Divisia Monetary Aggregates and Exchange Rate Forecasting

By

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Abstract

Divisia monetary aggregates have been shown to be an improvement on the simple-sum monetary aggregates used by policy makers in the great majority of central banks in the world. Since Barnett (1978, 1980) derived the User Cost Price and produced the theoretically correct from of aggregation, Divisia monetary aggregates have helped solve some of the difficult problems in the profession. One such problem is forecasting exchange rates. Since Meese & Rogoff (1983) convincingly argued that no model could outperform a driftless random walk in predicting exchange rates, there have been many papers which have tried to find some forecasting methodology that could beat the random walk, at least for certain forecasting periods. In particular, Wright (2008) introduced Bayesian Model Averaging as a tool to forecast exchange rates and Lam et al. (2008) compared Bayesian Model Averaging and three structural models to a benchmark model (the random walk), both studies obtaining positive results. Also, Carriero et al (2009) found positive results using a Bayesian Vector Auto-regression model. Barnett & Kwag (2005) availed themselves of the User Cost Price and Divisia monetary aggregates and included them as variables in the Flexible Price Monetary model, Sticky Price and Hooper Morton models to show that they have greater forecasting power than the random walk when the aforementioned variables replace the interest rate and simple sum monetary aggregates (respectively). Specifically, the authors worked with the US dollar/British pound exchange rate. This dissertation aims to extend three different experiments. The first chapter compares Purchasing Power Parity, Uncovered Interest Rate, Sticky Price, Bayesian Model Averaging, and Bayesian Vector Auto-regression models to the random walk benchmark in forecasting exchange rates between the Paraguayan Guarani and the US Dollar, the Brazilian Real and the Argentinian Peso. The second, follows Barnett and Kwag's work, applied to the US dollar/Euro exchange rate, but also includes an Uncovered Interest-rate Parity model. I use the Root Mean Square Error, Direction of Change statistic, and the Diebold-Mariano statistic to compare the forecasting power of the models in Chapters 1 and 2. In the first chapter, results indicate that in shorter forecasting horizons Bayesian Model Averaging, and Bayesian Vector Auto-regression models perform better than the random walk and that structural models outperform the random walk at longer horizons. In the second chapter, results indicate that Uncovered Interest-rate Parity with the User Cost Price instead of the interest rate improves on the random walk forecasts in every time horizon. In view of the results in Chapter 2, Divisia monetary aggregates for the country of Paraguay are calculated in the last chapter with the aim to examine their performance against simple-sum monetary aggregates in estimating money demand. Results suggest that Divisia monetary aggregates are superior to simple-sum aggregates in money demand estimation.

Acknowledgements

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Chapter 1

Comparing Exchange Rate Forecastability: the Paraguay Case

1.1 Introduction

Exchange rate forecasting is a complicated matter. It has been the subject of many studies which have yielded promising results only to be subsequently refuted by others. A definitive model or framework remains elusive. In particular, since Meese and Rogoff (1983) argued that no model outperforms a driftless random walk in forecasting exchange rates, researchers have been forced to go back to the drawing board to come up with more solid alternatives. For instance, Lothian & Wu (2011) show that Uncovered Interest Parity (UIP) has remarkable forecasting power in longer time horizons. But even recently Cheung et al. (2017) have reinforced the idea that no model can consistently beat a random walk.

The objective of the present work is to make a small contribution to the literature by expanding on the results obtained by Wright (2008) and extending the framework of Lam et al (2008). Both studies used Bayesian Model Averaging (BMA) to forecast the exchange rates of the U.S. Dollar (USD) with respect to several other currencies and then compare them to the performance of a benchmark model, namely, the driftless random walk. In particular, Lam et al added three more models and compared them to the random walk as well. These models are Purchasing Power Parity (PPP), the aforementioned UIP and Sticky Price (SP). They are well-known models in the literature and have been extensively discussed. It is this latter approach that is followed for this paper but with the addition of one more model: a Bayesian Vector Auto-regression (BVAR) model with a Minnesota prior. As shown for instance by Carriero et al. (2009), BVAR models perform well in the short run. I evaluate the performance of each model according to the Root Mean Square Error (RMSE) ratio, Direction of Change (DoC) ratio and the Diebold-Mariano (DM) statistic.

