Socially Optimal Contracting Between a Regional Blood Bank and Hospitals

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Abstract

Motivated by the operational challenges faced by a Regional Blood Bank (RBB) in distributing the blood (and related products) among the hospitals in its service area, we study socially optimal contracting decisions of an RBB serving multiple hospitals. The supply of blood can be uncertain and insufficient to satisfy the total orders from hospitals. In the face of supply uncertainty, hospitals tend to over-order to compensate for a potential supply shortfall. When all hospitals inflate their order quantities, overall allocation of blood suffers and ultimately inflates the total cost of blood, which in turn lightens the wallets of patients. We model the blood bank as a social planner with the objective of minimizing the total cost of shortages and outdates throughout the supply chain. We assume hospitals need some economic incentive to report demand honestly when multiple hospitals compete for limited blood supply. We show that if the blood bank offers a suitable per unit subsidy for every unit of shortage experienced by a hospital, then hospitals would be induced to report demand to the blood bank without inflation. We also investigate whether or not a consignment contract can be consistent with uninflated ordering by hospitals. Finally, we perform a detailed numerical study based on real demand and supply data to investigate the impact of over-ordering by hospitals on shortages and outdates in the supply chain, and the role of the appropriate contracts and allocation policies in obviating the concomitant social costs.

Key words and phrases: Contracts design, Perishable goods, Blood supply chain.

1 Introduction

The motivation for this paper is our realization - through our interactions with the managers at a regional blood bank - that the efficiency with which blood banks operate, and therefore the total social cost of the blood, depends both on logistics and on the contracts enforced by the blood bank for payment of blood by the hospitals it serves. The logistics of blood banks has been well studied for many decades. However, the problem of contract design in the specific context of blood banking has not - to the best of our knowledge - been studied in detail. In this paper, we model the contracting relationship between a regional blood bank and the hospitals in its service area, and derive some results that we hope have interesting policy implications for blood bank operations.

The regional blood bank we studied has two general contractual models with hospitals - a consignment model and a no-returns model. For hospitals where the blood bank provides most or all of their blood inventory, the consignment model is common. Many community blood centers operate under the consignment model. The hospital pays for a unit of blood only if it is removed from stock to be used

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Transfusable Blood Components Summary

Whole Blood	Red Blood Cells	Platelets	Plasma	Cryoprecipitated AHF
COLOR OF THIS BLOOD COMPONENET				
Red	Red	Colorless	Yellowish	White
BLOOD COMPONENT SHELF LIFE				
21/35 Days*	Up to 42 Days*	5 Days	1 Year	1 Year
STORAGE CONDITIONS				
Refrigerated	Refrigerated	Room temperature with constant agitation to prevent clumping	Frozen	Frozen
KEY USES OF THIS BLOOD TYPE				
• Trauma • Surgery	• Trauma • Surgery • Anemia • Any blood loss • Blood disorders, such as sickle cell	• Cancer treatment • Organ transplant • Surgery	• Burn patients • Shock • Bleeding disorders	• Hemophilia Von Willebrand Disease (most common hereditary coagulation abnormality) Rich source of Fibrinogen

*Shelf life of whole blood and red cells varies based on the type anticoagulant used.

Figure 1: Main blood components, shelf life, and use (Red Cross, 2015).

for a patient. The blood bank also provides full credit for outdated products. A consignment policy of this sort creates an incentive for hospitals to order more blood than they need. An unintended consequence is that blood units - particularly those with shorter lifespans - spoil raising the cost of blood for the entire supply chain. The central question that we address in the current paper whether it is possible for the blood bank to devise simple equitable contracts that can coax accurate order quantities from hospitals. The no-returns model is often adopted when the regional blood bank functions as a supplemental - as opposed to the primary - provider. In this model, the hospitals are fully responsible for blood that outdates on their shelves. Typically, small rural hospitals operate under this model. Our research suggests that the no-returns model, accompanied by an appropriate clause to deal with shortages, should be more widely adopted by blood banks.

For each of 365 days of a year, we obtained data on order quantities for a specific blood product (platelets) at three depots and the total supply available at the regional blood bank supplying the depots (the depots in turn serviced hospitals in their specific local areas).

We note some salient facts about the data. We observed a supply shortfall on 2/3 of the days of the year. Moreover, the order quantities at the depots had high variability while the variability in supply

Metric	Statistic
$\frac{\text{Total annual supply from the RBB}}{\text{Total annual order quantity at the depots}}$	0.75
Supply shortfall,i.e., $Probability(Supply < total\, order\, quantity)$	0.67
Coefficient of variation of total order quantity	[0.5,1.5]
Coefficient of variation of supply	0.4

Table 1: Aggregate statistics.

	Order Quantity at Depot-1	Order Quantity at Depot-2	Order Quantity at Depot-3	Total Supply	Total Order Quantity
Mean	13.07	97.05	19.2	97.88	129.32
Std.dev	9.55	52.82	29.16	40.78	71.94
Std.dev/Mean	0.73	0.544	1.52	0.42	0.56

Table 2: Summary statistics.

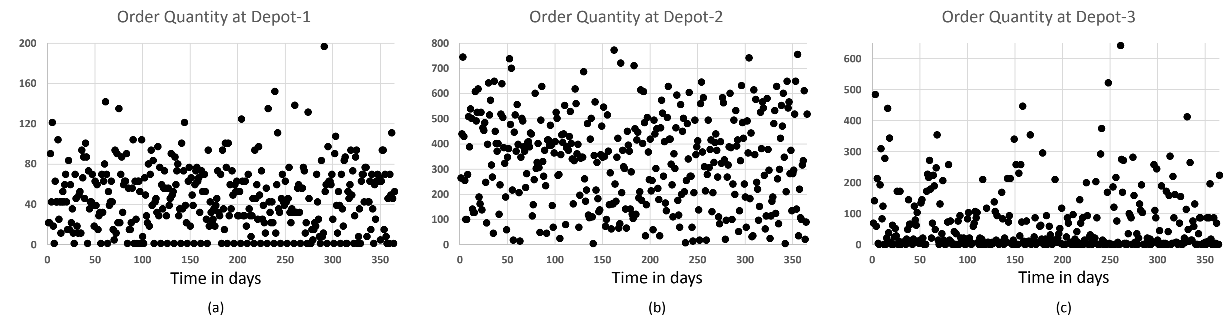


Figure 2: Time phased data on order quantity at each depot.

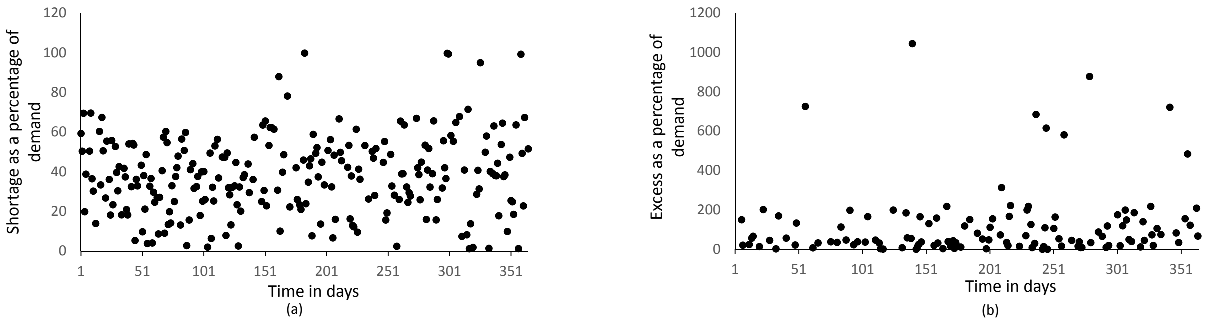


Figure 3: Time phased data on total excess and shortage in each period.

was less than that in order quantities. Table 1 provides the aggregate statistics while Table 2 provides more detailed statistics on order quantites and supply. Figures 2 and 3 graphically illustrates the time phased data. In Figure 3 we provide the data on the total excess and the shortage in the system.

Blood banks often operate under conditions of constrained supply. This implies that over time, there will be many occasions when the blood bank must ration a limited stock of blood products amongst the hospitals in its service area in an equitable manner. Under constrained supply conditions, there is the temptation for a rational but self-interested hospital to game the system by exaggerating its demand for blood with the intent of increasing its share of the limited supply that is rationed out. This practice has been documented in the medical literature; for instance, Williamson and Devine (2013) call for 'standard protocols to minimize so-called just-in-case ordering behavior' by hospitals (page 1871). Such gaming leads to a loss of welfare for the blood bank supply chain as a whole, and imposes a social cost. In this paper, we shall attempt to find simple contracting rules that induce hospitals to order responsibly and honestly, and also allow the blood bank to allocate their limited supplies equitably.

The topology of the blood supply chain has three hierarchical levels: the hospital level, the regional blood center level, and the supply chain level (Beliën and Forcé, 2012). The total demand for blood at a hospital reflects the decisions made by transfusion medicine specialists in the hospital, and the temporal variation in the demand of a hospital for blood depends on both the type of blood product (see Figure 1) and the type of hospital. Generally speaking, there are dozens of blood products being kept at a blood bank at any given time, but the most common are packed red blood cells (which have a stable and constant demand and have a shelf-life of 6 weeks), platelets - either single-donor or pooled, depending on the blood bank (which have a variable demand depending on what type of patients are being treated in the hospital and have a shelf-life of 3-5 days), cryoprecipitate (variable demand, shelf-life 1 year), and fresh frozen plasma (variable demand, shelf life 1 year)[1]. A regional blood center in the USA typically has a few hundred hospitals spanning a few contiguous states in its service area. The regional blood center produces blood, and allocates it to local depots or community blood centers, and these entities directly service hospitals. The regional blood center and the hospitals sign a supply contract to determine the terms of blood delivery - its price and mode of payment. It is this specific nexus of the supply chain that we focus on in this paper. The statistics on the (i) total collection of whole blood and red blood cells and (ii) transfusion (i.e., observable demand) in U.S.A. are given in Figure 4. U.S. Department of Health and Human Services (2011) also provides details on other blood products and corresponding transfusion. The Figure 5 provides a snapshot on the number of blood units outdated and unaccounted for in year 2011. When a hospital experiences a short supply of blood, the regional blood center requisitions additional units from other sources at a higher price. However, Figure 6 indicates that, even in the presence of all these measures, the hospitals' operations are significantly affected by shortages. Figure 7 illustrates the national blood shortage. More national level data on blood supply –

[1] Information is obtained through personal communication with practitioners.

Table 3-1. Estimated 2011 Collection and Transfusion by US Blood Centers and Hospitals for Whole Blood (WB) and Red Blood Cells (RBC) (expressed in thousands of units)

	Blood Centers	Hospitals	2011 Combined Total	±95% CI	% of Total Collections/ Transfusions	2008 Total	% Change 2008-2011
Collections							
WB Allogeneic (excluding directed)	12,659	927	13,586*	188	86.4	15,047	-9.7
WB Autologous	79	34	113*	11	0.7	253	-55.5
WB Directed	23	22	45*	9	0.3	61	-25.9
RBC Apheresis	1,925	52	1,978*	24	12.6	1,926	2.7
Total Supply	14,686	1,036	15,721*	200	100.0	17,286	-9.1
Rejected on Testing	92	10	102*	5	0.7	127	-19.4
Rejected for Other Reasons	964	66	1,030	32	6.6	--	--
Available Supply (minus Rejected on Testing)	14,594	1,026	15,619*	198	99.3	17,159	-9.0
Available Supply (minus all Rejected Units)	13,630	960	14,589	187	92.8	--	--
Transfusions							
Allogeneic (excluding Directed)†	176	13,507	13,684*	553	99.3	14,782	-7.4
Autologous	2	63	65*	12	0.5	159	-59.4
Directed (to designated patient)	0	37	37	18	0.3	73	-49.7
Total	178	13,607	13,785*	557	100.0	15,014	-8.2
Outdated WB/RBCs	171	199	370*	29	2.4	447	-17.3

*Significantly different from 2008 data.
†Including pediatric units transfused.

Figure 4: Blood collection and transfusion statistics (U.S. Department of Health and Human Services, 2011).

Table 4-2. Outdated Components as a Percentage of the Total Number of Units of Each Type Processed* for Transfusion in 2011.

