7.50	6.47	197.4	12.99	22.50	3.0	110.	3000.	10.43	567.2
10	10	2	C	18WF50	30.00	14.71	800.6	15.13	90.00	3.0	110.	3000.	21.37	2370.0
10	10	2	D	18WF50	30.00	14.71	800.6	15.13	55.50	3.0	110.	3000.	21.37	2035.6
11		5	C	16WF45	28.00	13.24	583.3	13.56	84.00	2.5	110.	3750.	17.13	1716.6
11		5	D	16WF45	28.00	13.24	583.3	13.56	47.48	2.5	110.	3750.	17.13	1428.7
12	6	1	C	16WF50	25.83	14.70	655.4	13.62	114.00	2.5	110.	3000.	22.32	1975.9
12	6	1	D	16WF50	25.83	14.70	655.4	13.62	47.07	2.5	110.	3000.	22.32	1481.6
13	4	1	C	21WF96	25.50	28.21	2088.9	16.07	102.00	2.5	110.	3000.	24.24	4539.3
13	4	1	D	21WF96	25.50	28.21	2088.9	16.07	49.04	2.5	110.	3000.	24.24	3568.8
13	4	3	C	14WF30	25.50	8.81	289.6	12.43	102.00	2.5	110.	3000.	18.73	1043.0
13	4	3	D	14WF30	25.50	8.81	289.6	12.43	54.73	2.5	110.	3000.	18.73	875.7
14	31	1	A	21WF55	38.67	16.18	1140.7	17.40	116.00	2.5	145.	3000.	62.16	4092.2
										4.5	110.	3000.		

Table 2. Tee-Beam Properties Continued

Blg.	Floor	Loc.	Code	Beam	Span ft.	A_s^* in^2	I^* in^4	y_b in.	b in.	t in.	w_c pcf	f_c' psi	w lb/in.	I_t in^4
14	31	1	B	21WF 55	38.67	16.18	1140.7	17.40	80.22	2.5	145.	3000.	62.16	3795.2
										4.5	110.	3000.		
14	31	5	A	21WF 73	38.67	21.46	1600.3	17.62	116.00	2.5	145.	3000.	63.66	5318.0
										4.5	110.	3000.		
14	31	5	B	21WF 73	38.67	21.46	1600.3	17.62	80.30	2.5	145.	3000.	63.66	4902.1
										4.5	110.	3000.		
15	7	1	C	16WF 36	23.25	10.59	446.3	13.43	112.00	2.5	110.	3000.	20.83	1486.4
15	7	1	D	16WF 36	23.25	10.59	446.3	13.43	46.99	2.5	110.	3000.	20.83	1147.8
15	7	3	C	36WF300	56.73	88.17	20290.0	23.86	279.00	2.5	110.	3000.	69.44	36660.2
15	7	3	D	36WF300	56.73	88.17	20290.0	23.86	56.66	2.5	110.	3000.	69.44	24963.5
16	3	1	C	21WF 62	47.60	24.23	1872.2	18.22	96.00	5.0	145.	3000.	47.16	6071.5
16	3	1	D	21WF 62	47.60	24.23	1872.2	18.22	88.24	5.0	145.	3000.	47.16	5951.8
17	2	1	C	18WF 50	34.50	14.71	800.6	14.00	112.00	5.0	115.	3000.	41.45	2340.2
17	2	1	D	18WF 50	34.50	14.71	800.6	14.00	87.50	5.0	115.	3000.	41.45	2224.1
17	2	4	C	16 B 31	28.00	9.12	372.5	12.92	112.00	5.0	115.	3000.	39.86	1279.3
17	2	4	D	16 B 31	28.00	9.12	372.5	12.92	85.53	5.0	115.	3000.	39.86	1216.2
17	2	7	C	24WF 68	28.00	20.00	1814.5	16.85	375.00	5.0	115.	3000.	130.46	5731.4
17	2	7	D	24WF 68	28.00	20.00	1814.5	16.85	88.96	5.0	115.	3000.	130.46	4484.3
18		1	A	18WF 50	40.50	20.71	1185.0	19.26	121.00	2.5	145.	3000.	82.03	5766.4
										5.0	145.	3000.		
18		1	B	18WF 50	40.50	20.71	1185.0	19.26	87.50	2.5	145.	3000.	82.03	5381.9
										5.0	145.	3000.		
18		4	A	12WF 27	30.17	12.97	333.1	15.98	71.28	2.5	145.	3000.	48.54	2190.2
										5.0	145.	3000.		
18		4	B	12WF 27	30.17	12.97	333.1	15.98	71.28	2.5	145.	3000.	48.54	2190.2
										5.0	145.	3000.		

Table 2. Tee-Beam Properties Continued

Blg.	Floor	Loc.	Code	Beam	Span ft.	A_s* in^2	I* in^4	y_b in.	b in.	t in.	w_c pcf	f_c' psi	w lb/in.	I_t in^4
19	1	4	C	16 B 31	32.00	9.12	372.5	11.42	96.00	3.5	145.	3000.	30.78	1092.4
19	1	4	D	16 B 31	32.00	9.12	372.5	11.42	61.52	3.5	145.	3000.	30.79	1010.3
20	3	2	C	16 B 31	36.00	9.12	372.5	12.92	144.00	5.0	110.	3000.	48.42	1322.3
20	3	2	D	16 B 31	36.00	9.12	372.5	12.92	85.52	5.0	110.	3000.	48.42	1200.3
20	3	4	C	16WF 36	36.00	10.59	446.3	12.92	144.00	5.0	110.	3000.	48.84	1508.3
20	3	4	D	16WF 36	36.00	10.59	446.3	12.92	86.99	5.0	110.	3000.	48.84	1370.5

* Includes cover plate, if any.

Table 2. Tee-Beam Properties Continued

Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Frequency Theor.	Frequency Actual M	Frequency Actual H	Mech Amplitude Theor.	Mech Amplitude Actual	Human Amplitude Theor.	Human Amplitude Actual	Equivalent Bms Mech	Equivalent Bms Human
1	2	1	21-12M	C	16 B26	30.00	7.23			.01141	.0034	.01832		3.36	
	2	1	21-12M	D	16 B26	30.00	7.15			.01153	.0034	.01857		3.40	
	2	2	21-9 M	C	16 B31	30.00	7.03			.00891	.0018	.01442		4.95	
	2	2	21-9 M	D	16 B31	30.00	6.78			.00925	.0018	.01511		5.14	
	2	3	21-2 M }	C	16 B31	30.00	7.03		7.7	.00891	.0010	.01442	.0070	8.91	2.06
	2	3	21-3 L	D	16 B31	30.00	6.78		7.7	.00925	.0010	.01511	.0070	9.25	2.16
2	4	1	22-25M	C	24WF84	37.58	6.40			.00312		.00516			
	4	1	22-25M	D	24WF84	37.58	5.66			.00312		.00598			
3	R	2	32-2 L	C1	16 B26	40.00	4.93		6.3	.00913		.01579	.0072		2.19
	R	2	32-2 L	D1	16 B26	40.00	4.68		6.3	.00962		.01673	.0072		2.33
	R	1	32-3 L	C2	31WF68	54.00	5.02		4.6	.00644		.01111	.0059		1.89
	R	1	32-3 L	D2	31WF68	54.00	4.83		4.6	.00670		.01161	.0059		1.97
	2	3	31-1 M }	C2	31WF76	54.00	5.51			.00516	.0030	.00878	.0048	1.72	1.83
	2	3	31-2 L	D2	31WF76	54.00	5.33			.00534	.0030	.00912	.0048	1.78	1.90
	2	4	31-12M	C	16WF40	30.00	7.36		9.9	.00726	.0030	.01160	.0052	2.42	2.23
	2	4	31-13L	D	16WF40	30.00	7.16		9.9	.00747	.0030	.01202	.0052	2.49	2.31
	2	5	31-8 M	C1	18WF45	40.00	5.64	7.3		.00697	.0028	.01181		2.48	
	2	5	31-8 M	D1	18WF45	40.00	5.42	7.3		.00725	.0028	.01237		2.59	
	2	6	31-10L	C1	18WF45	40.00	5.64		7.5	.00697		.01181	.0052		2.27
	2	6	31-10L	D1	18WF45	40.00	5.42		7.5	.00725		.01237	.0052		2.38

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies

							Frequency			Mech Amplitude		Human Amplitude		Equivalent Rms	
Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Theor.	Actual		Theor.	Actual	Theor.	Actual	Mech	Human
								M	H						
3	2	7	32-6 M	C2	31WF76	54.00	5.51		7.1	.00516	.0026	.00878	.0040	1.99	2.19
	2	7	32-7 L	D2	31WF76	54.00	5.33		7.1	.00534	.0026	.00912	.0040	2.05	2.28
	2	8	32-12M	C1	18WF45	40.00	5.64			.00697	.0024	.01181	.0041	2.90	2.88
	2	8	32-13L	D1	18WF45	40.00	5.42			.00725	.0024	.01237	.0041	3.02	3.02
4	2	1	41-6 M	C	16WF36	30.00	7.77	10.0		.00924	.0026	.01452	.0056	3.55	2.59
	2	1	41-7 L	D	16WF36	30.00	7.45	10.0		.00963	.0026	.01534	.0056	3.70	2.74
	2	2	41-1 M }	C	16WF36	30.00	7.77	8.6	7.5	.00924	.0032	.01452	.0054	2.89	2.69
	2	2	41-2 M												
	2	2	41-3 L	D	16WF36	30.00	7.45	8.6	7.5	.00963	.0032	.01534	.0054	3.01	2.84
5	10	1	41-18M	C	16WF36	31.00	7.77	10.3	12.5	.00924	.0046	.01452	.0060	2.02	2.42
	10	1	41-19L	D	16WF36	31.00	7.45	10.3	12.5	.00963	.0046	.01534	.0060	2.09	2.55
	10	2	41-10M	C	16WF36	31.00	7.77			.00924	.0042	.01452		2.20	
	10	2	41-11L	D	16WF36	31.00	7.45			.00963	.0042	.01534		2.29	
6	7	1	41-22M	C	12 B19	25.00	8.12	7.4	7.5	.01436	.0068	.02724	.0090	2.11	2.48
	7	1	41-23L	D	12 B19	25.00	7.81	7.4	7.5	.01495	.0068	.02345	.0090	2.20	2.61
	7	2	41-30M	C	16WF45	25.42	8.60			.00458	.0046	.00696	.0030	1.00	2.32
	7	2	41-31L	D	16WF45	25.42	7.52			.00526	.0046	.00834	.0030	1.14	2.78
7	1	1	42-2 L	C	24WF68	25.00	13.05			.00228		.00284	.0014		2.03
	1	1	42-2 L	D	24WF68	25.00	11.61			.00257		.00341	.0014		2.44

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies (Cont.)

Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Frequency Theor.	Frequency Actual M	Frequency Actual H	Mech Amplitude Theor.	Mech Amplitude Actual	Human Amplitude Theor.	Human Amplitude Actual	Equivalent Bms Mech	Equivalent Bms Human
8	4	1	42-5 M	C	16 B26	28.50	7.47			.01035	.0068	.01645	.0070	1.52	2.35
	4	1	42-6 L	D	16 B26	28.50	7.18			.01077	.0068	.01732	.0070	1.58	2.48
	4	2	42-3 M	C	16 B26	28.50	7.56		10.8	.00527	.0054	.00835	.0062	0.98	1.35
	4	2	42-3 L	D	16 B26	28.50	6.90		10.8	.00529	.0054	.00941	.0062	0.98	1.52
	4	3	42-7 M	C	18WF45	20.00	13.91			.00278	.0010	.00335	.0028	2.78	1.20
	4	3	42-8 L	D	18WF45	20.00	12.26			.00318	.0010	.00410	.0028	3.18	1.46
	11	4	42-13M	C	16 B26	20.00	15.48			.00725	.0054	.00820	.0024	1.34	3.42
	11	4	42-14L	D	16 B26	20.00	14.94			.00752	.0054	.00870	.0024	1.39	3.62
10	10	1	62-6 M	C	14 B22	7.50	169.98			.00229	.0046	.00082	.0032	0.50	0.26
	10	1	62-7 L	D	14 B22	7.50	151.15			.00290	.0046	.00103	.0032	0.63	0.32
	10	2	62-4 M	C	18WF50	30.00	13.49			.00912	.0046	.01117	.0058	1.98	1.93
	10	2	62-5 L	D	18WF50	30.00	12.51			.00988	.0046	.01261	.0058	2.15	2.17
	10	4	62-12M	C	18WF50	30.00	13.49	11.2	12.5	.00912	.0050	.01117	.0068	1.82	1.64
	10	4	62-13L	D	18WF50	30.00	12.51	11.2	12.5	.00988	.0050	.01261	.0068	1.98	1.86
11		3	63-4 M	C	16WE45	28.00	14.72			.01111	.0050	.01295	.0050	2.22	2.59
		3	63-5 L	D	16WF45	28.00	13.43			.01225	.0050	.01504	.0050	2.45	3.00
		5	63-12M	C	16WF45	28.00	14.72			.01111	.0042	.01295		2.65	
		5	63-13L	D	16WF45	28.00	13.43			.01225	.0042	.01504		2.92	
12	6	1	63-16M	C	16WF50	25.83	16.26			.00830	.0072	.00913	.0020	1.15	4.56
	6	1	63-17L	D	16WF50	25.83	14.08			.00969	.0072	.01159	.0020	1.35	5.79

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies (Cont.)

Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Frequency			Mech Amplitude		Human Amplitude		Equivalent Bms	
							Theor.	Actual M	Actual H	Theor.	Actual	Theor.	Actual	Mech.	Human
13	4	1	63-20M	C	21WF96	25.50	24.26			.00491	.0040	.00425	.0020	1.22	2.13
	4	1	63-21L	D	21WF96	25.50	21.51			.00566	.0040	.00526	.0020	1.41	2.63
	4	3	64-5 M	C	14WF30	25.50	13.23	14.7	15.2	.01249	.0072	.01547	.0088	1.73	1.76
	4	3	64-6 L	D	14WF30	25.50	12.12	14.7	15.2	.01370	.0072	.01777	.0088	1.90	2.02
14	31	1	64-9 M	A	21WF55	38.67	6.26			.00537	.0010	.00893	.0020	5.37	4.47
	31	1	64-10L	B	21WF55	38.67	6.02			.00558	.0010	.00934	.0020	5.58	4.67
	31	5	64-19M	A	21WF73	38.67	7.05			.00465	.0016	.00751	.0026	2.91	2.89
	31	5	65-1 L	B	21WF73	38.67	6.77			.00484	.0016	.00791	.0026	3.03	3.04
15	7	1	65-9 M	C	16WF36	23.25	18.02			.00883	.0040	.00913	.0058	2.21	1.57
	7	1	65-10L	D	16WF36	23.25	15.83			.01017	.0040	.01136	.0058	2.54	1.96
	7	3	65-21M	C	16WF300	56.73	8.23			.00248	.0010	.00382	.0016	2.48	2.39
	7	3	65-22L	D	16WF300	56.73	6.79			.00301	.0010	.00492	.0016	3.01	3.07
	7	4	65-15M	C	16WF36	23.25	18.02			.00883	.0094	.00913	.0018	0.94	5.07
	7	4	65-16L	D	16WF36	23.25	15.83			.01017	.0094	.01136	.0018	1.08	6.30
16	3	1	71-1 M	C	21WF62	47.60	5.77	9.2	8.4	.00624	.0030	.01053	.0070	2.08	1.51
	3	1	71-2 L	D	21WF62	47.60	5.72	9.2	8.4	.00630	.0030	.01065	.0070	2.10	1.53
	3	3	71-7 M	C	21WF62	47.60	5.77	9.0	8.0	.00624	.0026	.01053	.0044	2.40	2.39
	3	3	71-8 L	D	21WF62	47.60	5.72	9.0	8.0	.00630	.0026	.01065	.0044	2.42	2.42
17	2	1	71-9 M }	C	18WF50	34.50	7.28	7.5		.00774	.0038	.01241	.0054	2.03	2.30
	2	1	71-10												

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies (Cont.)

Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Frequency Theor.	Frequency Actual M	Frequency Actual H	Mech Amplitude Theor.	Mech Amplitude Actual	Human Amplitude Theor.	Human Amplitude Actual	Equivalent Rms Mech	Equivalent Rms Human
17	2	1	71-11L	D	18WF50	34.50	7.10	7.5		.00794	.0038	.01282	.0054	2.09	2.38
	2	3	72-4 M	C	18WF50	34.50	7.28			.00774	.0042	.01241	.0040	1.84	3.11
	2	3	72-5 L	D	18WF50	34.50	7.10			.00794	.0042	.01282	.0040	1.89	3.21
	2	4	72-6 M	C	16 B31	28.00	8.33	7.5	8.3	.00864	.0036	.01326	.0038	2.40	3.49
	2	4	72-7 L	D	16 B31	28.00	8.12	7.5	8.3	.00887	.0036	.01373	.0038	2.46	3.62
	2	6	72-10M	C	16 B31	28.00	8.33			.00864	.0042	.01326	.0050	2.06	2.66
	2	6	72-11L	D	16 B31	28.00	8.12			.00887	.0042	.01373	.0050	2.11	2.75
	2	7	72-12M	C	24WF68	28.00	9.75			.00225	.0014	.00324	.0018	1.61	1.80
	2	7	72-13L	D	24WF68	28.00	8.62			.00255	.0014	.00386	.0018	1.82	2.14
18		1	72-14M	A	18WF50	40.50	5.89			.00413	.0054	.00694	.0024	0.76	2.89
		1	72-15L	B	18WF50	40.50	5.69			.00427	.0054	.00723	.0024	0.79	3.01
		3	72-20M	A	18WF50	40.50	5.89			.00413	.0004	.00694	.0004	10.3	17.3
		3	72-21L	B	18WF50	40.50	5.69			.00427	.0004	.00723	.0004	10.7	18.1
		4	72-22M	A	12WF27	30.20	8.51		9.8	.00645	.0058	.00982	.0046	1.11	2.14
		4	72-23L	B	12WF27	30.20	8.51		9.8	.00645	.0058	.00982	.0046	1.11	2.14
		10	72-26M	A	18WF50	40.50	5.89		7.15	.00413	.0022	.00694	.0038	1.88	1.83
		10	73-1 L	B	18WF50	40.50	5.69		7.15	.00427	.0022	.00723	.0038	1.94	1.91
		12	73-6 M	A	18WF50	40.50	5.89		6.9	.00413	.0024	.00694	.0036	1.72	1.93
		12	73-7 L	B	18WF50	40.50	5.69		6.9	.00427	.0024	.00723	.0036	1.78	2.01
19	1	4		C	16 B31	32.00	6.71			.01221		.01999			
		4		D	16 B31	32.00	6.45			.01270		.02098			

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies (Cont.)

Blg.	Floor	Loc.	Tape No.	Code	Beam	Length	Frequency Theor.	Frequency Actual M	Frequency Actual H	Mech Amplitude Theor.	Mech Amplitude Actual	Human Amplitude Theor.	Human Amplitude Actual	Equivalent Bms Mech	Equivalent Bms Human
20	3	2	91-7 L	C	16 B31	36.00	4.65		5.0	.00999		.01739	.0091		1.91
		2	91-15L	D	16 B31	36.00	4.43		5.0	.01049		.01836	.0091		2.02
		4	91-11L	C	16WF36	36.00	4.94			.00931		.01609	.0062		2.59
		4	91-16L	D	16WF36	36.00	4.71			.00977		.01698	.0062		2.74

M - Mechanical Impact

L - Human Impact, Lenzen

A - Topping included; effective slab width taken as the sum of one-half the distances to adjacent beams.

B - Topping included; effective slab width as limited by AISC specification.

C - Same as A except no topping.

D - Same as B except no topping.

1 - Cover plates on beam.

2 - Castellated beam.

Table 3. Summary of Theoretical and Measured Amplitudes and Frequencies (Cont.)

DISCUSSION OF RESULTS

It was possible to calculate a critical damping ratio for only 22 of the 49 locations tested. For most of these cases the number of cycles necessary for the amplitude to reach one-fifth of the first amplitude was less than five. (It is to be noted that damping percent as given by Fig. 3 using the number of cycles to one-fifth initial amplitude shown in Table 1 and that calculated by Eq. (13) and shown in Table 1 may not agree. In calculating the damping ratio the portion of the response record which produced the most consistent ratios of successive amplitudes was used. This portion did not include the first amplitude in many instances.)

In the steel joist phase of the project, it was determined that a human feels only the initial impact, no vibration, if the floor is damped to a small amplitude prior to five cycles. The floors tested in building 19 and 20 are the only floors where the amplitude of the vibration was significant after five cycles. Buildings 19 and 20 were the only buildings in which the occupants had complained of annoying vibrations prior to the tests. Comparison of the tapes of these buildings and any of the others shows clearly the difference in the floor response. (Data was not taken where the impactor and recorder were at midspan and over a beam for Building 19. However, several tapes are included in Appendix A showing the response for other locations.)

Of the buildings for which the fire-proofing method is known only Buildings 3, 16, 19, and 20 used other than the spray-on method. Comparison of the critical damping ratios, the number of cycles to one-fifth first amplitude, and of the response records for these floors with the other floors indicates that spray-on fireproofing may be an important factor in damping considerations. Further studies in this area are necessary before a final recommendation can be justified.

Comparison of the theoretical and measured frequencies indicates that the effect of the different methods of calculating the moment of inertia for the tee-beam section is negligible. It is recommended that the effective slab width be taken as the sum of one half the distances to adjacent beams for the moment of inertia term in Eq. (5). Calculating the moment of inertia term in this manner will reflect the additional stiffness obtained when using larger beam spacings. It is felt that shear lag is not as important in vibration analysis as it is in static cases. It is also recommended that any topping concrete be included in the composite section.

Comparison of the theoretical and measured amplitudes indicates that a single tee-beam model is not realistic for this calculation. As a conservative estimate the number of tee-beams effective in resisting the impacts can be taken as two. It is thought that a more accurate method for predicting the initial amplitude will result from the isotropic plate study. The results of the tee-beam model analysis can then be used to calculate the number of tee-beams effective in resisting the impacts.

Figure 11 is a plot of the number of effective tee-beams for mechanical impact versus the number for human impact. Considering the variation in the field conditions, it can be assumed that the number of effective tee-beams is the same for both mechanical and human impacts.

The experimental data from Table 3 is plotted on the modified Reiher and Meister curves of Fig. 12. The results are inconclusive. The research team's attempt to subjectively rate each of the floors has been

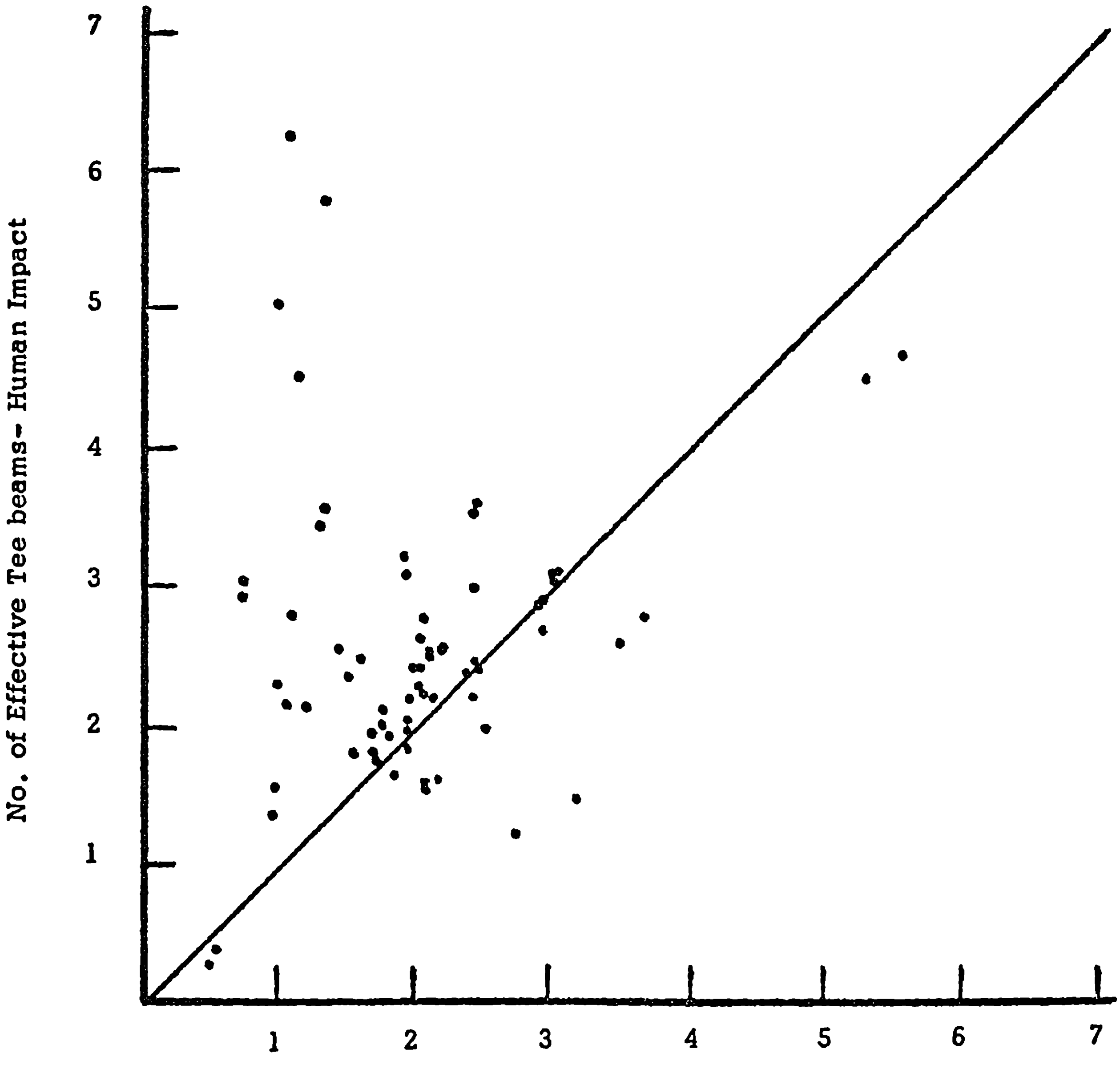


Figure 11. Number of Effective Tee-Beams from Mechanical Impact Versus the Number from Human Impact.

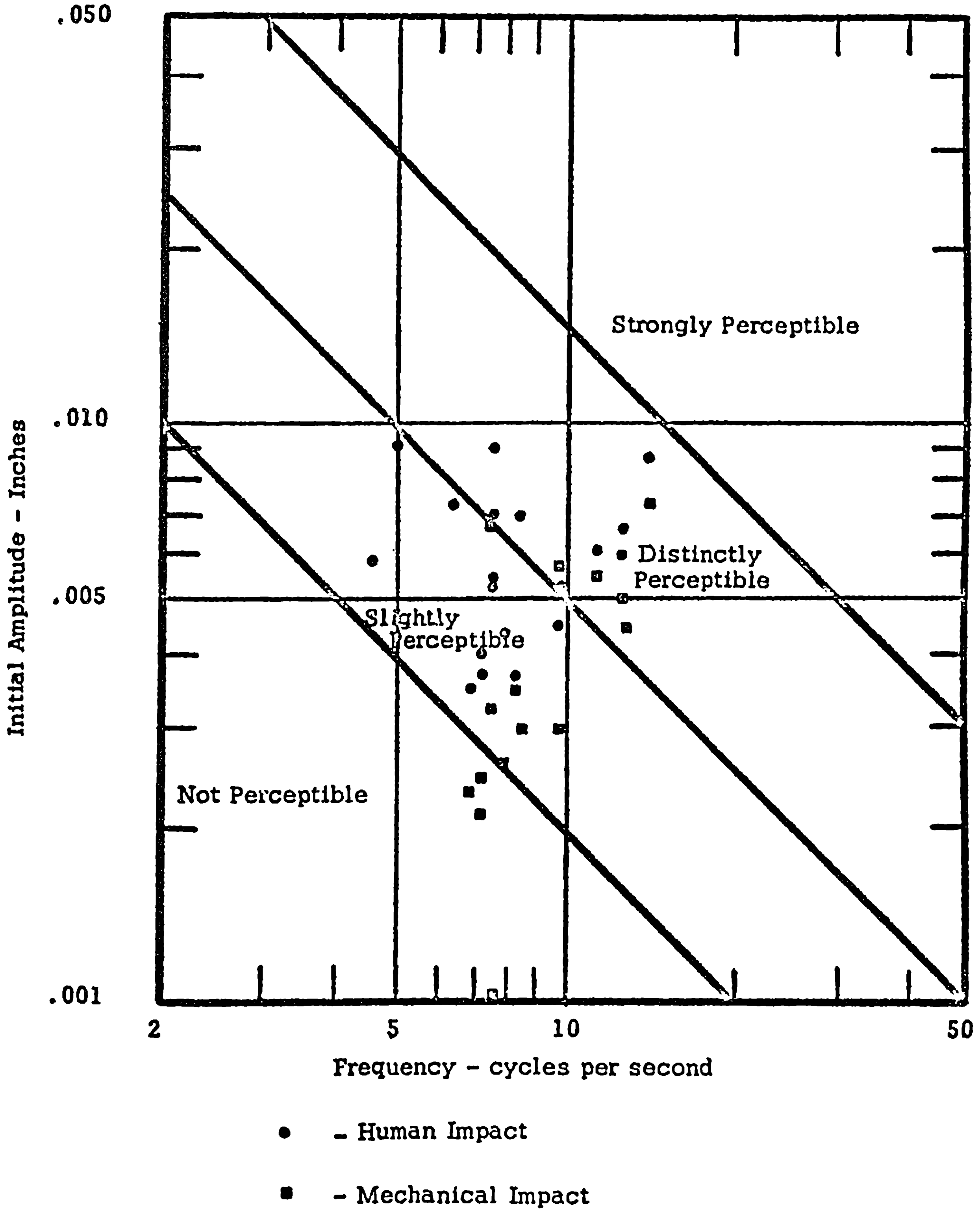


Figure 12. Experimental Data Plotted on Modified Reiher and Meister Human Response Curve.

discarded since it is felt that their ratings were colored by previous experience with joist floors where the vibration is much more perceptible. Therefore, a criteria for mathematically describing an annoying vibration cannot be established from this study. However, a conservative design will result from the use of the modified Reiher and Meister curves if sufficient damping is present.

Finally, comparison of the data taken from the various types of floor construction tested failed to indicate any significant difference in the vibrational characteristics of the floor systems.

