



# The macroeconomic implications of deficit financing under present bias<sup>☆</sup>

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## ABSTRACT

We examine how present bias affects deficit, inflation, and welfare in an economy where the deficit is funded by a seigniorage tax. In a hyperbolic discounting economy, reduced money holdings due to the desire for immediate consumption cause a decline in the sustainable deficit limit. To meet the targeted deficit, the government must raise seigniorage tax collection, especially with present bias. This results in increased inflation rates and higher welfare costs associated with the deficit for hyperbolic discounting individuals.

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## 1. Introduction

Governments face the challenge of determining a sustainable deficit level to finance public expenditures. Often, they resort to printing money, resulting in inflation and the imposition of a seigniorage tax on nominal asset holdings. Research suggests that consumers exhibit hyperbolic time discounting, showing a preference for current consumption over saving (Thaler, 1981).

This paper examines the effects of present bias on maximum deficit levels, inflation rates, and welfare using a three-period monetary overlapping generations (OLG) model with quasi-hyperbolic discounting (QHD) preferences provided by Laibson (1997). The findings reveal that the QHD economy has a lower maximum deficit level compared to exponential discounting (ED) economies due to QHD consumers holding less real money balances. The government's ability to introduce a deficit corresponds to the revenue derived from the inflation tax on real money balances, which serves as the tax base for the seigniorage tax. Furthermore, the study demonstrates that, under present bias, the government must increase seigniorage tax rates to monetize the same deficit level due to reduced aggregate money holdings. Consequently, the welfare cost of a deficit is higher in the QHD

economy compared to the ED economy. These results underscore the importance of considering empirically relevant preferences when analyzing the impact of policies on macroeconomic outcomes and welfare. Our paper shares similarities with the works of Lahiri and Puhakka (1998), Bunzel (2006), who explore the impact of introducing habit persistence on deficit levels. While habit preferences enable governments to sustain higher deficit levels, the introduction of present bias leads to contrasting outcomes in deficit levels.

Section 2 describes the model. Section 3 provides an analysis of the government deficit. The final section concludes the study.

## 2. Model

Time is discrete, indexed from 0 to infinity. Each period, a representative agent is born and lives for three periods. We assume that there is no population growth. The agent receives perishable endowments denoted as  $\omega = (\omega_1, \omega_2, \omega_3)$  throughout their lifetime. Their consumption profile over time is represented by  $(c_{1,t}, c_{2,t+1}, c_{3,t+2})$  at dates  $t$ ,  $t + 1$ , and  $t + 2$ , respectively. Agents engage in goods exchange using fiat money to transfer wealth across time. The agent demands  $m_{1,t}$  amount of money today and  $m_{2,t+1}$  in the next period.

Following Laibson (1997), we assume that consumers have QHD preferences, where their utilities in the first and second periods are described by

$$u(c_{1,t}) + \beta\delta [u(c_{2,t+1}) + \delta u(c_{3,t+2})] \quad (1)$$

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and

$$u(c_{2,t+1}) + \beta \delta u(c_{3,t+2}), \tag{2}$$

respectively, where  $u(\cdot)$  satisfies standard conditions. Here,  $\beta$  represents the degree of present bias and  $\delta$  represents the long-run time discount factor. We assume  $\beta \in (0, 1]$  and  $\delta \in (0, 1]$ .

The QHD preferences capture time inconsistency, where the optimal plans made by the first-period self are suboptimal for the second-period self due to the presence of different discount factors between the second and third periods, as illustrated in Eqs. (1) and (2).

The representative agent born in time  $t$  chooses  $(c_{1,t}, c_{2,t+1}, c_{3,t+2})$  and  $(m_{1,t}, m_{2,t+1})$  to maximize (1), subject to the following budget constraints:

$$\begin{aligned} p_t c_{1,t} &\leq p_t \omega_1 - m_{1,t}, \\ p_{t+1} c_{2,t+1} &\leq p_{t+1} \omega_2 + (m_{1,t} - m_{2,t+1}), \\ p_{t+2} c_{3,t+2} &\leq p_{t+2} \omega_3 + m_{2,t+1}, \end{aligned} \tag{3}$$

and  $(c_{1,t}, c_{2,t+1}, c_{3,t+2}) \geq 0$ , where  $p_t$  is the price of the consumption good at date  $t$ .