	WB/RBCs	Whole-Blood Derived Platelets (both individual concentrates and pools)	Apheresis Platelets	Plasma	Cryoprecipitate (both individual concentrates and pools)	Granulocytes	All Components
Outdated Total	375,986	301,724	321,070	128,759	56,344	103	1,183,986
Processed/Produced	15,842,412	1,762,163	2,515,696	5,925,800	1,690,093	2,607	27,738,770
Percent Oudated	2.4%	17.1%	12.8%	2.2%	3.3%	4.0%	4.3%
Reported Wasted	†	48,697	24,724	108,316	51,548	45	233,330
Percent Wasted	--	2.8%	1.0%	1.8%	3.1%	1.7%	0.8%
Transfused	13,785,000	993,000	1,973,000	3,882,000	1,094,000	3,360	21,730,360
Unaccounted	1,681,426	418,742	196,901	1,806,725	488,201	(901)	4,824,424

*Number reported as processed or produced by a facility; this may differ slightly from the number reported as collected.
†These data were not collected in 2011 survey.

Figure 5: Statistics on outdated blood units (U.S. Department of Health and Human Services, 2011).

blood availability for 0–1 days, 1–3 days, and 3+ days – can be obtained from America's Blood Centers (2016).

In this work we take the perspective of a regional blood bank and develop easy-to-implement contracts to achieve equitable and efficient allocation of blood to the hospitals in its catchment area. The contract design for a particular blood product is driven by demand characteristics as well as shelf life. Motivated by our discussion with practitioners, we consider two settings: (i) a single period setting (Sec-

Table 8-1. Cancellation of Elective Surgeries by US Hospitals, 1997-2011*

Year	% Hospitals with Cancellation of ≥1 Day	Range of Days	Median No. of Days	Number of Patients Affected
1997	8.6	1-21	2	Not determined
1999	7.4	1-150	2	568
2001	12.7	1-63	2	952
2004	8.4	1-39	2	546
2006	6.9	1-120	3	412 (721 weighted)
2008	4.4	1-100	2	151 (325 weighted)
2011	3.3	1-14	2	173 (433 weighted)

*All data is unweighted.

Figure 6: Cancellation of surgeries due to shortage of blood products (U.S. Department of Health and Human Services, 2011).

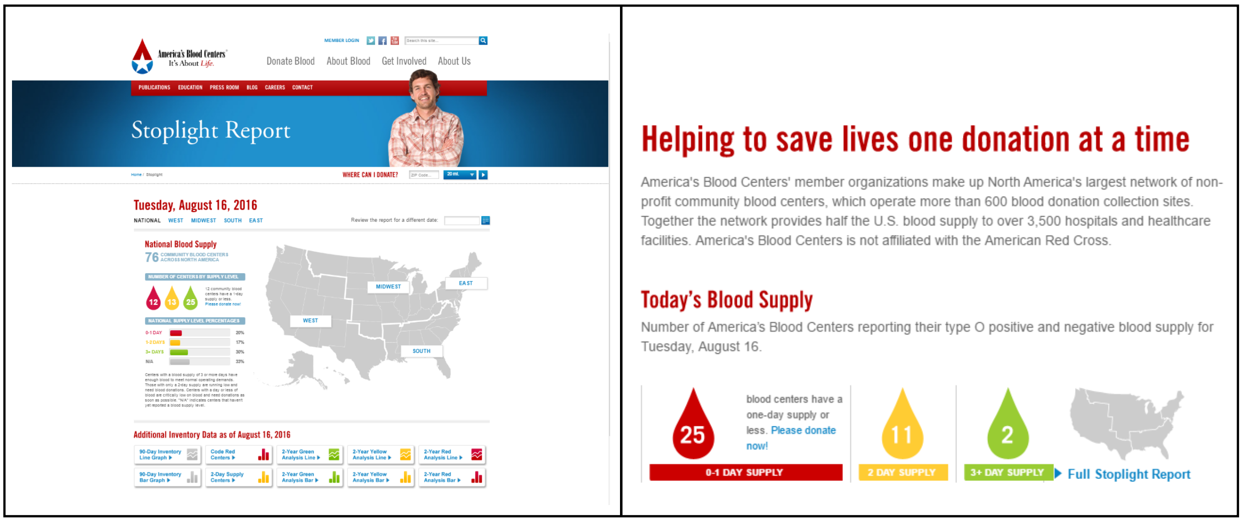


Figure 7: Statistics on blood shortages (America's Blood Centers, 2016).

tion 4.1) with deterministic demand, and (ii) a single period setting with random demand (Section 4.2). Then in Section 4.4 we model a situation in which a consignment contract can be shown to achieve the socially optimal outcome. In Section 5 we describe several numerical experiments that we performed - based on real demand and supply data - to investigate the impact of over-ordering by hospitals on shortages and outdates in the supply chain, and the role of the appropriate contracts and allocation policies in obviating the concomitant social costs. In Section 6 we summarize the contributions of our paper and discuss some practical implications of our findings.

2 Literature Review

The literature on blood inventory/distribution management, to the best of our knowledge, does not model the specific nexus between the RBB and the hospitals. In contrast, contract design is the sole focus of our paper. The capacity allocation literature in supply chain management assumes the contract designer (i.e., the manufacturer) to be a self-interested expected profit maximizer who allocates capacity to competing retailers. The contract designer in our paper (the RBB) is a social planner seeking to minimize the societal cost of expected excess and shortage. To the best of our knowledge, our paper is the first to explicitly model contracting between a blood center and hospitals.

The theme of our paper is contracting in a blood supply chain, but the effective allocation of blood from the regional blood bank to hospitals is implicit in contract design. We first briefly review research on the distribution of blood products. Prastacos (1978) studies a distribution system consisting of a central blood bank and a number of local depots each of which services a number of hospitals in its catchment area and faces random demand in every period. The regional blood bank receives a random quantity of fresh units of blood product at the beginning of a period and decides the quantity and age composition of product to be allocated to each depot. At the end of every period, all the good units at the depots may be transported back to the central warehouse for reallocation in the next period (the rotation model) or else the depots keep their ending inventories (the retention model). Prastacos derives optimal single-period policies with the objective of minimizing the sum of expected shortage and outdating costs in both the rotation and retention models. Prastacos (1981) derives some properties of the optimal average cost policy, in the space of stationary policies, over an infinite horizon with stationary demand and supply. Zhou, Leung, and Pierskalla (2011) take a closer look at the inventory management of platelets that have shelf life of five days and an effective life span of three days (when the transportation time and the handling and operational time is netted out). They assume that a regular order is placed at the beginning of a period and an expedited order (order-up-to-level policy) can be placed during the period. They prove the existence and the uniqueness of an optimal policy that minimizes the expected cost. Duan and Liao (2013) study a supply chain with a highly perishable good and use simulation optimization to obtain optimum inventory policies, while Duan and Liao (2014) analyze an inventory model specifically for the red blood cell supply chain. Civelek et al. (2015) study a discrete-time inventory system for blood platelets where there is demand for products of different ages. A substitution cost is incurred when the demand for a certain-aged item is satisfied with a different-aged item. They develop a policy that protects the newest items against excessive substitution to satisfy the demands for older products. Williamson and Devine (2013) give an insightful discussion of challenges in blood supply chain management from the point of view of both suppliers and demand centers. They stress that 'decisions made by individual hospital blood banks ultimately affect the entire system's ability to meet demand, so considering the whole picture is important to the successful balance between adequate supply and minimum wastage'; this is one of the important assumptions that underpin the modeling approach in our paper. Osorio et al. (2016) present an integrated simulation-optimization model wherein simulation is used to represent the flows through the supply chain, and an integer programming model optimizing over a rolling planning horizon is used to support daily decisions, such as the required number of donors. We also direct the interested readers to reviews by Beliën and Forcé (2012) and Prastacos (1984) for comprehensive reviews on blood inventory management.

Next, we mention some papers that study models with uncertain supply or random yield. Motivated by the frozen seafood industry, Khang and Fujiwara (2000) consider a setting with deterministic demand and random supply limit, corresponding to a random harvest. They characterize the optimal replenishment policy and establish a sufficient condition for the myopic policy to be optimal. Yang (2004) studies inventory control for a raw material dual-sourced from one perfectly reliable and one random supplier, and converted to a finished product with random demand. Li and Zheng (2006) study the joint inventory replenishment and pricing problem for production systems with random demand and yield in the framework of a periodic-review model, and show that the optimal policy is of the threshold type (it is optimal to produce if and only if the starting inventory in a period is below a threshold value). Bollapragada, Rao, and Zhang (2004) study an assembly system where the end-product has a random demand and each component has a random supply capacity. They optimize the component base-stock levels in order to minimize the total procurement cost subject to a minimum type-1 service level for the end-product. Pan and So (2010) consider an assemble-to-order product that has a one-time, random demand. One of the components of the product is subject to random yield; the authors analyze how this randomness in component supply affects the optimal pricing and production decisions. Lin and Chen (2010) model the problem of maximizing the total net profit of the perishable commodity supply chain, the optimal orders placed to suppliers, and the allocation to retailers in the face of supply uncertainty. In our setting, we are also faced with random demand and random supply. Further, in our case there is information asymmetry between the players, which facilitates the need for a contract design to induce truthful information from the hospitals.

We now turn to contract design for perishable goods. Although there are numerous papers in the operations management literature that are dedicated to contracting, we did not find many papers dedicated specifically to contracting for blood products. However, we mention some noteworthy papers dealing with contracting for perishable goods. There are several papers that discuss returns policies and supply chain coordination. Pasternack (1985) investigates the pricing decisions and return policies of a manufacturer who produces a short shelf product. Retail demand is stochastic while retail price is fixed. Pasternack shows that neither a policy that allows unlimited returns with full refund nor a one that does not allow returns is optimal. Webster and Weng (2000) investigate a setting where a manufacturer sells a short life-cycle product to a risk-neutral retailer. They study returns policies where the retailer's expected profit is increased and the manufacturer's profit is at least as large as when no returns are allowed. The work by Wang and Webster (2009) considers a scenario where the retailer liquidates the left over perishable goods inventory via clearance pricing. Two different types of markdown policies - percent markdown and quantity markdown - are analyzed. They show that both methods can coordinate the supply chain. Milner and Rosenblatt (2002) investigate a two-period setting where the buyer adjust the order quantity, by incurring a penalty after observing the demand for the first period. They compare the model and the results with a contract that does not allow for order adjustment. The works by Chung and Erhun (2013) and Milner and Rosenblatt (2002) combine contract design with order adjustment. Chung and Erhun (2013) consider a two-period setting where the supplier needs to consider the existence of old and young products in making decisions. They study a two-level wholesale price contract, a two-level buy-back contract, and a buy-back contract with channel rebates (the term 'two-level' implies age-based price differentiation). When we compare the contractual forms studied in these papers with the contractual forms that we study, we note that ours is the only paper to study subsidy contracts, whereas penalty contracts have indeed been studied in the published literature. Both our paper and the extant literature on perishable goods study returns policies; in our paper we treat the special case of a consignment contract, whereas more general forms of returns policies are studied in Pasternack (1985).

There is a vast stream of literature on contracting of non-perishable goods in operations and supply chain management. The reader is referred to the excellent survey papers by Tsay and Agrawal (2004) and Cachon and Lariviere (2003) for a comprehensive review of this research. Within this extant literature, our work relates most closely to the capacity allocation problems under asymmetric information. The classic work by Maskin (1989) develops an optimal auction mechanism for multi-unit auctions such as the capacity allocation problems in supply chains. Cachon and Lariviere (1999b) consider the problem of capacity allocation problem of a manufacturer faced with demands from multiple retailers each having private information about demand. In the presence of a capacity shortage a retailer might deliberately inflate its order quantity. Cachon and Lariviere develop allocation rules under which truth telling is an equilibrium strategy. They show that the lexicographic allocation rule, where a retailer's allocation is independent of its order quantity, is able to elicit private information. Various researchers have extended the modeling framework of Cachon and Lariviere (1999b). Cachon and Lariviere (1999a) propose a two-period game to model the turn-and-earn allocation observed in the automotive industry. Deshpande and Schwartz (2002) develop optimal pricing and allocation rules for a manufacturer selling to multiple retailers with private information about demand. Mallik and Harker (2004) extend the framework of Cachon and Lariviere (1999b) to include multiple manufacturers/suppliers, while Mallik (2007) studied contracting over multiple parameters in presence of multiple manufactures and multiple retailers. In addition to capacity allocation problems, elicitation of private information has also been studied in other related areas of supply chain management. For example, Corbett et al. (2004) develop contracts for a supplier that elicit information about a buyers cost structure. Ozer and Wei (2006) develop contracts for a supplier to elicit truthful demand forecasts from buyers. Finally, Zhang and Zenios (2008) consider a very general multi-period principal agent problem that induces truthful state revelation by each agent and maximizes the principal's payoff.