SUMMARY OF CONCLUSIONS

The conclusion reached from this study can be summarized as follows:

1. Human perceptibility to vibration is dependent on the duration of the vibration. Damping in a structural system is, therefore, significant.
2. For the types of construction considered in this report, spray-on fire-proofing seems to provide sufficient damping to prevent annoying vibrations.
3. Room partitions and dividers securely attached to the floor slabs are effective in preventing annoying vibrations mainly because of increased damping.
4. It was not possible to discern any difference in the vibrational characteristics of the various types of floor systems tested.
5. The single tee-beam model is sufficiently accurate to predict the natural frequency of the system if (a) the structural slab and non-structural topping are considered to act compositely with the steel beam, (b) the effective slab width is taken as the sum of one-half the distances to adjacent beams.
6. The tee-beam model is not sufficiently accurate to predict the first maximum aplitude of a floor system.
7. The number of tee-beams effective in resisting either the human or mechanical impacts is the same.
8. The number of tee-beams effective can be conservatively taken as two.
9. With the available data, a criterion cannot be established to define the limit of annoying vibrations. However, if sufficient damping is present the modified Reiher and Meister curves can be used for

ACKNOWLEDGEMENTS

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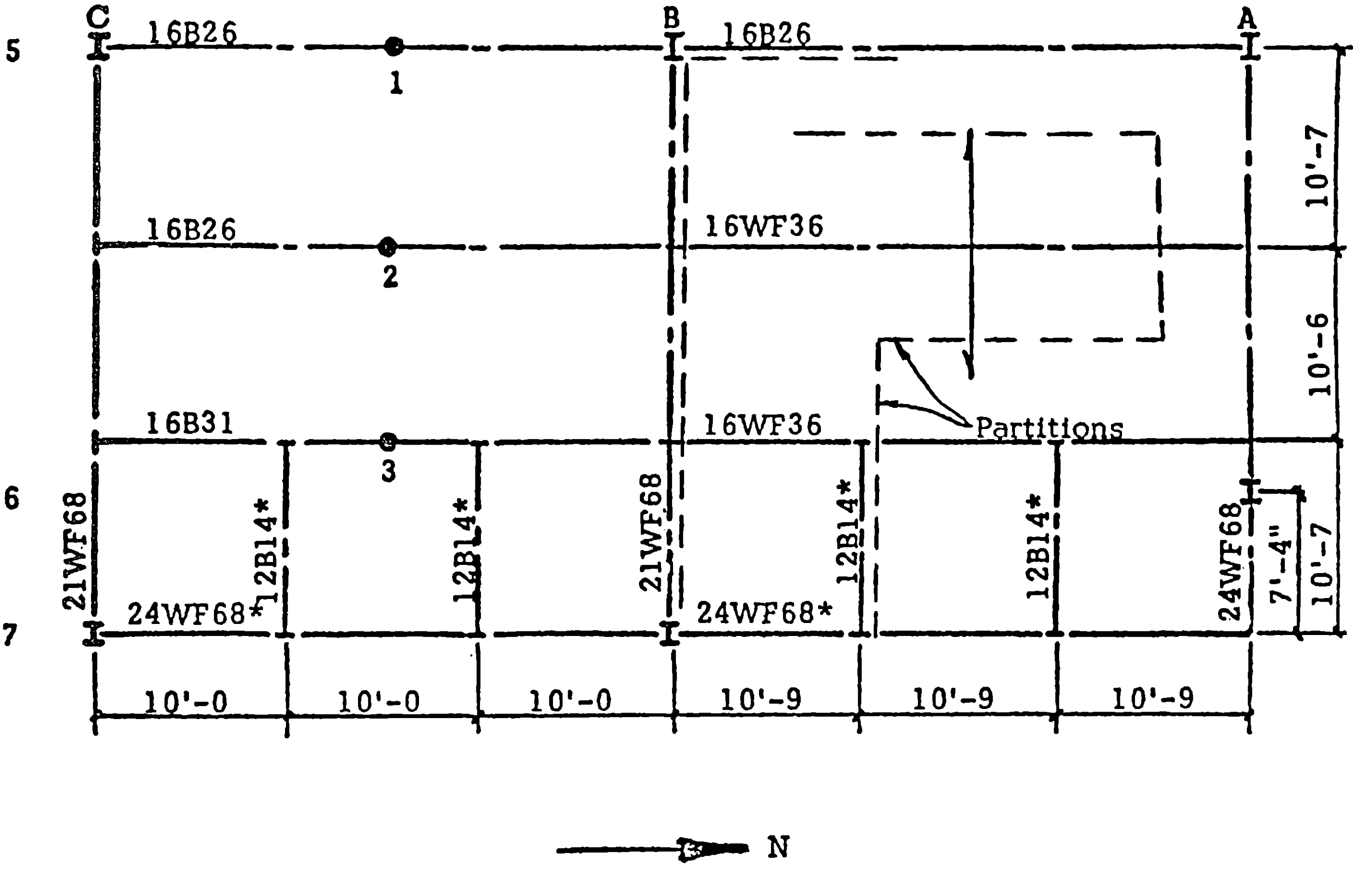
NOTES

NOTES

APPENDIX A

PARTIAL FLOOR PLANS
AND
RESPONSE RECORDS



Partial Framing Plan - 2nd Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite construction.

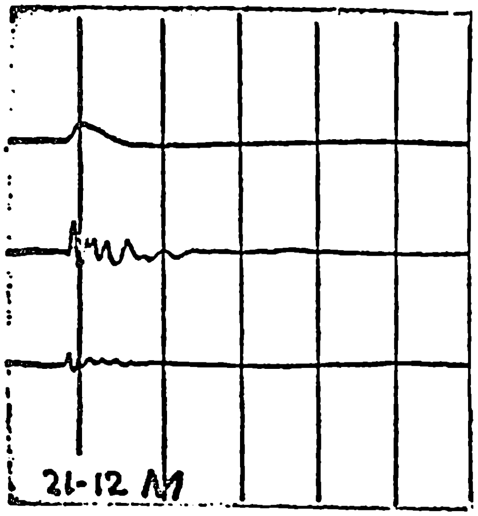
5" Lightweight (110 pcf) concrete slab

A36 Steel

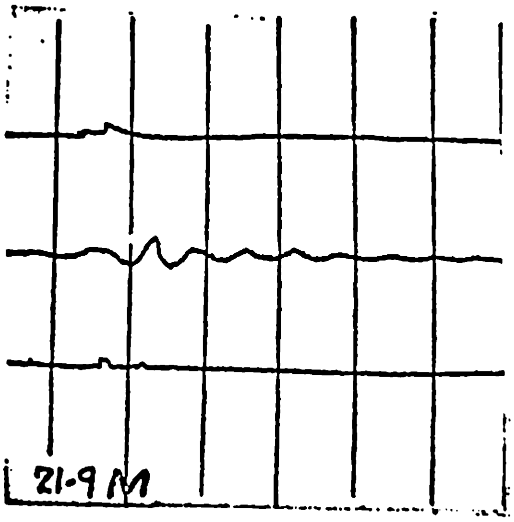
Type 4 centering material

Spray-on fireproofing

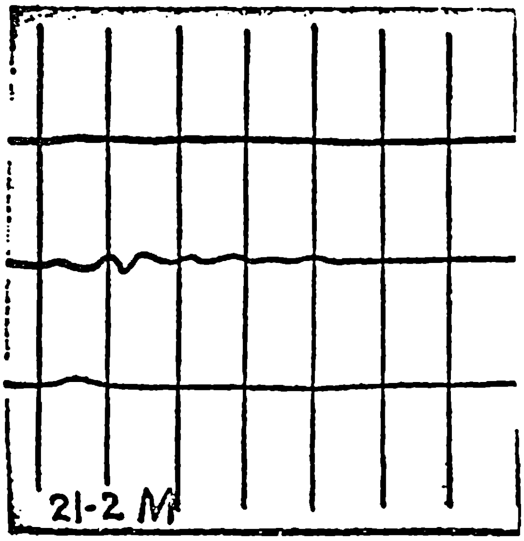
BUILDING 1.



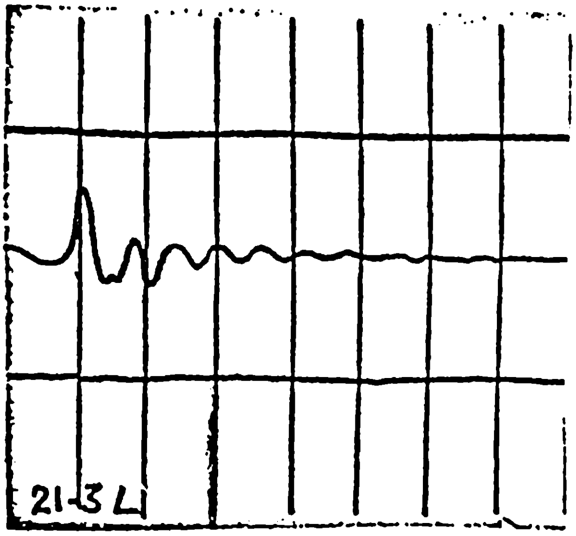
Location 1
Mechanical Impact



Location 2
Mechanical Impact

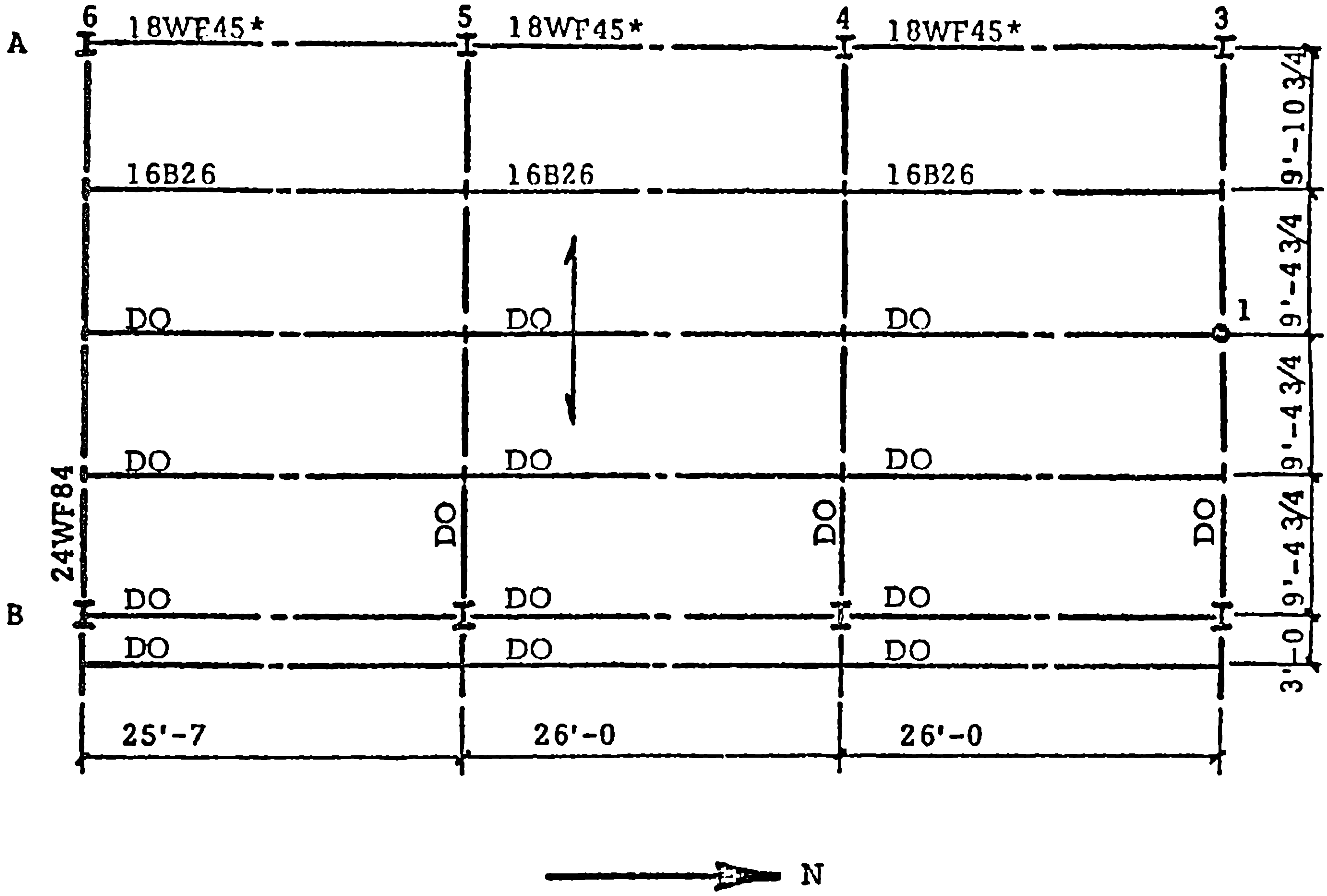


Location 3
Mechanical Impact



Location 3
Human Impact

2nd Floor - BUILDING 1.



Partial Framing Plan - 4th Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite construction.

5" Lightweight (110 pcf) concrete slab

A36 Steel

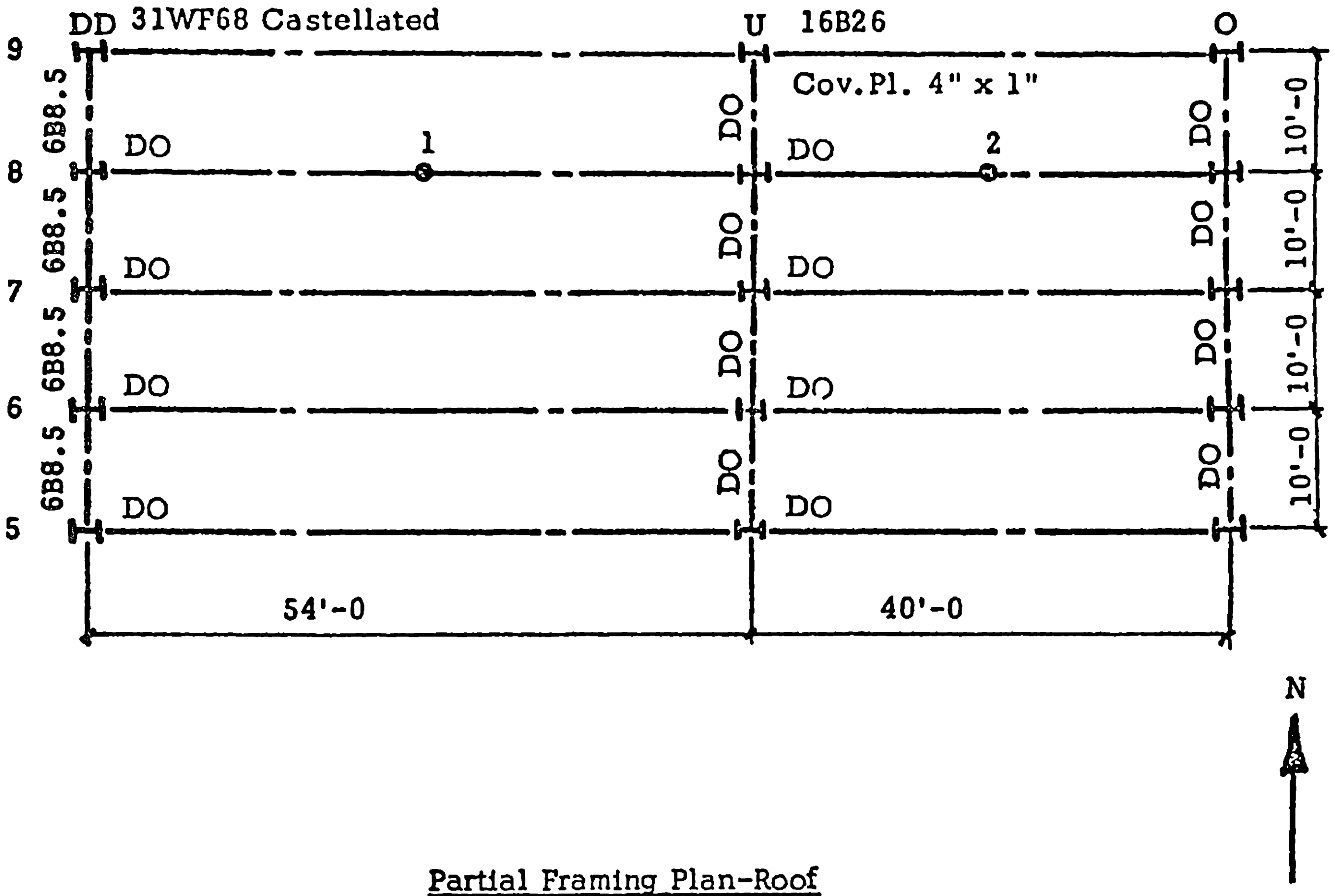
Type 4 centering material

Spray-on fireproofing

BUILDING 2.

22-25 M
NO TRACE

4th Floor - BUILDING 2.



Partial Framing Plan-Roof
(No Scale)

Composite construction

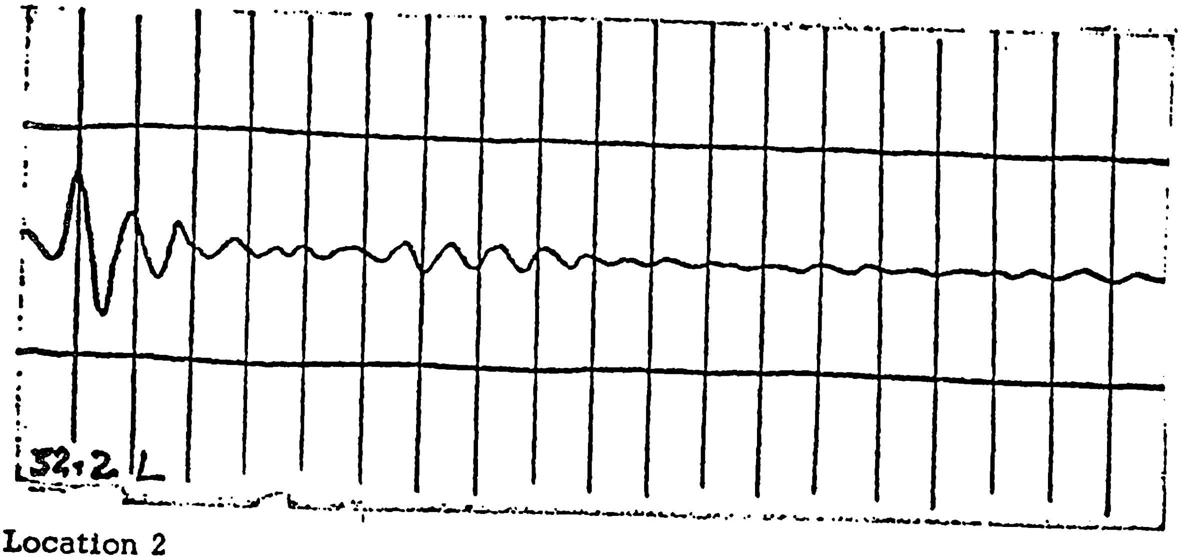
4" Concrete slab

A36 Steel

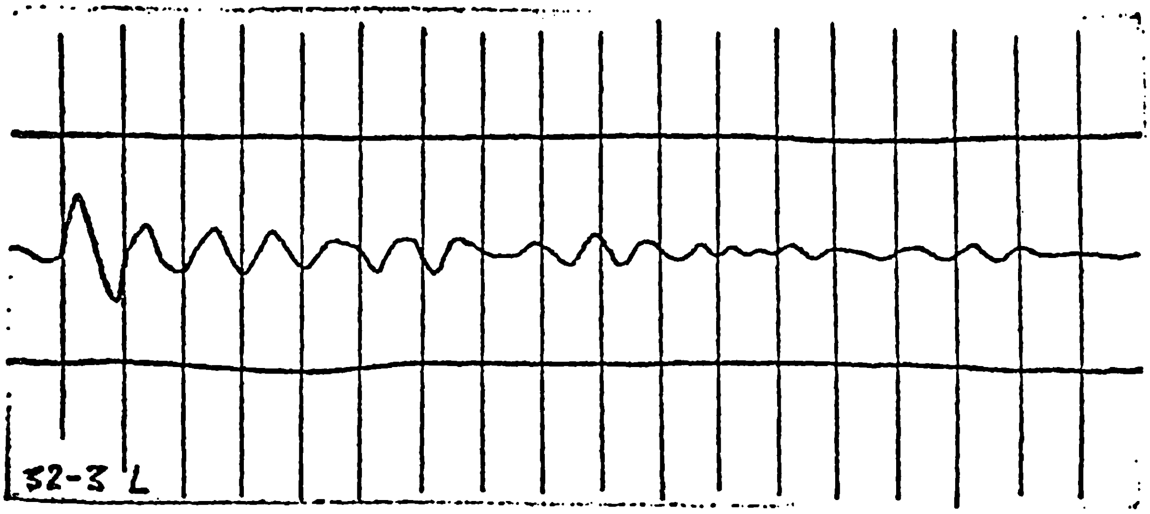
No Centering material

No fireproofing

BUILDING 3.

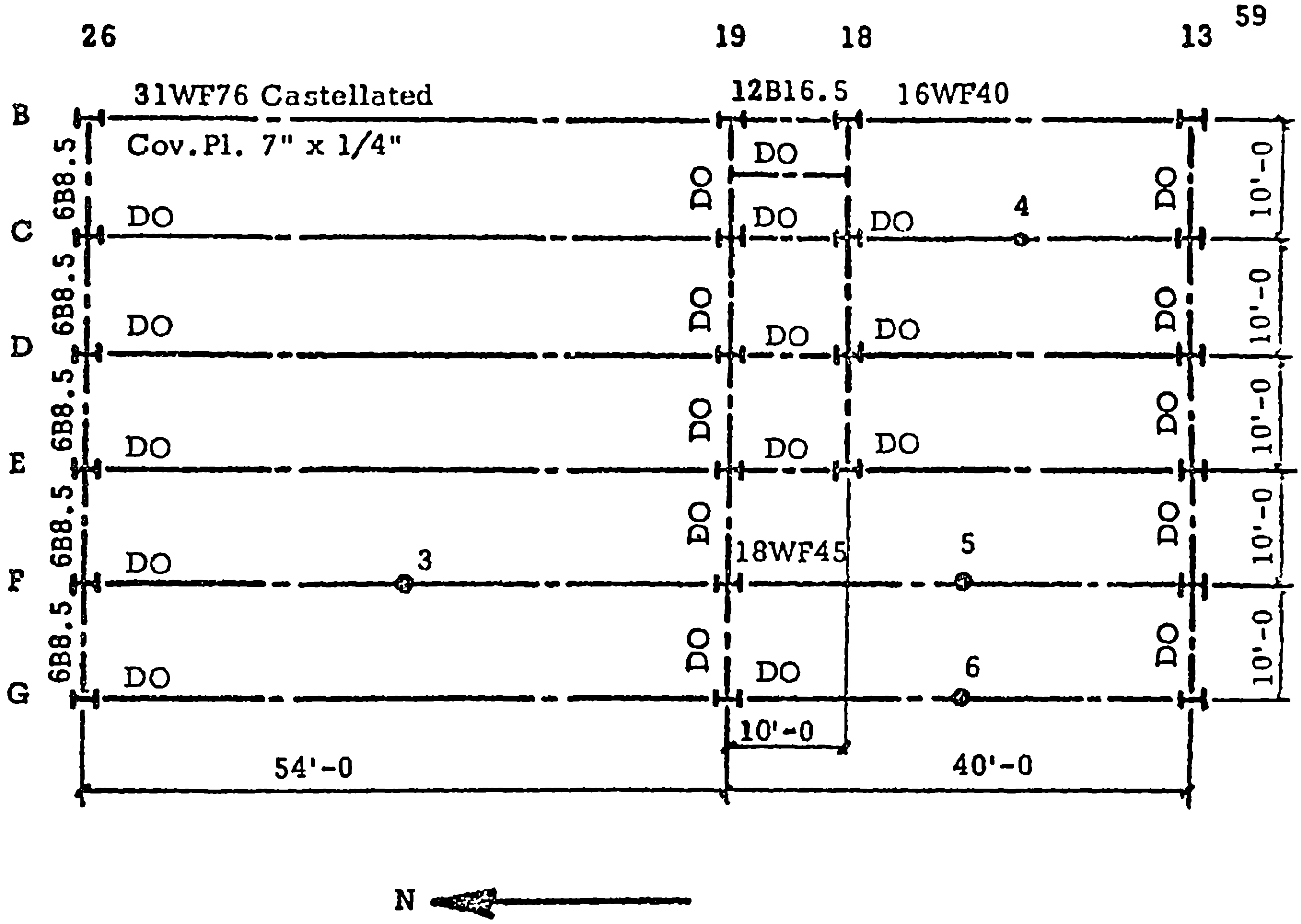


Location 2
Human Impact



Location 1
Human Impact

Roof - BUILDING 3.



Partial Framing Plan - 2nd Floor
(No Scale)

Composite construction

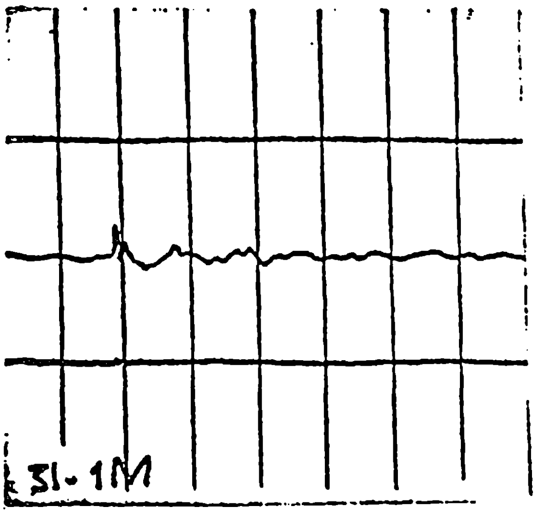
4" Concrete slab

A36 Steel

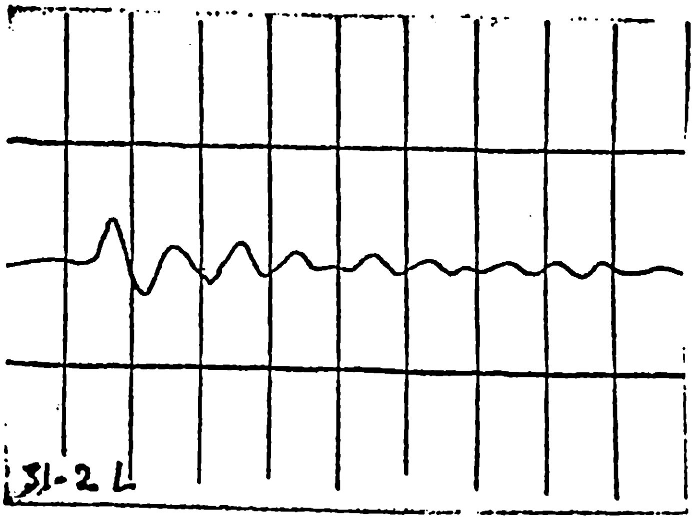
No Centering material

No fireproofing

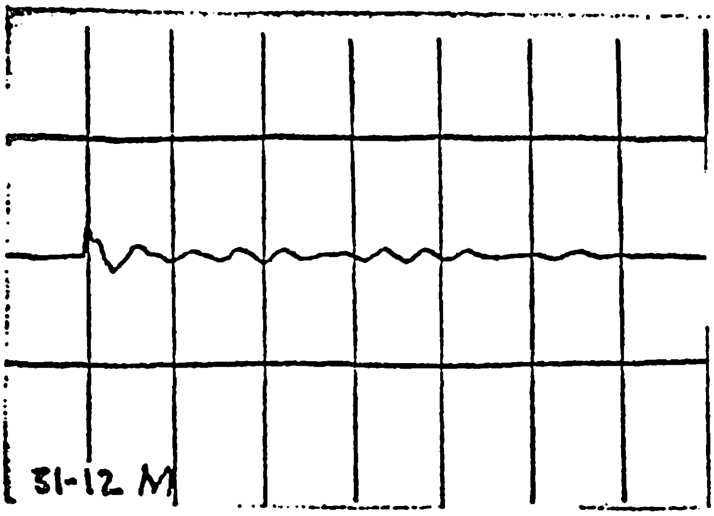
BUILDING 3.



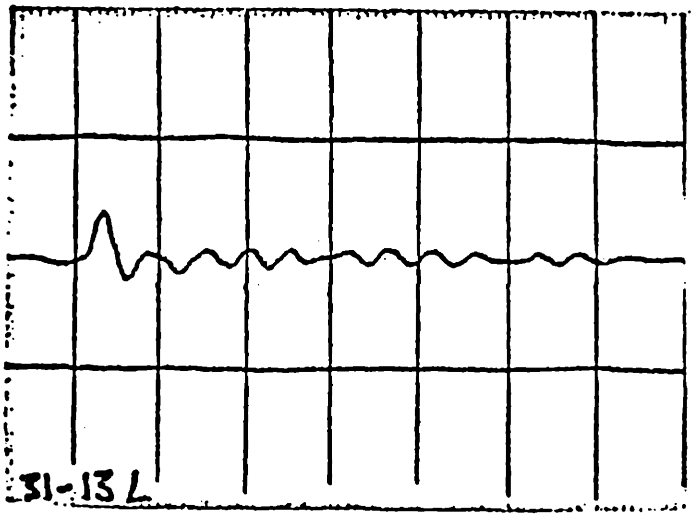
Location 3
Mechanical Impact



Location 3
Human Impact

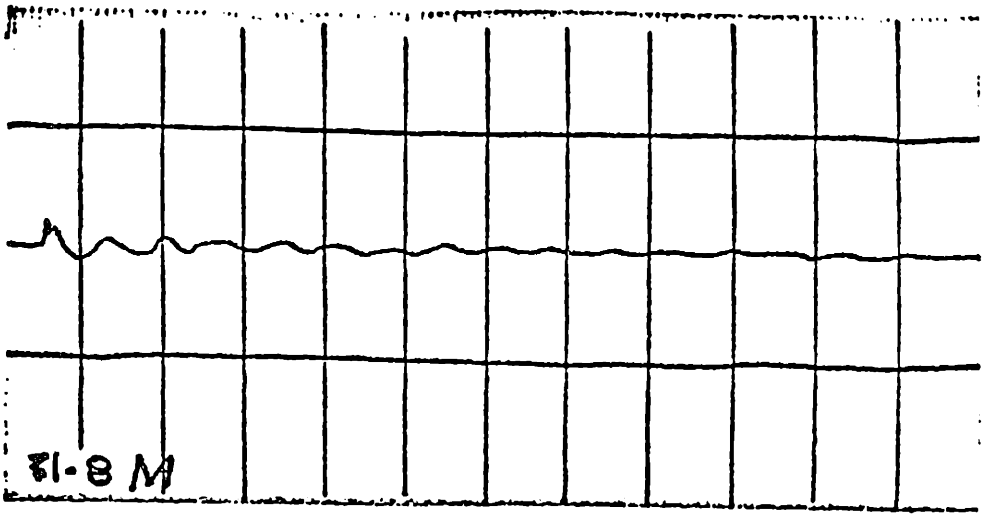


Location 4
Mechanical Impact

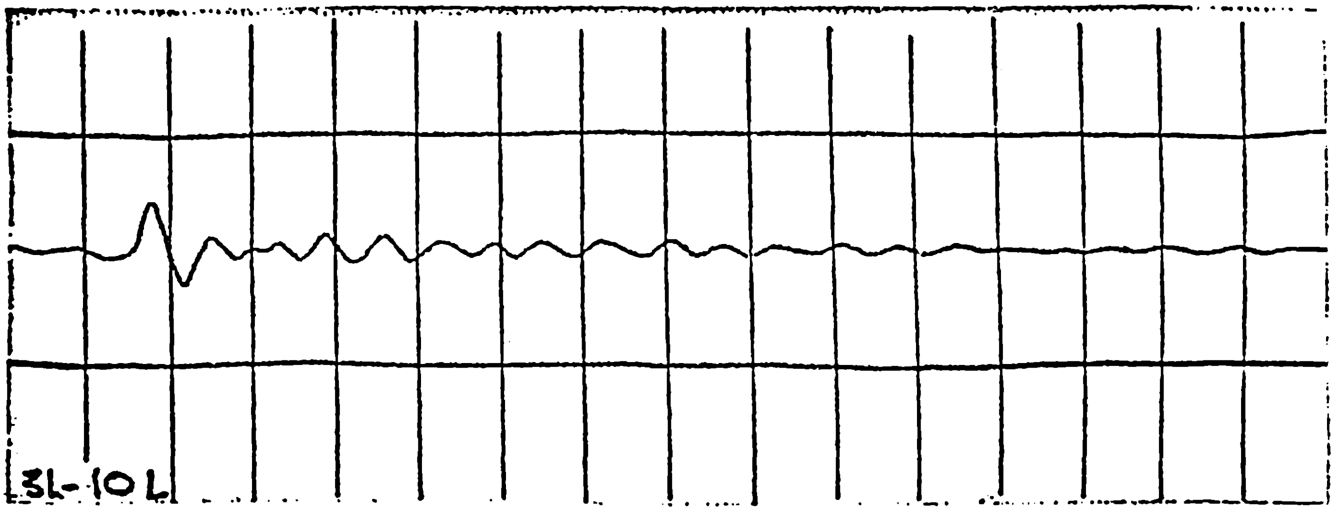


Location 4
Human Impact

2nd Floor - BUILDING 3.