We replace nominal money holdings at time  $t$  with real balances, represented by  $a_{1,t} = m_{1,t}/p_t$  and  $a_{2,t+1} = m_{2,t+1}/p_{t+1}$ . The gross real return on holding fiat money is denoted as  $R_t = p_t/p_{t+1}$ , and the gross inflation rate  $\pi_t$  is defined as  $p_{t+1}/p_t$ . With these notations, we can rewrite the budget constraints (3) as follows:

$$\begin{aligned} c_{1,t} &\leq \omega_1 - a_{1,t}, \\ c_{2,t+1} &\leq \omega_2 + R_t a_{1,t} - a_{2,t+1}, \\ c_{3,t+2} &\leq \omega_3 + R_{t+1} a_{2,t+1}. \end{aligned} \tag{4}$$

We assume a log-utility function and focus on sophisticated consumers who account for the present bias of their future selves and make decisions through backward induction.<sup>1</sup> We provide closed-form solutions for the optimal consumption profile.

**Proposition 1.** Given  $u(c) = \ln(c)$ ,

$$\begin{aligned} c_{1,t} &= \frac{\omega_1 + \omega_2/R_t + \omega_3/(R_t R_{t+1})}{1 + \beta\delta + \beta\delta^2}, \\ c_{2,t+1} &= \frac{\beta\delta(1 + \delta)}{1 + \beta\delta} R_t c_{1,t}, \\ c_{3,t+2} &= \beta\delta R_{t+1} c_{2,t+1}. \end{aligned} \tag{5}$$

**Proof.** See the appendix.  $\square$

Let us define the total money supply at time  $t$  as  $m_t = m_{1,t} + m_{2,t}$ . Following Lahiri and Puhakka (1998), the government acts as a net borrower, while consumers are net lenders. The government introduces new money at date  $t$ , denoted as  $\Delta m_t = m_t - m_{t-1}$ , with its real value being  $\Delta m_t/p_t$ . This real money supply is utilized to finance the real government deficit,  $D_t$ , which is used for purchasing exogenously provided public goods, represented as  $g_t$ . Therefore, we have  $\Delta m_t/p_t = D_t = g_t$ . The money market clearing condition at time  $t$  is given by

$$m_t = \Delta m_t + m_{t-1}. \tag{6}$$

Let  $A_t$  be the aggregate stock of real money outstanding, defined as  $A_t = a_{1,t} + a_{2,t}$ . With this notation, we can express the market clearing condition in real terms at time  $t$  as

$$A_t = D_t + R_{t-1} A_{t-1}. \tag{7}$$

<sup>1</sup> However, we extend the results to include CRRA preferences and naive consumers in the appendix.

The good market clearing condition at time  $t$  is given by

$$c_{1,t} + c_{2,t} + c_{3,t} + g_t = \omega_1 + \omega_2 + \omega_3. \tag{8}$$

In our model, agents have perfect foresight regarding equilibrium prices. The detailed definition of a competitive monetary equilibrium is provided in the appendix.

### 3. Equilibrium analysis

The asset market clearing condition (7) simplifies to

$$A(R^*) = D^* + R^* A(R^*) \tag{9}$$

in the steady state, where  $*$  represents steady state variables, and  $A(R^*)$  shows the relationship between  $A^*$  and  $R^*$ .

The steady-state deficit  $D(R^*)$  is a function of  $R^*$ :

$$D(R^*) = (1 - R^*) A(R^*). \tag{10}$$

This relationship represents a monetary Laffer curve. Expansionary monetary policy through money creation leads to inflation, with  $\pi^* > 1$ . This decreases the value of real money holdings for consumers, resulting in  $R^* < 1$  (since  $\pi^* = 1/R^*$ ). Thus,  $(1 - R^*)$  can be interpreted as the seigniorage tax rate,  $A^*$  is the tax base, and  $(1 - R^*) A(R^*)$  represents the tax revenue. The government utilizes the seigniorage tax revenue to purchase  $g^*$ , which equals  $D^*$ .