2.1 Focus of the Present Paper

We consider a decentralized setting and model the blood bank as a social planner with the objective of minimizing the total cost of shortages and outdates throughout the supply chain. We assume that the hospitals have the objective of minimizing their own costs; therefore the hospitals are sure to report their demand for blood and their current stock on blood inventories with complete accuracy only if they are given sufficient economic incentive to do so. Specifically, we address the following questions:

1. Can the RBB devise a contract that induces rational hospitals to report their demands without inflation?

2. Should consignment contracts be used by blood banks? Under what conditions? This is a pressing question, in view of the prevalence of consignment contracts in blood supply chain practice.

3. Does the effectiveness of the proposed contracts depend on demand and supply conditions? Are there any patterns to this variation?

We answer the first question in affirmative in Section 4. The second question is analyzed in Section 4.4 while the last one is investigated using a computational experiment based on real-world data in Section 5. To the best of our knowledge, this is the first paper to address these questions through analytical modeling.

3 Model Setting

As elaborated in Section 1 of our paper, a regional blood bank produces and stores dozens of blood products with differing shelf-life and demand characteristics. Packed red blood cells often exhibit a stable, deterministic demand. Specialty hospitals (e.g., cancer research centers) performing only scheduled surgeries/procedures also exhibit relatively stable demand for blood products. Blood platelets, often used on trauma patients and often requisitioned by large hospitals with Level I Trauma Centers, exhibit highly variable demand pattern. This necessitates us to consider the deterministic demand scenario and the stochastic demand scenario separately. The single period model with deterministic demand is presented in Section 4.1. The model formulation for single period stochastic demand case is in Sections 4.2.

In accord with the standard game theoretic modeling paradigm, our fundamental assumption is that a self-interested economic agent will share information only if it can be credibly demonstrated that it is in its interest to do so; the challenge is to design mechanisms that create incentives for economic agents to disclose private information honestly and accurately. Each hospital is concerned with minimizing its own expected costs whereas the blood bank, as a social planner, is interested in minimizing the social cost – the expected sum of the shortage costs and outdating costs – accruing at all the hospitals taken together; the smaller the value of these costs, the smaller is the cost borne by society at large. It is important to clarify that we are not assuming that every hospitals deliberately sets out to distort information and harm society as a whole. Rather, we are assuming that it is a natural tendency for hospitals to take a myopic view of the blood supply chain, and this has unintended adverse consequences for society at large.

We consider a decentralized setting where single regional blood bank (RBB) serves n distinct hospitals, denoted $H_1, H_2, ..., H_n$. We define a sequential game between the RBB and the hospitals. The RBB is the leader of the game. In accordance with our real-world observations, the RBB signs a contract with each hospital it serves and the allocation of blood is done accordingly. The sequence of events in the game is as follows.

1. At the start of the period, the RBB announces the terms of the contract, dealing with the price and the allocation policy (we spell out the details regarding each part of the contract further on).

2. The demand of hospital i is a random variable q_i (for $i = 1, 2, ..., n$), which is observed only by hospital i. Each hospital also observes a random signal of the *sum of the orders placed by the other hospitals*, which we denote by $\tilde{q}_{-i}$.

3. All the hospital's observe a random signal S of the RBB's supply. The RBB privately observes the realization $\hat{S}$.

4. Hospital H_i orders a quantity Q_i (for $i = 1, 2, ..., n$).

5. An allocation rule is a map from the vector of order quantities and the supply to a vector of quantities supplied to hospitals. The RBB makes allocation decisions as per the following allocation rule. If $\sum_i Q_i \leq \hat{S}$, then each H_i is allocated its order quantity Q_i using the existing supply. If $\sum_i Q_i > \hat{S}$, then the RBB allocates a quantity $\hat{Q}_i \leq Q_i$ such that $\sum_i \hat{Q}_i = \hat{S}$ from its existing supply as per the announced allocation rule. That is, the blood received by H_i from the existing stock is $\min\{Q_i, \hat{Q}_i\}$.

6. The deficit quantities $max\{Q_i - \hat{Q}_i, 0\}$ are supplied separately to H_i at a higher unit price which is specified in the contract. That is to say that the RBB takes the responsibility of satisfying any deficits in the current period by arranging for another regional blood bank to supply the deficit quantities directly to the hospitals at a higher price per unit.

7. If a hospital requires more blood than the quantity Q_i requisitioned by H_i at the outset, then, the RBB will have it delivered at a higher unit price. This event will occur when demand realizes at the hospitals at a later point (after the initial allocations are made by the RBB), and the realized demand exceeds Q_i.

8. Excess blood at each H_i must be disposed of at a per unit cost of e.

3.1 Inter-Relationships Between the Random Variables and Information Structure

We assume that corresponding to each $i = 1, ..., n$ there exists a random variable q_i that models the demand of H_i during the upcoming period. Corresponding to each $i = 1, ..., n$ there also exists a random variable $\tilde{q}_{-i}$; this random variable models the random signal that H_i observes about the sum of the order order quantities of H_j, $i \neq j$. Note that we do not assume any distributional relationship between any pair $\tilde{q}_{-i}, \tilde{q}_{-k}$ where $i \neq k$, other than their mutual independence. For the sake of notational convenience, we assume that all the hospitals observe the same random signal S about the supply at the RBB, the realization of which is observed privately by the RBB. All these random variables are assumed to be mutually independent; consequently, only the marginal distributions of these random variables play a role in the analysis.

An important point is that our model assumptions imply that *each hospital H_i can estimate the probability of a supply shortage in a given period.* If all the hospitals know the distributions of the order signals $\tilde{q}_{-i}$ and the distribution of the RBB supply S, then each hospital can compute the shortage probability using the analytical method given in detail in our proofs (please see the Appendix). On the other hand, even if none of the hospitals H_i know the distribution of $\tilde{q}_{-i}$ for $j \neq i$, the rules governing the RBB's allocation policy and the subsidy contract ensure that each hospital will be aware of a supply shortage, if it occurs. Thus, over time, each hospital can empirically estimate the probability of a supply shortage before deciding on its order quantity. In other words, we can replace the assumption that for all i hospital H_i knows the distributions of the random variables $\tilde{q}_{-i}$ with the assumption that *the hospitals can estimate the shortage probability before they place their orders.* Thus, we obtain two sets of theorems; Theorem 2 and Theorem 4 are valid even when H_i does not receive any signals $\tilde{q}_{-i}$ of other hospitals' order quantities, or a supply signal S, whereas the remaining theorems assume that these signals are received.

Next we discuss the costs that we use to frame the objectives of the players in our model.

Price of blood: The RBB charges a per unit price p for each unit sold to a hospital from its own supply. Thus H_i pays $p\hat{Q}_i$ (for $i = 1, 2, ..., n$) to the RBB for the units supplied to it and the RBB collects a total revenue of $p \sum_i \hat{Q}_i$ on account of the units it supplies from its own stock. The unit price of blood covers the processing cost and includes a surcharge to cover the labor cost and the overhead cost of running a blood bank.

Shortage cost: χ denotes the average surcharge per unit of blood when blood has to be requisitioned from an external source by the RBB. Thus the price of each unit supplied to a hospital from an outside source is $p + \chi$. Note that the unit cost χ may be affected by the ability of the hospital to secure emergency supplies from the regular donors (Civelek et al., 2015; Fontaine et al., 2009; Haijema et al., 2009). Also this cost includes possible social costs (e.g. rescheduling surgeries) as well as ordering costs.

Outdating cost: Blood left over at the end of the period is outdated and must be disposed. We let the per unit outdating (disposal) cost of blood be e. This includes the disposal cost of the outdated units, as well as the associated social cost.

We collect frequently used notation in Table 3.

Notation/Parameters	Description
H_i	Hospital i $(i = 1, 2, ..., n)$
p	Per unit purchasing cost of blood
χ	Per unit premium for outside supply
e	Per unit outdating cost
S	Random variable representing supply at RBB
$\hat{S}$	Realized supply at RBB
R_i	Deterministic (i.e., Observed) Demand at hospital i (privately known)
q_i	Random variable representing demand at H_i
$\tilde{q}_{-i}$	The random signal of the sum of orders placed by all the other hospitals observed by H_i
$\Omega(S, Q_1, Q_2, \ldots, Q_n)$	$\Omega : R_+^{n+1} \to R_+^n$ Allocation policy of RBB
Decision Variables	
Q_i	Order quantity of $H_i, i = 1, 2, \ldots, n$
$\hat{Q}_i$	Allocated quantity to H_i from RBB's supply
s	Per unit subsidy for blood supplied from an external source

Table 3: Frequently used notation.

4 Analytical Results

First in Section 4.1 we formulate a model in which each hospital observes its demand realization R_i (for $i = 1, 2, ..., n$) before placing its order quantity. This models the case when the hospital is requisitioning blood supply for its planned surgeries during the upcoming period. Each hospital also observes the random order signals $\tilde{q}_{-i}$ and S. In the following subsection (Section 4.2) we formulate a model in which each hospital observes only its own demand distribution, as opposed to its demand realization, at the time when the order quantities are submitted to the RBB. This models the case when the hospital is requisitioning blood supply speculatively for random demand that may arise during the upcoming period (for instance, blood demanding events that are emergencies or accidents). In both cases we show that a shortage-subsidy contract achieves the desired outcome.

4.1 Deterministic Demand

We remark that in line with the convention in the operations management literature, we use the phrase 'deterministic demand' to capture the case when each hospital observes its demand realization before placing its order. Let R_i be the demand at H_i. We propose a subsidy contract that induces truthful demand requisitions from the hospitals. Let Ω denote the blood allocation policy. Ω is a map from the n-vector of orders and supply to the n-vector of quantities supplied. That is to say that $\Omega(S, Q_1, Q_2 \ldots, Q_n) = \{(\hat{Q}_1, \hat{Q}_2, \ldots, \hat{Q}_n) : \hat{Q}_i \leq Q_i, \}$, where Q_i is the quantity requested by H_i and $\hat{Q}_i$ is the quantity delivered to H_i from the RBB's own supply. We formally define a subsidy contract as follows.

SUBSIDY CONTRACT $\mathcal{SC}$:

1. Allocation Rule: If $\hat{S} \geq \sum_i Q_i$, then $\hat{Q}_i = Q_i$. If $\hat{S} < \sum_i Q_i$, then $\hat{Q}_i$ is a non-decreasing function of Q_i. All positive deficit quantities $max\{Q_i - \hat{Q}_i, 0\}$ will be supplied by an external source to H_i.
2. Purchasing cost of blood: Each hospital pays a unit price of p for blood supplied directly by the RBB and a unit price of $p + \chi$ for blood supplied by an external source.
3. Each hospital is responsible for incurring the per unit outdating cost e.
4. The blood bank supplies a per unit subsidy s for every unit of blood supplied by an external source.

Note that the subsidy covers a portion of the extra cost of blood that results from the external shipment that is requisitioned by the RBB to fill hospital demands in any given period. Thus the hospitals pay for blood supplied directly by the RBB at the rate of p per unit and they pay the external supplier at the rate of $p+\chi$ per unit for blood supplied by the external source. The RBB applies a subsidy of s per unit of external supply when it bills the hospital, so the hospital ends up paying a net price of $p+\chi-s$ per unit for blood supplied by the external source.

Next we formulate the objective functions of the players in the presence of the contract $\mathcal{SC}$.

Objective function of hospital H_i

Each hospital H_i chooses Q_i so as to minimize its own expected cost. Let $\mathcal{C}_{H_i}(Q_i, Q_{-i})$ be the total expected cost incurred by H_i in ordering Q_i, given the orders Q_{-i} of the other hospitals. Then each H_i orders Q_i so as to minimize

$$\mathcal{C}_{H_i}(Q_i, Q_{-i}) = \Big\{ e \max\{0, Q_i - R_i\} + \underset{S,\tilde{q}_{-i}}{E} \big[p\hat{Q}_i + (p + \chi - s) \max\{0, Q_i - \hat{Q}_i\} \big] \Big\}$$

Note that the expectation in H_i's objective function is taken with respect to the distribution of S and $\tilde{q}_{-i}$. The first term in the objective function is deterministic. Since the order quantity is Q_i and the total supply by the RBB (with its own supply and from outside sources) is R_i, the term $\max\{0, Q_i - R_i\}$ captures the number of units outdates at the end of the period. Thus the first term of the objective function represents the outdating cost. The second term captures the total purchase cost for the hospital.