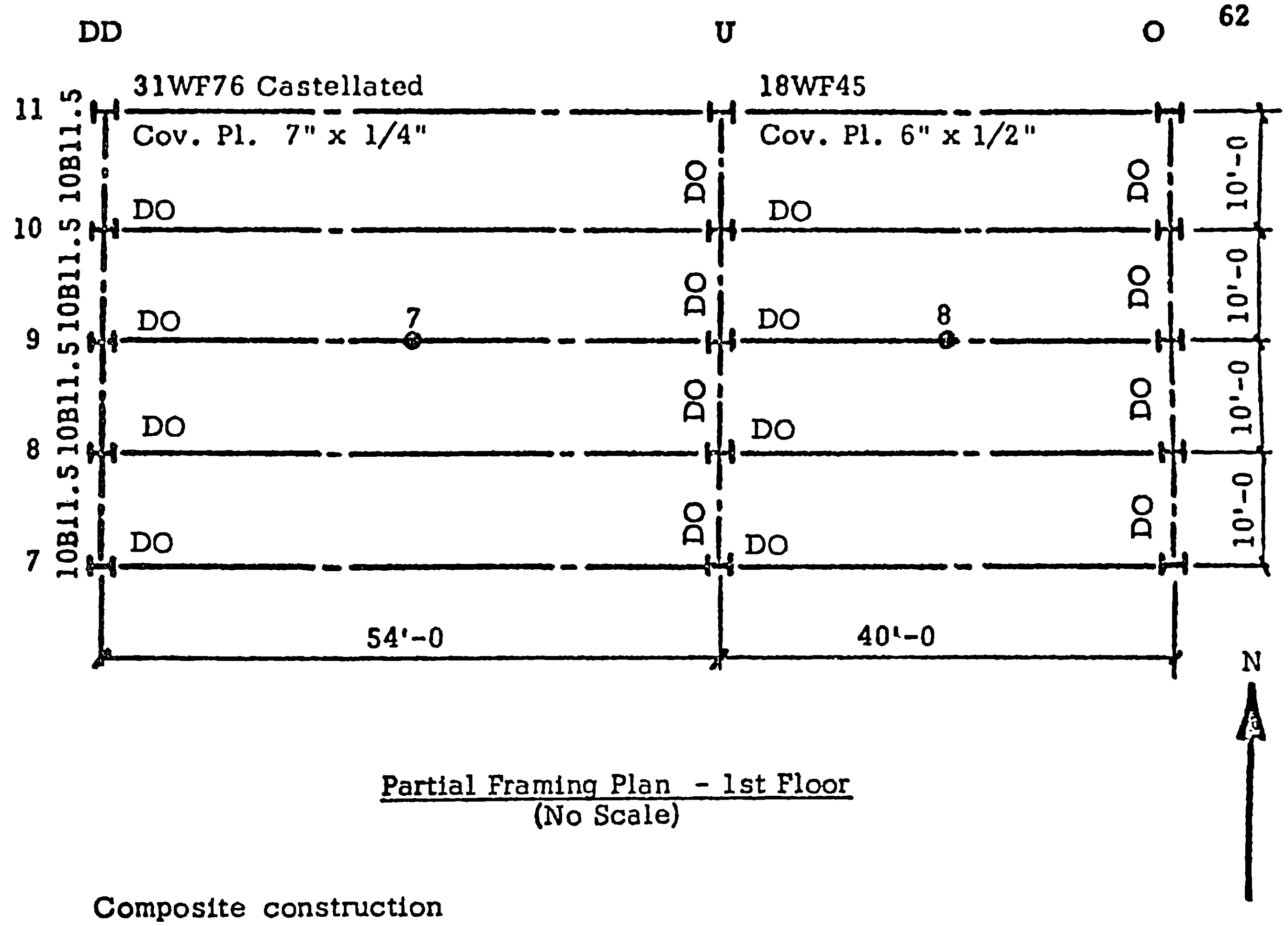


Location 5
Mechanical Impact



Location 6
Human Impact

2nd Floor - BUILDING 3.



Partial Framing Plan - 1st Floor
(No Scale)

Composite construction

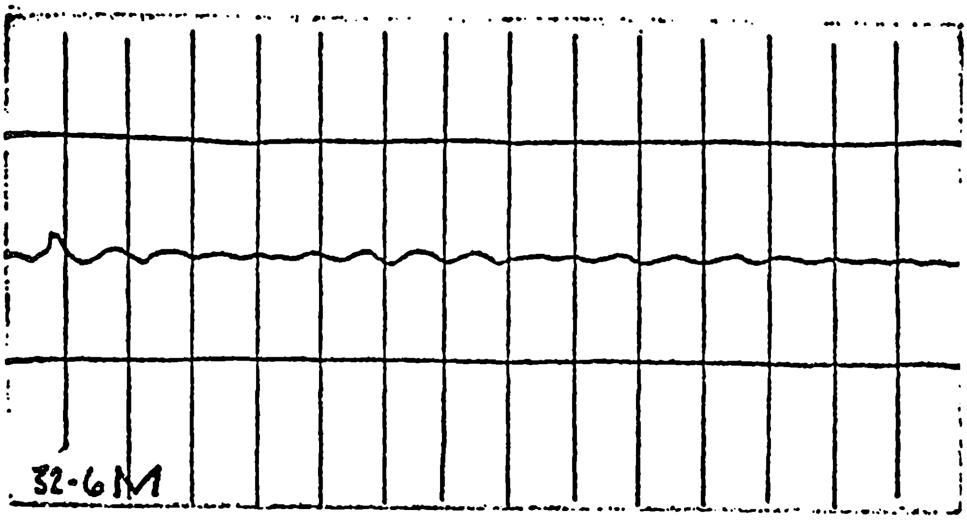
4" Concrete slab

A36 Steel

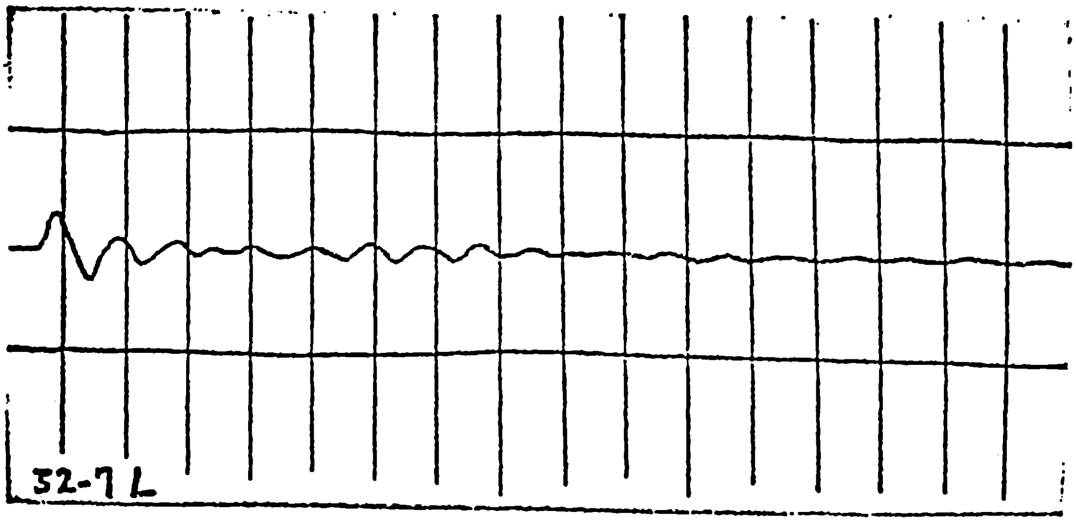
No Centering material

No fireproofing

BUILDING 3.

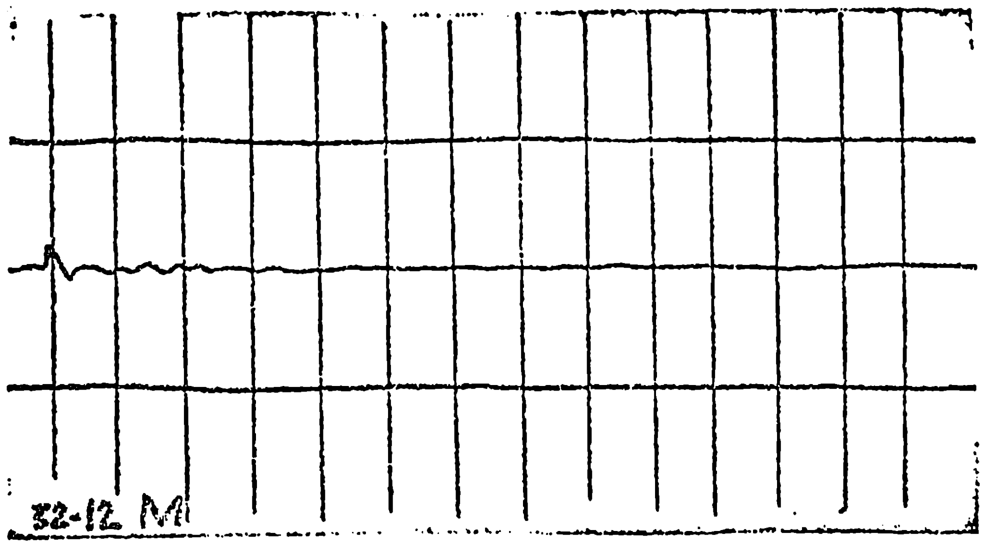


Location 7
Mechanical Impact

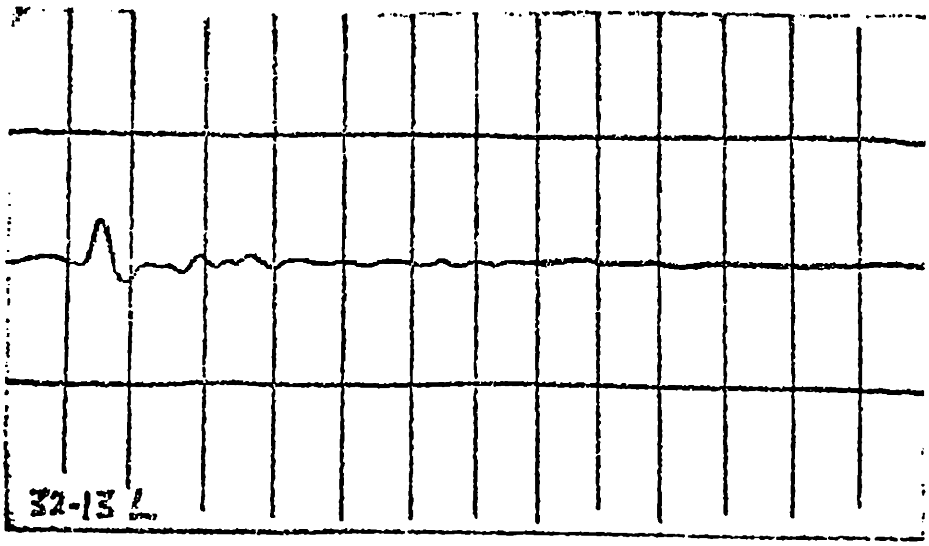


Location 7
Human Impact

1st Floor - BUILDING 3.

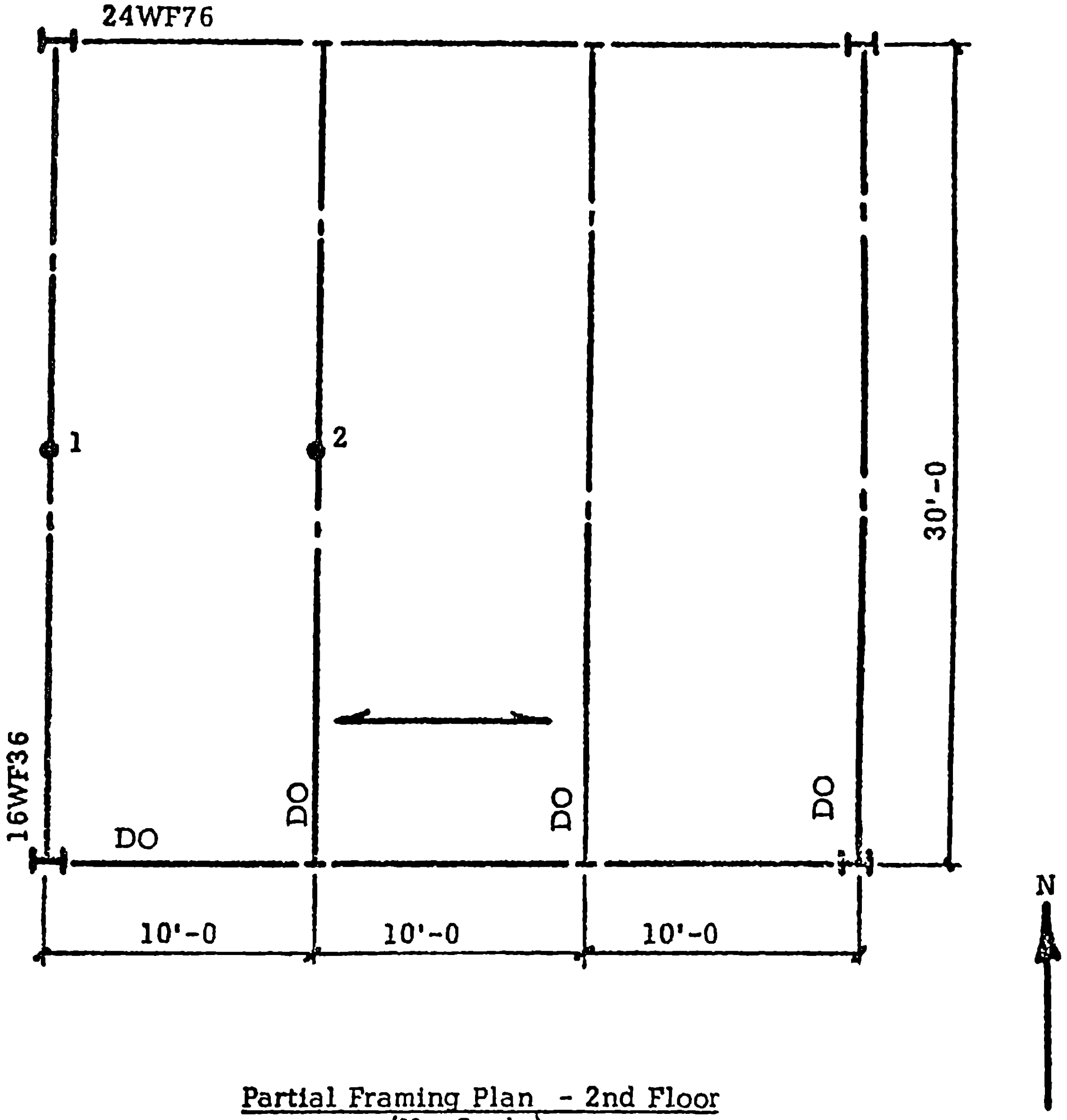


Location 8
Mechanical Impact



Location 8
Mechanical Impact

1st Floor - BUILDING 3.



Partial Framing Plan - 2nd Floor
(No Scale)

Composite construction

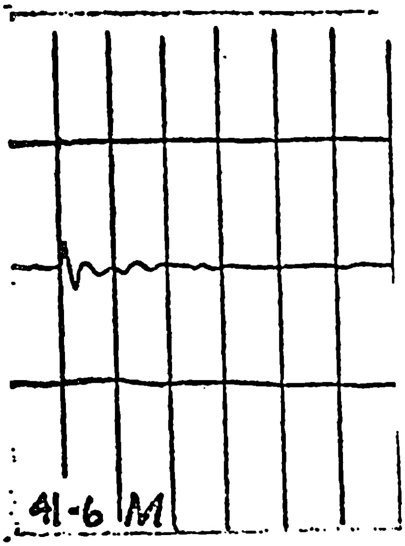
4 1/2" Lightweight (110pcf) concrete slab

A36 Steel

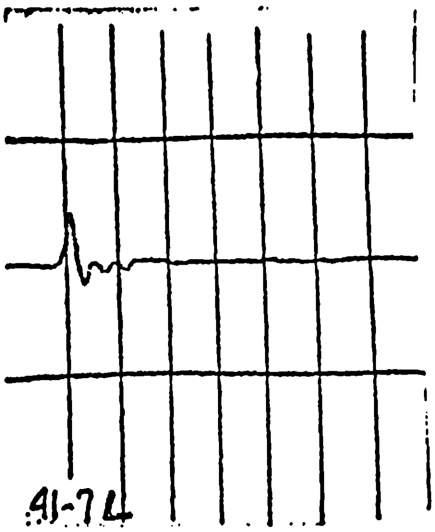
Type 4 Centering material

Fireproofing not known

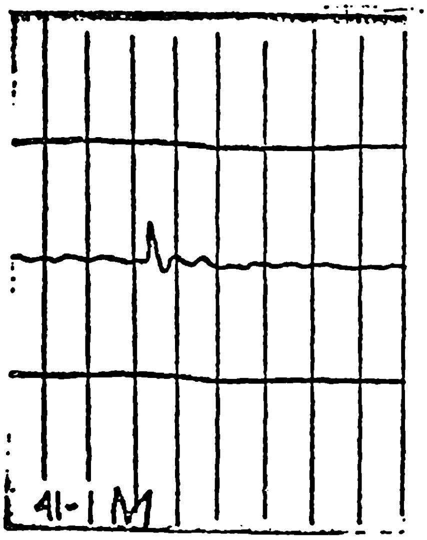
BUILDING 4.



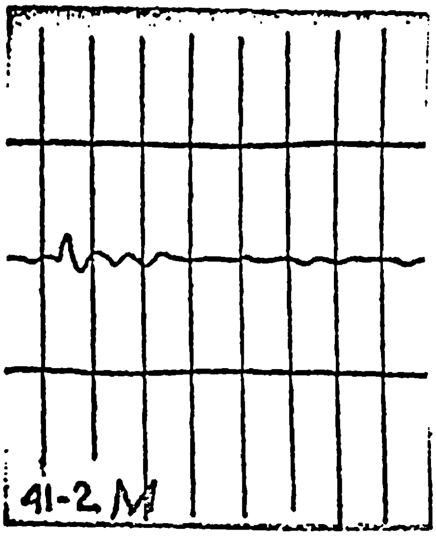
Location 1
Mechanical Impact



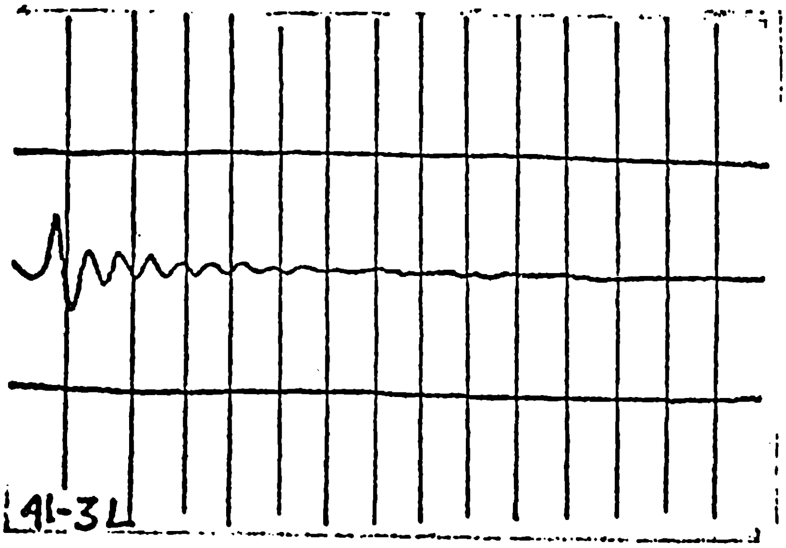
Location 1
Human Impact



Location 2
Mechanical Impact

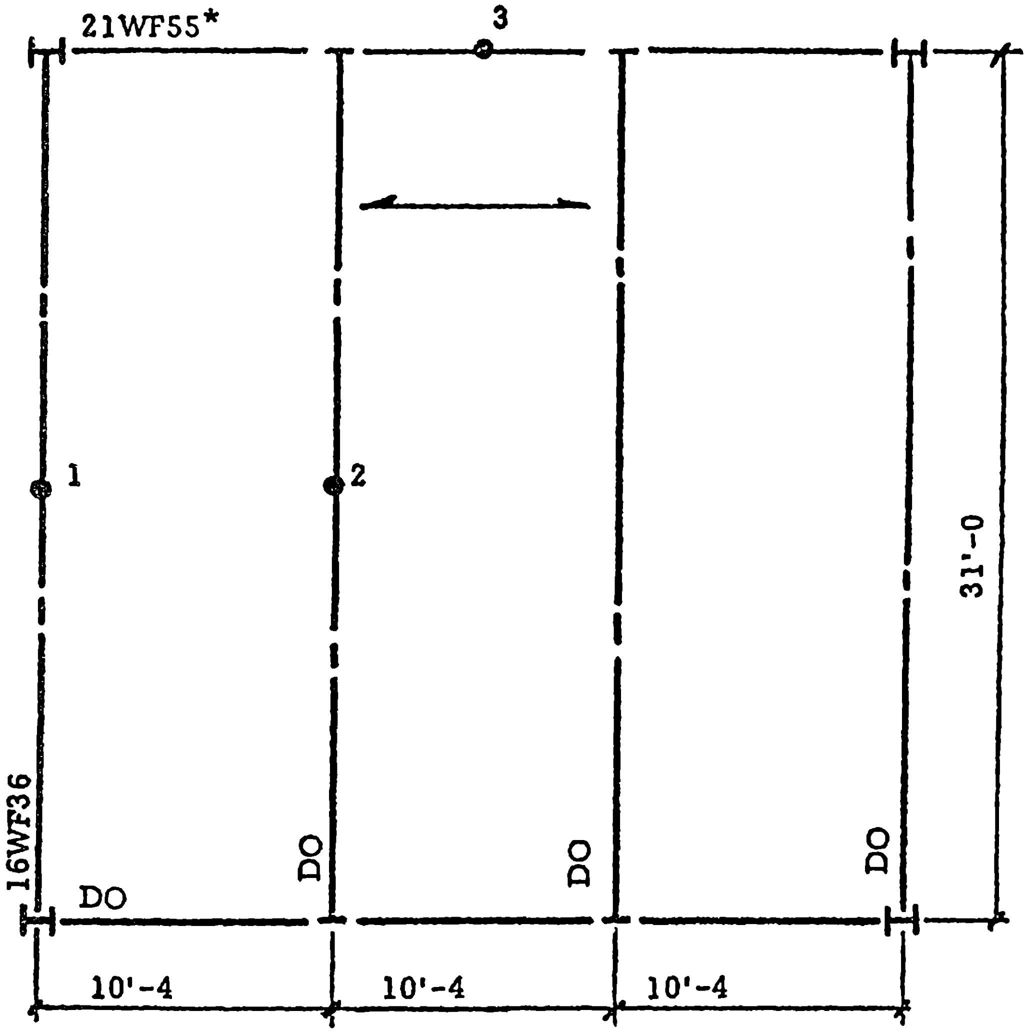


Location 2
Mechanical Impact



Location 2
Human Impact

2nd Floor - BUILDING 4.



Partial Framing Plan - 10th Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite construction.

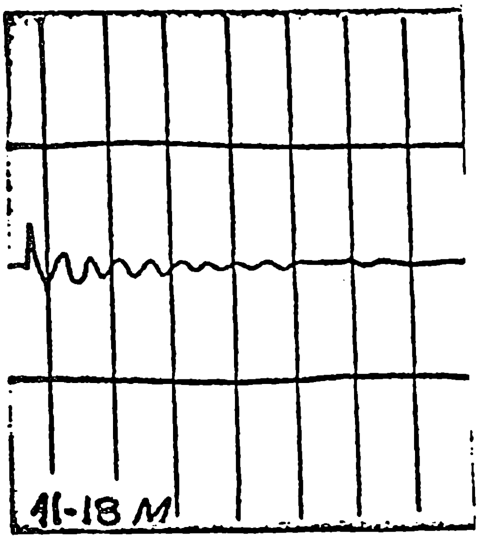
4 1/2" Lightweight (110pcf) concrete slab

A35 Steel

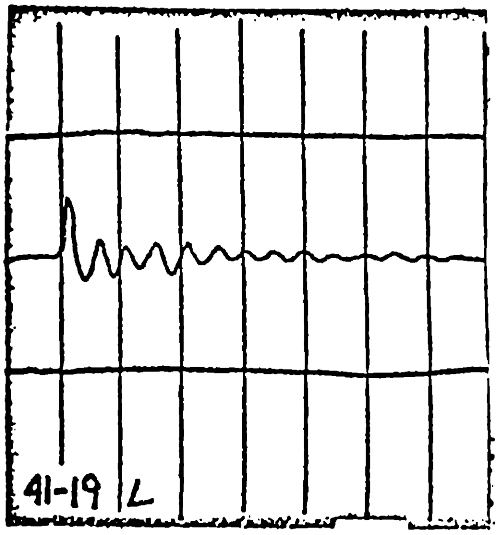
Type 4 centering material

Fireproofing not known

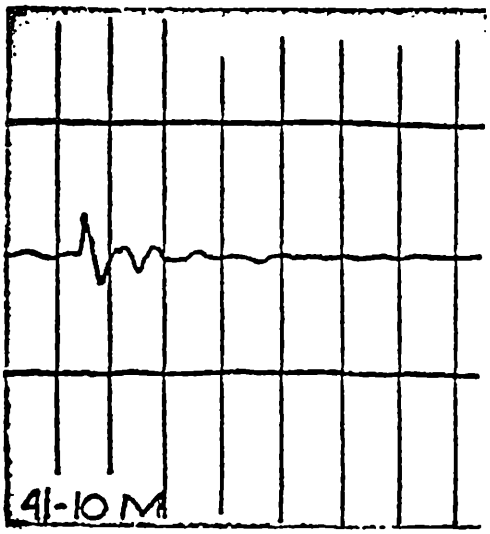
BUILDING 5.



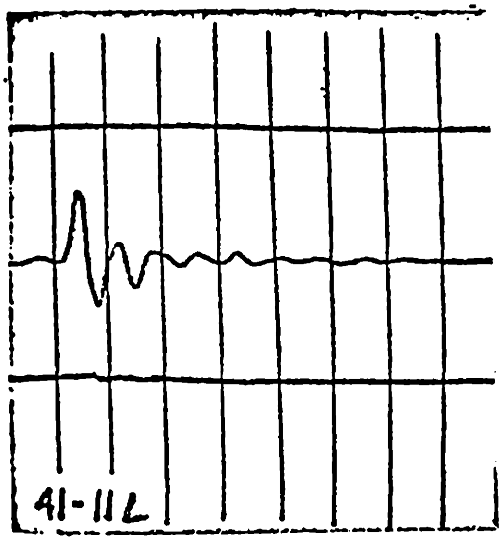
Location 1
Mechanical Impact



Location 1
Human Impact

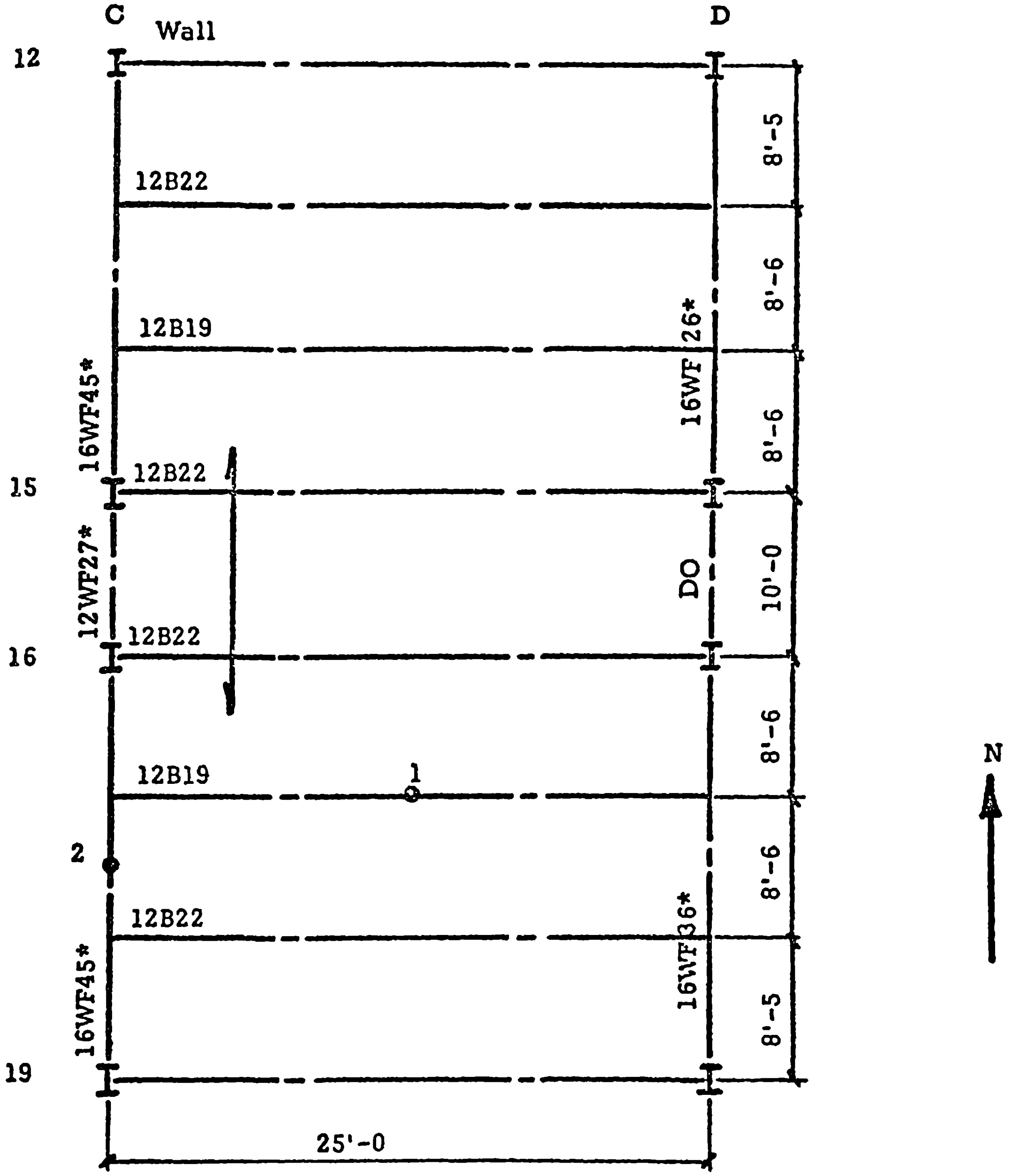


Location 2
Mechanical Impact



Location 2
Human Impact

10th Floor - BUILDING 5.