More specifically, I have used the above mentioned models to forecast exchange rates between the Paraguayan Guarani (PYG) and the USD, the Brazilian Real (BRL) and the Argentinian Peso (ARS) using monthly data. Unlike Wright, I do not separate my variables into a financial and a macroeconomic data set in order to estimate monthly and quarterly exchange rates, respectively – all variables are monthly. Lam et al only produced forecasts based on quarterly data. The forecasting periods are 3, 6, 9, and 12 months ahead. The results are encouraging and in line with Wright's and Lothian and Wu's works, as well as Carriero et al: under the RMSE criterion, in the cases of the USD and BRL, BMA and BVAR outperform all other models in the 3-month and 6-month horizons; UIP outperforms all other models in the subsequent horizons; and, results are similar under the DoC criterion. In the case of Argentina, PPP and SP appear to fare far better under both criteria, owing perhaps to Argentina's recent and complex inflation history. Under the DM criterion, forecasts are statistically significant improvements in the 3-month horizon in the cases of BMA and BVAR for the PYG/USD, and only in the case of BVAR for PYG/BRL. UIP produces improved forecasts that are statistically significant only beyond 36 periods ahead for both exchange rates. As for the Argentinian peso, BVAR and SP forecasts are the statistically significant improvements over the random walk.

The rest of the paper proceeds as follows: in section 2, I discuss the previous literature related to exchange rate forecasting; in section 3, I describe the models and the reason for their choice; in section 4, I describe the data and their sources; in section 5, I present the results, briefly discussing them and I suggest possible further research; section 6 concludes.

1.2 Literature Review

Exchange rate forecasting models have been around for some time now. Models such as Purchasing Power Parity (PPP) and Uncovered Interest-Rate Parity (UIP) have been thoroughly analyzed time and again (see, for instance, Balassa (1964) and the aforementioned Lothian and Wu paper). Dornbusch (1976) proposed a Sticky Price (SP) model based on monetary fundamentals and Frankel (1979) further developed this framework by emphasizing the role of expectations. However, Meese and Rogoff (1983) wrote a seminal paper in which they argued that no exchange rate model can outperform a driftless random walk in out-of-sample forecasting. Since then, Mark (1995) proposed that at longer horizons a monetary fundamentals model could provide with better out-of-sample forecasts. This model has been subject to criticism by Kilian (1999) and Faust et al. (2003) where they argue that improvements occur only with a two-year window and disappear afterwards. Interestingly, Kilian & Taylor (2003) finds that ESTAR models are helpful in explaining real exchange rate behavior.

Authors have had some success in forecasting using large data sets as in Stock & Watson (2002) for the Index of Industrial Production, and Bernanke & Boivin (2003) for inflation. Moreover, Stock & Watson (2004) have used the combination of forecast methods to approximate output growth with encouraging results (the Bernanke and Boivin paper also utilizes forecast combination methods for inflation measures). It is worth noting that forecast combination methods originate with Bates & Granger (1969).

Bayesian Model Averaging (BMA) was first introduced by Leamer (1978) and was further developed by Raftery et al. (1997) and Hoeting et al. (1999). BMA was first used for exchange rate forecasting by Wright (2008) and subsequently by Lam et al (2008). Both papers find that BMA produces improvements in out-of-sample forecasts when compared with a driftless random walk.

BVAR was used in forecasting as far back as Litterman (1986). Sarantis (2006) showed that a BVAR model outperforms a random walk in forecasting daily exchange rates. Banbura et al. (2007) used BVAR for forecasting employment, CPI and the Fed Funds Rate with positive results for first-quarter predictions. Recently, Beckmann et al. (2018) have used VAR-based models with Bayesian estimation methods for exchange rate forecasting.