Since the RBB only supplies a quantity $\hat{Q}_i$ from its own supply, the hospital pays $p\hat{Q}_i$ to secure this quantity. Since any unfulfilled portion of the demand is satisfied using outside sources, the per unit price paid by the hospital for that quantity is $(p+\chi-s)$. Consequently, $(p+\chi-s)\max\{0, Q_i - \hat{Q}_i\}$ is the total amount paid to secure this quantity. Recall that since the allocation policy employed by the RBB is a function of its supply as well as the order quantities placed by other hospitals we take the expectation with respect to S and $\tilde{q}_{-i}$.

Objective function of the RBB

Recall that once S realizes, the RBB observes whether there is sufficient supply to satisfy all the demands or not. If there is sufficient supply, the RBB allocates each hospital the quantity demanded by it. If there is insufficient supply, the RBB issues the external supplier the quantities to be supplied to each hospital. Therefore, the objective of the RBB is to design a subsidy contract that ensures that the hospitals report their true demands. Recall that the RBB minimizes the total social cost $\mathcal{C}_{social}$. Now we can formulate RBB's problem. We use the random variable $\hat{q}_i$ to denote the RBB's estimate of H_i's order quantity. We need to define the random variables $\hat{q}_i$ to define the RBB's objective function precisely, but we shall see that the distribution of $\hat{q}_i$ does not play any role in devising the subsidy contract (please see Remark 3 below for further discussion of this point).

$$\begin{aligned} \underset{s,\hat{Q}_i, i=1,2,\ldots,n}{\text{Minimize}}\ \mathcal{C}_{social} \quad &= \quad \sum_{i=1}^{n} \underset{\hat{q}_i}{E}\Big[e\max\{0, Q_i - \hat{q}_i\}\Big] + \sum_{i=1}^{n} \underset{\hat{q}_1,\hat{q}_2,\ldots,\hat{q}_n}{E}\Big[(p+\chi)\max\{0, Q_i - \hat{Q}_i\}\Big] \\ \text{s.t.} \quad & \quad \mathcal{C}_{H_i}(R_i, Q_{-i}) \le \mathcal{C}_{H_i}(Q_i, Q_{-i}), \forall Q_i \in \mathbb{R}^+, \forall i = 1,2,\ldots,n. \end{aligned}$$

Note that the RBB receives order quantities $Q_i, i = 1,2,\ldots,n$ from hospitals. Given these quantities, the RBB minimizes the total social cost – sum of outdating cost due to excess at each hospital and the shortage cost due to ordering extra units from an external source. Observe that the subsidy payment, which is an internal transfer between the RBB and the hospitals, does not enter the objective function. The expectation of each component (associated with H_i) of the first term of the objective function (total outdating cost) is taken with respect to the distribution of $\hat{q}_i$ while the expectation of each component in the second term is taken with respect to the random vector $\hat{q}_1, \hat{q}_2 \ldots, \hat{q}_n$.

Remark 1: It is apparent from the formulation of the hospital's objective function that in case of a supply shortage, each hospital would need to know the precise allocation rule chosen by the RBB in order to compute its expected cost. We show in the proof of our first major result (Theorem 1 below) that so long as the allocation rule belongs to the family of rules spelled out in Subsidy Contract $\mathcal{SC}$ (that is, allocations that are increasing functions of order quantity), the hospitals can deduce that it is in their own interest to not inflate their order quantities.

Remark 2: It is worthwhile to note that if the per-unit subsidy $s = \chi$, then the objective function of H_i reduces to $\mathcal{C}_{H_i}(Q_i, Q_{-i}) = e\max\{0, Q_i - R_i\} + pQ_i$. Then it is easy to see that the optimal order quantity that minimizes the total cost of H_i is $Q_i^* = R_i$. Note that the subsidy cost s does not appear in the RBB's objective function, $\mathcal{C}_{social}$. Thus a per-unit subsidy $s = \chi$ induces truthful demand reporting from each hospital, and is also socially optimal. However, the blood bank would like to minimize their subsidy cost. Finding the minimal subsidy is a formidable task. Hence, we will focus on a smaller per-unit subsidy than $s = \chi$ that still elicits truthful information from the hospitals and social optimality.

Remark 3: In devising the appropriate subsidy contract, the knowledge that the RBB has about the beliefs of the hospitals about supply and total orders are crucial because the RBB has to anticipate the reasoning that the hospitals will carry out in order to determine its optimal order quantity Q_i. The RBB's private knowledge about individual hospital orders or about its own supply distribution does not matter as far as contract design is concerned *unless the RBB uses such knowledge to influence the hospitals' beliefs about supply and demand.* Indeed, we argue that it would be a good policy decision for the RBB to give credible signals to all the hospitals about aggregate demand and supply, based on its superior private knowledge. Such information sharing will lead to accurate belief-formation among the hospitals and eventually lead to the minimization of total social cost.

Our first major result below specifies a subsidy contract that provides just the right economic incentive for the hospitals to report demand honestly.

Theorem 1 *A shortage-subsidy contract with per unit subsidy $s = max(\chi - e - p, 0)$ induces a truth-revealing Bayesian Nash equilibrium under* **each** *of the following conditions:*

(a) For each $i = 1, 2, ..., n$ we have that $S \geq_{st} R_i + \tilde{q}_{-i}$;

(b) For each $i = 1, 2, ..., n$ we have that $S \geq_{icv} R_i + \tilde{q}_{-i}$ and $\tilde{q}_{-i}$ has a decreasing density function.

Recall that $p + \chi$ is the premium price per unit to obtain blood from an external source. Applying this observation to Theorem 1 we obtain the following corollary.

Corollary 1 *If the conditions of Theorem 1 hold and if in addition the premium price is less than twice the regular price, then no subsidy is needed to induce truthful demand reporting by hospitals in equilibrium.*

Allocation policies: Recall that the RBB needs to specify the rule whereby it will allocate units to the hospitals if supply falls short of total demand. In our definition of the subsidy contract $\mathcal{SC}$ we only state that the allocation is non-decreasing in the hospitals' order quantities. Thus, RBB can choose from a variety of allocation policies that fall under above category. For example, proportional allocation is one such rule. Our equilibrium result holds irrespective of the specific allocation rule chosen from the class of permissible allocation rules. We remark that our proof of Theorem 1 does make use of the assumption that a feasible allocation is non-decreasing in the hospitals' order quantities, and that this fact is known to the hospitals.

Corollary 2 *The outcome of the subsidy contract $\mathcal{SC}$ valid under any allocation policy that ensures the allocation to H_i is a non-decreasing function of the order quantity Q_i. That is to say that the exact allocation rule does not play a role in achieving a truth-revealing equilibrium, so long as the allocation rule is non-decreasing in Q_i for all i.*

We make a few additional comments about the scope and significance of the result in Theorem 1.

- The total subsidy burden borne by the blood bank depends on demand and supply conditions. Both conditions (a) and (b) hold when supply is adequate relative to total demand. When supply is relatively tight the blood bank bears a greater financial burden in terms of the total subsidy it provides to hospitals (Theorem 2 spells out this insight in a precise form). This contractual form is not one that a profit-maximizing or cost-minimizing firm would consider adopting.

- It is important to clarify that the hospitals do not have any incentive to mis-state the number of shortages they experience. This is because the subsidy does not cover the cost of the blood itself; rather, it covers the extra cost that hospitals incur as a result of the emergency shipments needed to fill the shortages. As a result, it would financially hurt the hospitals to over-report shortages. The total quantity Q_i requested by hospital i will be completely filled by the blood bank (for all i); in case of a shortage, a portion of the order quantity will be filled by the outside supplier that the blood bank has a prior arrangement with (this is actually the case with the blood bank we studied). That is to say that if a hospital generates a fictitious shortage of one unit, it will pay $p + e + \chi - s > 0$ for that unit. It is clear that there is no incentive for a hospital to behave in such a manner.

- We note that a simple, but socially unappealing, allocation mechanism induces a truth-revealing dominant equilibrium in our game (see Cachon and Lariviere (1999b)). The mechanism is to sequence the hospitals in some arbitrary manner *before* they make their orders and fill the orders of the hospitals completely from the supply available at the RBB in that sequence; the shortages are filled from the external source at a premium price. The mechanism design literature defines such a rule as a *dictatorial rule* (see Vohra (2001), page 8). Clearly, the arbitrariness of a dictatorial rule makes it unattractive in our setting.

Our next result finds a truth-inducing subsidy contract without imposing any distributional constraints on supply and demand.

Define $\nu^i := S - [R_i + \tilde{q}_{-i}]$ and $\theta = P\{\nu^i < 0\}$. Then we have the following theorem.

Theorem 2 *Suppose there are no constraints on the demand and supply distributions. Then a shortage-subsidy contract with per unit subsidy $s = max\big([\frac{\theta}{1-\theta}]\chi - e - p, 0\big)$ induces a truth-revealing Bayesian Nash equilibrium. θ is the shortage probability as defined above.*

Observe from the statement of Theorem 2 that the subsidy required increases with the shortage probability θ. If the estimated subsidy budget appears to be too high for the RBB to sustain its own operations, we suggest that the blood bank should consider raising the price of blood. In this connection, we recall the standard precept from economics that the price of a good reflects the balance between supply and demand. However, rather than directly making price a decision variable, our model allows the blood bank to consider varying the price of blood parametrically and choosing the price that best fits with the budget.

Let us apply the preceding theorems to real data. Recall from our real-world data that

$$Probability(Supply < Total\,Order\,Quantity) = 0.67.$$

We can use Theorem 2 to obtain an upper bound on the per unit subsidy required. Setting $e = 0.1p$ and $p = 500$ in the formula for s in Theorem 2 we get $s = max\big([\frac{\theta}{1-\theta}]\chi - e - p, 0\big) \leq \max\big([\frac{0.67}{1-0.67}]\chi - 550, 0\big) \approx \max\big(2\chi - 550, 0\big)$. Observe that the subsidy is positive only when the surcharge is more than USD275 (that is when emergency supply cost exceeds USD775).

4.2 Random Demand

In this subsection we investigate the case where, in contrast to the assumption underlying the results in the previous subsection, each hospital does not observe its demand realization before deciding the order quantity. We use the phrase ‘random demand’ to capture this situation. Suppose the demand at H_i is a random variable q_i. Note that q_i does *not* realize at H_i at the start of the period when the hospitals have to submit their order quantities to the RBB. Recall that only H_i knows the distribution of q_i; also each hospital i observes a (potentially noisier) order signal $\tilde{q}_{-i}$. The RBB observes $\tilde{q}_{-i}, i = 1, 2, \ldots, n$.

Under conditions of unlimited supply, suppose H_i's order quantity is Q_i^*. However, under conditions of limited supply, there is the temptation for each hospital to inflate its order quantity above Q_i^*. The task of the RBB is to set the per unit subsidy high enough that each H_i is induced to report the order quantity Q_i^* it would order without any 'shortage gaming'.

Let $\mathcal{C}_{H_i}(Q_i, Q_{-i})$ be the total expected cost incurred by H_i in ordering Q_i, given the orders Q_{-i} of the other hospitals. Then the expected cost function of H_i is given by

$$\begin{aligned}\mathcal{C}_{H_i}(Q_i, Q_{-i}) \quad = \quad & \Big\{ \underset{S,\tilde{q}_{-i}}{E}\left[p\hat{Q}_i + (p + \chi - s)\max\{0, Q_i - \hat{Q}_i\}\right] \\ & \qquad + \underset{q_i}{E}\big[e \max\{0, Q_i - q_i\} + (p + \chi - s)\max\{0, q_i - Q_i\}\big] \Big\}\end{aligned}$$

The terms associated with the first expectation operator correspond to the cost of obtaining Q_i from RBB (using RBB's own supply as well as from outside sources) while the terms associated with the second expectation operator capture the excess and shortage costs that materialize after the realization of the actual demand. The RBB's problem formulation is as follows:

$$\begin{aligned}\underset{s,\hat{Q}_i,i=1,2,\ldots,n}{\text{Minimize}} \ \mathcal{C}_{social} \quad = \quad & \sum_{i=1}^{n} \underset{\hat{q}_1,\hat{q}_2,\ldots,\hat{q}_n}{E}\Big[(p+\chi)\max\{0, Q_i - \hat{Q}_i\}\Big] \\ & \qquad + \sum_{i=1}^{n} \underset{\hat{q}_i}{E}\Big[e \max\{0, Q_i - \hat{q}_i\} + (p+\chi)\max\{0, \hat{q}_i - Q_i\}\Big] \\ \text{s.t.} \qquad & \mathcal{C}_{H_i}(Q_i^*, Q_{-i}) \leq \mathcal{C}_{H_i}(Q_i, Q_{-i}), \forall Q_i \in \mathbb{R}^+, \forall i = 1, 2, \ldots, n.\end{aligned}$$

Note that the terms associated with the first expectation operator pertain to costs associated with the fulfillment of the order quantity, while the terms associated with the second expectation operator account for the discrepancy between order quantity and actual demand at the hospital after it realizes. We have the following result.