Partial Framing Plan - 7th Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite

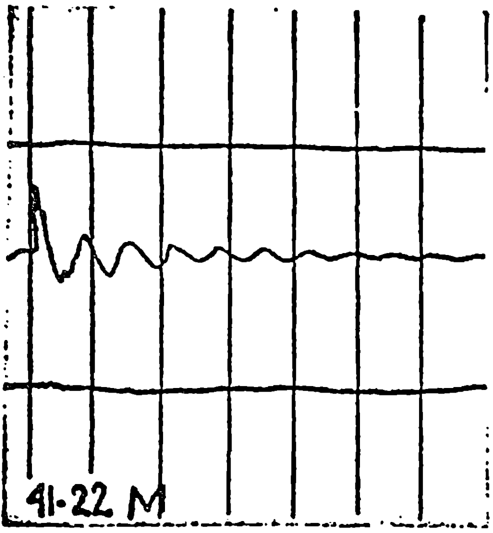
4" Lightweight (110pcf) concrete slab

A36 Steel

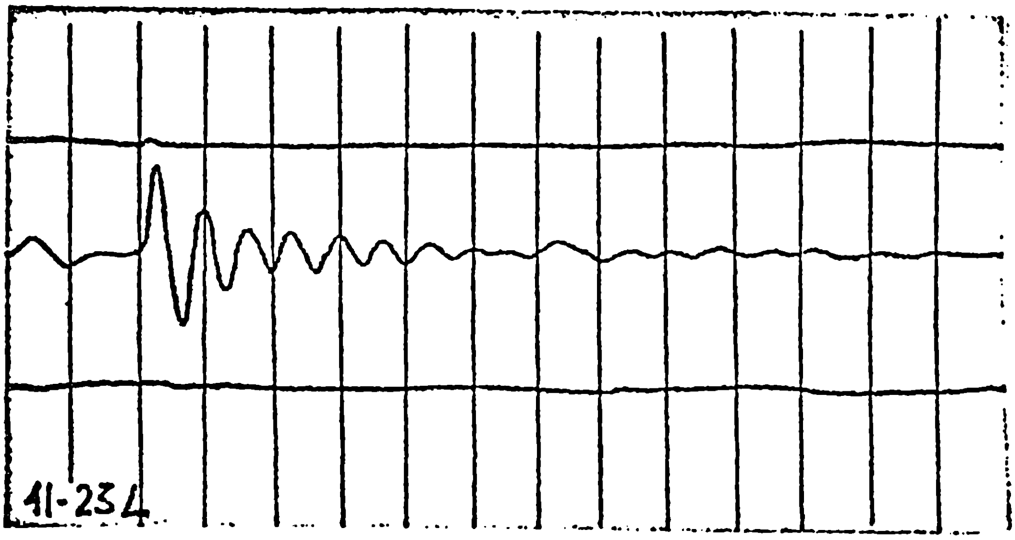
Type 4 centering material

Fireproofing not known

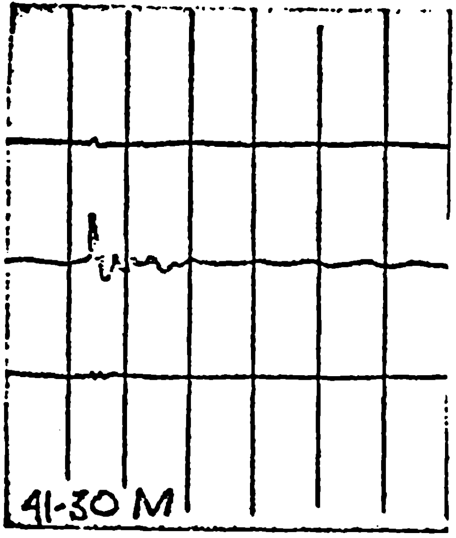
BUILDING 6.



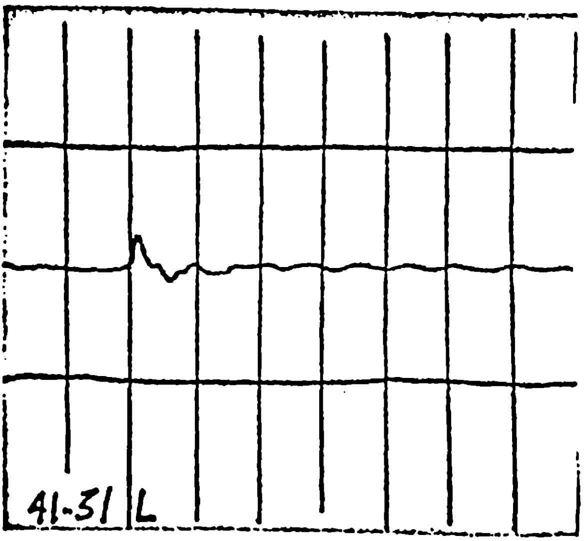
Location 1
Mechanical Impact



Location 1
Human Impact

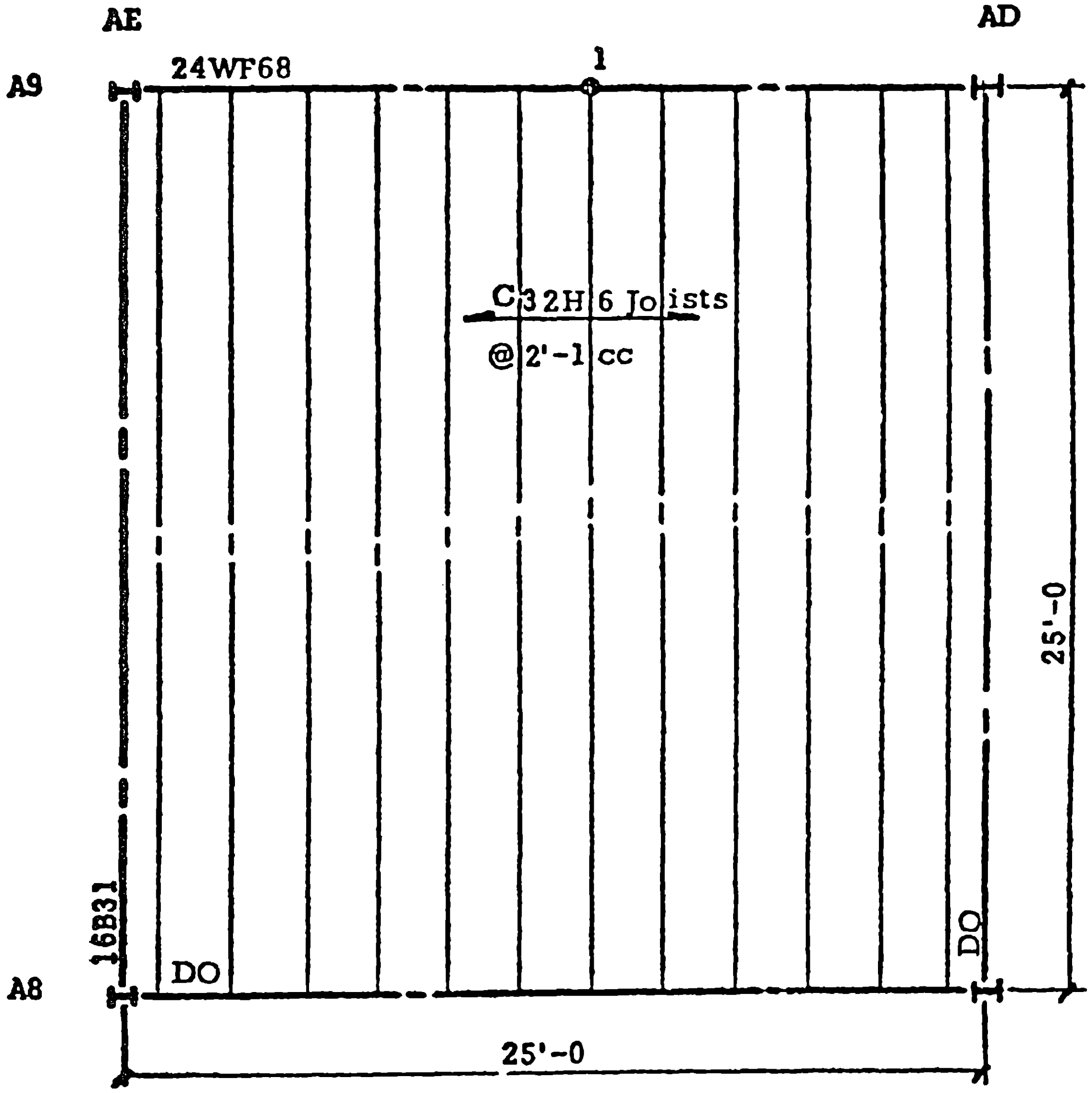


Location 2
Mechanical Impact



Location 2
Human Impact

7th Floor - BUILDING 6.



Partial Framing Plan - 1st Floor
(No Scale)

Non-composite construction

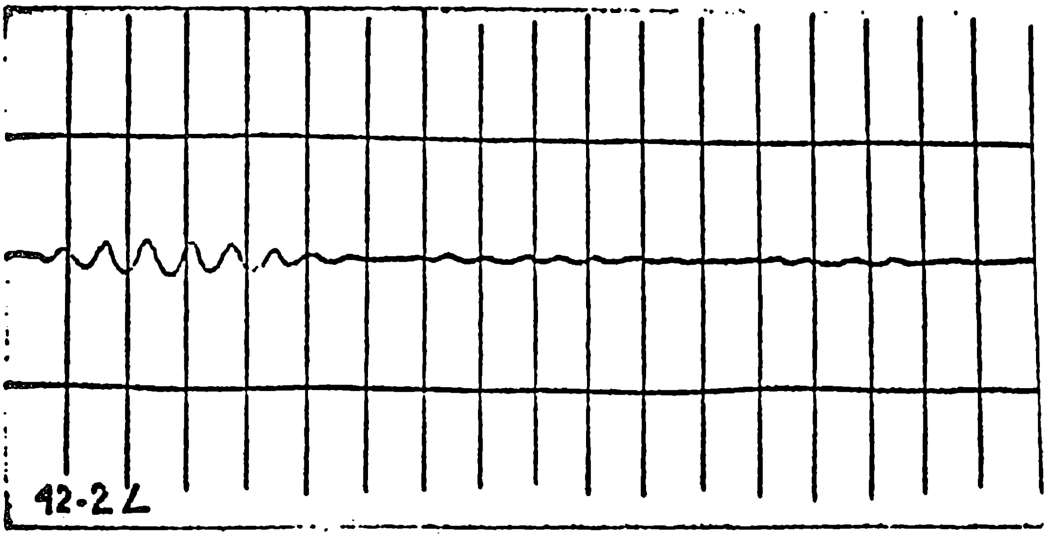
4" Concrete slab

A36 Steel

Type 4 centering material

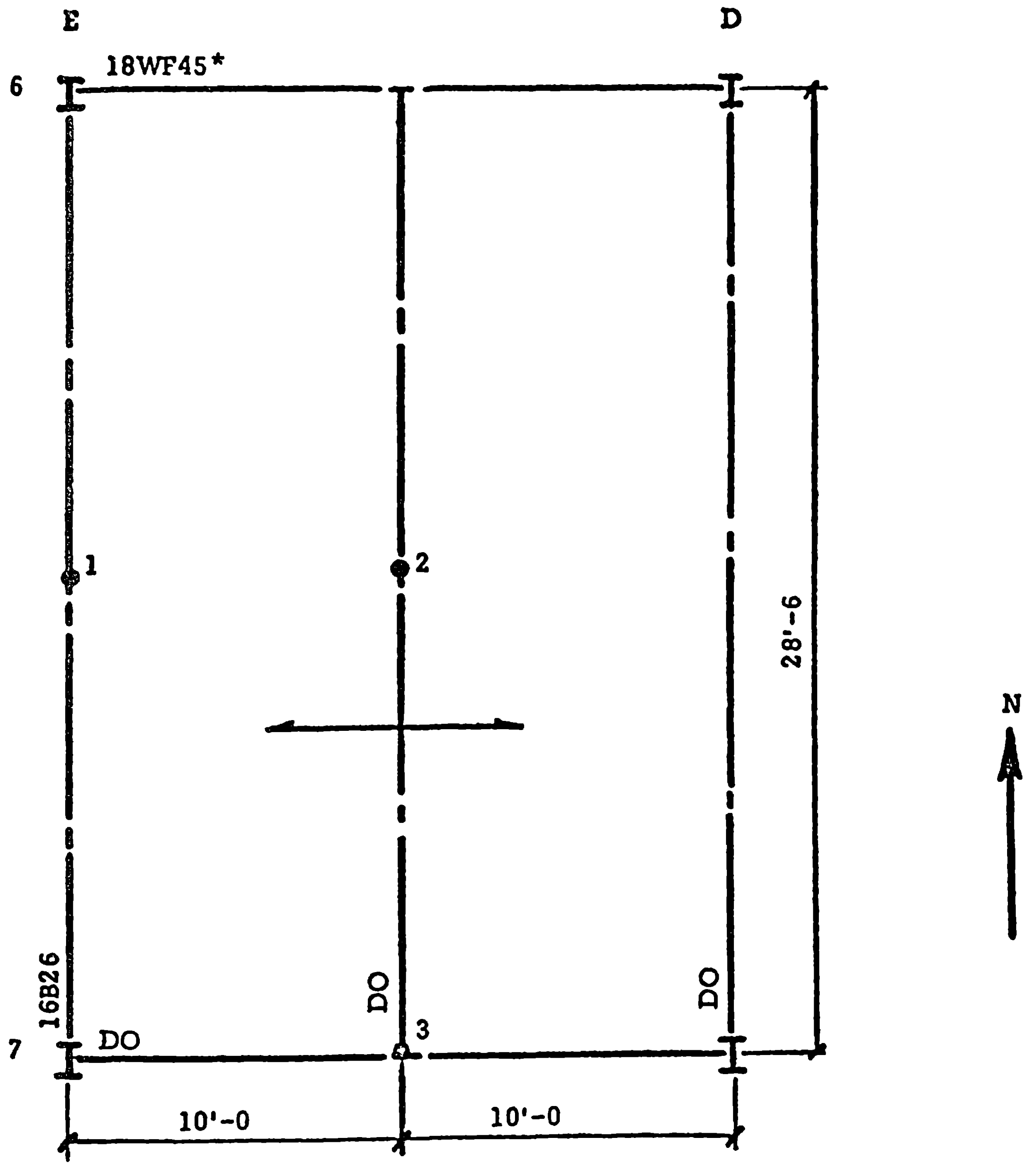
Fireproofing not known

BUILDING 7.



Location 1
Human Impact

1st Floor - BUILDING 7.



Partial Framing Plan - 4th Floor
(No Scale)

Non-composite beams marked (*) in plan, other composite construction

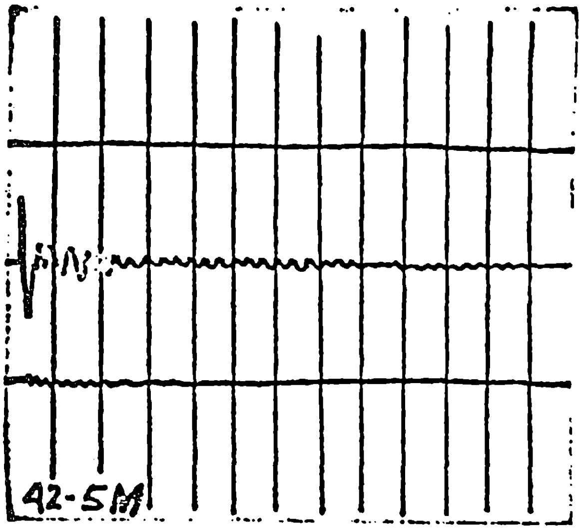
4 1/2" Lightweight (110pcf) concrete slab

A36 Steel

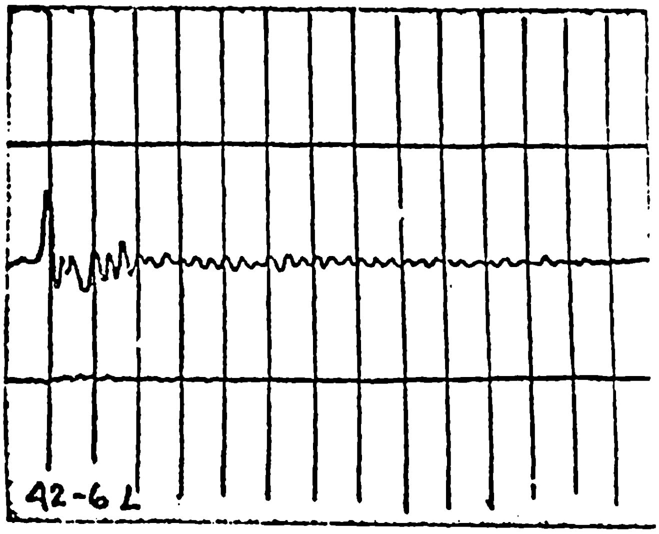
Type 4 centering material

Fireproofing not known

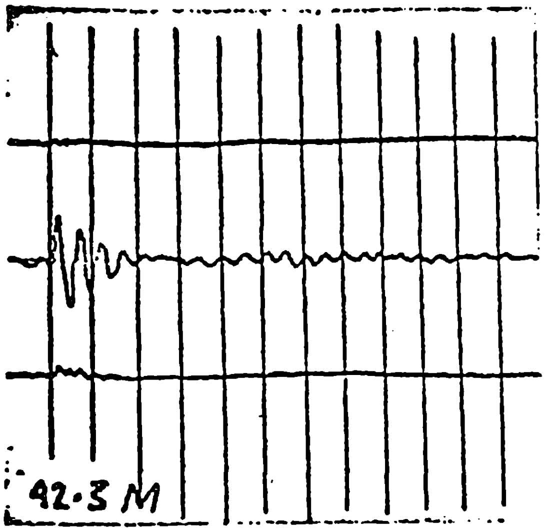
BUILDING 8.



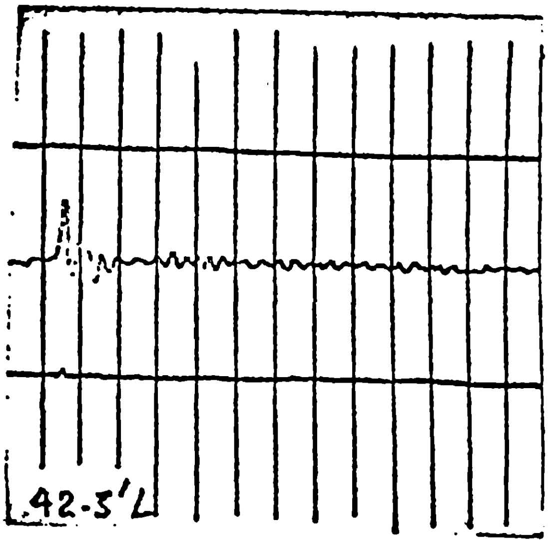
Location 1
Mechanical Impact



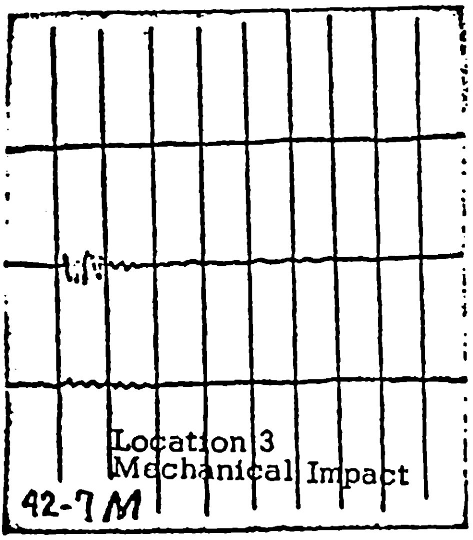
Location 1
Human Impact



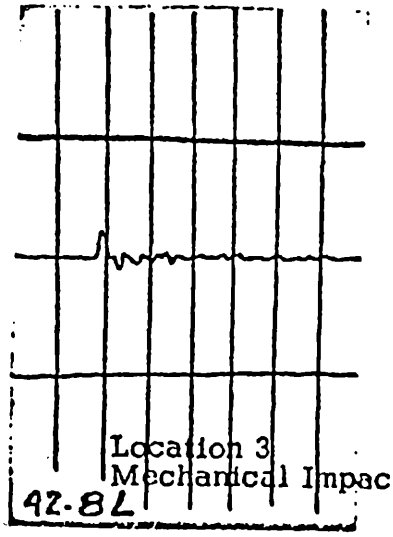
Location 2
Mechanical Impact



Location 2
Human Impact

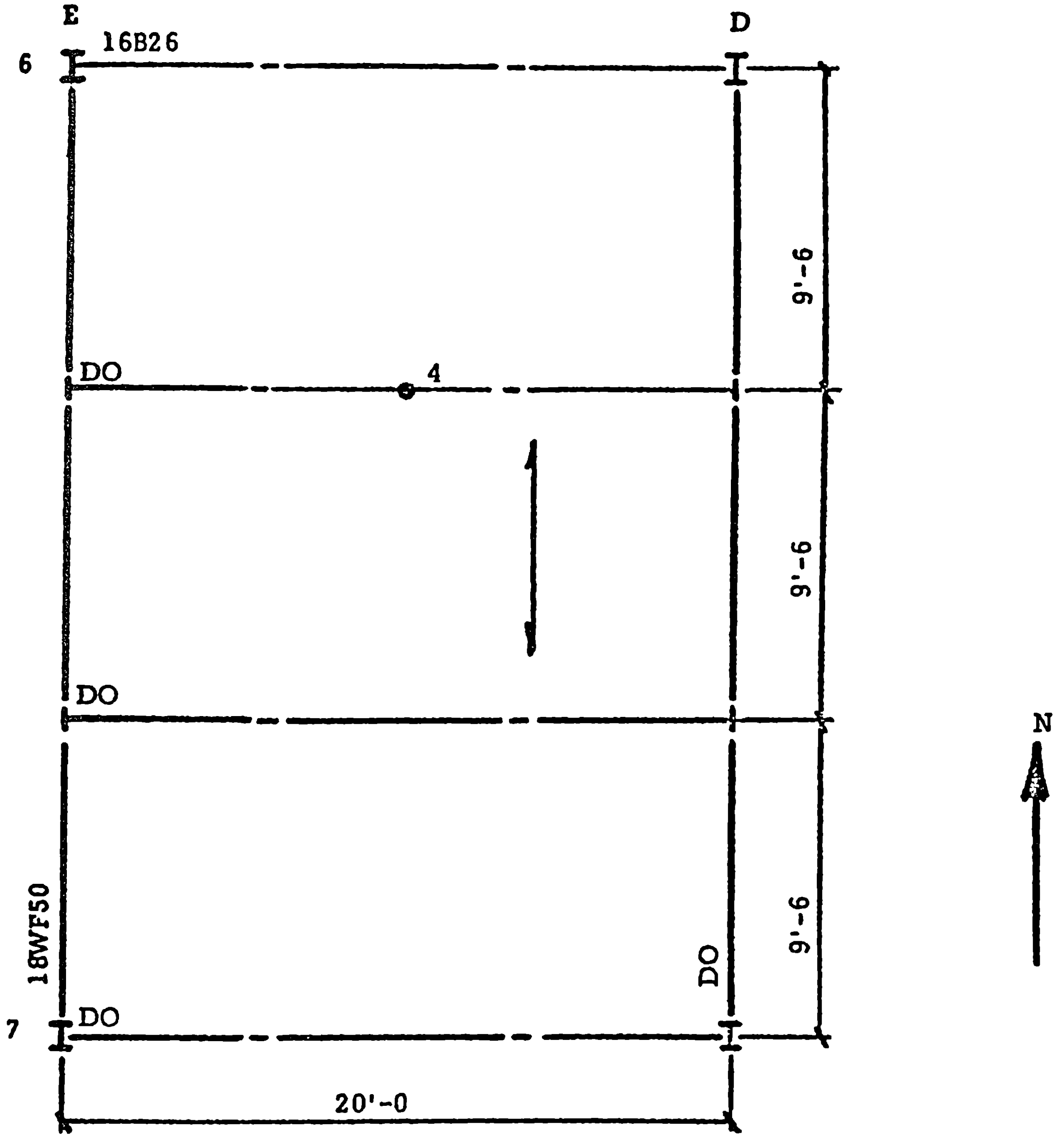


Location 3
Mechanical Impact



Location 3
Mechanical Impact

4th Floor - BUILDING 8.



Partial Framing Plan - 11th Floor
(No Scale)

Non-composite construction

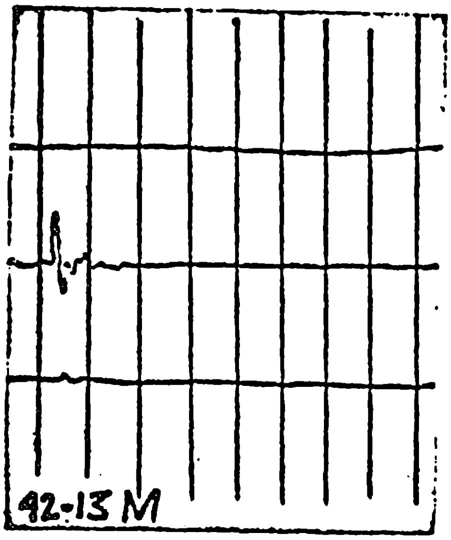
4 1/2" Concrete slab

A36 Steel

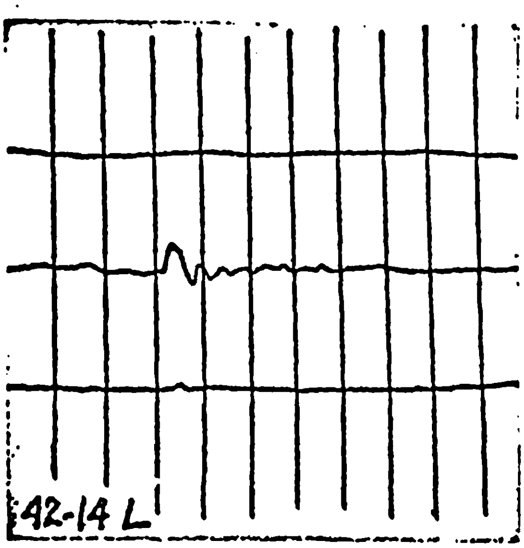
Type 4 centering material

Fireproofing not known

BUILDING 8.

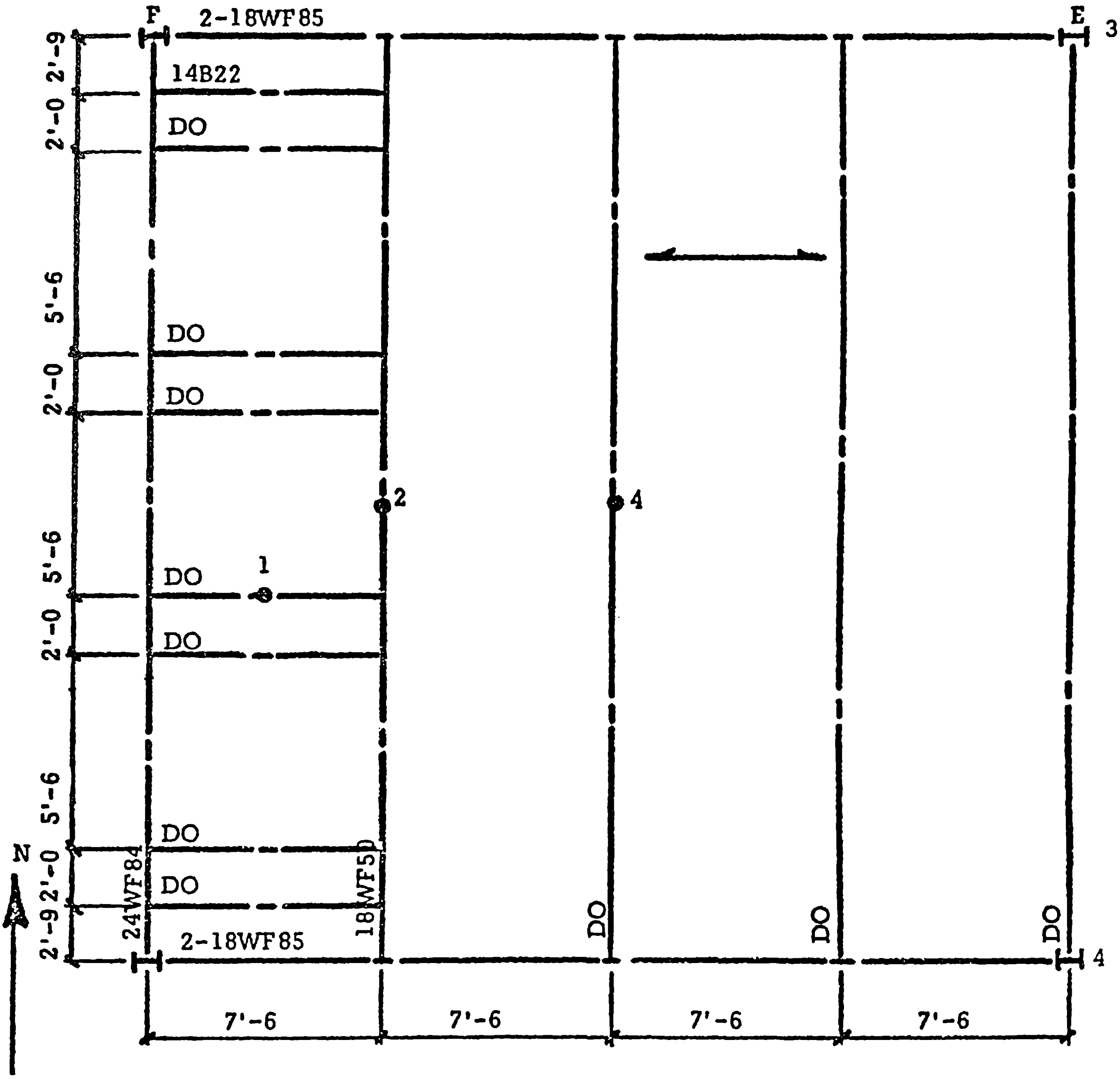


Location 1
Mechanical Impact



Location 1
Mechanical Impact

11th Floor - BUILDING 8.



Partial Framing Plan - 10th Floor
(No Scale)

Non-composite construction

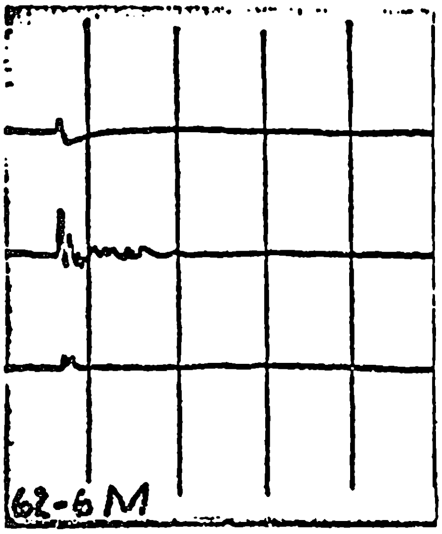
3" Lightweight (110 pcf) concrete slab

A36 Steel

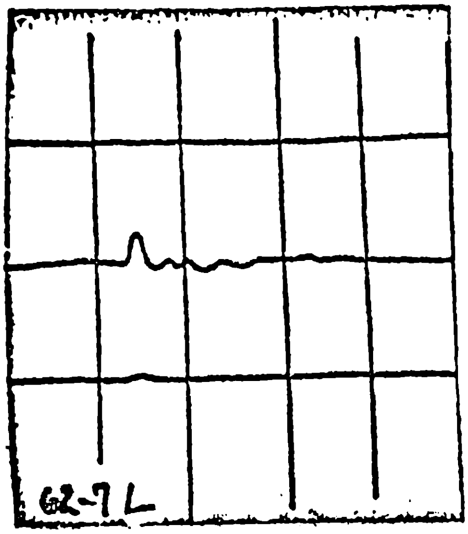
Type 1 centering material

Spray-on fireproofing

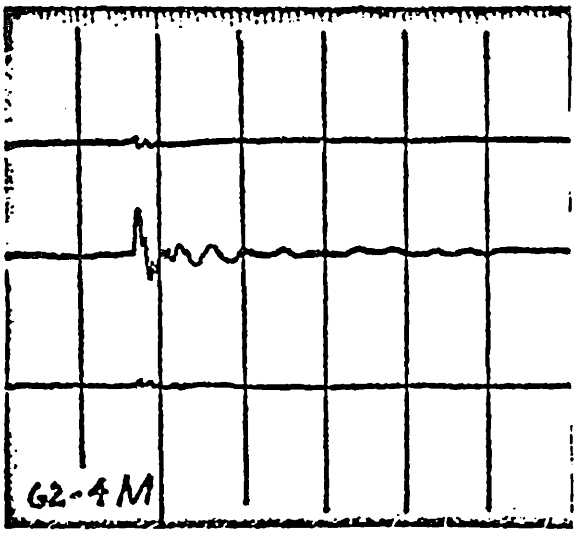
BUILDING 10.