1.3 Methodology

This section discusses the four different models that are used to estimate the PYG/USD, PYG/BRL and PYG/ARS exchange rate forecasts and their respective specifications. In the choice of models I have followed Lam et al (2008) and, partly, Cheung et al (2017). I will also explain how I have evaluated the performance of each model.

Purchasing Power Parity

PPP is well-known and – as mentioned in the previous section – widely discussed theoretical model which gives a clear and intuitive explanation for exchange rate determination. The PPP model is expressed in the following manner:

$$\ln e_t = \ln p_t - \ln p_t^* \tag{1.1}$$

where e_t is the nominal exchange rate, p_t is the domestic price and p_t^* is the foreign price. These are, of course, price indexes and not price levels.

The PPP specification used here involves an error-correction restriction and no short-run dynamics so that the variation from the exchange rate is a correction of the deviation from a long-run equilibrium in the previous period. The form of the equation is then:

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{p}_t) + \epsilon_t \tag{1.2}$$

where $\tilde{p}_t$ is the relative price level of the domestic economy relative to the foreign one, h is the forecast horizon and ϵ_t is the error term.

Uncovered Interest-rate Parity

UIP is another model that has been studied repeatedly as an approximation to forecasting exchange rates. This model entails the no-arbitrage condition that the expected return of the exchange rate h periods ahead is equal to the interest rate differential, which can be expressed thus:

$$E_t(\ln e_{t+h} - \ln e_t) = i_t - i_t^* \tag{1.3}$$

where E_t is the expectation and i_t and i_t^* are the domestic and foreign interest rates, respectively.

In a similar specification as the one above for the PPP model, also including an error-correction restriction, we write the equation as:

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{i}_t) + \epsilon_t \tag{1.4}$$

Here the $\tilde{i}_t$ is the relative interest rate (domestic to foreign).

Sticky Price

For this model we follow Frankel (1979) and expand the PPP framework so that exchange rates now are also determined by money supply, output and interest rates. This is given by the equation

$$\ln e_t = \ln m_t - \ln m_t^* - \phi(\ln y_t - \ln y_t^*) + \lambda(\ln i_t - \ln i_t^*) + \beta(\ln \pi_t - ln \pi_t^*) \tag{1.5}$$

where m_t and m_t^*, y_t and y_t^*, i_t and i_t^*, and π_t and π_t^* are, respectively, domestic and foreign money supply, domestic and foreign output, domestic and foreign interest rates and domestic and foreign current long-run expected rates of inflation.

As in the above cases, we use a restrictive error correction form of the model:

$$\ln e_{t+h} - \ln e_t = \alpha_0 + \alpha_1(\ln e_t - \beta_0 - \beta_1 \ln \tilde{m}_t - \beta_2 \ln \tilde{y}_t - \beta_3 \ln \tilde{i}_t - \beta_4 \ln \tilde{p}_t) + \epsilon_t \tag{1.6}$$

Here $\tilde{m}_t$, $\tilde{y}_t$, and $\tilde{i}_t$ are domestic to foreign relative money demand, output and short-term interest rates, respectively. Notice that we have replaced long-run expected rates of inflation with the only proxy available - relative prices. The reason for this choice will be explained in the next section.

Bayesian Model Averaging

BMA is a forecasting method that utilizes large data sets and many different models. Say there are M_i models, *i=1,...n* each of which has a parameter θ_i. I do not know which model is the true model but assume that one of them is. I assume the *ith* model is the true model based on some prior belief $P(M_i)$. Posterior probabilities are computed starting from a prior about which model is the true one. So if D is the data, we have

$$P(M_i|D) = P(D|M_i)P(M_i)/\sum_{j=1}^{n} P(D|M_j)P(M_j) \tag{1.7}$$

where

$$P(D|M_i) = \int P(D|\theta_i, M_i)P(\theta_i|M_i)d\theta_i \tag{1.8}$$

Here M_i's marginal likelihood is $P(D|M_i)$, $P(\theta_i|M_i)$ is the prior density of the parameters and the likelihood is given by $P(D|\theta_i, M_i)$. The forecasts from each of the different models are then weighted by their respective posteriors. The model is assumed to be linear. And so, one has