When  $D^* = 0$  (since  $g^* = 0$ ), the money market clearing condition (9) reduces to

$$A(R^*) = R^* A(R^*). \tag{11}$$

This equation indicates two types of stationary equilibria: (i)  $R^* = 1$  and  $A(1) > 0$ , and (ii)  $R_{I_0}^* > 0$ , where  $A(R_{I_0}^*) = 0$ . The former case corresponds to outside money, where fiat money held by households has a positive value. The latter case represents inside money, where aggregate savings are zero. We denote  $R_{I_0}^*$  as the steady-state interest rate for the inside money case when  $D^* = 0$ . These two steady states determine the boundary points of a monetary Laffer curve.

For the remainder of the analysis, we assume zero last-period income due to retirement ( $\omega_3 = 0$ ) and normalize the total endowment to 1 ( $\omega_1 + \omega_2 = 1$ ). We focus on the Samuelson economy, where  $A(1) > 0$ . We derive a closed-form solution for the positive steady-state interest rate  $R_{I_0}^*$  as presented below.

**Lemma 1.**  $R_{I_0}^*$  is given by

$$\begin{aligned} 0 < R_{I_0}^* &= \frac{-(1 + \beta\delta)\omega_1 + \beta\delta\omega_2 + \sqrt{[(1 + \beta\delta)\omega_1 + \beta\delta\omega_2]^2 + \frac{4(1 + \beta\delta)}{1 + \delta}\omega_1\omega_2}}{2\beta\delta\omega_1} < 1. \end{aligned} \tag{12}$$

**Proof.** See the appendix.  $\square$

This result suggests that  $R_{I_0}^*$  decreases as  $\beta$  increases. Within the range of  $R^* \in [R_{I_0}^*, 1]$ , a positive or zero deficit can be sustained. To understand the properties of the deficit function, we analyze its characteristics in the following lemma.

**Lemma 2.**  $D(R^*)$  satisfies

$$\left. \frac{\partial D(R^*)}{\partial R^*} \right|_{R^*=1} < 0, \tag{13}$$

$$\left. \frac{\partial D(R^*)}{\partial R^*} \right|_{R^*=R_{I_0}^*} > 0, \tag{14}$$

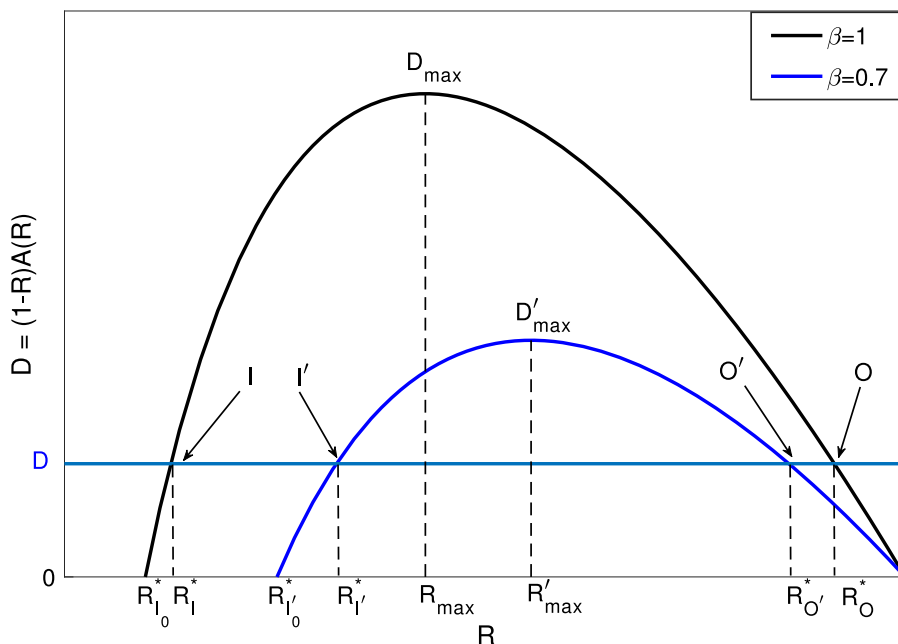


Fig. 1. Monetary Laffer curves at  $(\omega_1, \omega_2, \omega_3) = (0.7, 0.3, 0)$  and  $\delta = 0.7$ .

and

$$\frac{\partial^2 D(R^*)}{\partial R^{*2}} < 0 \tag{15}$$

for  $R^* \in [R_{10}^*, 1]$ .