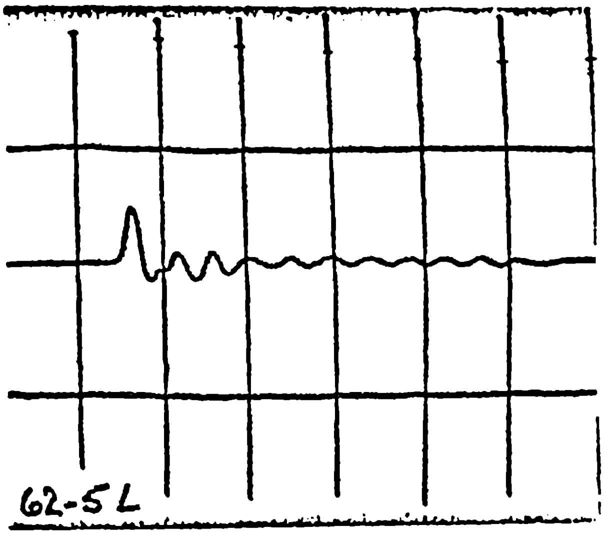
Location 1
Mechanical Impact



Location 1
Human Impact

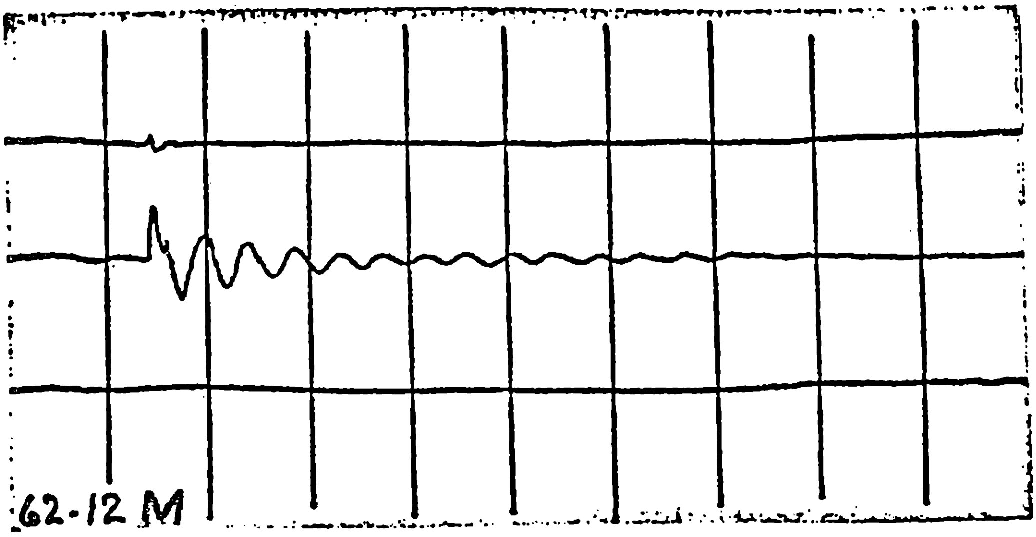


Location 2
Mechanical Impact

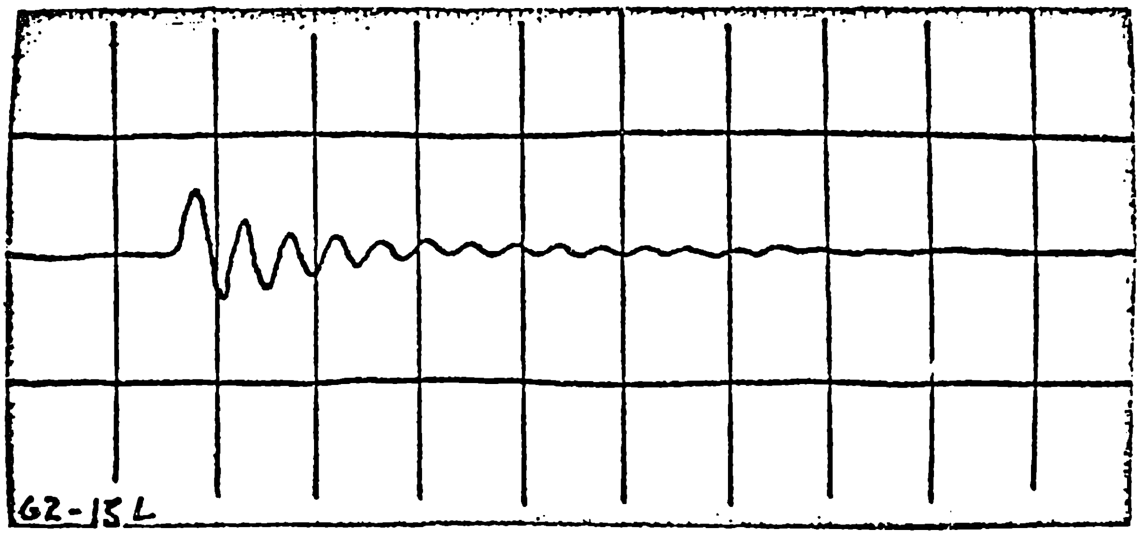


Location 2
Human Impact

10th Floor - BUILDING 10.

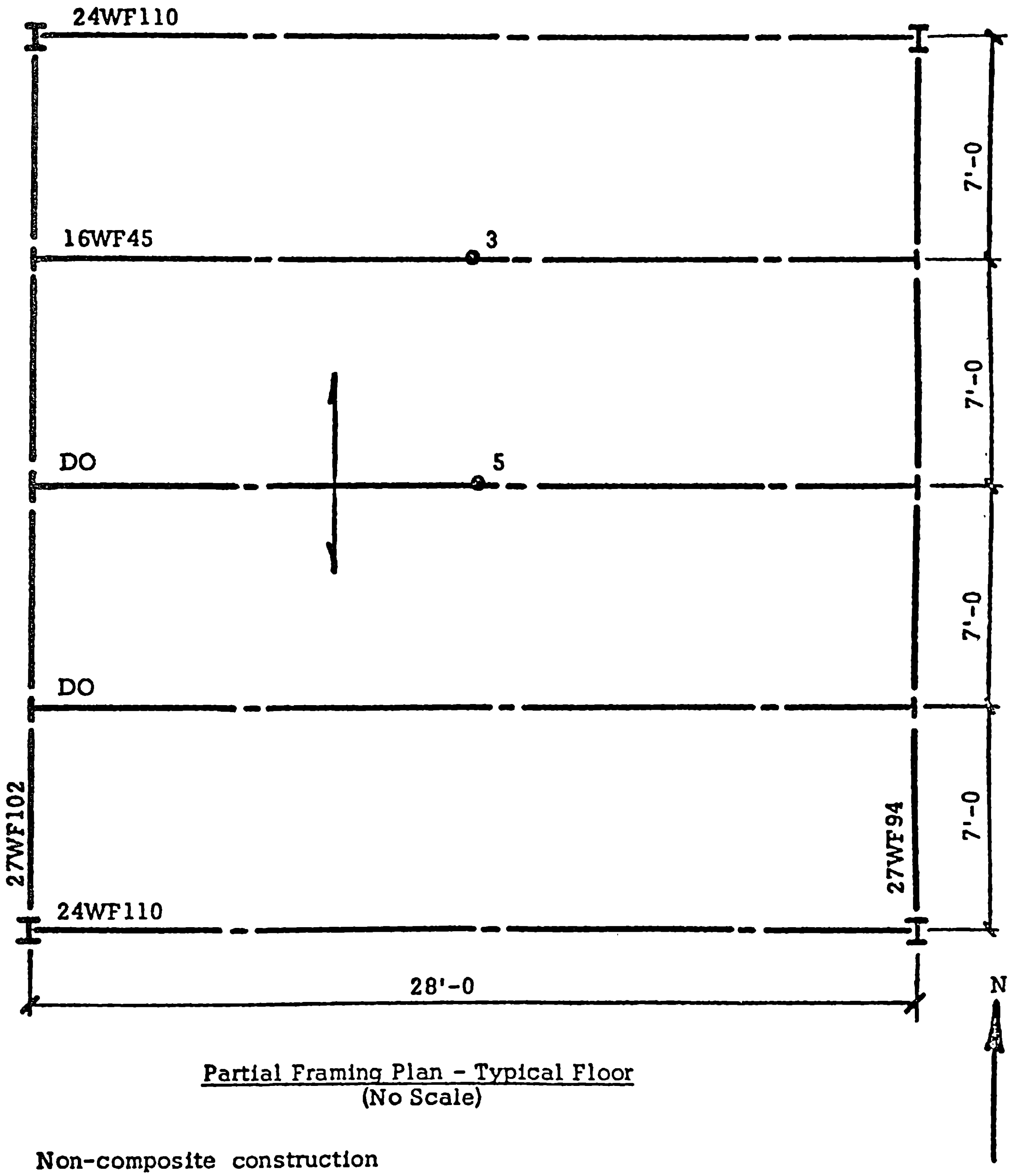


Location 4
Mechanical Impact



Location 4
Human Impact

10th Floor - BUILDING 10.



Partial Framing Plan - Typical Floor
(No Scale)

Non-composite construction

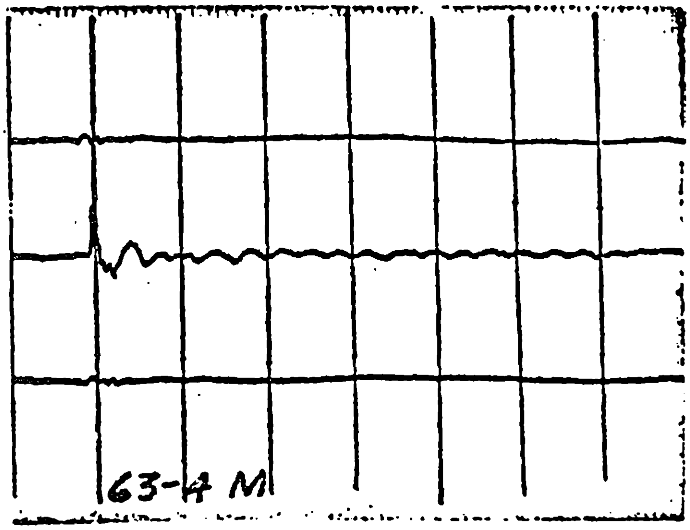
2 1/2" Lightweight (110 pcf) concrete slab

A36 Steel

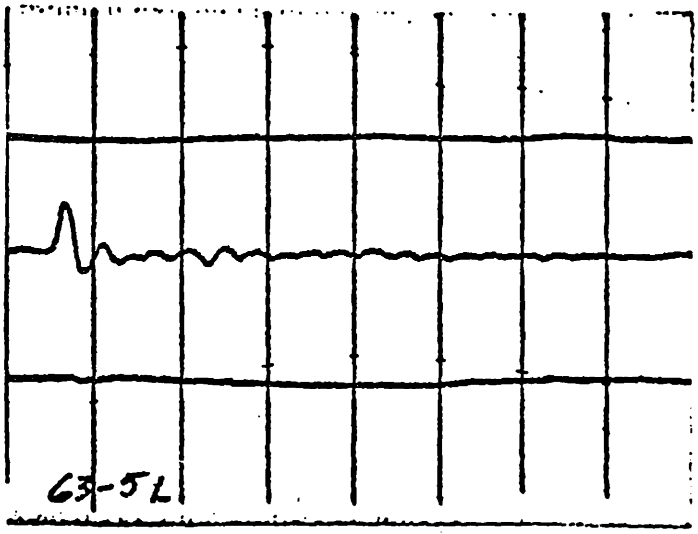
Type 2 centering material

Spray-on fireproofing

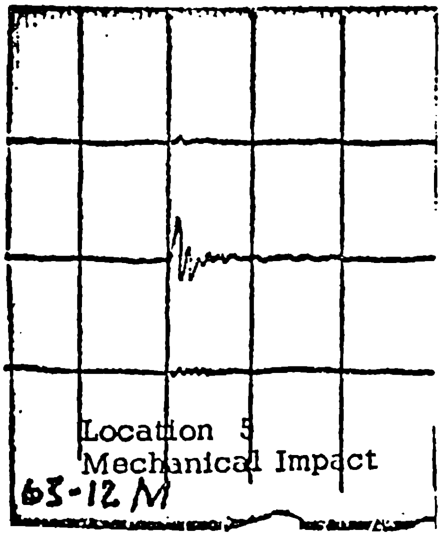
BUILDING 11



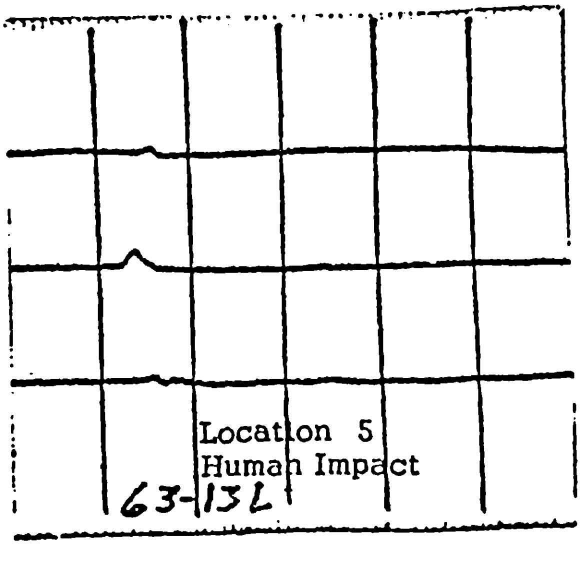
Location 3
Mechanical Impact



Location 3
Human Impact

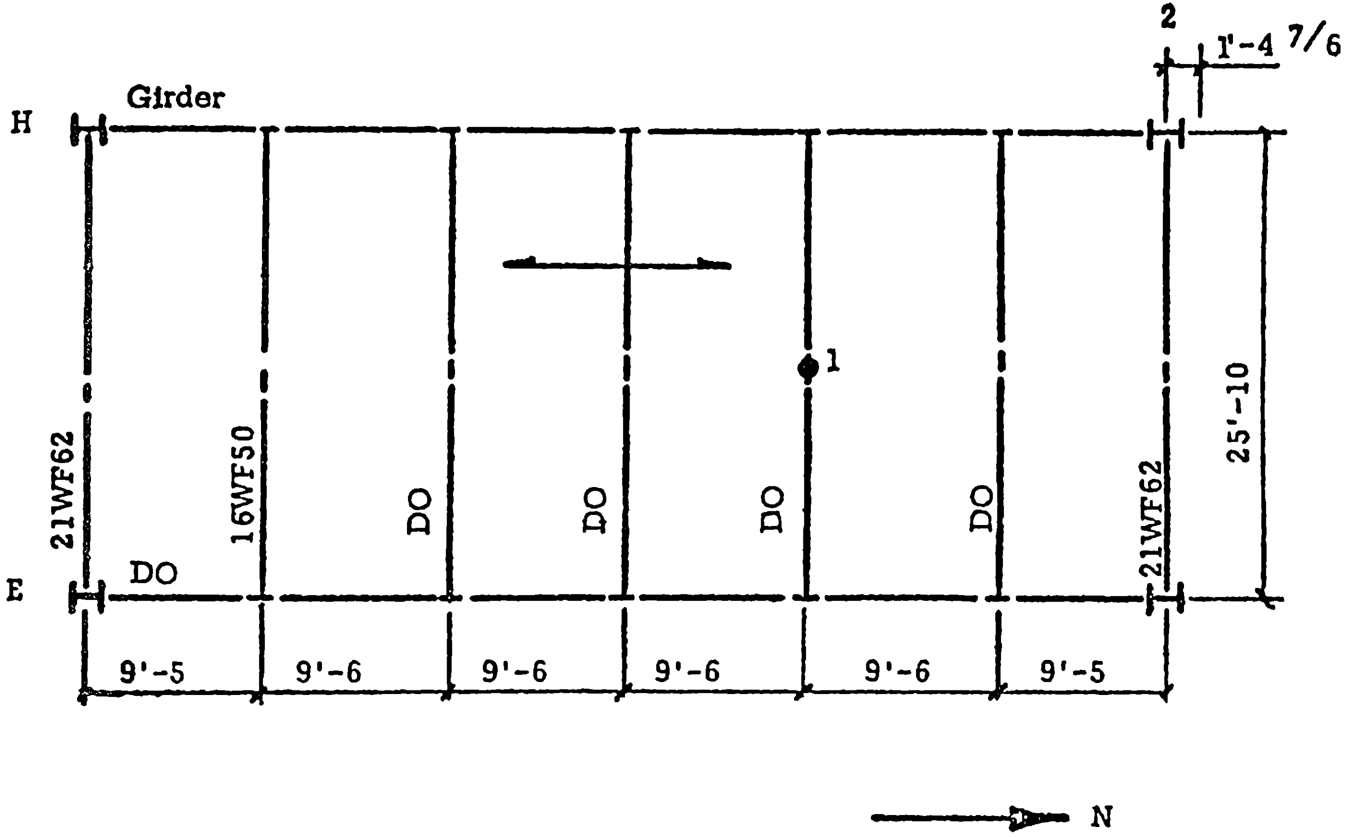


Location 5
Mechanical Impact



Location 5
Human Impact

Typical Floor - BUILDING 11.



Partial Framing Plan - 6th Floor
(No Scale)

Non-composite construction

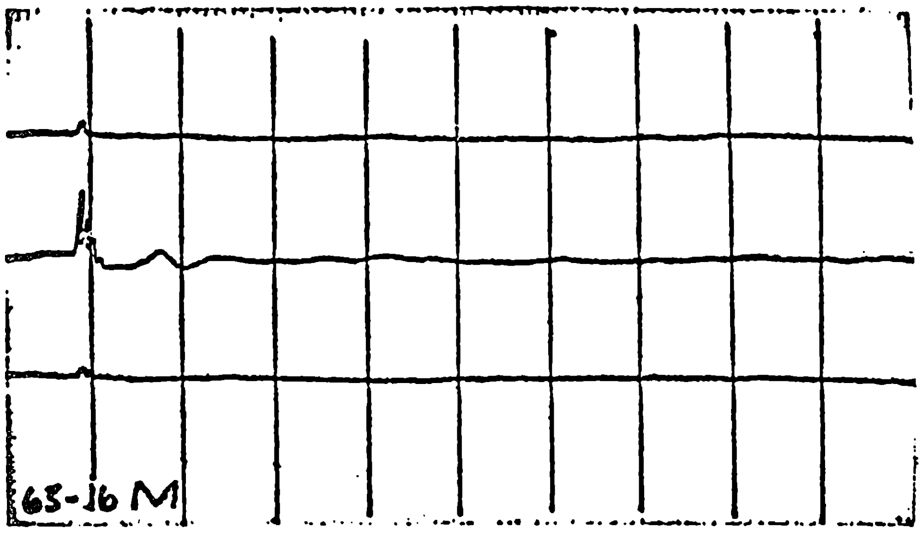
2 1/2" Lightweight (110pcf) concrete slab

A36 Steel

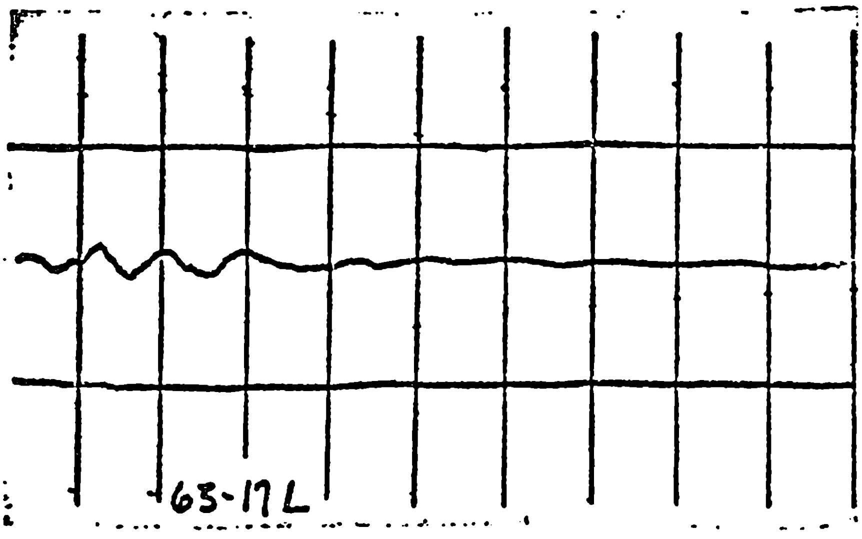
Type 1 centering material

Spray-on fireproofing

BUILDING 12.

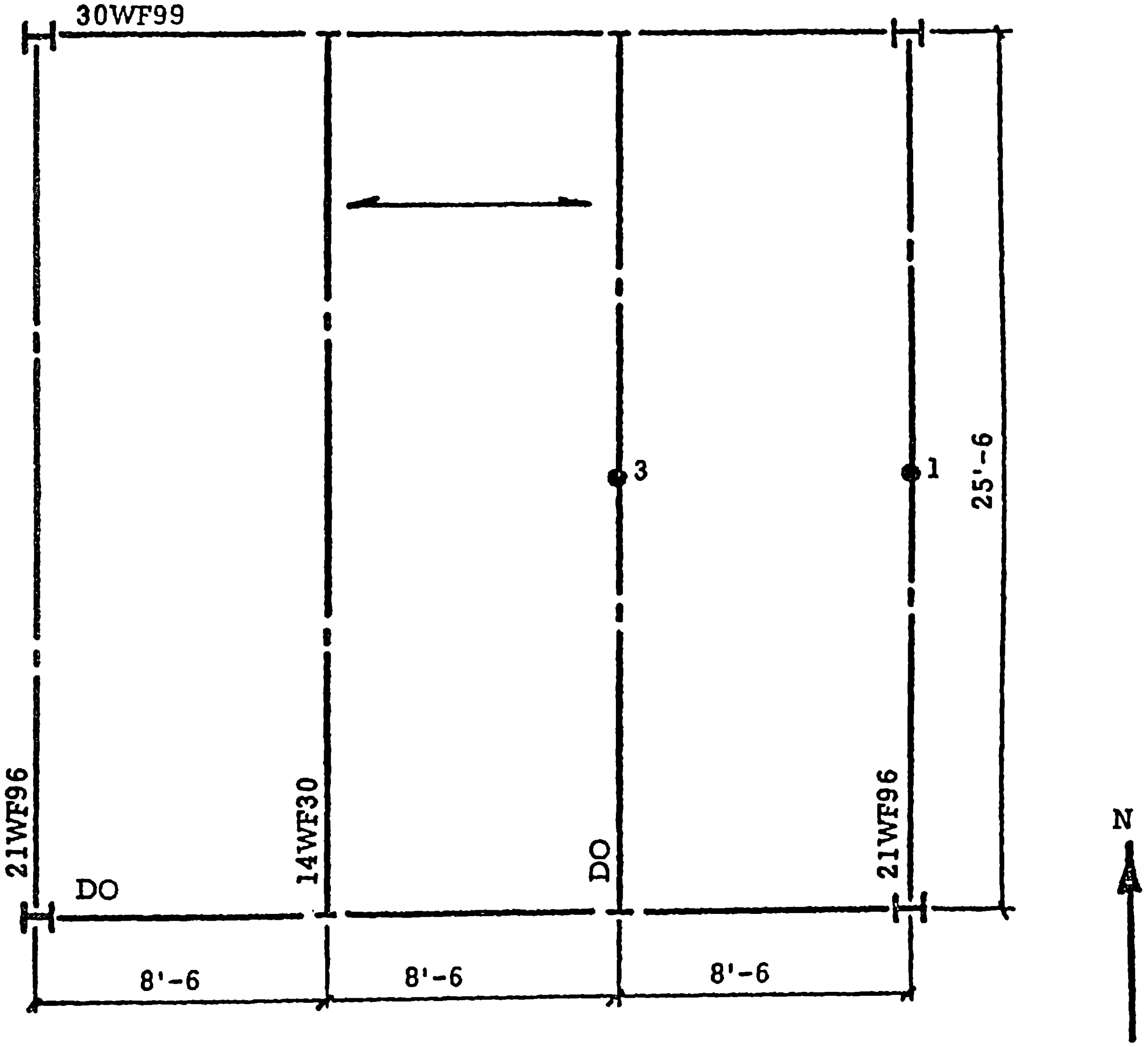


Location 1
Mechanical Impact



Location 1
Human Impact

6th Floor - BUILDING 12.



Partial Floor Plan - 4th Floor
(No Scale)

Non-composite construction

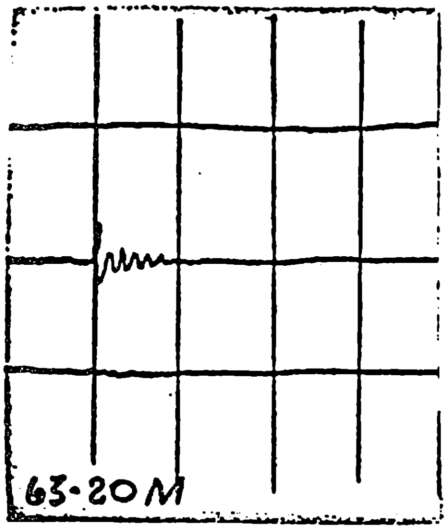
2 1/2" Lightweight (110pcf) concrete slab

A36 Steel

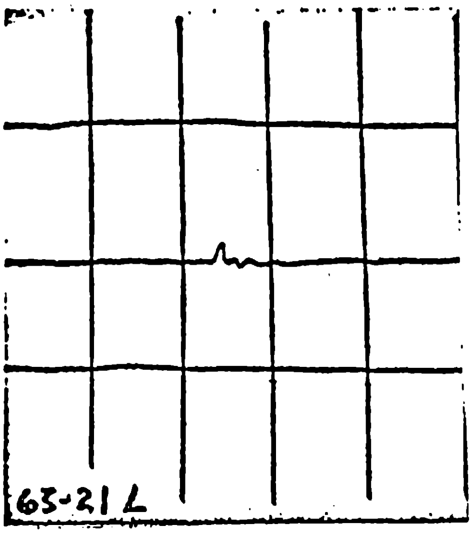
Type 1 centering material

Spray-on fireproofing

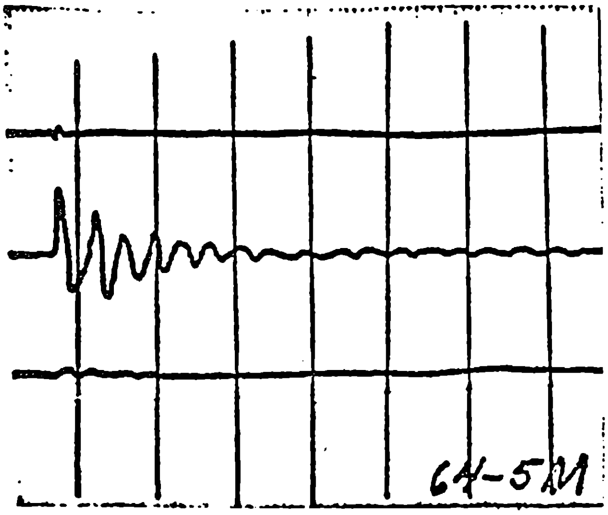
BUILDING 13.