Theorem 3 *(a) For each $i = 1, 2, ..., n$ suppose (i) $S \geq_{st} q_i + \tilde{q}_{-i}$ and (ii) Q_i^* is greater than or equal to the median of the demand distribution of H_i, for all i. Then in equilibrium, the hospitals will order their uninflated quantities Q_i^* when the RBB provides a per unit shortage subsidy $s = max(\chi - \frac{e+p}{2}, 0)$.*
(b) Suppose (i) in (a) hold and Q_i^* is greater than or equal to the 67th percentile of the demand distribution of H_i, for all i. Then in equilibrium, the hospitals will order their uninflated quantities Q_i^* when the RBB provides a per unit shortage subsidy $s = max(\chi - e - p, 0)$.
(c) Both (a) and (b) above hold if condition (i) is replaced by the conditions $S \geq_{icv} q_i + \tilde{q}_{-i}$ and $q_i + \tilde{q}_{-i}$ has a decreasing density function, and the remaining conditions are unchanged.

Theorem 3 states that if we make the same assumptions as we did in the case of deterministic demand together with the reasonable condition that hospitals would order at least the 50th percentile of their demand distribution if they were guaranteed to receive their entire order quantity at unit price p, a higher unit shortage subsidy is needed to suppress order inflation by hospitals compared with the case when the hospitals face predictable demand conditions. However, if the hospitals order at least the 67th percentile of their demand distribution under unlimited supply, then precisely the same unit subsidy that suppresses order inflation by hospitals in the deterministic case accomplishes the same end in the case of random demand. Notice that the assumption that hospitals would order at least the 67th percentile of their demand distribution says that hospitals have a target service level of at least 67% with respect to the availability of blood; in reality we would expect the target service level to be much higher than 67%.

As in the deterministic demand case, we note that the informational assumptions behind this result are quite modest. In order to make equitable allocations in case of a supply shortage the RBB will require some information about the hospital demand distributions, but we do not need a hospital to have distributional knowledge of the demand at other hospitals to find the appropriate subsidy contract for the RBB. Instead, as in the case of deterministic demand, the main distributional assumption that drives our result is information at the aggregate level.

The next theorem spells out the truth-inducing subsidy contract as a function of the shortage probability. Define $\nu^i := S - [q_i + \tilde{q}_{-i}]$ and let $\theta = P\{\nu^i < 0\}$. Then we have the following.

Theorem 4 *(a) Suppose there are no constraints on the demand and supply distributions. Then provided Q_i^* is at least equal to the median of the demand distribution of H_i for all i, a shortage-subsidy contract with per unit subsidy $s = max\big([\frac{\theta}{1-\theta}]\chi - \frac{e+p}{2}, 0\big)$ induces a truth-revealing Bayesian Nash equilibrium.*

(b) Suppose there are no constraints on the demand and supply distributions. Then provided Q_i^* is at least equal to the 67th percentile of the demand distribution of H_i for all i, a shortage-subsidy contract with per unit subsidy $s = max\big([\frac{\theta}{1-\theta}]\chi - e - p, 0\big)$ induces a truth-revealing Bayesian Nash equilibrium. Note that θ is the shortage probability as defined above.

4.3 Summary of the Deterministic and Random Demand Models

In this section, we first summarize the results obtained under observed and random demand models (Tables 4 and 5). Then we illustrate the differences between the two models from the perspective of the supply chain partners, the RBB, and the hospitals. Finally, we examine the sensitivity of the proposed shortage subsidy to parameters e and χ.

Conditions	Shortage Subsidy, s
$S \geq_{st} R_i + \tilde{q}_{-i}$	$\max(\chi - e - p, 0)$
$S \geq_{icv} R_i + \tilde{q}_{-i}$ $\tilde{q}_{-i}$ has a decreasing density function	$\max(\chi - e - p, 0)$
Probability of shortage is θ	$\max\left(\frac{\theta}{(1-\theta)}\chi - e - p, 0\right)$

Table 4: Summary of results: deterministic demand model

Conditions	Shortage Subsidy, s	
	$Q_i^* \geq$ median of $H_i, \forall i$	$Q_i^* \geq 67^{th}$ percentile of $H_i, \forall i$
$S \geq_{st} q_i + \tilde{q}_{-i}$	$\max\left(\chi - \frac{(e+p)}{2}, 0\right)$	$\max(\chi - e - p, 0)$
$S \geq_{icv} q_i + \tilde{q}_{-i}$ $q_i + \tilde{q}_{-i}$ has a decreasing density function	$\max\left(\chi - \frac{(e+p)}{2}, 0\right)$	$\max(\chi - e - p, 0)$
Probability of shortage is θ	$\max\left(\frac{\theta}{(1-\theta)}\chi - \frac{(e+p)}{2}, 0\right)$	$\max\left(\frac{\theta}{(1-\theta)}\chi - e - p, 0\right)$

Table 5: Summary of results: random demand model

Note that we consider two decentralized settings in which the RBB is the first mover. The differences between these two models from the perspective of the players, the RBB, and the hospitals, are summarized in Table 6. Let us take a closer look at the impact of parameters e and χ on the per unit shortage subsidy that the RBB would need to provide. Figure 8 illustrates the variation of unit shortage subsidy with $\frac{e}{\chi}$, under the conditions of Theorem 1 and Theorem 2. To generate the graphs, we set $p = 500$ and $\theta = 0.67$. Note that when $e \geq \chi$ the shortage subsidy is always zero. Thus we only consider scenarios where $e < \chi$. Figure 8 (a) corresponds to the subsidy under observed and random demand cases under the first set of conditions shown in the summary tables above. Figure 8 (b) corresponds

Player	Model	
	Deterministic Demand	Random Demand
Hospital i	observes own demand realization R_i and order distribution for $\tilde{q}_{-i}$ observes supply distribution	observes own demand distribution q_i and order distribution for $\tilde{q}_{-i}$ observes supply distribution
RBB	observes supply before allocation observes order distributions for $\tilde{q}_{-i}, \forall i$ contract to extract order quantities R_i	observes supply before allocation observes order distributions for $\tilde{q}_{-i}, \forall i$ contract to extract order quantities Q_i^*

Table 6: Differences between the two models from the perspective of the players

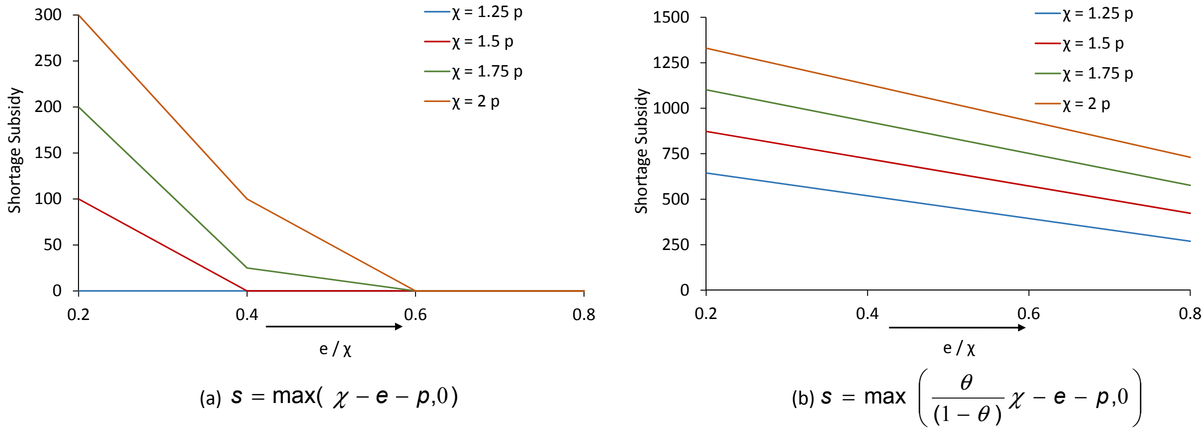


Figure 8: Change in shortage subsidy with e and χ: (a) $s = \max(\chi - e - p, 0)$, and (b) $s = \max(\frac{\theta}{(1-\theta)}\chi - e - p, 0)$. We set $p = 500$ and $\theta = 0.67$.

to the case where the shortage probability $\theta = 0.67$. That is to say that Figure 8 (a) applies when the RBB has more information about demand compared with the situation depicted in Figure 8 (b).

We make some observations based on these figures.

(i) In both Figure 8(a) and 8(b) we see that for any fixed value of χ, the subsidy is decreasing in e. This is because the higher the cost of outdating, the lower the incentive for order inflation; hence a lower subsidy is sufficient to induce truthful ordering.

(ii) Observe from both figures that when e/χ is fixed, an increase in χ leads to an increase in the unit subsidy required. This can be explained by the fact that the higher the cost of shortage, the greater the incentive for order inflation; thus, the subsidy needs to be higher in order to motivate the hospitals to order truthfully.

(iii) Suppose that the RBB decides to calculate its shortage subsidy based on the shortage probability since the conditions in Theorem 1 do not apply. Then compared with the unit subsidy that would be needed under the conditions spelled out in Theorem 1:

- the unit subsidy needed is higher;
- as excess cost e increases, the relative reduction in subsidy is lower;
- the relative increase in the subsidy is lower when (1) χ/p is higher, and (2) e/χ lower.

Next we study conditions under which a consignment contract may induce truthful ordering.

4.4 Can a Consignment Contract be Socially Optimal?

Typically, some form of consignment contract is commonly used in regional blood banking practice. It is apparent that a complete consignment contract without some checks and balances opens the door for a hospital to order arbitrarily large quantities with impunity, to the detriment of the entire supply chain. If the RBB has complete access to all the hospitals' inventory records (which we call the **verifiability assumption**), then we shall show that a variation of the complete consignment contract may indeed be truth inducing. In some cases, the assumption of verifiability may hold in practice. For example, the State of New York mandates the disposal of outdated blood at hospitals to be done within 72 hours of the expiration. Furthermore, the outdated units need to be transferred to a different storage location within 24 hours of expiration and required to be destroyed within 72 hours of expiration. Due to the strict regulations on the handling of expired blood units many hospitals choose to return these units to the collection facilities. The time period allowed for returning the expired units to collection facilities, i.e., blood banks, is one week (State Government Sites, 2015). Similar guidelines may be observed in other states. Under these circumstances, our assumption of complete verifiability of inventory may be realistic.

We define a shortage-subsidy consignment contract to be the subsidy contract defined in the previous subsection together with the clause that the hospitals pay only for blood that they actually use (in particular, the hospitals are not responsible for disposing excess units, if any). We have the following result.

Theorem 5 *(a) A shortage-subsidy consignment contract with per unit subsidy* $s = \max(0, \chi - \frac{p}{2})$ *induces a truth-revealing Bayesian Nash equilibrium provided (i) the verifiability assumption is satisfied, (ii)* $r^* \geq 0.5$*, (iii)* $S \geq_{st} q_i + \tilde{q}_{-i}$*.*
(b) A shortage-subsidy consignment contract with per unit subsidy $s = max(0, \chi - p)$ *induces a truth-revealing Bayesian Nash equilibrium provided* $r^* \geq 0.67$ *and (i), (iii), and (iv), above hold.*
(c) Both (a) and (b) above hold if condition (iii) is replaced by the following conditions: $S \geq_{icv} q_i + \tilde{q}_{-i}$ *and for each* i *we have that* $(q_i + \tilde{q}_{-i})$ *has a decreasing density function, and the remaining conditions are unchanged.*

This theorem implies that *provided hospitals are responsible for a non-zero portion of the cost of units that the blood bank has to procure from an external source,* a consignment contract together with a suitable subsidy scheme can induce truthful ordering by hospitals.

5 Numerical Study Based on Real Data

It is a worthwhile exercise to try and estimate the beneficial impact of the recommended subsidy contracts on social cost. In this section we utilize the real world data described in Section 1 to generate test instances which we use to get some insights into the interplay between contractual mechanism and social cost.