$$y = X_i\beta_i + \epsilon \tag{1.9}$$

where y is the vector of exchange rates (in this case), X_i are the predictors, β_i are parameters and ϵ is the mean zero, i.i.d. error with variance σ^2 and $\theta_i = (\beta_i', \sigma^2)$. Strict exogeneity is assumed of all regressors. As for the coefficients, the prior is of mean zero. The structure of their variance is given by Zellner's g so that

$$B_i|g \sim N(0, \sigma^2(1/gX_i'X_i)^{-1}) \tag{1.10}$$

where the hyperparameter g is set to the default "unit information prior" g=n (the number of models).

The forecasting model is then given by

$$\ln e_{t+h} - \ln e_t = \beta_i' X_{i,t} + \epsilon_t \tag{1.11}$$

where $X_{i,t}$ is the vector of regressors at time t for model i. For each model I have a forecast $\tilde{\beta}_i' X_{i,t}$ where $\tilde{\beta}_i'$ is the posterior mean of β_i. Each model is weighted by their posterior probabilities so that the forecast is given by $\sum_{i=1}^{n} P(M_i|D)\tilde{\beta}_i' X_{i,t}$ where $P(M_i|D)$ is the posterior probability of the ith model and D is the data set. Following Wright and Lam et al I consider the following variables as potential predictors from a monthly data set: (i) short-term interest rates and relative short-term interest rates, (ii) log of output and log of relative output (domestic to foreign), (iii) log of money supply and log of relative money supply (domestic to foreign), (iv) log of price levels and log of relative price levels (domestic to foreign), (v) oil price, and for the particular case of Paraguay, (vi) soy price. This gives a total of 2^6 possible models.

Bayesian Vector Auto-regression

The BVAR model with a Minnesota prior was introduced in the aforementioned paper by Litterman and, as previously described, has been widely used in forecasting. If the model is as follows

$$y = (I_m \otimes X)\alpha + \epsilon, \epsilon \sim (0, \Sigma_\epsilon \otimes I_T) \tag{1.12}$$

then y and ϵ are $mT \times 1$ vectors of dependent variables and errors, respectively, and where m is the number of variables and T, the time periods. I_m is the identity matrix, X is the matrix of independent variables and α is a $ml \times 1$ vector where l is the number of lags.

More specifically, $\alpha = \bar{\alpha} + \xi_\alpha$ with $\xi_\alpha \sim N(0, \Sigma_\alpha)$, where in the Minnesota prior $\bar{\alpha} = 0$ except $\bar{\alpha}_{1i} = 1, i = 1, ..., m$, Σ_α is diagonal and each element $\sigma_{ij,l}$ (equation i, variable j, and lag l) is as follows

$$\sigma_{ij,l} = \phi_0 / h(l), i = j \tag{1.13}$$

If j is endogenous, then

$$\sigma_{ij,l} = \phi_0 \times \phi_1 / h(l) \times (\sigma_j / \sigma_i)^2, i \neq j \tag{1.14}$$

And if j is exogenous, then

$$\sigma_{ij,l} = \phi_0 \times \phi_2 \tag{1.15}$$

In this case $\phi_0, \phi_1, \phi_2, (\sigma_j/\sigma_i)^2$ and $h(l)$ are, respectively, hyperparameters, a scaling factor, and a function of lags l. Note that ϕ_0 measures the tightness of the first lag's variance, ϕ_1 is the relative tightness of any other variables, and ϕ_2 is the relative tightness of exogenous variables. Finally, $h(l)$ is a measure of the relative tightness of the variance of the lags.

The error correction model follows a similar process to the one laid out for the SP model, using the same variables. The number of lags is 1.