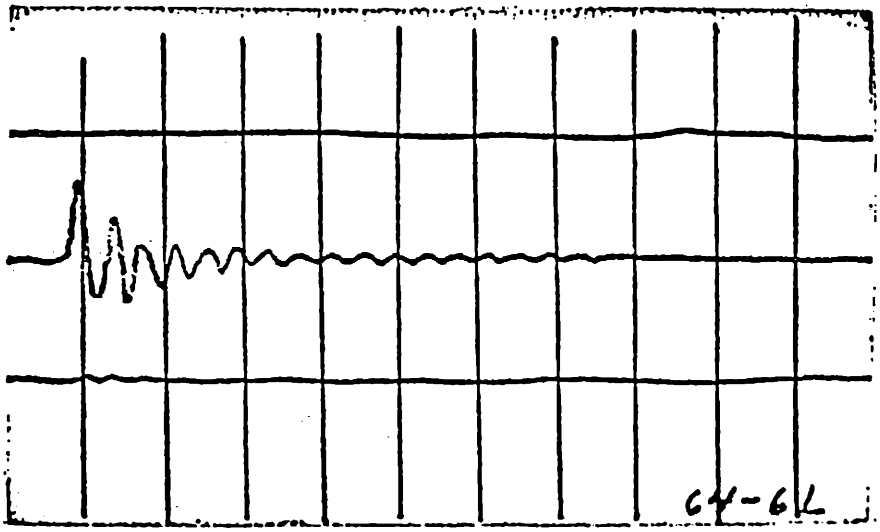
Location 1
Mechanical Impact



Location 1
Human Impact

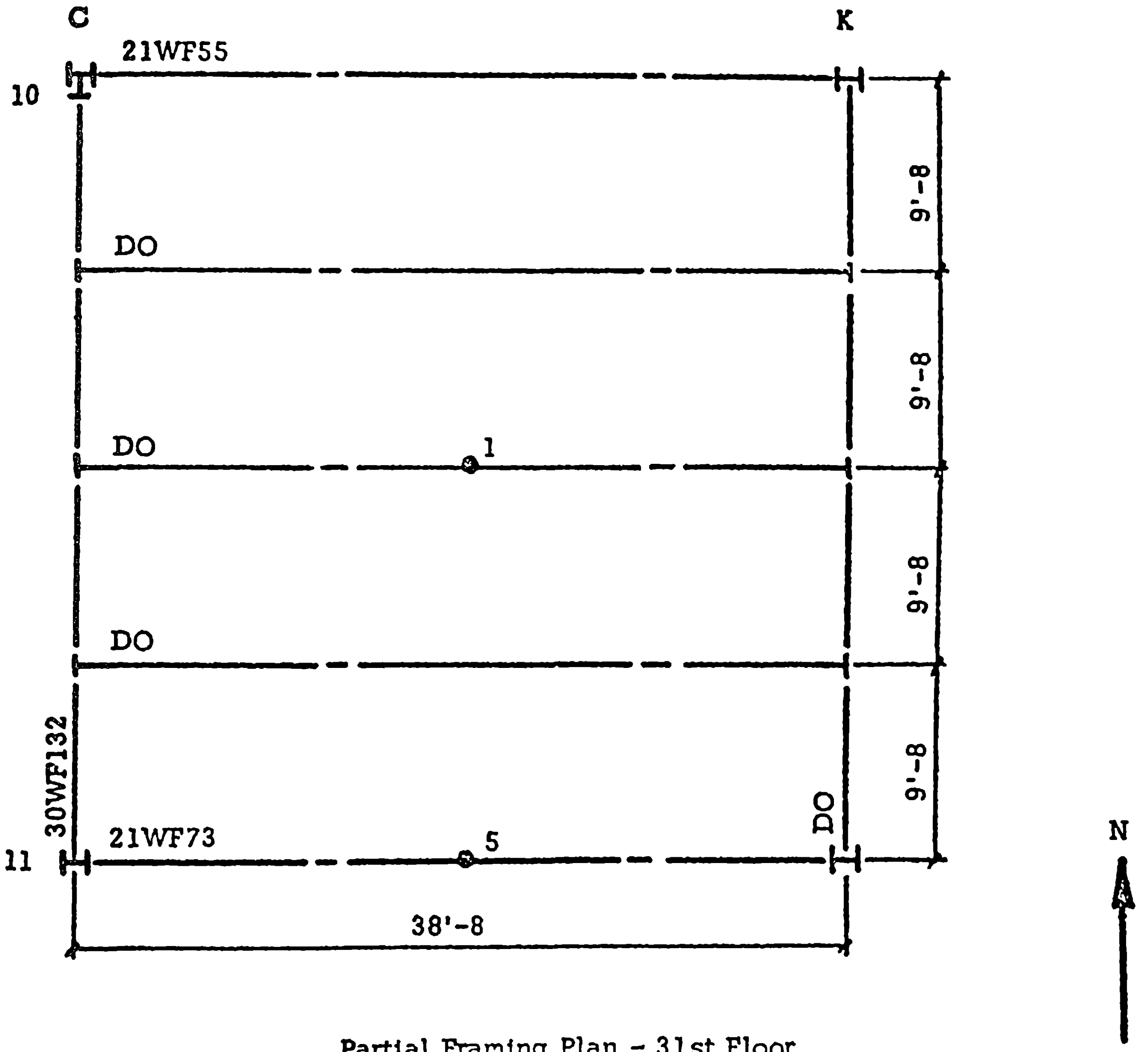


Location 3
Mechanical Impact



Location 3
Human Impact

4th Floor - BUILDING 13



Partial Framing Plan - 31st Floor
(No Scale)

Composite construction

2 1/2" Concrete topping

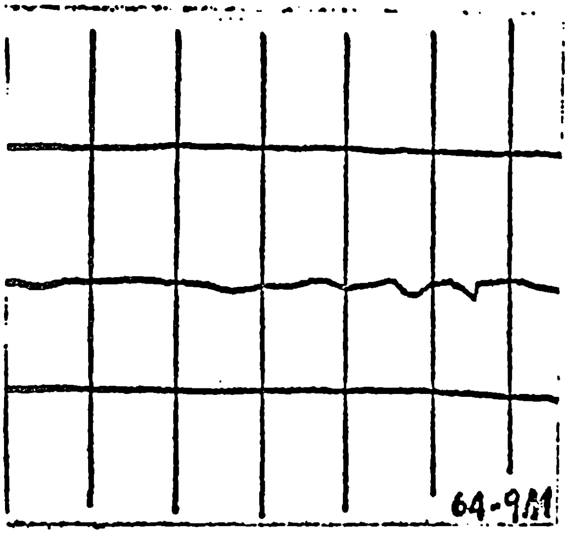
4 1/2" Lightweight (110pcf) concrete slab

A36 Steel

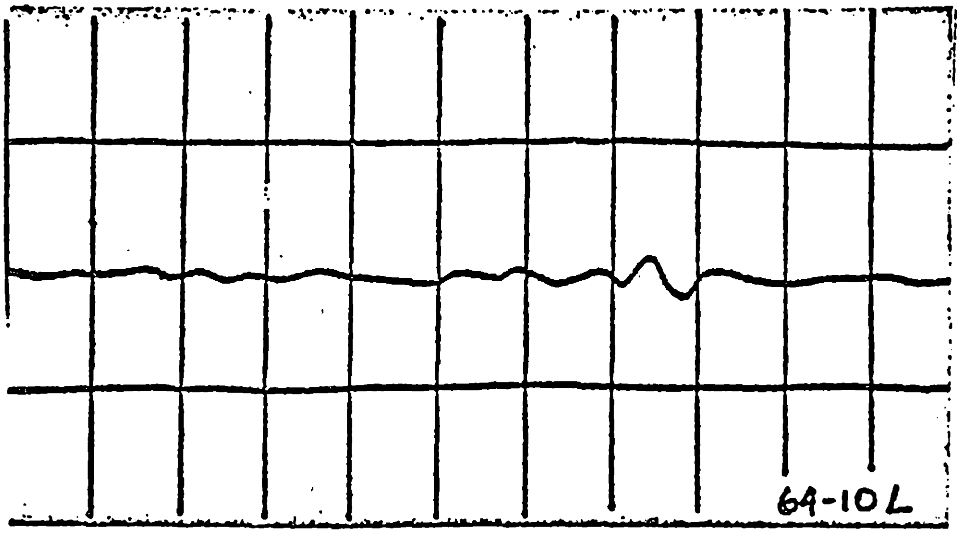
No centering material

Spray-on fireproofing

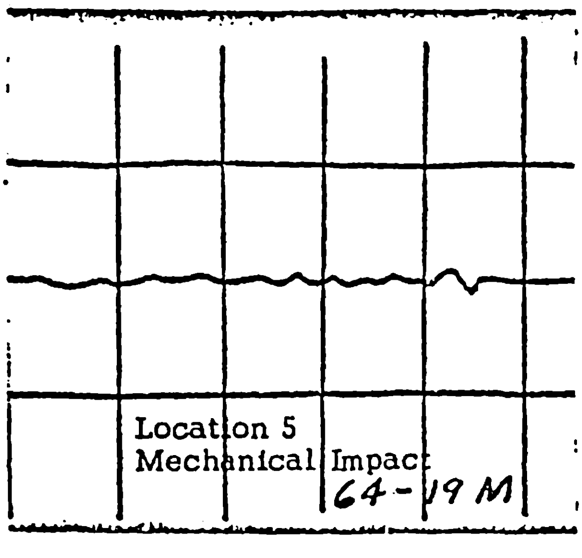
BUILDING 14.



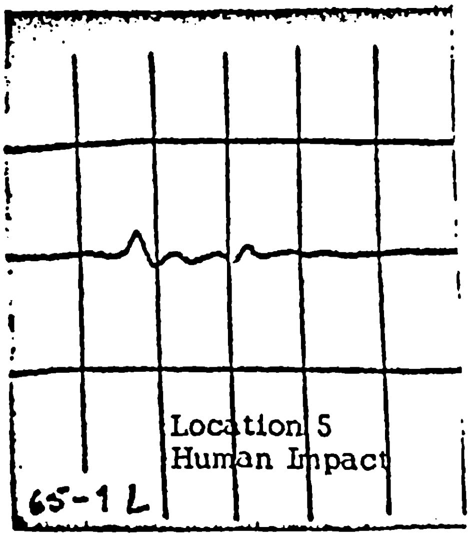
Location 1
Mechanical Impact



Location 1
Human Impact

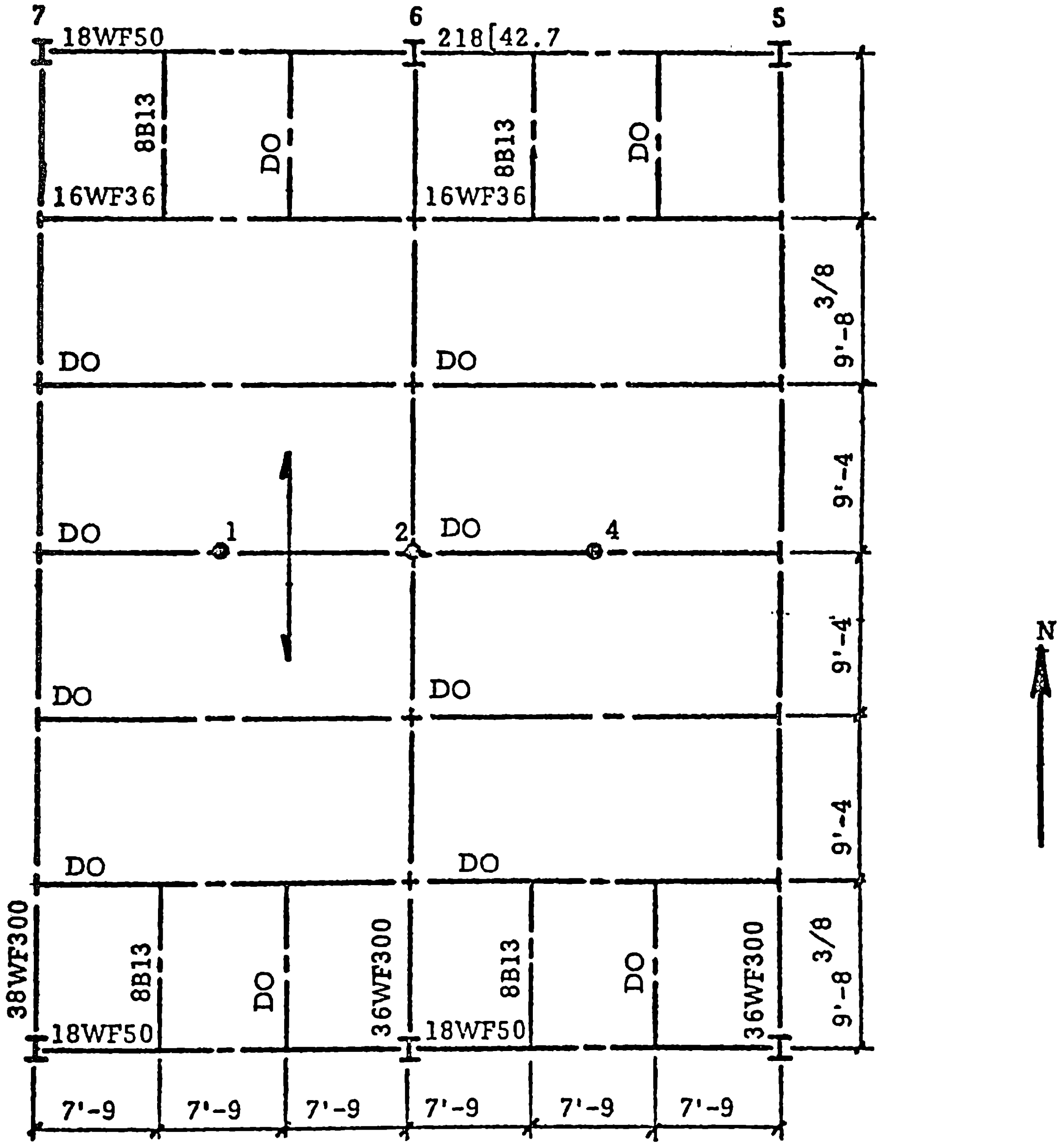


Location 5
Mechanical Impact



Location 5
Human Impact

31st Floor - BUILDING 14.



Partial Framing Plan - 7th Floor
(No Scale)

Non-composite construction

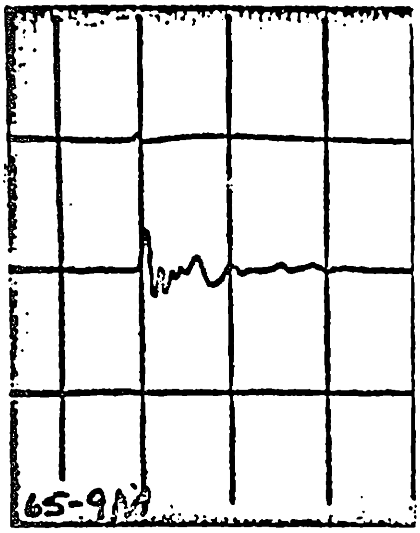
2 1/2" Lightweight (110pcf) concrete slab

A36 Steel

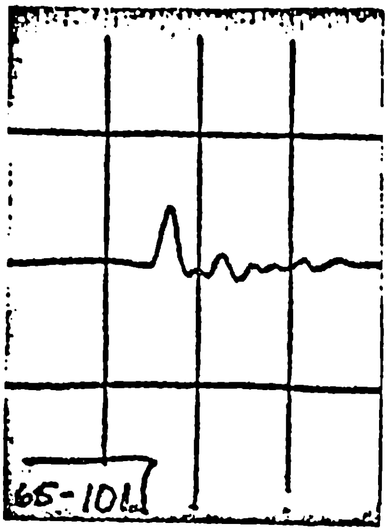
Type 1 centering material

Spray-on fireproofing

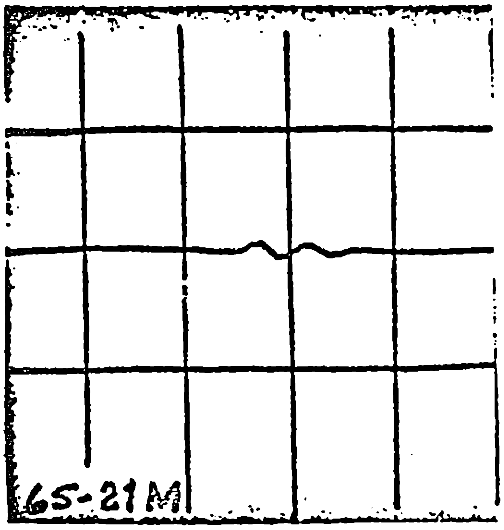
BUILDING 15.



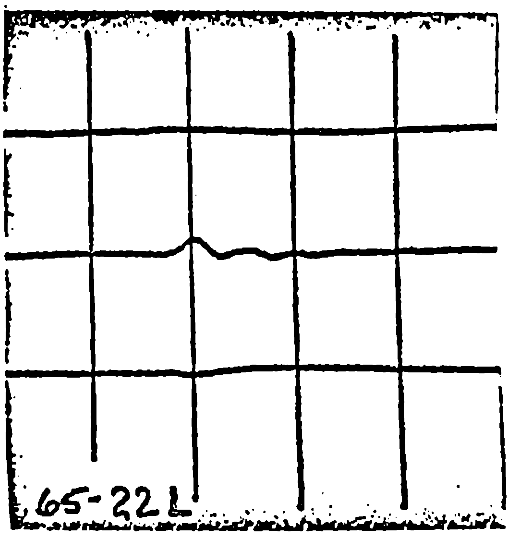
Location 1
Mechanical Impact



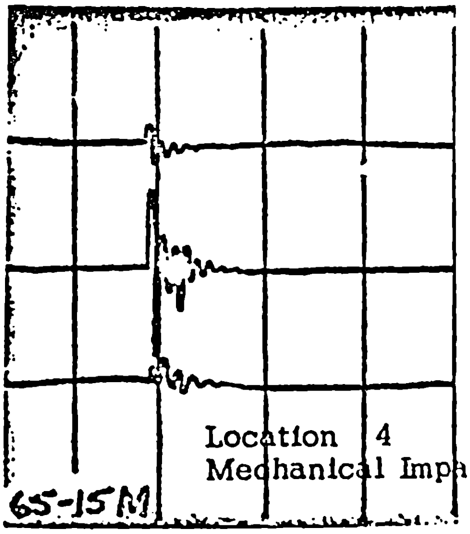
Location 1
Human Impact



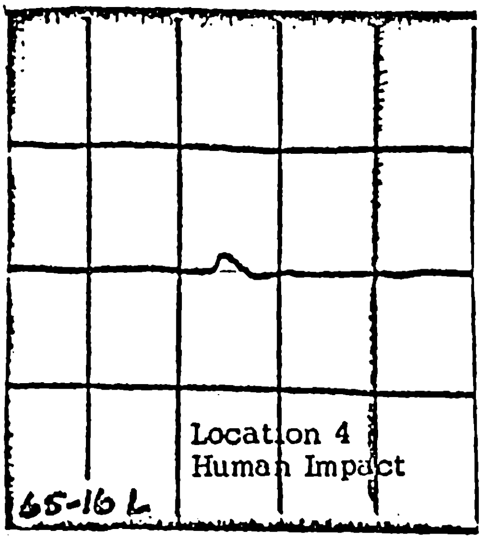
Location 3
Mechanical Impact



Location 3
Human Impact

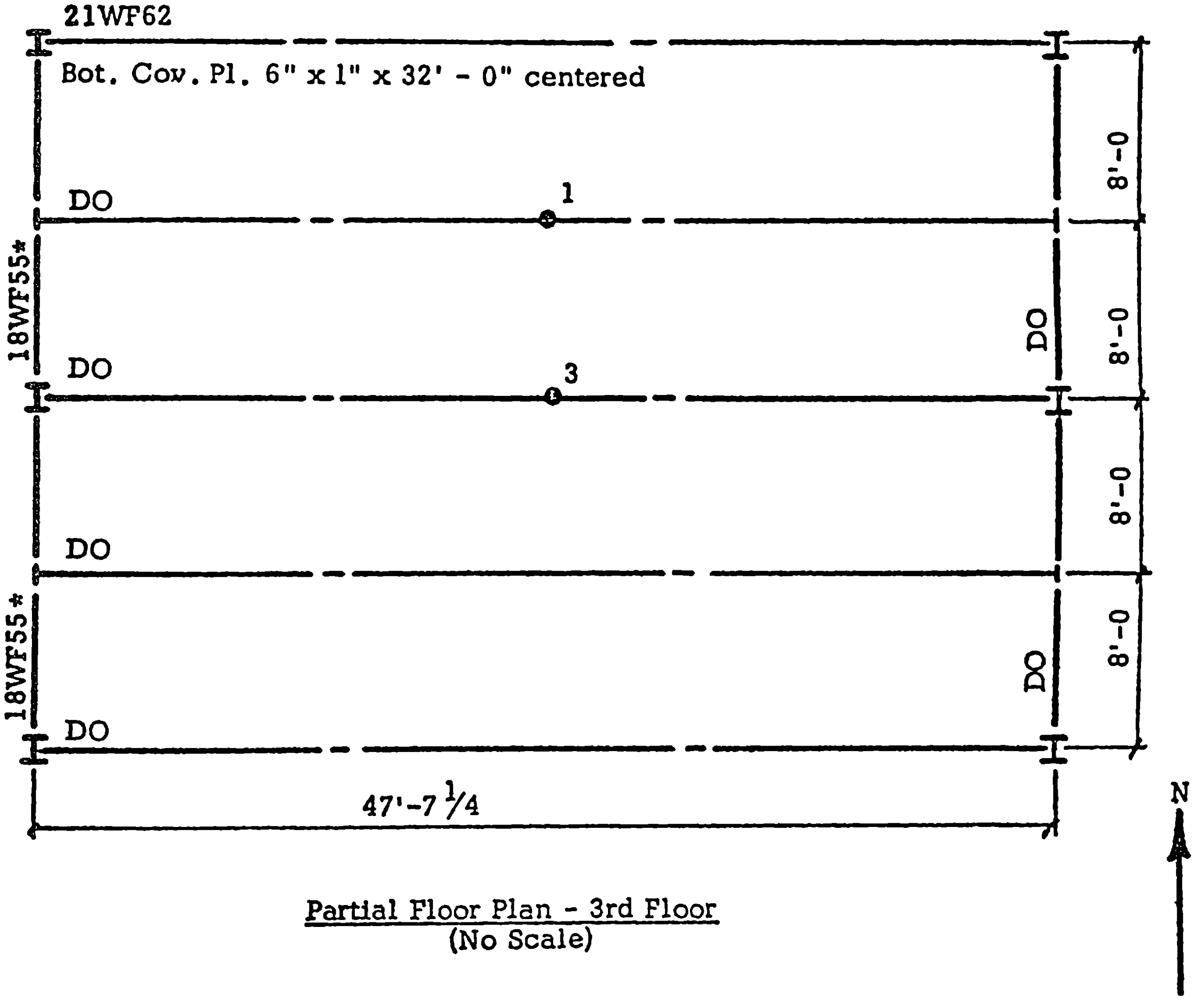


Location 4
Mechanical Impact



Location 4
Human Impact

7th Floor - BUILDING 15.



Partial Floor Plan - 3rd Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite construction.

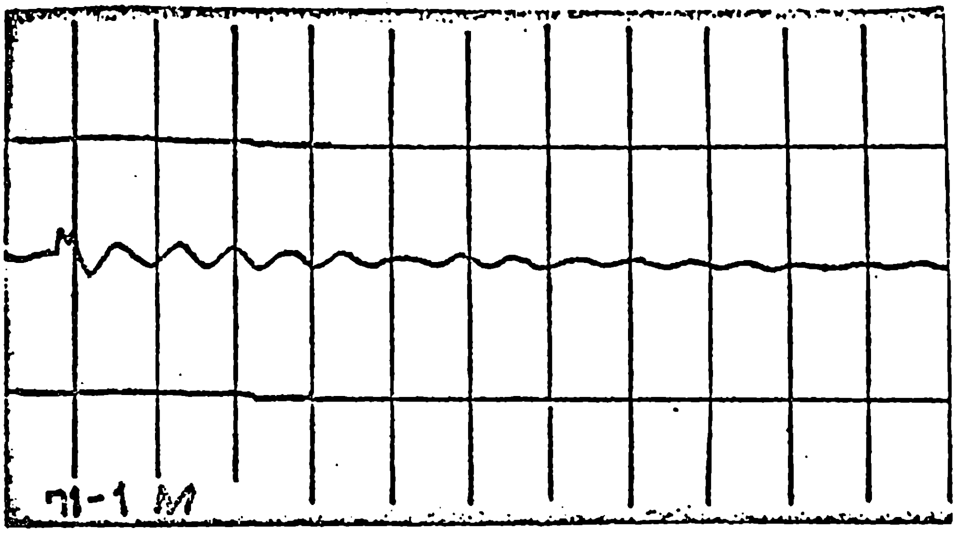
5" Concrete slab

A7 Steel

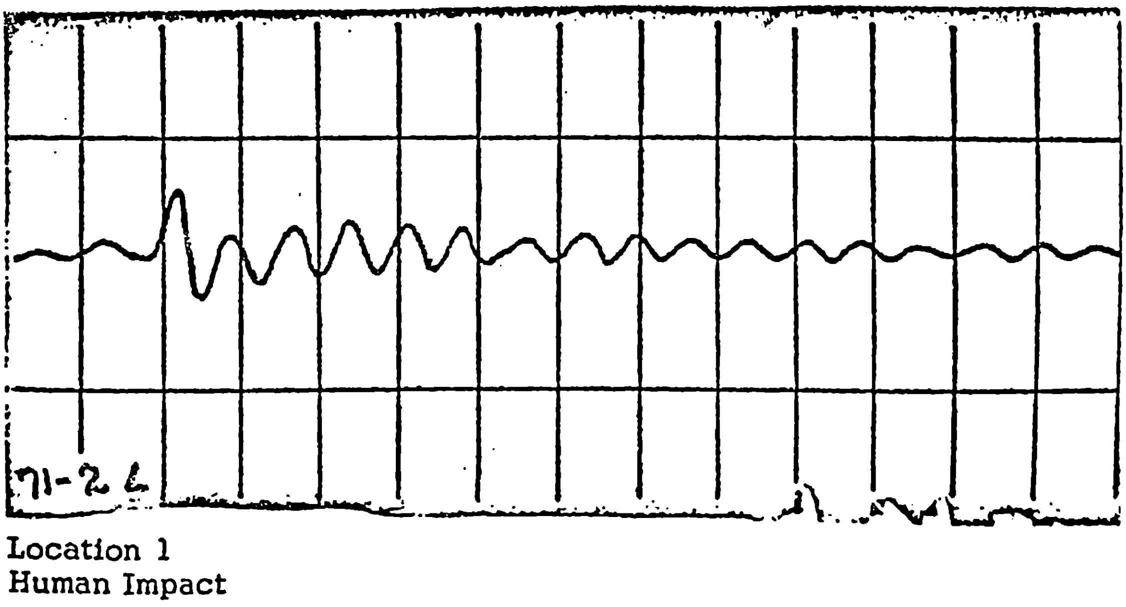
No centering material

Lath and plaster fireproofing

BUILDING 16.

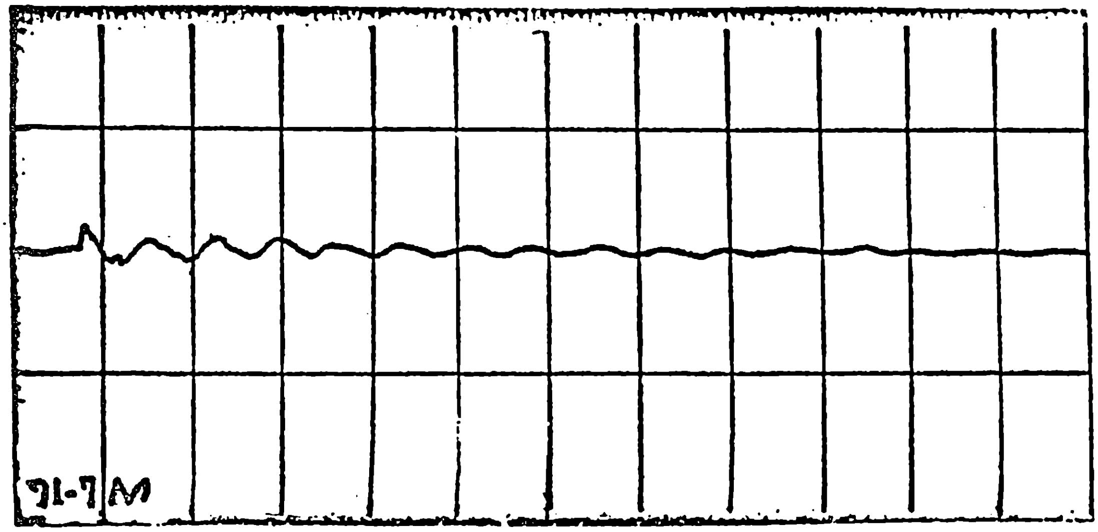


Location 1
Mechanical Impact

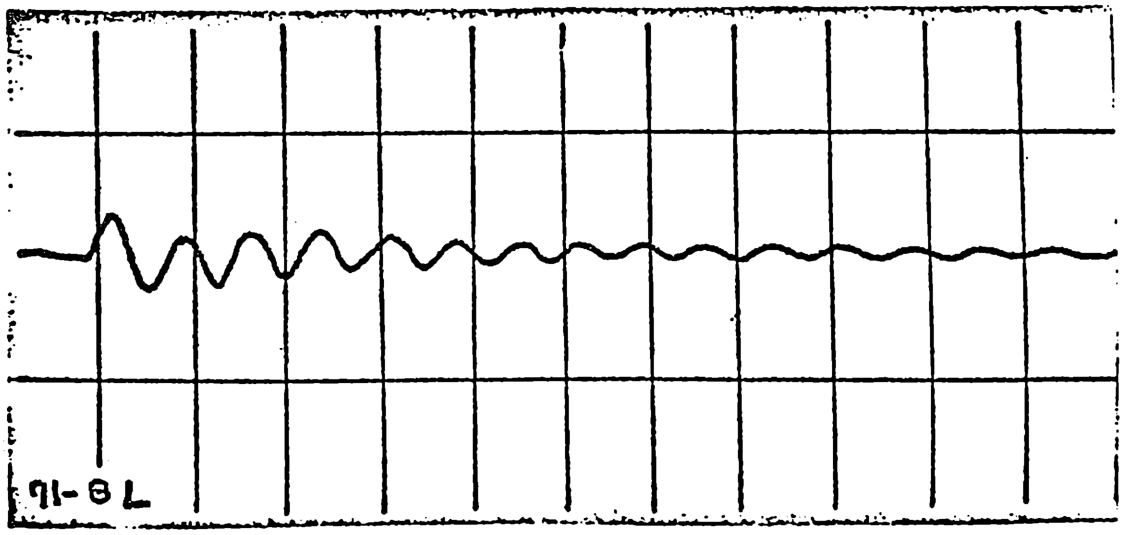


Location 1
Human Impact

3rd Floor - BUILDING 16.

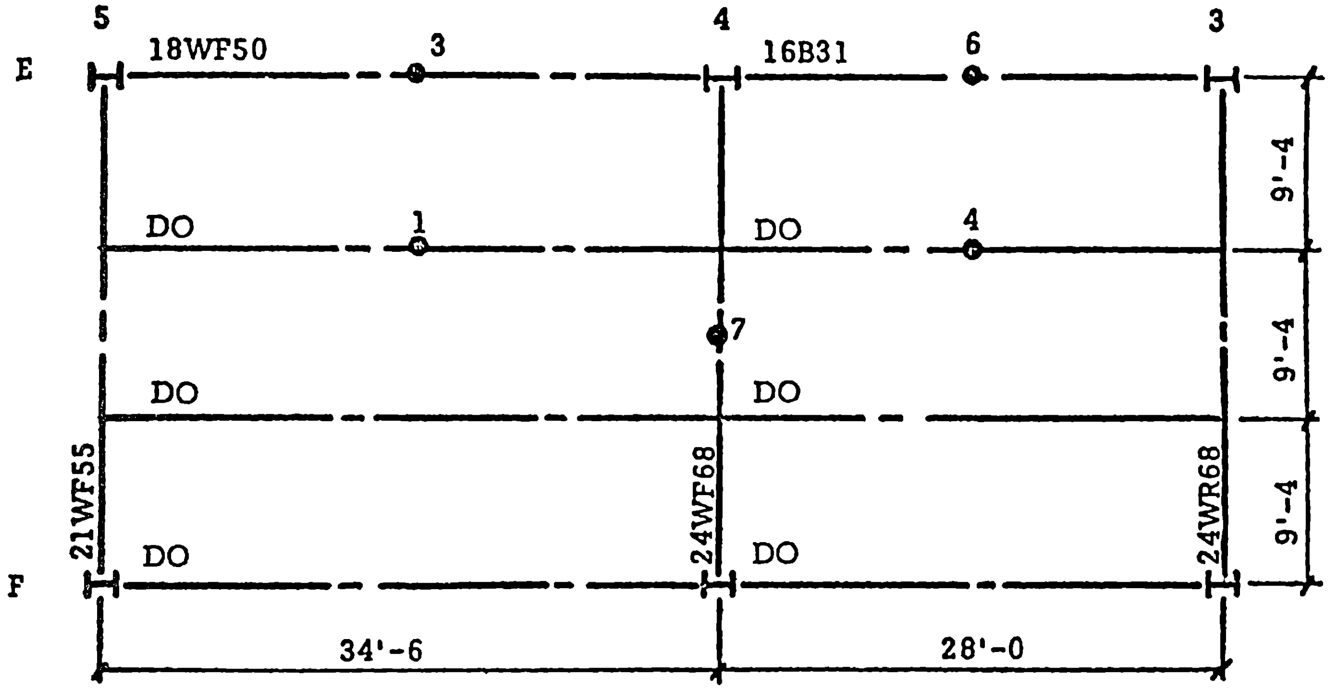


Location 3
Mechanical Impact



Location 3
Human Impact

3rd Floor - BUILDING 16.



Partial Framing Plan - 2nd Floor
(No Scale)

Composite construction

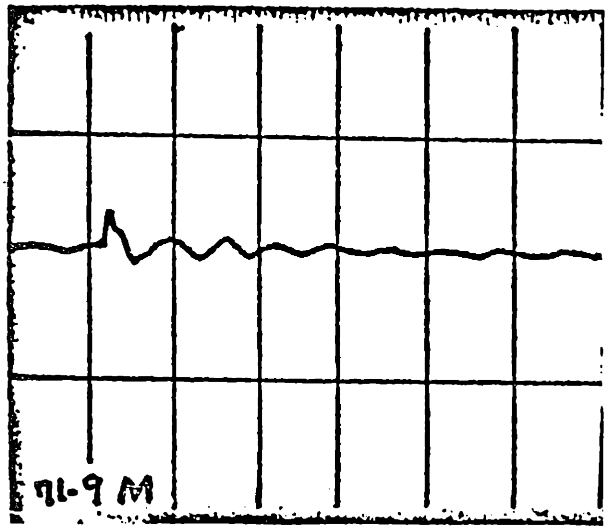
5" Lightweight (110pcf) concrete slab

A36 Steel

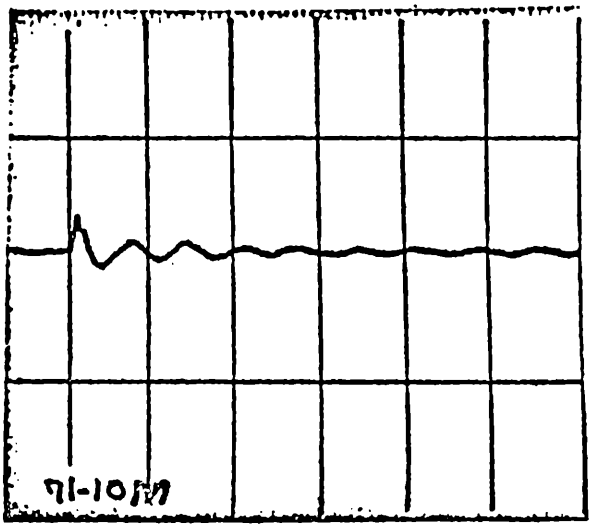
Type 4 centering material

Spray-on fireproofing, hung acoustic ceiling

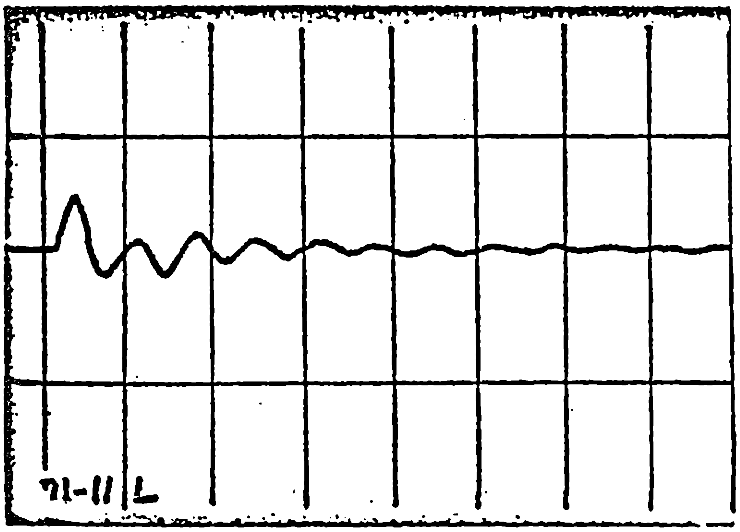
BUILDING 17.