First, we take a closer look at our data and try to identify the distributions that best describe the demand at depots and the supply at the RBB. Note that we only observe the order quantities at depots. In identifying the demand distributions, we assume that the order quantities are indicative of true demand. We used the Kolmogorov-Smirnov test and the Chi-Squared test to identify the best fit. Our analysis revealed the following. The demand at depot 1 and 2 can be described using Generalized-Extreme-Value distribution. Despite out best efforts, we could not find a distribution that describes the demand at depot 3 sufficiently closely to pass tests of statistical significance. However, Generalized-Extreme-Value distribution is the best fit among all the tested distributions. Thus we use the General-Extreme-Value distribution to generate our demand data. The supply at RBB also fits a Generalized-Extreme-Value distribution, therefore, used for the generation of supply at RBB in performing the computations. Recall that this distribution is characterized by three parameters: (i) μ, location, (ii) σ, scale, and (iii) k, shape (Coles, 2001). The PDF of Generalized-Extreme-Value distribution is $\frac{1}{\sigma}t(x)^{k+1}e^{-t(x)}$ where,

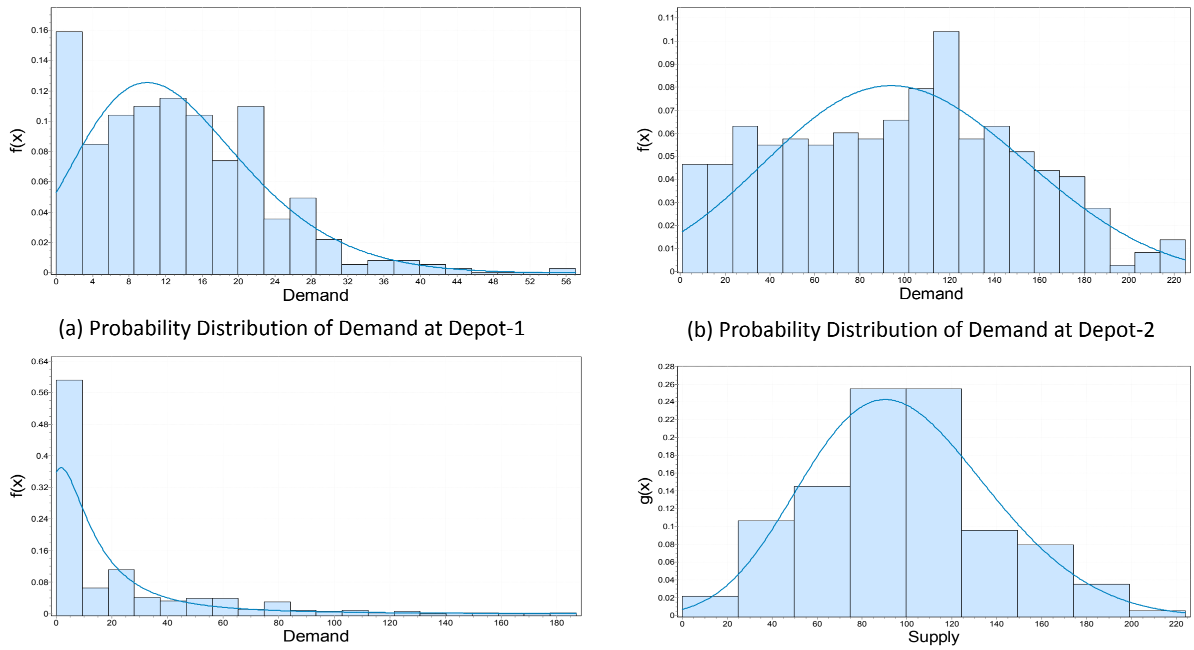
$$t(x) = \begin{cases} [1 + \frac{(x-\mu)k}{\sigma}]^{-\frac{1}{k}} & \text{if } k \neq 0 \\ e^{\frac{(x-\mu)}{\sigma}} & \text{if } k = 0 \end{cases}$$

Below we provide the test statistics together with histograms.

	μ (Location)	σ (Scale)	k (Shape)	Kolmogorov-Smirnov		Chi-Squared	
				Statistic	p-value	Statistic	p-value
Demand at Depot-1	9.0307	8.4078	-0.10754	0.06843	0.06253	-	-
Demand at Depot-2	77.869	53.201	-0.2708	0.04424	0.45968	13.834	0.08619
Demand at Depot-3	5.1621	10.177	0.45318	-	-	-	-
Supply at RBB	82.054	38.507	-0.19727	0.04087	0.56141	13.128	0.10752

Table 7: Generalized-Extreme Value distribution parameters, test statistics, and p-values for demand and supply

Next we discuss the goals of our experiment (Section 5.1). Then we provide the details of the data generation and finally summarize the results from our experiment in Sections 5.2 and 5.3, respectively.



(a) Probability Distribution of Demand at Depot-1

(b) Probability Distribution of Demand at Depot-2

(c) Probability Distribution of Demand at Depot-3

(d) Probability Distribution of Supply at RBB

Figure 9: Histograms and probability distributions of demand at depots and the supply at RBB.

5.1 Goals of the Experiment

Our main goal is to illustrate the impact of proposed contracts in reducing the shortage and excess at hospitals. Ideally, we would like to observe no excess at any hospital. To understand the impact of various supply and demand conditions, we design experiments to achieve the following.

1. Impact of demand on social cost: Recall that each hospital has its own demand distribution. Thus, the outcome of the contract may depend on the nature of the demand distributions. For example, the effectiveness of the contract may differ when the hospitals are similar in terms of their demand distributions as opposed to when they significantly differ in this respect.

2. Impact of order inflation on social cost: In the absence of a contract, the hospitals inflate their order quantities resulting in undesirable shortage and excess at hospitals. In this experiment we vary the level of order inflation to observe its impact on shortages and excesses at the hospitals.

3. Similarly, the ratio of supply to total demand also plays a role in determining the impact of the contract on social cost.

4. Quantifying the subsidy: Since we use subsidy contracts to achieve the desired outcome, it is worthwhile to determine the amount of subsidy needed under varying supply conditions.

We note that the goal of this numerical experiment is to understand the impact of eliciting truthful information from the hospitals on shortages and excesses. The generation of our test beds is to achieve this objective. We clarify that we do not use the sufficient conditions specified in our analytical results in this numerical analysis. Next we discuss the data generation process.

5.2 Data Generation

In our experiment, we consider a single period setting. Since our data is for platelets with a very short shelf-life, a single period model is appropriate here. The base line values for the experiment are determined based on the analysis above. Note that the real world data was only for 3 depots. However, in out experiment we consider 10 demand centers. Thus the supply needs to be appropriately adjusted to obtain meaningful insights. To achieve this, we inflate the value of location parameter μ of the supply distribution such that under the baseline setting we achieve $supply/demand = 0.75$. The table below summarizes these values.

Parameter	Baseline Value
Number of hospitals	$n = 10$
Supply at RBB	$\mu_s = 350$ $\sigma_s = 100$ $k_s = -0.2$
Demand at hospital i	$\mu_i \sim U[5, 100]$ $\sigma_i \sim U[5, 60]$ $k_i \sim U[-0.5, 0.5]$
Percentage of order inflation	$U[0, 10]$

Table 8: Baseline values of the parameters

Test Bed 1: To observe the impact of demand on social cost, we fix the values of σ_i and k_i at their base values and vary μ_i. Specifically, we draw μ_i from the following: (i) $[5, 25], [25, 45], [45, 65], [65, 85], [85, 105]$, and (ii) $[5, 100], [15, 90], [25, 80], [35, 70], [45, 60]$. For each case we run $10,000$ instances and report the average value. Thus we have $10,000 \times (4 + 4) = 80,000$ instance in the test bed.

Test Bed 2: In this experiment we draw the percentage of order inflation by each hospital from the following: (i) $[0, 10], [10, 20], [20, 30]$, and (ii) $[10, 20], [5, 25], [0, 30]$. All the other parameters are set at their base values. For each scenario we create $10,000$ instance and report the average outcome, thus, the number of instances in this test bed is $60,000$.

Test Bed 3: To observe the impact of change in mean supply on the social cost, we fix the demand distributions for the hospitals and vary the values of the supply distribution as follows. We fix the σ_s

and k_s at their base values. Then we set $\mu_s \in \{100, 200, 300, 400, 500\}$. For each choice of μ_s we run 10,000 instances and report the average resulting in 50,000 instances in this test bed.

Test Bed 4: To quantify the subsidy, we generate the demand distributions using the base setting given above. Then we generate the parameters for the supply distribution such that $E[S] \geq E[\sum_{i=1}^{10} q_i]$. Note that the mean of Generalized-Extreme-Value distribution is given by $\mu + \sigma \frac{\Gamma(1-k)-1}{k}$. In this experiment we let $\frac{E[S]}{E[\sum_{i=1}^{10} q_i]} \in \{1.0, 1.1, 1.2, 1.3, 1.4\}$. We obtain the average for each case over 10,000 instances resulting in a total of 50,000 instances in the test bed.

In testing each instance in each test bed we perform following steps. Note that in the last experiment we avoid third and fourth steps.

1. First we randomly create the demand distributions.

2. We draw the supply from the specified distribution.

3. Then we consider the case where there is no contract.

   – In this case each hospital reports its inflated order quantity which is calculated by multiplying the randomly drawn percentage order inflation with corresponding μ_i. Note that since σ_i and k_i are fixed, inflating μ_i translates to inflating the mean value.

   – We allocate the supply among the hospitals based on the inflated order quantities. In this case we assume that the RBB considers the inflated value to be the true location parameter of the distribution. The allocation is performed in such a way that the Type-1 service level at each hospital to be the same.

4. Next we consider the case where the contract is in place.

   – In this case each hospital reports its true location parameter μ_i as the order quantity.

   – The supply is allocated in such a way that the Type-1 service level at each hospital to be the same..

5. Now we draw the realized demand vector and calculate the total shortage and excess due to above two allocations separately.

5.3 Observations and Insights

To facilitate the discussion in this Section we define following two metrics.

$$\Lambda_S = \left[1 - \frac{\text{Total shortage of blood in the presence of the contract}}{\text{Total shortage of blood in the absence of a contract}}\right] \times 100$$

$$\Lambda_E = \left[1 - \frac{\text{Total excess of blood in the presence of the contract}}{\text{Total excess of blood in the absence of a contract}}\right] \times 100$$

Note that Λ_s (resp., Λ_E) captures the percentage decrease in total shortage (resp., excess) due to the

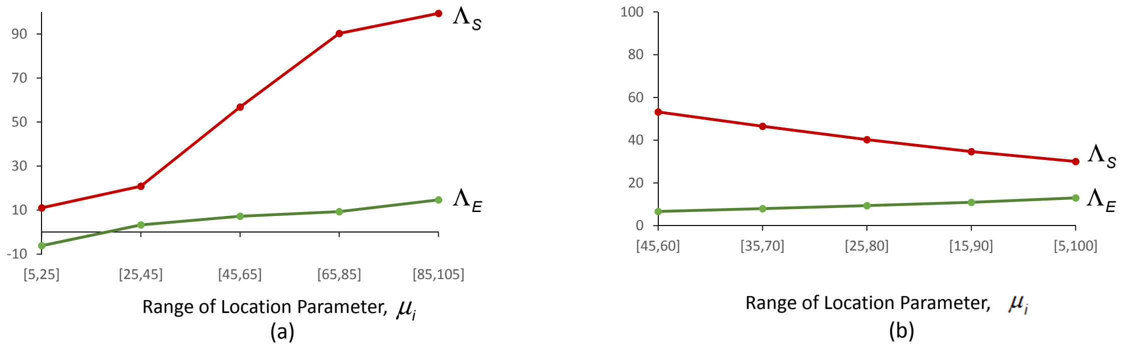


Figure 10: Impact of change in mean demand on percentage change in total shortage/excess: (a) increase in average mean and (b) increase in range of mean.

implementation of the proposed contract. Figure 10 illustrates the impact of change in demand on the above measures. Observe that in Figure 10 (a) we increase the mean of the uniform distribution used to draw the values of μ_i while keeping the range the same. This eventually translates into increasing the mean of the demand distributions at the hospitals while keeping the distribution of the size (in terms of mean demand) of the 10 hospitals relative to one another the same. Observe from the figure that the contract's ability to better allocate the limited supply increases with mean demand. In other words, when the mean demand from the hospitals is higher, the contract becomes more efficient in managing both the shortage as well as the excess in the system.