Out-of-Sample Performance Evaluation: Root Mean Square Error Ratio, Direction of Change, and the Diebold-Mariano Statistic

In this paper I use what is called in the literature a rolling regression in order to produce the predicted forecasts. I first pick an in-sample period for which the models are first estimated and then exchange rates are forecast for the out-of-sample period. The sample is then updated to the following period until there are no more out-sample-sample observations.

In order to evaluate how well each model is performing I have compared each one to a benchmark model which in this case is the driftless random walk given by

$$\ln e_{t+h} - \ln e_t = \epsilon_t \tag{1.16}$$

Following Meese and Rogoff's methodology, I take the expectation of the random walk so that it becomes a martingale, i.e. the predictor of the exchange rate h periods ahead is whatever the exchange rate is at time t.

First, I use the root mean square error (RMSE) of each of the four models and divide it by the RMSE of the random-walk. A ratio of less than one indicates that the model is performing better than the random-walk and vice-versa. I assess the out-of-sample performance of each model 3, 6, 9 and 12 months ahead.

The second method of evaluation is the Direction of Change (DoC) ratio where I measure the proportion of times each model correctly predicts whether the actual exchange rate increases or decreases. Assuming that the expected value of random walk predicting the right DoC is 0.5, values above 0.5 indicate that a model is outperforming the random walk. The higher the proportion is, the better the model is performing.

The third method is the statistic produced by Diebold & Mariano (1995), which allows for the comparison of forecasts in terms of whether the difference between two forecasts for the same forecasting period is statistically significant and whether the improvement is statistically significant (and thus, one forecast is "better" than another). If $g(e_{it}) = e_{it}^2$ is the loss function of a forecast error, the loss differential function is defined as $d_t = g(e_{1t}) - g(e_{2t})$. If d_t is zero, then the forecasts under examination are equally accurate. Under the null, the expected value of d_t is zero. The DM statistic itself takes the form

$$DM = \bar{d}/\sqrt{2\pi \hat{f}_{d(0)}/T} \tag{1.17}$$

where $\bar{d}$ is the sample mean of the loss differential function and $\hat{f}_{d(0)}$ is a consistent estimate of the spectral density. Under the null, $DM \to N(0,1)$. The null is rejected if $|DM| > z_{\alpha/2}$.

1.4 Data

The data for this study are monthly series of the above mentioned variables starting in January, 1994 up to July, 2017 for the PYG/USD exchange rate; January, 1994 to December, 2016 for Brazil; and, January, 1997 to December, 2016 for Argentina. The reason I chose to start the series at these particular dates is that before January 1994 there were scarce to none monthly Paraguayan data available and in the case of Argentina, because there was no monthly exchange rate data available prior to 1997. The choice of in-sample and out-of-sample data was done based on the conclusion of the Paraguayan Stand-by agreement with the International Monetary Fund (IMF) in 2004 regarding the payment of its foreign debt.Therefore, the out-of-sample prediction period starts in January 2005.

The choice of short-term interest rates for the SP model was done not only following Frankel's methodology but also out of necessity: it is the only interest rate that has been consistently reported since 1994. I should also point out that these are interest rates on the bonds that the Central Bank of Paraguay (BCP) trades with Paraguayan private banks as a monetary policy tool. For the same reason, they are the interest rates used in the UIP model. On a similar note, the choice of Consumer Price Index (CPI) as a proxy for expected inflation was also done following Frankel and out of necessity: of all the possible proxies used by Frankel, it was the only one available. The proxy for monthly Paraguayan, Argentinian and Brazilian GDP were, respectively: the Monthly Economic Activity Index (IMAEP) produced by the BCP which tracks the performance of the most relevant Paraguayan industries; the Monthly Estimator of Economic Activity (EMAE) produced by the National Institute of Statistics and Census (INDEC) in Argentina; and, the Index of Monthly Monetary Activity (IBC-Br) produced by the Central Bank of Brazil (BCB). Finally, soy prices were included in the BMA model because they are the main Paraguayan export and the soy market is highly dolarized.