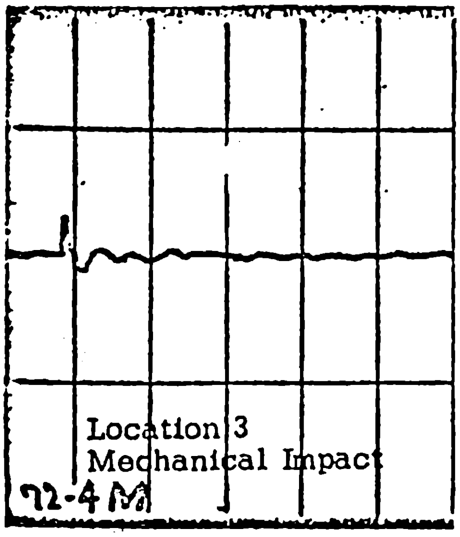
Location 1
Mechanical Impact



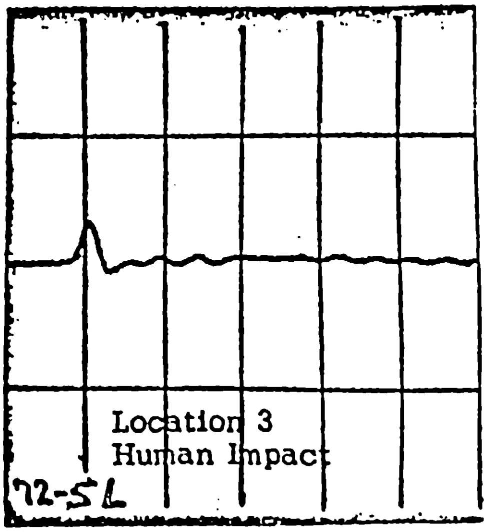
Location 1
Mechanical Impact



Location 1
Human Impact

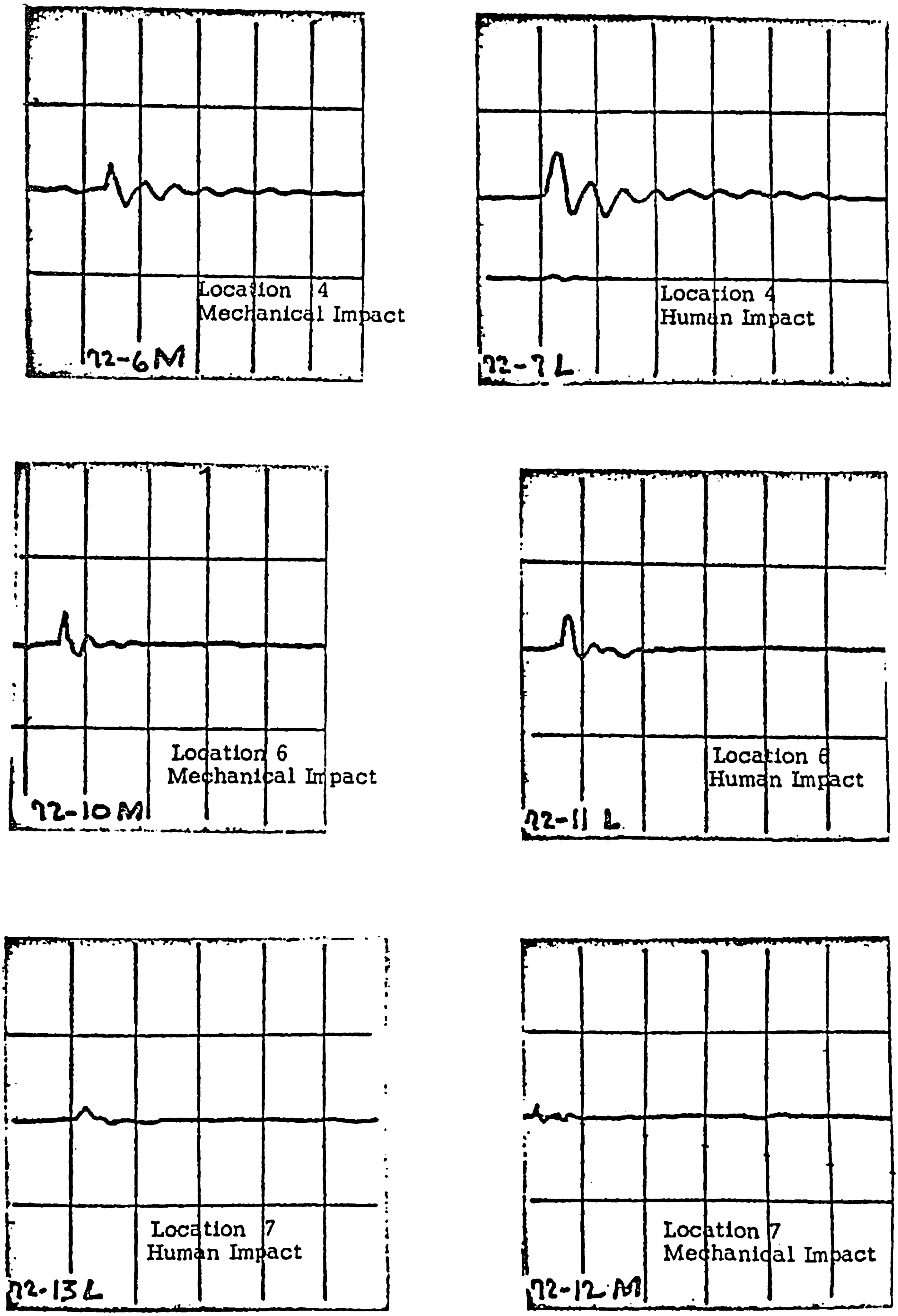


Location 3
Mechanical Impact

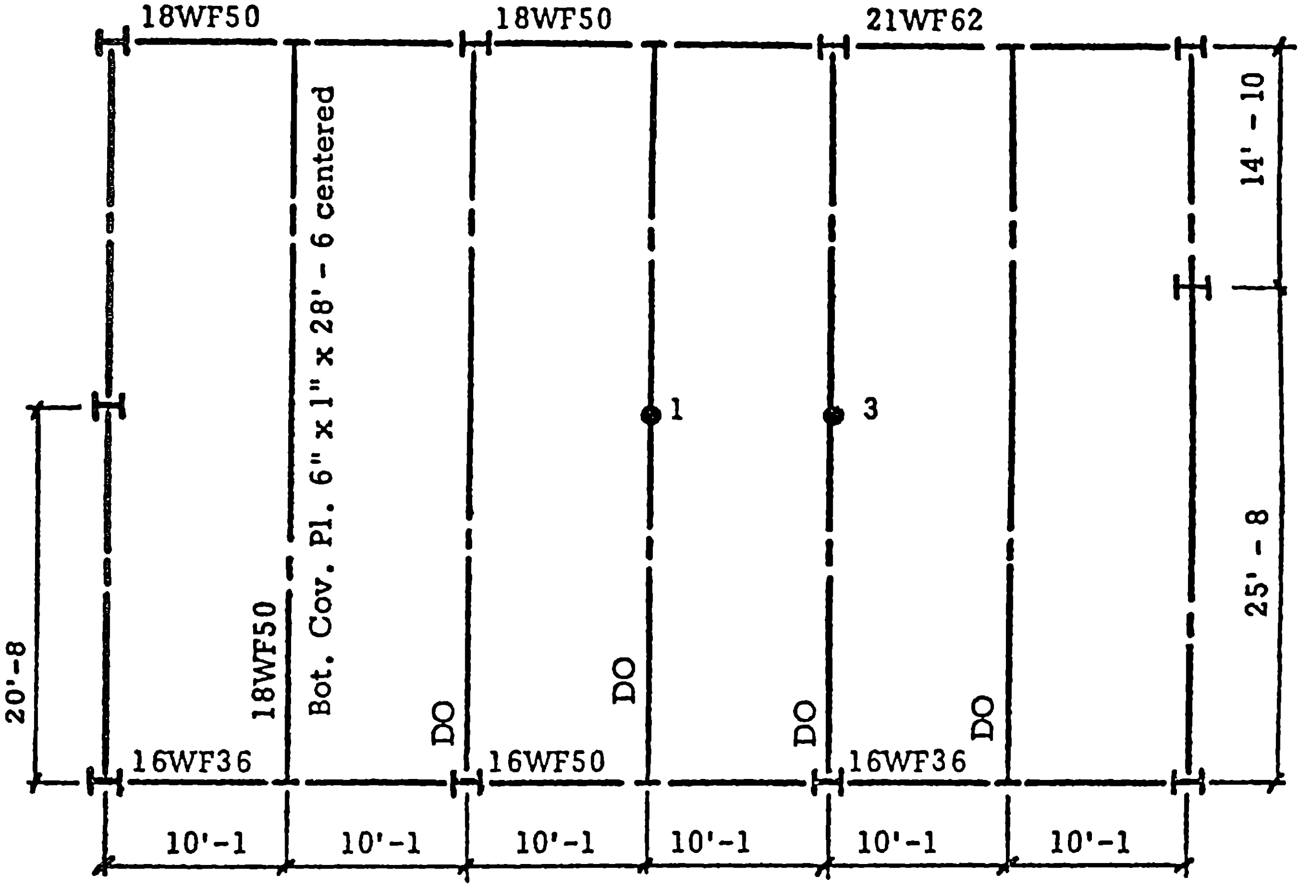


Location 3
Human Impact

2nd Floor - BUILDING 17.



2nd Floor - BUILDING 17.



Partial Framing Plan - Typical Floor
(No Scale)

Non-composite beams marked (*) in plan, all other composite construction.

2 1/2" Concrete topping

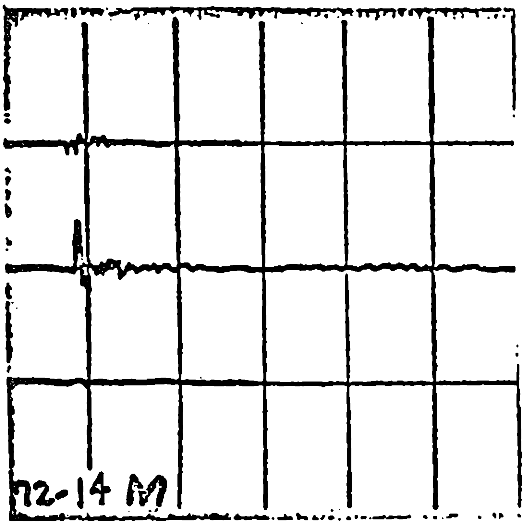
4" Concrete slab

A7 Steel

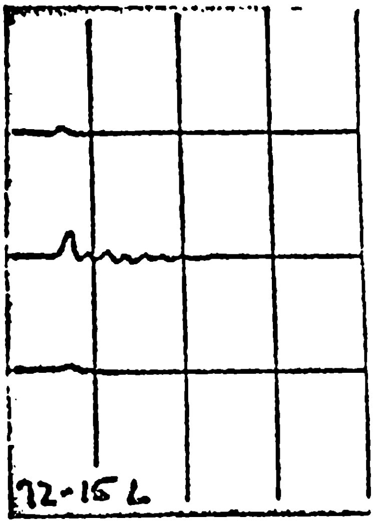
No centering material

Spray-on fireproofing

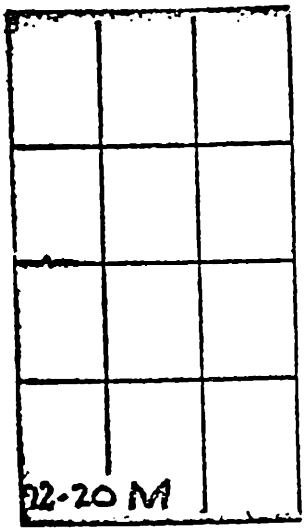
BUILDING 18.



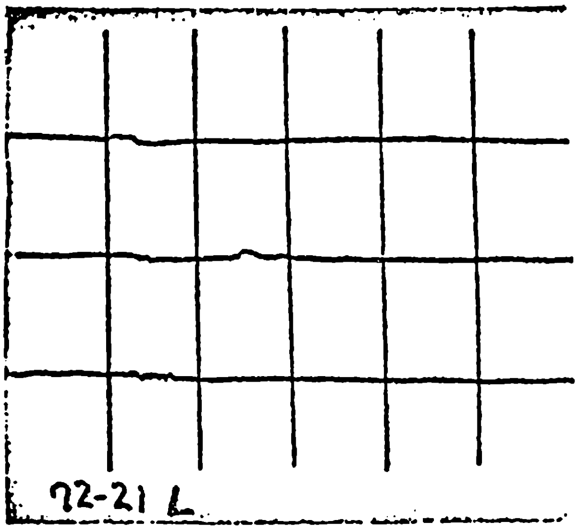
Location 1
Mechanical Impact



Location 1
Human Impact

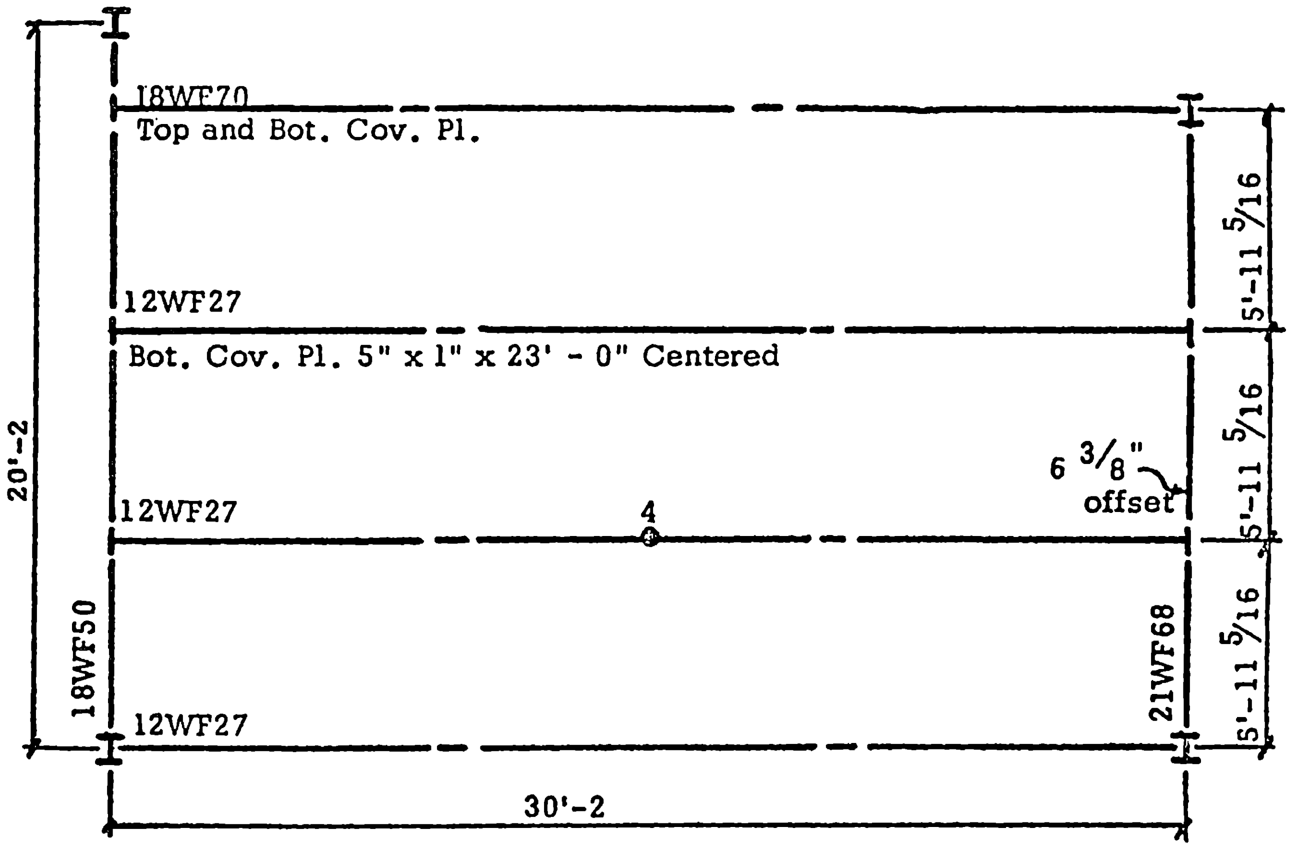


Location 3
Mechanical Impact



Location 3
Human Impact

Typical Floor - BUILDING 18.



Partial Framing Plan - Typical Floor
(No Scale)

Non-composte beams marked (*) in plan, all other composite construction

2 1/2" Concrete topping

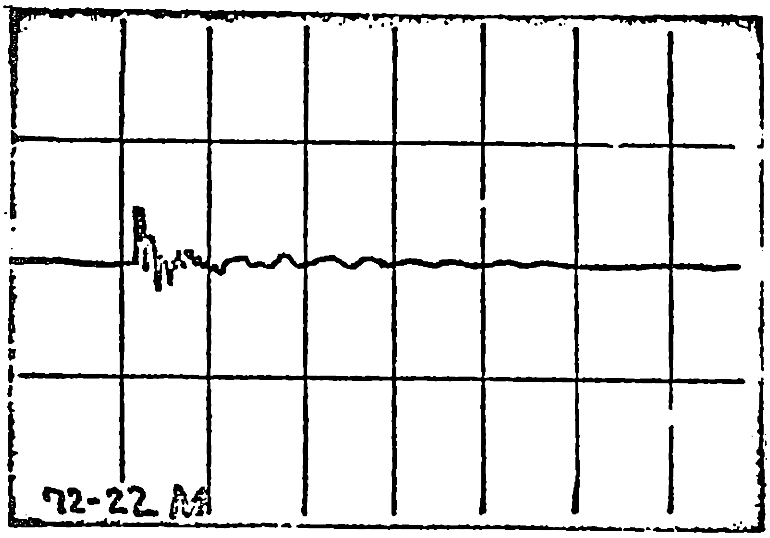
4" Concrete slab

A7 Steel

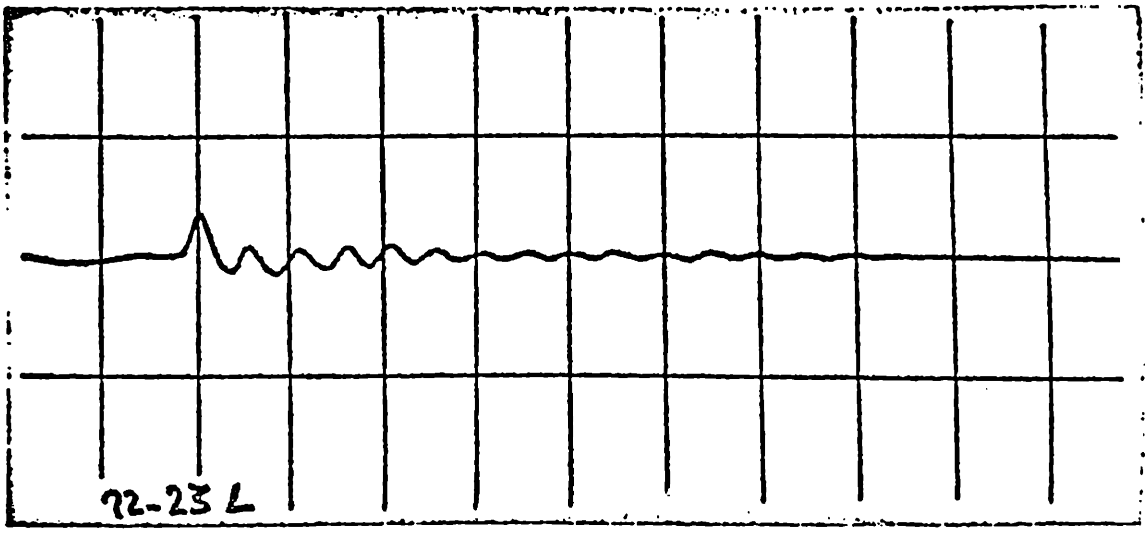
No centering material

Spray-on fireproofing, hung plaster ceiling

BUILDING 18.

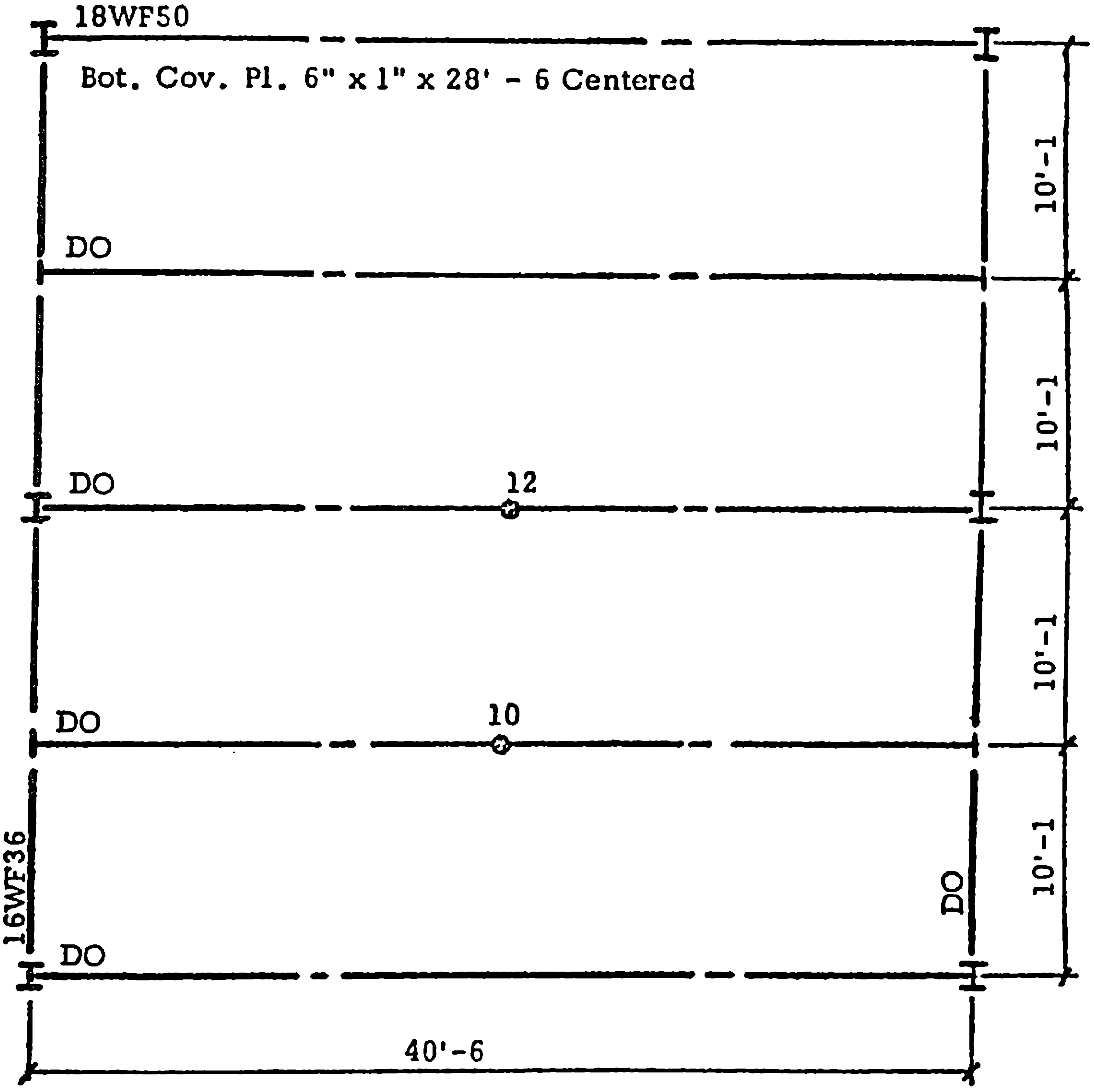


Location 4
Mechanical Impact



Location 4
Human Impact

Typical Floor - BUILDING 18.



Partial Framing Plan - Typical Floor

Composite construction

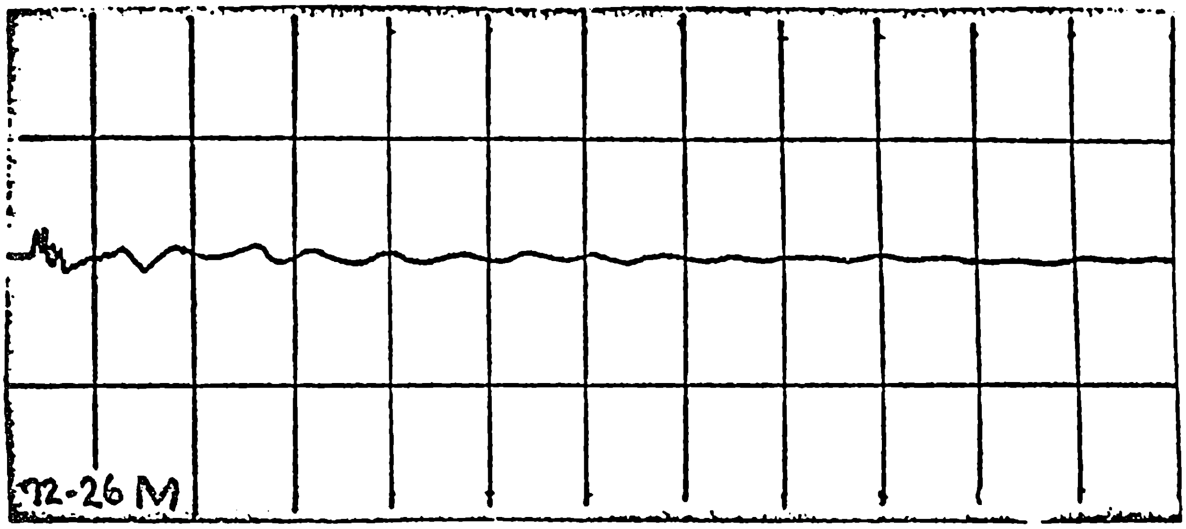
2 1/2" Concrete topping

4" Concrete slab

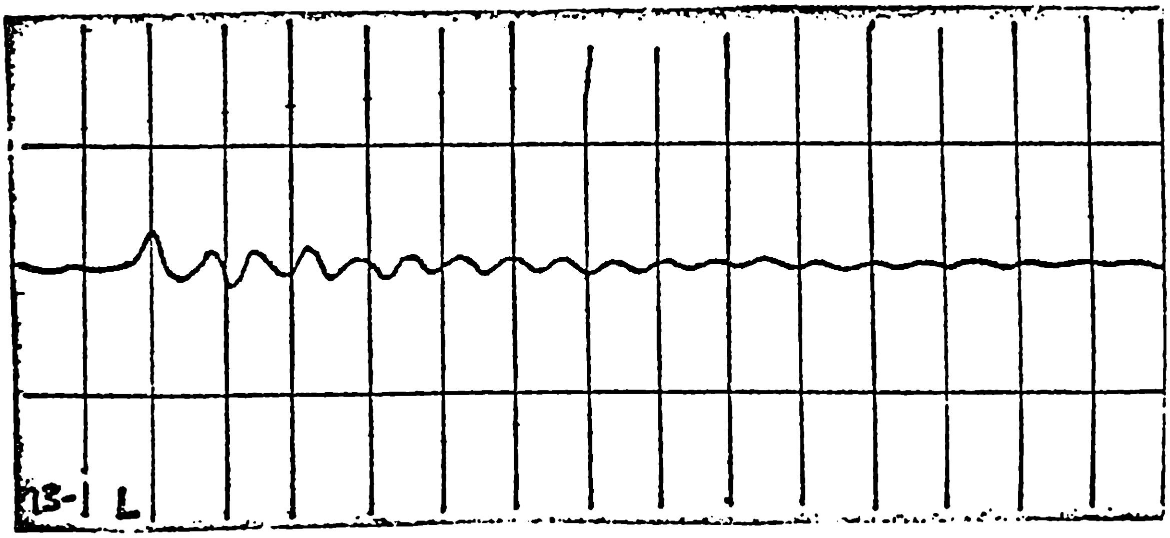
No centering material

Spray-on fireproofing

BUILDING 18.

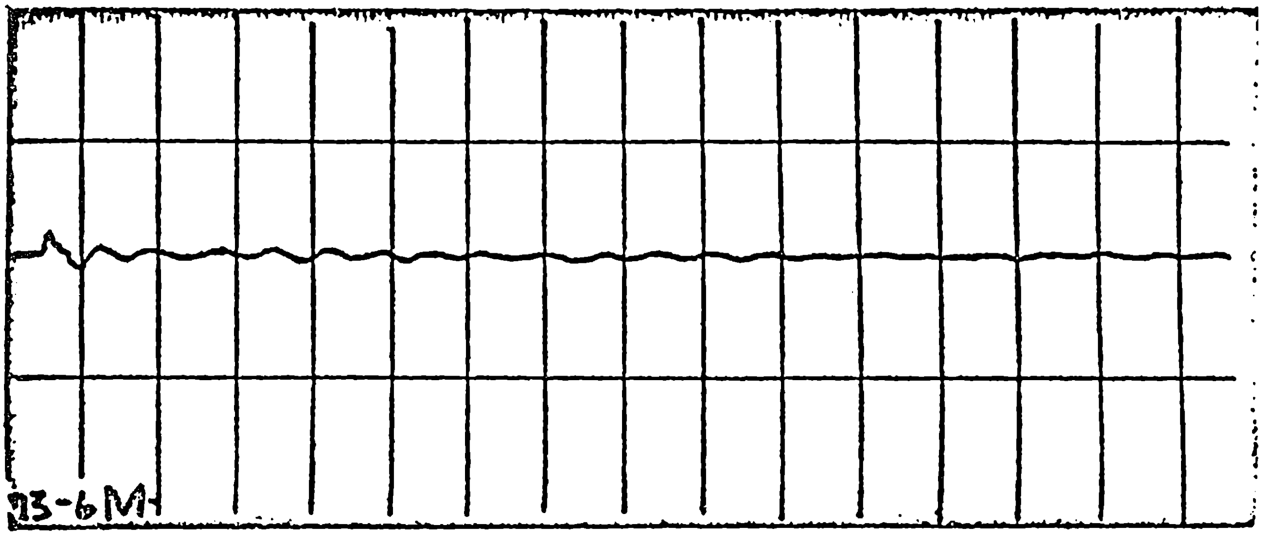


Location 10
Mechanical Impact

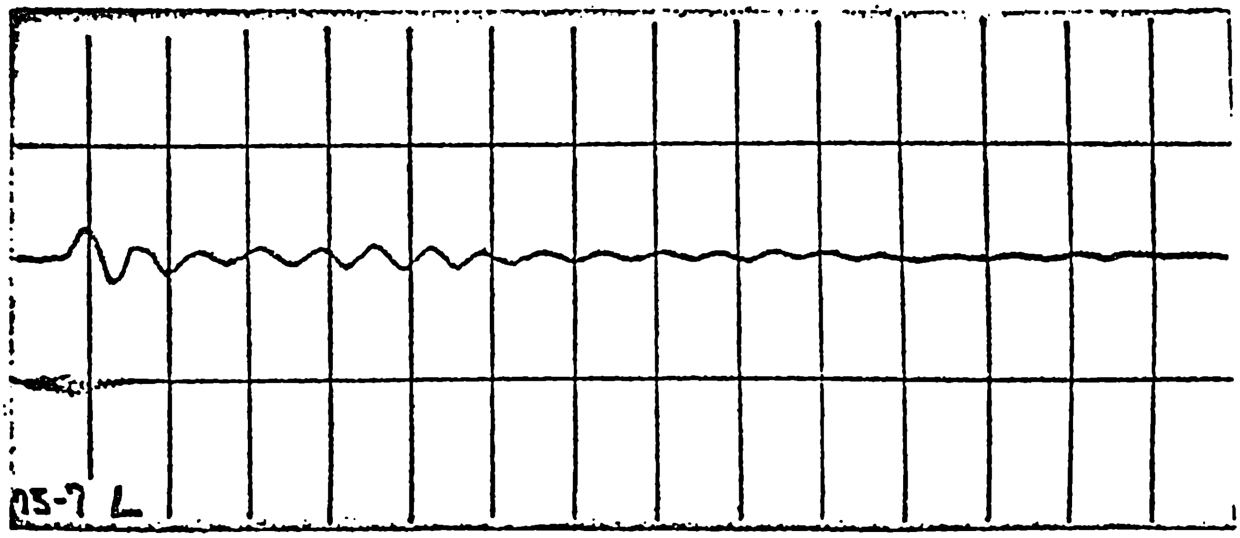


Location 10
Human Impact

Typical Floor - BUILDING 18.