Now consider Figure (b) which illustrates the impact of the contract on total shortage and excess with increase in heterogeneity among the hospitals. Observe that in this case we increase the range of the values μ_i. That is, as range increases, the μ_i values become relatively different from each other allowing the demand distributions to be more heterogenous. This translates to the case where we increase the heterogeneity of the pool of the hospitals. In a real world setting, we can consider a situation where the RBB services a set of similar hospitals (that is, hospitals with same demand profile in terms of the magnitude of demand) as opposed to a set of dissimilar hospitals (that is, a mixture of small and large hospitals). Observe form the figure that when the hospitals become more heterogeneous, the effectiveness of the contract decreases in terms of managing the shortage. However the contract's ability to manage the excess increases.

Observation 1: *(i) The contract becomes more effective in managing both shortage and excess as the supply becomes more stringent. (ii) The heterogeneity among hospitals (in terms of demand) strengthens our ability to contractually manage excess; however, it weakens our ability to manage shortage.*

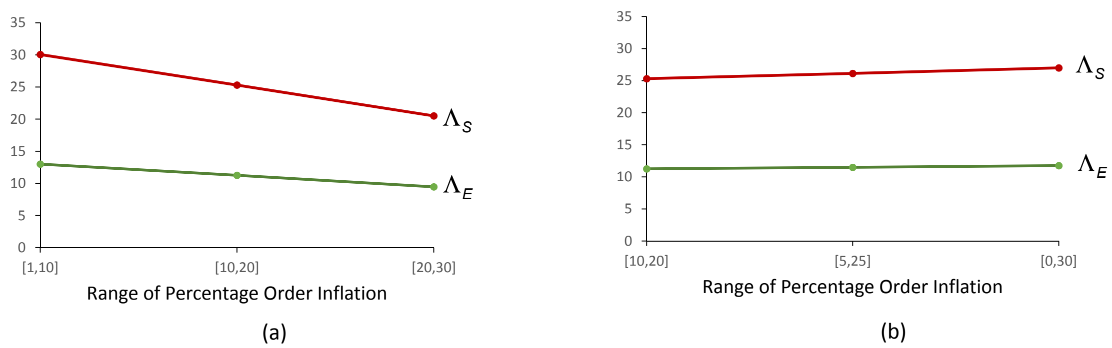


Figure 11: Impact of change in order inflation by hospitals on percentage change in total shortage/excess: (a) increase in average order and (b) increase in range of order inflation.

Let us now focus on the impact of order inflation in the absence of the contract on shortage and excess. Recall that in this experiment we fix the demand distribution at each hospital and the supply distribution at RBB. In Figure (a) we increase the average value of order inflation. Note that as the mean of percentage order inflation increases, the contracts ability to manage shortage and excess slightly decreases. Figure (b) illustrates the case where the hospital becomes heterogeneous in terms of the order inflation. Now we observe that the more significantly the percentage inflation by the hospitals varies, the more effective the contract becomes.

Observation 2: *As the percentage of order order inflation increases the effectiveness of the contract slightly reduces. However when the order inflation by the hospitals exhibits a significant variation the contract achieves its objective better.*

Figure 12 indicates the impact of increase in supply on the contract performance. In this experiment we keep the demand distributions and the order inflation at their base values. As observed from the figure, when there is extreme scarcity of supply ($Probability(Supply > Demand) = 0.002$ and $Supply = 0.27 \times Demand$), the contract has the ability to reduce the excess. However, the contract cannot effectively manage the shortage. When the supply is more realistic and increases, we observe that percentage reduction in shortage (resp., excess) due to the contract, Λ_S (resp., Λ_E) first increases

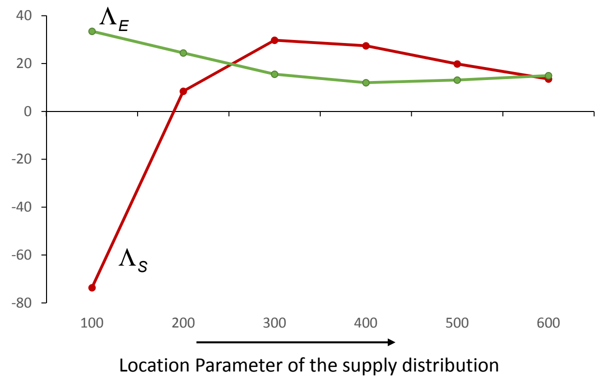


Figure 12: Impact of change in mean supply on percentage change in total shortage/excess.

significantly (resp., decreases) and then slightly decreases (resp., increases). This behavior is driven by the availability of ample supply. When the supply is very high, the RBB has the ability to better fulfill the demand. Naturally we expect the excess to increase.

Observation 3: *Estimating the subsidy budget.*

The table below presents the shortage of blood as a percentage of total demand. Note that when the expected supply is equal to the expected total demand, this percentage is about 30% and gradually decreases as the mean supply increases. To get an estimation on amount of subsidy needed in implementing the proposed contracts, let us set the disposal cost $e = 0$, and assume that the taregt service level at each hospital is at least 67%. Note that for any $e > 0$, the required amount of subsidy is lower than what is estimated. The cost of platelets is typically over USD 500 per unit (cost vary over time and suppliers). Thus in our calculation we let the price $p = 550$. Recall from Theorem 3 that, a unit subsidy of $s = max(\chi - p - e, 0)$ prevents order inflation. Note that s is positive only when the emergency supply cost, $p_{em} = p + \chi$, exceeds $2p$. Observe from the above results that the maximum

$\frac{E[S]}{E[\sum_i q_i]}$	$\frac{Shortage}{Demand}$	Dollar Value of Subsidy s per unit				
		$p_{em} = 2p$	$p_{em} = 2.25p$	$p_{em} = 2.5p$	$p_{em} = 2.75p$	$p_{em} = 3p$
1.0	0.30	0	41.25	82.50	123.75	165.00
1.1	0.26	0	35.75	71.50	107.25	143.00
1.2	0.22	0	30.25	60.50	90.75	121.00
1.3	0.18	0	24.75	49.50	74.25	99.00
1.4	0.13	0	17.88	35.75	53.63	71.50

Table 9: Estimation of subsidy per unit required under varying supply/demand conditions when $p = 550$.

shortage is around 30% when expected supply is equal to expected total demand and it monotonically reduces as the expected value of supply increases. Using above results, an RBB can estimate the budget requirement based on its supply, total demand from the depots, and the cost of emergency supplies.

For example, let us illustrate the budget required for the first case in our experiment where (i) average of total demand over 10,000 instances is 452 and (ii) shortage is 137 units. Let $p_{em} = 2.5p = 1,375$. In this case the total subsidy required is $82.50 \times 137 = 11,302.5$. Note that the total revenue for the blood bank in this case is $(452 - 137) \times 550 = 173,250$. Hence, we observe that the subsidy accounts for around 6.5% of the blood bank's revenue. Observe that when the emergency cost is lower than $2p$ the subsidy is 0.

6 Concluding Remarks

We modeled the problem of designing contracts between a regional blood bank and the hospitals in its service area. We were motivated by the high cost of blood shortages and outdates, a cost that is ultimately borne by society at large. It is clear that managing the blood supply chain efficiently requires good logistics practices. It is perhaps not as apparent that contract design also plays an important role in an efficient blood supply chain. Our aim in this paper was to create parsimonious models of the blood supply chain and design contracts geared towards creating incentives for honest information sharing, maximizing efficiency and fairness in allocation, and minimizing total waste.

Our main findings were as follows. Our benchmark behavioral assumption, in accord with the standard game theoretic paradigm, was that a rational and self-interested economic entity is certain to disclose information honestly only if it is given sufficient economic incentive to do so. This assumption precluded us from taking for granted that the hospitals would honestly share their demand estimates with the blood bank; rather, we assumed that they would be tempted to exaggerate their actual needs so as to garner a greater share of the available supply under constrained supply conditions. We showed that under these conditions a per unit shortage subsidy contract can induce a truthful Bayesian Nash equilibrium provided the subsidy is sufficiently generous to offset the financial incentive for hospitals to over order, particularly under constrained supply conditions. The more constrained the supply, the more generous the shortage subsidy is required to be. The shortage subsidy does not create an incentive to mis-report shortages because the subsidy applies only to the extra cost incurred by the blood bank in requisitioning emergency shipments of blood; in effect the hospitals share the emergency surcharge with the blood bank. However, in general the hospitals still pay more for every unit of blood requisitioned to offset a shortage than they pay for a regular unit of blood, under the shortage subsidy contract.

Let us compare and contrast our results with those in Cachon and Lariviere (1999b). Cachon and Lariviere (1999b) emphasize that in contrast to comparable work in the social choice literature, in their setup the supplier is self-interested. We note that our setup also differs from that of Cachon and Lariviere (1999b) in this critical facet. In their model, if a contracting mechanism is 'individually responsive' - in the sense that the received quantity is a non-decreasing function of the order requisitioned - then the retailers are sure to gain by inflating their orders. In our model too the blood bank designs an individually responsive contract mechanism. However, we counter the incentive of hospitals to inflate their orders by providing a sufficiently large subsidy. Of course, this pre-supposes a socially responsible mechanism designer, which takes our model outside the domain of applicability of the results in Cachon and Lariviere (1999b).

Let us note some key assumptions behind our work, as well as some limitations. We assume that the most significant social costs were connected with shortages and outdates of blood. While there can be no dispute about the extremely high human cost of blood shortages in hospitals, these costs may be blunted if hospitals have recourse to cheap outside options for their blood supply. To the best of our knowledge, finding alternative supplies of blood at short notice is not easy but our results certainly lose some of their force for a set of hospitals with ready access to multiple sources of blood at a reasonable cost. Our model does not study the interaction between hospitals on the one hand and two alternative suppliers of blood on the other; we suggest that this is an interesting problem for future research. Our own experience with blood banking in two large states in the USA is that hospitals sign on to supply contracts with either a community blood center or the Red Cross, but not both. According to an article in Slate (Engber , 2006), 'local hospitals work out contracts with regional suppliers or their local Red Cross facility. In general, they work with a single vendor, but they may shop around a bit to find the best prices'.

We derive Bayesian Nash equilibrium contracts. We cannot in general show the existence of dominant equilibrium contracts. Under severe supply constraints we would have to offer a generous subsidy contract to induce truth revealing behavior from hospitals. This presupposes that the regional blood bank has an external source of financial support so that it can tide over lean times, or else that it has the flexibility to raise the price of blood.

In our work, we model the social objective to be the total cost incurred by the society due to inefficient allocation of blood among the hospitals. The proposed social objective can also be translated into the number of patients affected due to inefficient allocation of blood. To achieve this, we need in-depth knowledge on types of patients using the considered blood type, the nature of the sickness, and the individual health records, to estimate the amount of blood needed by each patient. Although obtaining this detailed level information is not practical, the blood bank may use the average use of a blood type to estimate the impact of the contract (for example average use of red blood cells is around 3 pints per patient). In that case, we can show that the proposed contract still achieves the social optimum (we can obtain this result by setting the outdating cost to zero).

We learned in the course of our discussions with practitioners that a contract between a blood bank and a hospital typically does not depend on the type of blood product. However, contracts may vary based on the geographical location of the hospital. Developing different types of contracts based on the demand/supply characteristics of a particular type of a blood product may lead to an efficient management of the blood units. We suggest that modeling this is an interesting problem for future research.

Note: All the URL's that appear below are last accessed on January 18th, 2017.

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Appendix

Proof of Theorem 1(b): We remark that it is more convenient to prove part (b) before part (a). Please refer to the table below for definitions used in the proof. We break up the proof into two steps.

Notation	Definition
$\hat{f}(x)$ (resp.,$\hat{F}(x)$)	p.d.f (resp., c.d.f) of random variable $\tilde{q}_{-i}$
$g(y)$ (resp.,$G(y)$)	p.d.f (resp., c.d.f) of random variable $(S - R_i)$
$h(x, y)$	Joint density function of random variables $\tilde{q}_{-i}$ and $(S - R_i)$
$H(x/z)$	c.d.f of random variable Uz

Table 10: Notation used in the proof of Theorem 1

Step 1: Consider any specific hospital i, and fix the order quantities of all the other hospitals. For each i define

$$\nu^i := S - [R_i + \tilde{q}_{-i}].$$

We shall analyze H_i's estimate of $P\{\nu^i > 0\}$. Let $\hat{f}(x)$ be the density function of $\tilde{q}_{-i}$ and $g(y)$ be the density function of $(S - R_i)$. Also let $\hat{F}(x)$ and $G(y)$ be the distribution functions of $\tilde{q}_{-i}$ and $(S - R_i)$, respectively. We assume that both these distribution functions are continuous. We note the following simple lemma.