**Proof.** See the appendix.  $\square$

Based on the findings in Lemma 2,  $D(R^*)$  is strictly concave for  $R^* \in [R_{10}^*, 1]$ , and its first derivatives exhibit opposite signs at the boundary points. Thus, as the seigniorage tax rates  $(1 - R^*)$  increase from 0 ( $R^*$  decreases from 1)  $D(R^*)$  initially increases and then decreases, ultimately having a unique maximum within this range. In Fig. 1, we provide an illustrative example of  $D(R^*)$ . The graph includes two curves: one for the ED case with  $\beta = 1$  and another for the QHD case with  $\beta = 0.7$ .

We determine the maximum allowable size of the real deficit by identifying the unique maximizer of  $D(R^*)$  as  $R_{max}$ , where the condition

$$\left. \frac{\partial D(R^*)}{\partial R^*} \right|_{R^*=R_{max}} = -A(R_{max}) + (1 - R_{max}) \frac{\partial A(R_{max})}{\partial R_{max}} = 0 \tag{16}$$

is satisfied. The maximum deficit,  $D_{max}$ , is then given by  $(1 - R_{max})A(R_{max})$  with  $\pi_{max} = 1/R_{max}$ .

In the subsequent proposition, we present the findings regarding the impact of QHD preferences on  $D_{max}$ ,  $R_{max}$ , and  $\pi_{max}$ .

**Proposition 2.** The following properties hold for all  $\beta \in (0, 1]$ :

1.  $D_{max}$  is increasing in  $\beta$ ,
2.  $R_{max}$  ( $\pi_{max}$ ) is decreasing (increasing) in  $\beta$ .

**Proof.** See the appendix.  $\square$

The first property in Proposition 2 states that the presence of present bias leads to a decrease in the maximum deficit level. In a QHD economy, the function  $A(R^*)$  is lower compared to an ED economy at the same interest rates. Thus,  $D^*$  is uniformly smaller in the QHD economy for any  $R^* \in [R_{10}^*, 1]$ , resulting in a smaller  $D_{max}$  as well. The second property suggests that the

monetary Laffer curve reaches its maximum deficit at a higher interest rate (a lower inflation rate) in the QHD economy than in the ED economy. Therefore, as the interest rate (inflation rate) decreases (increases) from 1 (0), the level of deficit reaches its maximum earlier, and the interest rate (inflation rate) at which the maximum deficit is attained is higher (lower) in the QHD economy.

Next, we examine the impact of introducing present bias on the welfare effect of monetizing the deficit. To do so, we require a normative benchmark for welfare evaluation that applies to QHD consumers who evaluate welfare based on different lifetime utilities due to the evolving time discount factors. Following the existing literature (Laibson, 1997; Andersen and Bhattacharya, 2011; Guo and Krause, 2015; Kang, 2015), we adopt ED preferences to assess the actual well-being of QHD consumers:

$$U^{ED} = \ln(c_{1,t}) + \delta \ln(c_{2,t+1}) + \delta^2 \ln(c_{3,t+2}), \tag{17}$$

where the consumption profile  $(c_{1,t}, c_{2,t+1}, c_{3,t+2})$  is determined under the presence of present bias.

For the sake of tractability, we focus on the outside money case with  $R^* = 1$  and run a local welfare analysis considering an infinitesimal positive deficit at  $D^* = 0$ . Therefore, our focus is limited to the favorable side of the seigniorage Laffer curve, where increasing the inflation rate leads to higher seigniorage. For this, we compute the following cross-partial derivative of the  $i$ th-period utility with respect to deficit and present bias:

$$\begin{aligned} & \left. \frac{d}{d\beta} \left( \sum_{\tau=t}^{t+1} \frac{\partial \ln(c_{i,t-1+i})}{\partial R_\tau} \frac{\partial R_\tau}{\partial D^*} \right) \right|_{D^*=0, R^*=1} \\ &= \sum_{\tau=t}^{t+1} \left[ \underbrace{\frac{\partial}{\partial \beta} \left( \frac{\partial \ln(c_{i,t-1+i})}{\partial R_\tau} \right) \frac{\partial R_\tau}{\partial D^*}}_{\text{distortion effect}} + \underbrace{\frac{\partial^2 \ln(c_{i,t-1+i})}{\partial R_\tau^2} \frac{\partial R_\tau}{\partial \beta} \frac{\partial R_\tau}{\partial D^*}}_{\text{direct interest rate effect}} \right. \\ & \quad \left. + \underbrace{\frac{\partial \ln(c_{i,t-1+i})}{\partial R_\tau} \frac{\partial}{\partial \beta} \left( \frac{\partial R_\tau}{\partial D^*} \right)}_{\text{indirect interest rate effect}} \right] \tag{18} \end{aligned}$$

The first term in Eq. (18) represents the welfare effect of a distorted consumption profile due to changes in the interest rate, which depends on the degree of present bias. The second term illustrates how introducing a present bias affects the equilibrium interest rate at a given deficit level. The third term indicates how the equilibrium interest rate changes as the deficit increases, depending on the degree of present bias.