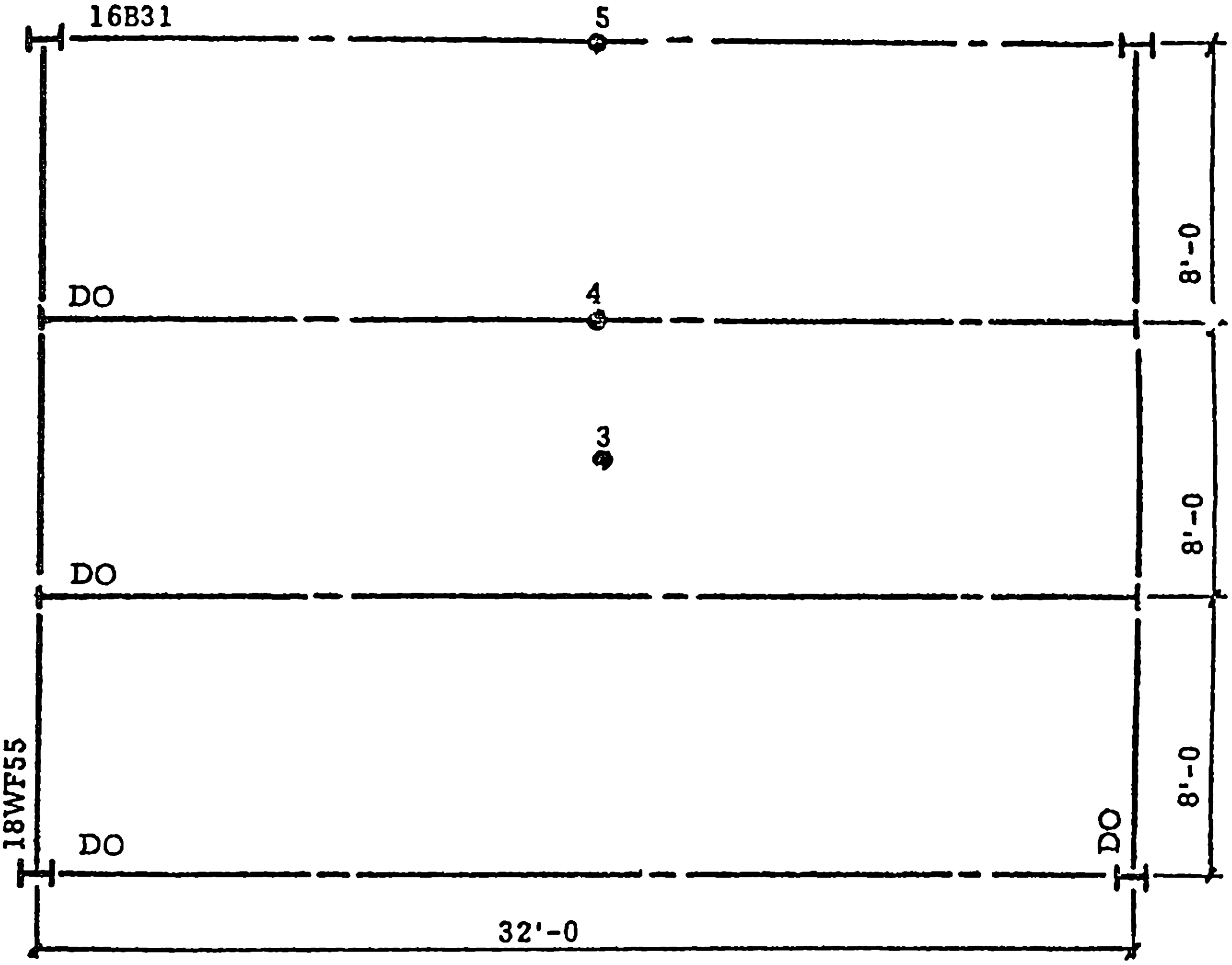


Location 12
Mechanical Impact



Location 12
Human Impact

Typical Floor - BUILDING 18.



Partial Framing Plan - 1st Floor
(No Scale)

Composite construction

3 1/2" Concrete slab

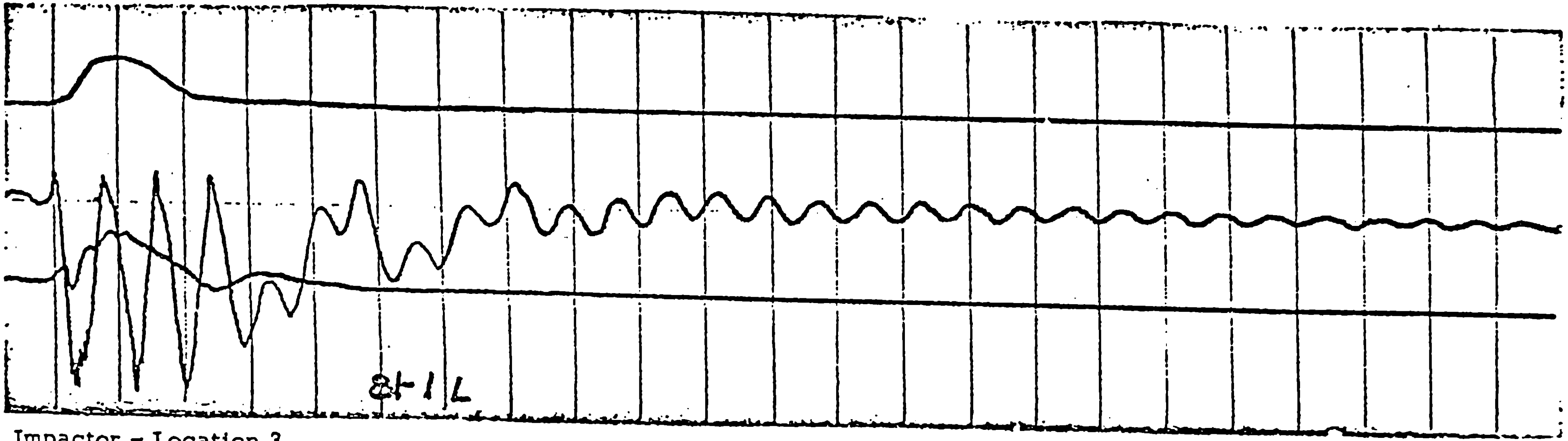
A36 Steel

No centering material

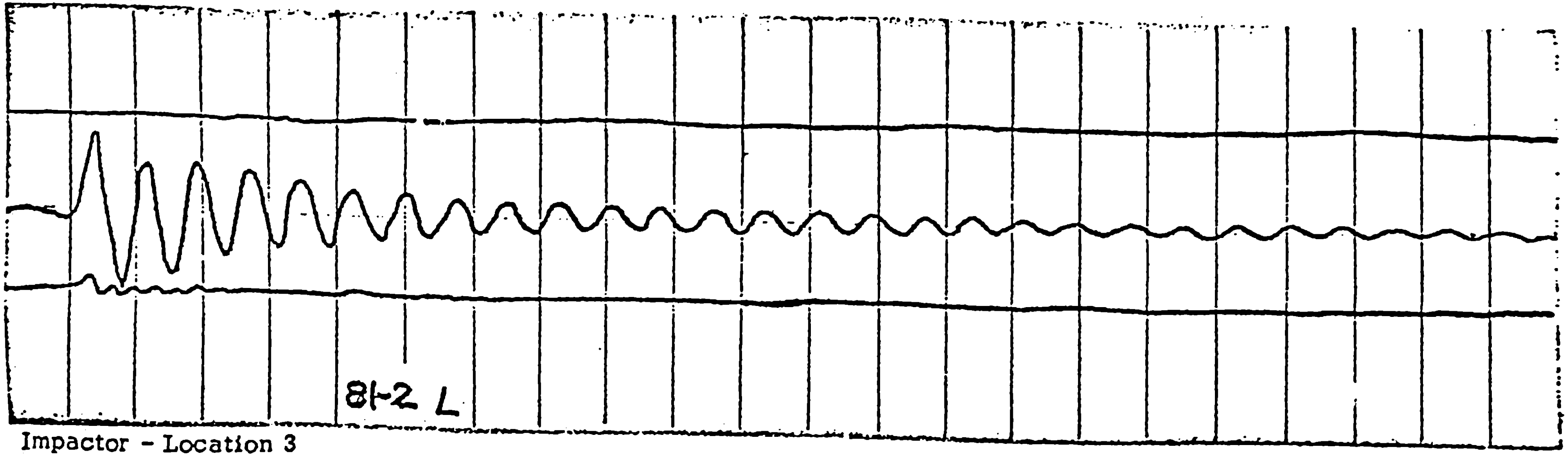
No fireproofing

BUILDING 19.

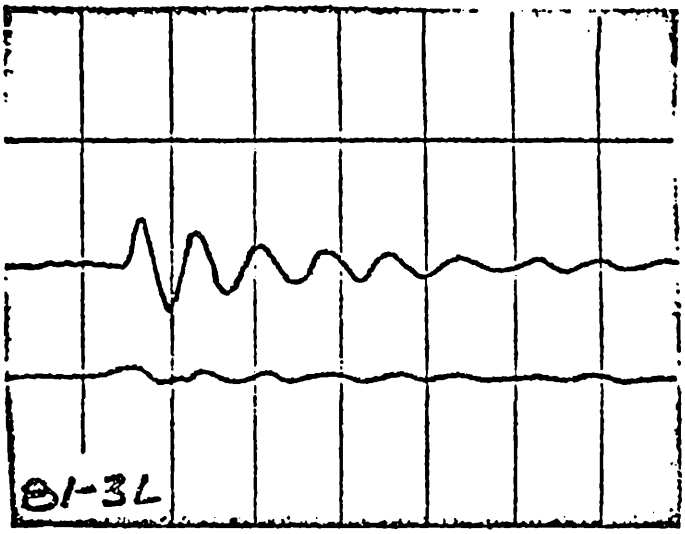
1st Floor - BUILDING 19.



Impactor - Location 3
Recorder - Location 3
Human Impact

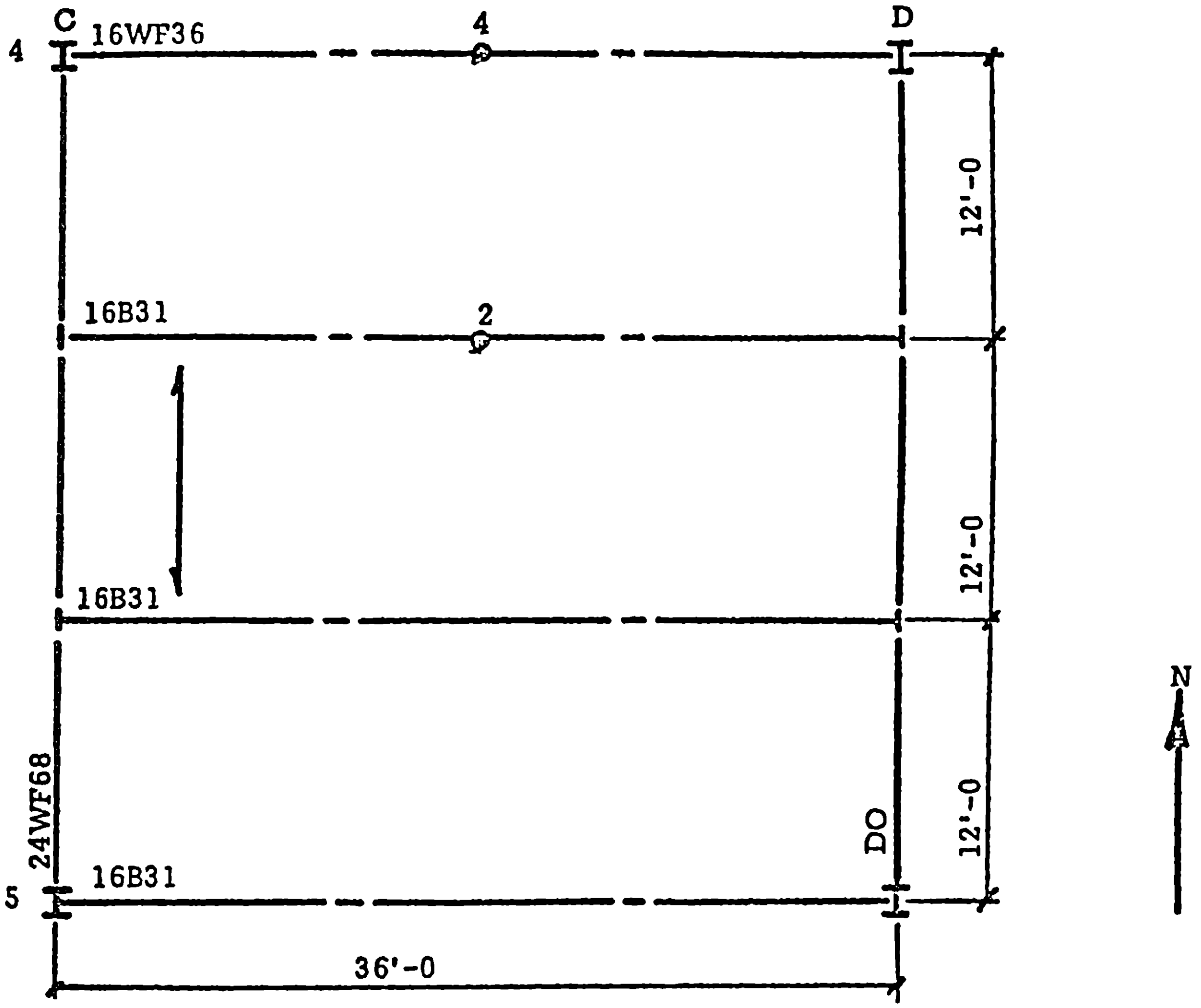


Impactor - Location 3
Recorder - Location 4
Human Impact



Impactor - Location 3
Recorder - Location 5
Human Impact

1st Floor - BUILDING 19.



Partial Framing Plan - 3rd Floor

Composite construction

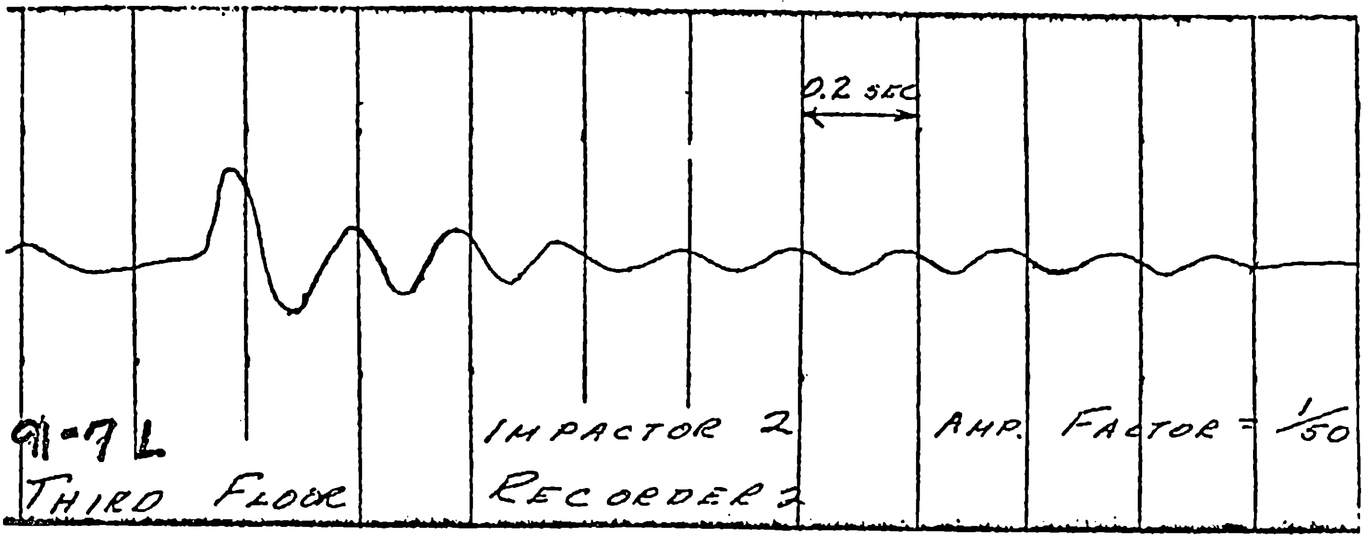
5" Lightweight (110pcf) concrete slab

A36 Steel

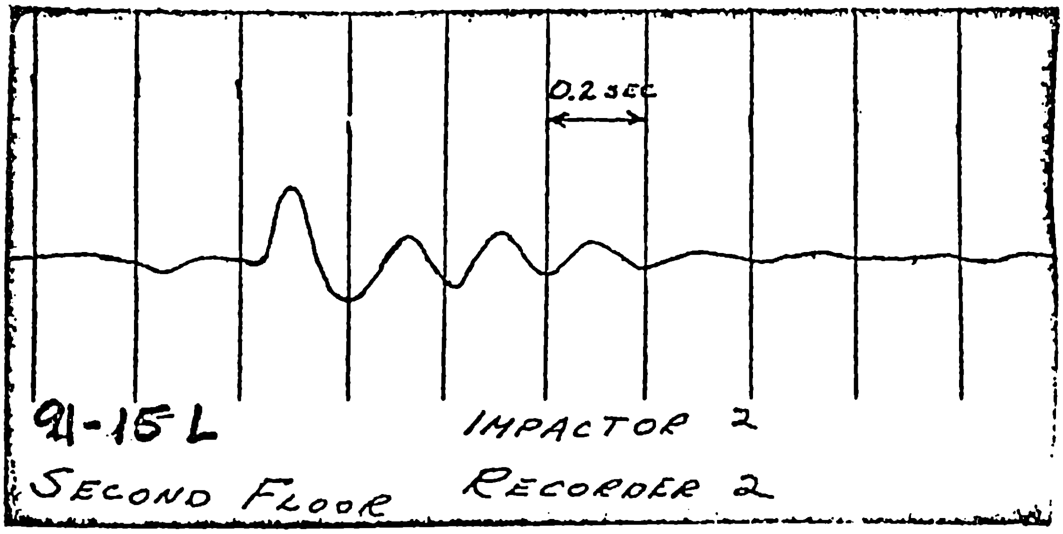
Type 4 centering material

No fireproofing

BUILDING 20.

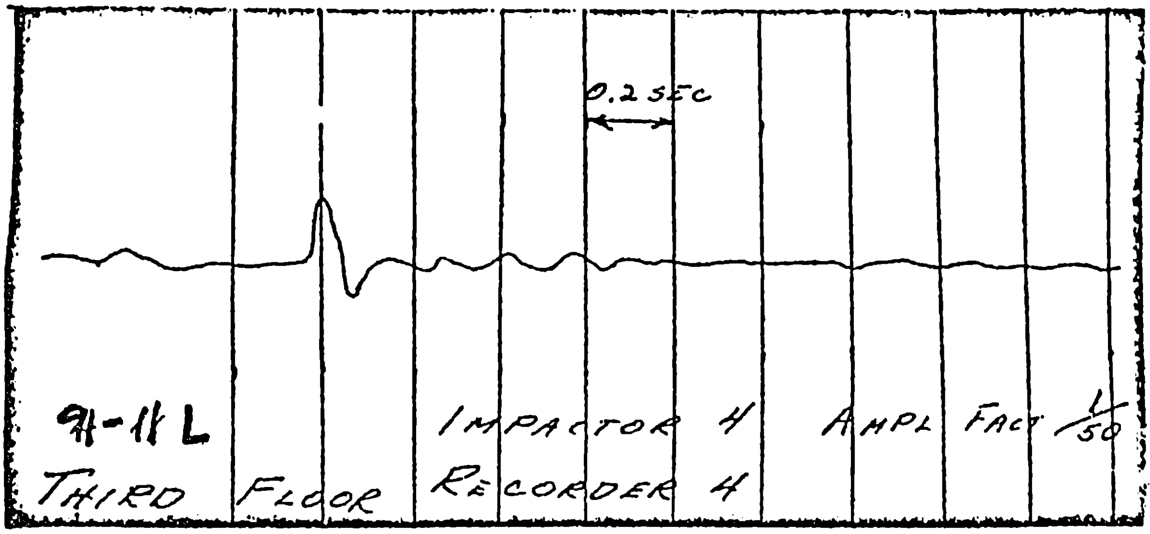


Location 2
Human Impact

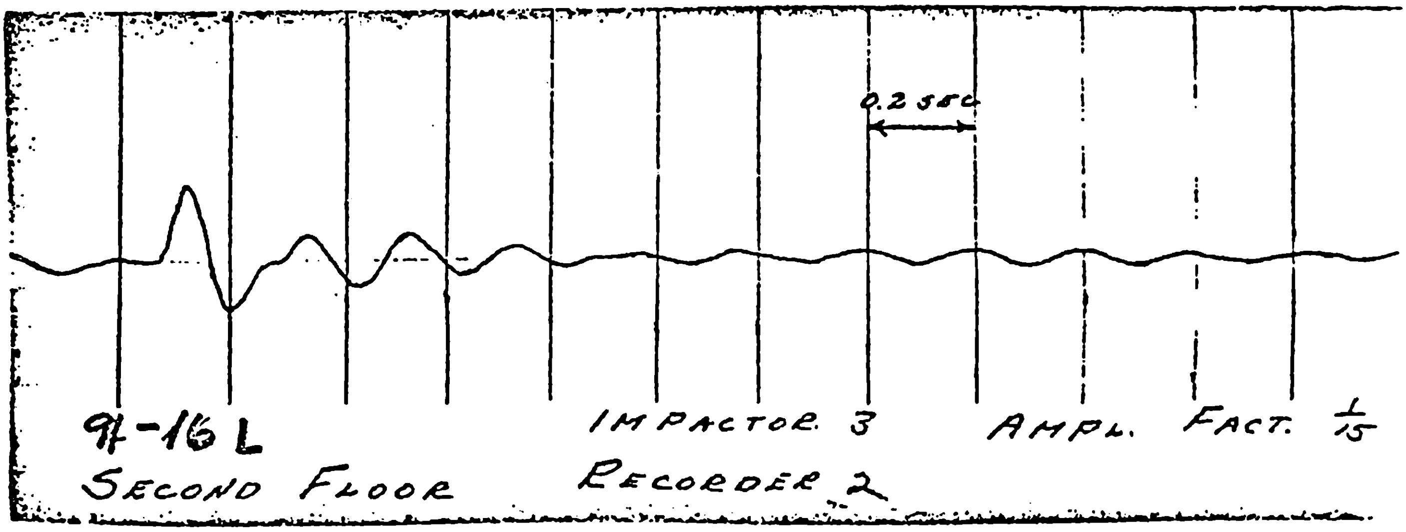


Location 2
Human Impact

2nd Floor - BUILDING 20.



Location 4
Human Impact



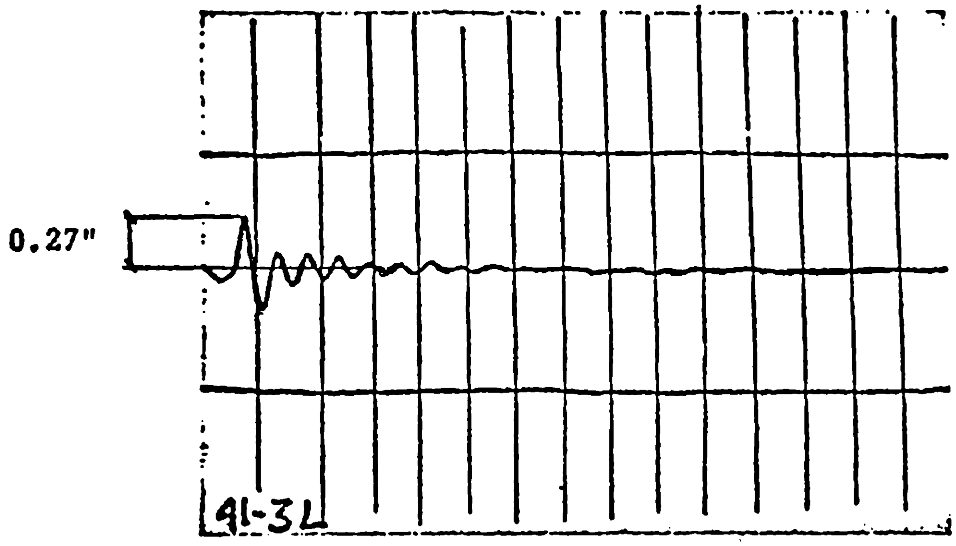
Location 4
Human Impact

2nd Floor - BUILDING 20.

APPENDIX B

SAMPLE REDUCTION
OF
TAPE RECORDS

SAMPLE REDUCTION OF TAPE RECORD



Building 4 Human Impact

Floor 2

Location 2

Frequency

$$f = \frac{5}{0.58} = 8.62 \text{ cycles/sec.}$$

First Amplitude

$$A_o = \frac{0.27}{50} = 0.0054 \text{ in.}$$

Damping

$$\zeta = \frac{1}{2\pi} \ln \left(\frac{A_n}{A_{n+1}} \right)_{avg}$$

$A_1 = 0.23$ $A_2 = 0.10$ $A_3 = 0.04$ $A_4 = 0.03$

$A_1/A_2 = 2.3$

$A_2/A_3 = 2.5$ $\left(\frac{A_n}{A_{n+1}}\right)_{avg} = \frac{2.3 + 2.5 + 1.3}{3} = 2.01$

$A_3/A_4 = 1.3$

$$\zeta = \frac{\ln (2.01)}{2\pi} \times 100\% = 11.1\%$$

APPENDIX C

SAMPLE THEORETICAL CALCULATIONS

Sample Theoretical Calculations

Building 3 Floor 1 - Location 8 Code D1

Span: $L = 40.0$ ft

Beam: 18 WF45

$A = 13.24 \text{ in}^2$

$I = 704.5 \text{ in}^4$

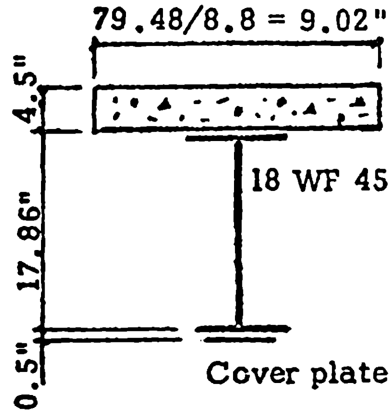
$b = 7.477$ in

Cover Plate : 6" x 1/2"

$A = 3.00 \text{ in}^2$

Slab: $W_c = 15016/\text{ft}^3$ $f'_c = 3000$ psi

Eff. Slab width = (16)(4.5) + 7.477 = 79.48 in.



From ACI 318-63

$$n = \frac{29{,}000{,}000}{W_c^{1.5}\,(33)\,\sqrt{f'_c}}$$

$$= \frac{29{,}000{,}000}{(150)^{1.5}(33)\,\sqrt{3000}} = 8.8$$

Weight per inch for tee-beam:

total width = 120.0 in.

$$w = \frac{45}{12} + \frac{10.2}{12} + \frac{(120.0)(4.5)(12)(150)}{(1728)(12)} = 51.50 \text{ lb/in}$$

C.G.:		A		y		Ay
Slab	(4.5)(9.02)	= 40.59	x	2.25	=	91.3
WF		= 13.24	x	13.43	=	178.0
Plate	(6)(0.5)	= 3.00	x	22.61	=	67.8
Σ		56.83				337.1

$$\bar{y} = \frac{337.1}{56.83} = 5.93 \text{ in.}$$

I_t :

		$\bar{I}$		Ad^2
Slab	$(9.02)(4.5)^3/12$	= 68.4	$(40.59)(5.93-2.25)^2$	= 550.0
WF		= 704.5	$(13.24)(13.43-5.93)^2$	= 744.5
Plate		-	$(3.00)(22.61-5.93)^2$	= 834.0
Σ		772.9		2128.5
		2128.5		
	I_t =	2901.4 in^4		

Frequency:

$$f_1 = \frac{1.57}{L^2}\left[\frac{gEI}{W}\right]^{1/2} = \frac{1.57}{(40)^2(144)}\left[\frac{(386)(29 \times 10^6)(2901.4)}{51.50}\right]^{1/2}$$

= 5.42 cycles/second

Initial Amplitudes:

$$\frac{L^3}{\pi^4 EI} = \frac{(40)^3(1728)}{\pi^4(29 \times 10^6)(2901.4)} = 1.348 \times 10^{-5}$$

a) Mechanical Impact - F = 794 lb t_d = 0.01 sec.

$$t_o = \frac{\pi}{\omega_1} = \frac{\pi}{2\pi(5.42)} = 0.0924 \text{ sec} > 0.01 \text{ sec.}$$

$$A_o = \frac{4F\,L^3}{\pi^4 EI} = (4)(794)(1.348 \times 10^{-5}) = 0.00726 \text{ in.}$$

b) Human Impact - F = 606 lb t_d = 0.05 sec.

$$\omega_1 t_d = (2\pi)(5.42)(0.05) = 1.703$$

$$t_o = \frac{2}{\omega_1}\tan^{-1}\omega_1 t_d \quad \frac{2}{(2\pi)(5.42)}\tan^{-1}(1.703)$$

= 0.061 sec > 0.05 sec.

$$A_o = \frac{2FL^3}{\pi^4 EI}\left[\frac{1}{\omega_1 t_d}\sqrt{2(1-\omega_1 t_d)\sin\omega_1 t_d-\cos\omega_1 t_d+(\omega_1 t_d)^2}\right]$$

$$= (2)(606)(1.348 \times 10^{-5})$$

$$\left[\frac{1}{1.703}\sqrt{2(1-1.703)\sin 1.703-\cos 1.703+(1.703)^2}\right]$$

$$= 0.01238 \text{ in.}$$