All the data pertaining to Paraguay, as well as oil and soy prices, were obtained from the Statistical Annex of the yearly Economic Report published by the BCP. M1 monetary aggregates, CPI and three-month Treasury bill interest rates for the US were retrieved from the Federal Reserve Bank of Saint Louis' Federal Reserve Economic Data (FRED). As for US monthly GDP data, they were obtained from the Macroeconomic Advisers data bank. The Brazilian data were taken from the BCB statistical bulletin, except the CPI which was retrieved from the Getulio Vargas Foundation site.

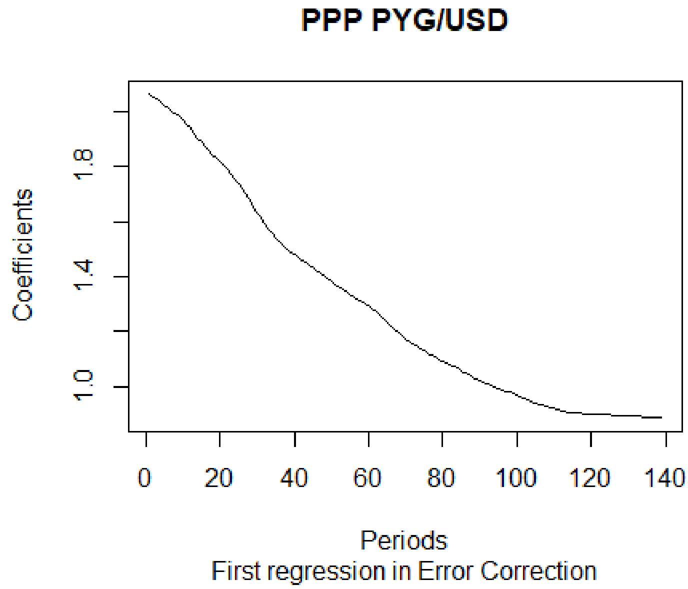
Argentinian data merit an observation. Its M1 aggregates were taken from the FRED, while interest rates came from the Central Bank of the Argentine Republic's (BCRA) site. But the CPI had to be constructed from four different sources: the original CPI series from INDEC (base year 2008), a second CPI series from INDEC (base year 2014), a third CPI series from INDEC (base year 2016) and a parallel series put together by the Argentine Congress, as the INDEC stopped producing its CPI series from November, 2015 to November, 2016. This series was retrieved from *Ambito.com*, the internet site which compiled it.