After taking the logarithm, the share and total income components of consumption decisions become additively separable. The share depends solely on  $\beta$  and  $\delta$ , while the total income depends only on endowments and interest rates, as described in Proposition 1. As a result, the cross-partial derivative of per-period utilities with respect to  $R_t$  and  $\beta$  should be zero, leading to no distortion effect. The direct interest rate effect is also insignificant. The conventional monetary steady state with  $D^* = 0$  is  $R^* = 1$  regardless of the value of  $\beta$ . Thus, a change in  $\beta$  does not affect  $R^*$  given  $D^* = 0$ . Only the indirect interest rate effect survives. Consequently, we have:

$$\frac{d^2 U^{ED}}{d\beta dD^*} \Big|_{D^*=0, R^*=1} = \left[ \frac{\partial \ln(c_{1,t})}{\partial R_t} + \delta \frac{\partial \ln(c_{2,t+1})}{\partial R_t} + \delta^2 \left( \frac{\partial \ln(c_{3,t+2})}{\partial R_t} + \frac{\partial \ln(c_{3,t+2})}{\partial R_{t+1}} \right) \right] \Big|_{D^*=0, R^*=1} \frac{\partial}{\partial \beta} \left( \frac{\partial R^*}{\partial D^*} \right). \tag{19}$$

Here,  $c_{1,t}$  and  $c_{2,t+1}$  depend solely on  $R_t$ , not  $R_{t+1}$  under the assumptions above. The bracket on the right is positive if  $A(1) > 0$ , as higher interest rates increase total consumption. We examine the effect of present bias on how monetizing the deficit impacts the real interest rate.

**Lemma 3.** At  $D^* = 0$  and  $R^* = 1$ , for all  $\beta \in (0, 1]$ ,

$$\frac{\partial}{\partial \beta} \left( \frac{\partial R^*}{\partial D^*} \right) \Big|_{D^*=0, R^*=1} > 0. \tag{20}$$

**Proof.** See the appendix.  $\square$

A small positive deficit reduces the gross real interest rate more in the QHD economy compared to the ED economy. This is due to a smaller tax base of aggregate savings in the QHD economy, requiring a higher seigniorage tax rate to finance government expenditures at the same level. Therefore, the presence of a present bias further decreases the real interest rate and increases the inflation rate. As a result, the QHD economy, with net savers, experiences a greater lifetime welfare loss due to a larger decrease in total consumption when financing the deficit, which is summarized in the following proposition.

**Proposition 3.** At  $D^* = 0$  and  $R^* = 1$ , for all  $\beta \in (0, 1]$ ,

$$\frac{d^2 U^{ED}}{d\beta dD^*} \Big|_{D^*=0, R^*=1} > 0. \tag{21}$$

**Proof.** See the appendix.  $\square$

In the appendix, we conduct a numerical evaluation of Eq. (21) for inflationary steady states where  $R^* < 1$  given  $D^* > 0$ , for different values of  $\beta$ .

#### 4. Conclusion

This paper investigates the impact of present bias on deficit, inflation, and welfare when the government finances the deficit through a seigniorage tax. We find that in an economy with QHD consumers, the seigniorage tax revenue is lower due to their tendency to hold less aggregate money holdings, driven by consumption temptation. This has two implications. Firstly, a higher inflation rate is needed to sustain the same deficit level in the presence of present bias. Secondly, the maximum sustainable deficit level at a steady state is lower in the QHD economy. Our welfare analysis shows that QHD consumers experience greater welfare loss as the deficit increases, as the government must implement a higher seigniorage tax, reducing the real value of money holdings and the overall level of consumption.

#### Data availability

No data was used for the research described in the article.

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#### Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.econlet.2023.111240>.

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