Lemma 1 *Let $H(x)$ be a continuous distribution function of a positive valued random variable. Then $\int_0^\infty H(x)dH(x) = \frac{1}{2}$.*

Proof: Integrating by parts, we get $\int_0^\infty H(x)dH(x) = H^2(x)|_0^\infty - \int_0^\infty H(x)dH(x)$ which, on rearranging, yields the claimed result. □

First, suppose $\tilde{q}_{-i}$ is a unimodal random variable with mode 0; since $\tilde{q}_{-i}$ is positive valued, this is equivalent to the assumption that $\tilde{q}_{-i}$ has a decreasing density function with mode 0. Then it follows from a standard result on unimodal random variables - see Chapter 1 of the monograph by Dharmadhikari and Kumar Joag-dev (1988) for a thorough account - that the random variable $\tilde{q}_{-i}$ can be written as UZ, where U is the uniform $(0, 1)$ random variable, and Z is a random variable independent of U. Since $\tilde{q}_{-i}$ is positive valued, Z must be positive valued too. Let us condition on Z. Given $Z = z > 0$, we have that $\tilde{q}_{-i} \sim Uz$ is uniformly distributed over $(0, z)$. Let $H(x|z)$ denote the distribution function of Uz; note that this is an increasing concave function of x on $[0, \infty)$. We have by hypothesis that

$$S \geq_{icv} R_i + \tilde{q}_{-i} \tag{1}$$

which implies that

$$S - R_i \geq_{icv} \tilde{q}_{-i}$$

since we obtain the latter from the former by adding the constant $-R_i$ to the random variable on each side (the icx ordering is closed under this operation, by Theorem 4.A.15 of Shaked and Shanthikumar

(2007) and it follows that this applies to icv ordering too). The distribution function of $UZ \sim \tilde{q}_{-i}$ is $\hat{F}(x)$. Since $H(x|z)$ is an increasing concave function, we have

$$\int_0^\infty H(x|z)d\hat{F}(x) \leq \int_0^\infty H(x|z)dG(x). \tag{2}$$

For ease of exposition and without loss of generality, let Z be a discrete random variable taking the value z with probability $p(z)$. Hence, we have

$$\sum_z H(x|z)p(z) = \hat{F}(x). \tag{3}$$

It follows directly from (2) that

$$\sum_z p(z) \int_0^\infty H(x|z)d\hat{F}(x) \leq \sum_z p(z) \int_0^\infty H(x|z)dG(x).$$

Interchanging the order of summation and integration - which, by Tonelli's Theorem, is valid when the functions are non-negative as in this case - this simplifies to

$$\int_0^\infty [\sum_z H(x|z)p(z)]d\hat{F}(x) \leq \int_0^\infty [\sum_z H(x|z)p(z)]dG(x). \tag{4}$$

Substituting (3) into (4) yields

$$\int_0^\infty \hat{F}(x)d\hat{F}(x) \leq \int_0^\infty \hat{F}(x)dG(x). \tag{5}$$

Applying Lemma 1, we have from (5) that

$$\int_0^\infty \hat{F}(x)dG(x) \geq \frac{1}{2}. \tag{6}$$

Define $h(x, y)$ as the joint density function of random variables $\tilde{q}_{-i}$ and $(S - R_i)$. Now, we integrate $h(x, y)$ over the region $y > x$ in the xy plane. Because of independence, we have $h(x, y) = \hat{f}(x)g(y)$. Now we have

$$\begin{aligned} P(\nu^i > 0) &= \int_0^\infty \int_x^\infty \hat{f}(x)g(y)dxdy = \int_0^\infty \Big\{ \int_x^\infty g(y)dy \Big\} \hat{f}(x)dx \\ &= \int_0^\infty [1 - G(x)]\hat{f}(x)dx = \int_0^\infty [1 - G(x)]d\hat{F}(x) \end{aligned}$$

So we have

$$\begin{aligned} P(\nu^i > 0) &= \int_0^\infty [1 - G(x)]d\hat{F}(x) \\ &= 1 - \int_0^\infty G(x)d\hat{F}(x) \end{aligned} \tag{7}$$

where we used the fact that $\int_0^\infty d\hat{F}(x) = 1$ to obtain the second identity. Now the following identity can be verified by integration by parts:

$$1 = \int_0^\infty \hat{F}(x)dG(x) + \int_0^\infty G(x)d\hat{F}(x). \tag{8}$$

Combining (9), (7), and (8) we obtain

$$P(\nu^i > 0) \geq \frac{1}{2}. \tag{9}$$

Now it is easy to see that (9) holds even when $\tilde{q}_{-i}$ is a random variable with a decreasing density function, and support $[a, \infty)$ or $[a, b]$, for some $b > a$, rather than $[0, \infty)$ or $[0, b]$. To see this, note that we can write $\tilde{q}_{-i}$ in the form $a + V$, where V is a positive valued unimodal random variable with mode 0. Further, we note that we can replace (1) by

$$S \geq_{icv} R_i + \tilde{q}_{-i} + a$$

and re-run the entire argument in the case when the mode was 0.

Step 2: Consider two cases, $\nu^i \geq 0$ and $\nu^i < 0$. If $\nu^i \geq 0$ and H_i over-orders by δ - that is, orders $R_i + \delta$ - then the extra cost borne by the hospital, relative to the case when it orders exactly its demand, is at least $\delta(e + p)$; note that the true value of the extra cost lies between $\delta(e + p)$ and $\delta(e + p + \chi)$, and depends on exactly how many of the extra units come from the RBB and from the external source. On the other hand, if $\nu^i < 0$ and H_i over-orders by δ then the shortage cost of the hospital relative to the case when it orders exactly its demand would be reduced by $Y(\delta)\chi$, where $Y(\delta)$ is equal to the additional units received by H_i because it ordered δ more than its true demand. Note that $0 \leq Y(\delta) \leq \delta$, since the hospital receives at most δ more than the quantity it would have received had it ordered exactly its demand.

Thus the expected cost of over-ordering by δ is greater than

$$P\{\nu^i \geq 0\}[\delta(e + p)] \tag{10}$$

while the expected benefit of over-ordering by δ is at most

$$P\{\nu^i < 0\}[\delta\chi]. \tag{11}$$

Suppose $R_i + \tilde{q}_{-i}$ has a decreasing density function for all i. Then it follows from Step 1 that $P\{\nu^i \geq 0\} \geq \frac{1}{2}$. Therefore it follows from (10) and (11) that if the RBB provides a unit subsidy of at least $\max(\chi - p - e, 0)$, the expected cost of over-ordering will exceed the expected benefit. The theorem follows. ■

Proof of Theorem 1(a): If $Y \geq_{st} X$, then it follows that $P(Y - X > 0) \geq \frac{1}{2}$. Hence we directly obtain the inequality

$$P(\nu^i > 0) \geq \frac{1}{2}$$

and can therefore proceed directly to Step 2 of the proof of part (a) and repeat the argument there. ■

Proof of Corollary 1 : It is easily checked that if $p + e > \chi$, the expected cost of over-ordering will exceed the expected benefit even without a subsidy. The condition we state in the corollary reduces to $p > \chi$, which implies that $p + e > \chi$; the corollary follows. ■

Proof of Corollary 2 : This corollary can verified by noting that in Step 2 of the proof of Theorem 1, we invoked a function $Y(\delta)$ that captures the additional units that a hospital receives when it over-orders by δ. If the allocation policy is non-decreasing in Q_i, then it follows that $0 \leq Y(\delta) \leq \delta$. We made use of this inequality in the proof of Theorem 1; nowhere else in the proof does the nature of the allocation policy come into play. ∎

Proof of Theorem 2 : For each i the expected cost of over-ordering by δ is greater than

$$P\{\nu^i \geq 0\}[\delta(e+p)]$$

while the expected benefit of over-ordering by δ is at most

$$P\{\nu^i < 0\}[\delta\chi]. \tag{12}$$

Note that $\theta := P\{\nu^i < 0\}$. Hence the expected cost of over-ordering by δ is greater than

$$[\delta(e+p)][1-\theta]$$

while the expected benefit of over-ordering by δ is at most

$$[\delta\chi][\theta].$$

The result now follows easily. ∎

Proof of Theorem 3 : (a) Let $\mathcal{E}_i$ denote the event

$$q_i \leq Q_i^*.$$

We have that for all hospitals H_i

$$P(\mathcal{E}_i) = r^*.$$

Define

$$\nu^i := S - [q_i + \tilde{q}_{-i}].$$

Note that we had defined ν_i earlier (in the deterministic demand case), but with hospital i demand in that case denoted by R_i rather than q_i. For each i let $\mathcal{G}_i$ denote the event $\{\nu^i \geq 0\}$. Then it follows that the expected cost when H_i orders δ more than Q_i^* is at least

$$P\{\mathcal{E}_i \cap \mathcal{G}_i\}[\delta(e+p)]. \tag{13}$$

Note that the events $\mathcal{E}_i$ and $\mathcal{G}_i$ are *not* independent, so $P\{\mathcal{E}_i \cap \mathcal{G}_i\} \neq P\{\mathcal{E}_i\}P\{\mathcal{G}_i\}$. We shall show that it is the case that

$$P\{\mathcal{E}_i \cap \mathcal{G}_i\} \geq P\{\mathcal{E}_i\}P\{\mathcal{G}_i\}. \tag{14}$$

We note that

$$\begin{aligned}
\mathcal{E}_i &:= \{\omega : q_i \leq Q_i^*\} \\
\mathcal{G}_i &:= \{\omega : q_i \leq S - \tilde{q}_{-i}\}
\end{aligned}$$

where ω denotes a generic point in the probability space of the random variable q_i. Therefore,

$$\begin{aligned} P\{\mathcal{E}_i \cap \mathcal{G}_i\} &= P\{\omega : q_i \le Q_i^*, q_i \le S - \tilde{q}_{-i}\} \\ &= P\{q_i \le Min(Q_i^*, S - \tilde{q}_{-i})\} \end{aligned}$$

while

$$P\{\mathcal{E}_i\}P\{\mathcal{G}_i\} = P\{q_i \le Min(Q_i^*, S - \tilde{q}_{-i})\}.P\{q_i \le Max(Q_i^*, S - \tilde{q}_{-i})\}.$$

Now (14) follows immediately.

It follows from (13) and (14) that the expected cost when H_i orders δ more than Q_i^* is at least

$$P\{\nu^i \ge 0\}r^*[\delta(p+e)]. \tag{15}$$

It is clear that the expected benefit of over-ordering by δ is at most

$$\delta\chi(1-r^*). \tag{16}$$

since over-ordering is beneficial only in the event that the demand at H_i exceeds Q_i^*, an event of probability $(1-r^*)$.

Further, we have assumed that $r^* > 1/2$ for all the hospitals. Therefore it follows from (15) and (16) that if the RBB provides of unit subsidy of at least $\max(\chi - (p+e)/2, 0)$, the expected cost of over-ordering will exceed the expected benefit. The theorem follows.

Remark: We showed in the proof of Theorem 1 that $P\{\nu^i \ge 0\} \ge \frac{1}{2}$; the same result holds in this case and the proof is almost unchanged. The main change is that the real number R_i is replaced by a random variable q_i, but since this random variable is independent of S and $\tilde{q}_{-i}$, the earlier proof goes through. The statement $S \ge_{icv} R_i + \tilde{q}_{-i}$ implies $S - R_i \ge_{icv} \tilde{q}_{-i}$ in the proof of Theorem 1(b) has to be replaced by the statement that $S \ge_{icv} q_i + \tilde{q}_{-i}$ implies $S - q_i \ge_{icv} \tilde{q}_{-i}$; this is true because of the mutual independence of the random variables (as spelled out in the previous sentence) and the closure of the 'icv' order under mixtures.

(b) Using the assumption that $r^* > 2/3$ we easily obtain that with a unit subsidy of at least $\max(\chi - p - e, 0)$, the expected cost of over-ordering will exceed the expected benefit.

(c) The stochastic ordering directly implies that $P(\nu^i > 0) \ge \frac{1}{2}$ holds; we can therefore proceed directly to Step 2 of the proof of part (b) and repeat the argument there. ■

Proof of Theorem 4 : The proof follows by combining the proof techniques of Theorem 2 and Theorem 3.

Proof of Theorem 5 : The same proof as that of Theorem 3 applies with $e = 0$ since in a consignment contract the hospitals do not bear the cost of outdated units. ■