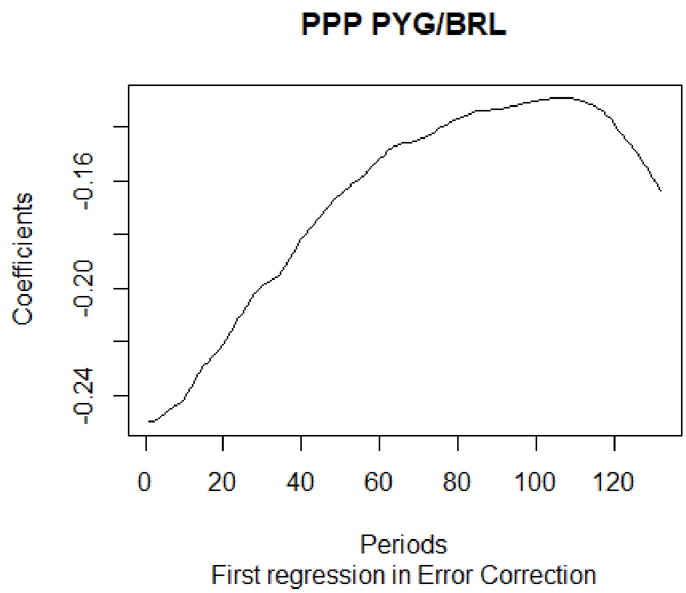
1.5 Results

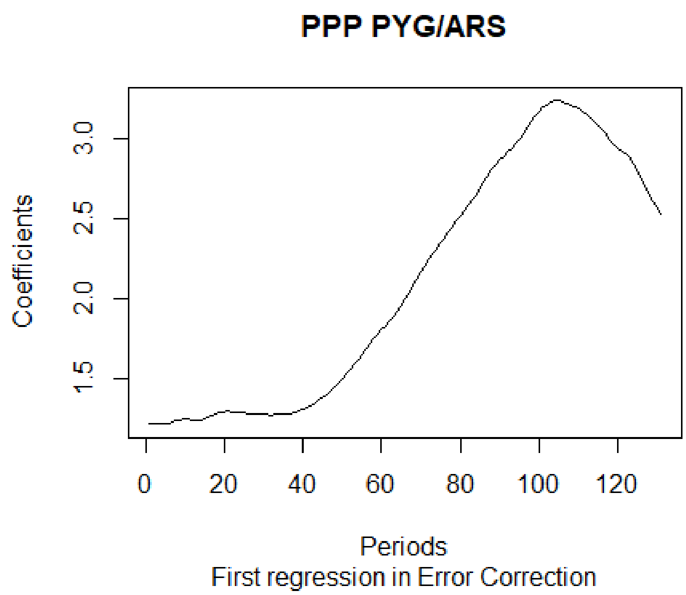
Coefficients in the PPP and UIP models

Before discussing the results themselves, the graphs below depict the behavior of the coefficients for the PPP and UIP models in the first regression of the error correction. As might be expected, the coefficients almost always differ from 1. The PPP coefficients for the PYG/USD exchange rate follow a neat downward trajectory ranging from 2.06 in the first period to 0.885 at its lowest in the last period. The coefficients for the PYG/BRL and PYG/ARS range from a low of -0.25 (in the first period) to a high of -0.13 (November, 2013) and from 1.22 (in the first period) to a high of 3.24 (September, 2015). The graphs also show that the latter two exchange rate coefficients behave somewhat similarly but differ greatly from the PYG/USD coefficients.

UIP coefficients for the PYG/USD exchange rate vary from a low of 0.039 (September, 2007) to a high of 0.17 in the first period. The PYG/BRL coefficients range from -0.022 (November, 2009) to 0.095 (March, 2007). As for the PYG/ARS coefficients, they hit a low of 0.163 (December, 2005) and high of 0.316 in the last period.






PPP PYG/ARS
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2.5
3.0
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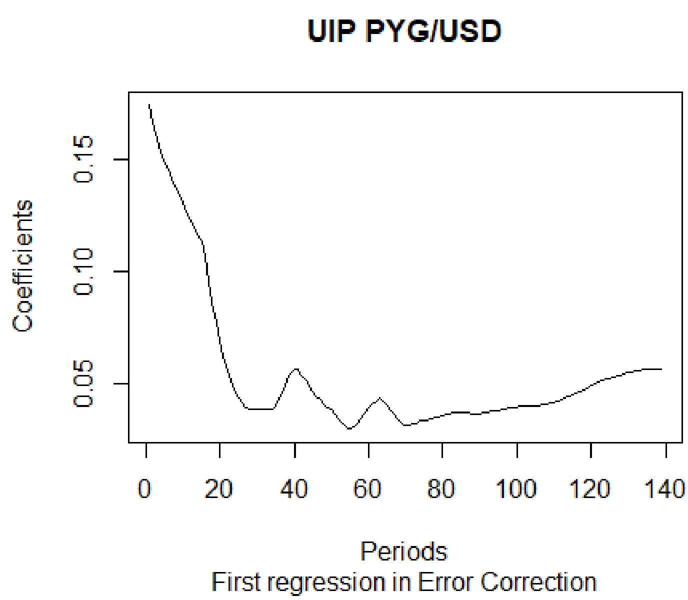
20
40
60
80
100
120
Periods
First regression in Error Correction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0.15
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80
100
120
140
Periods
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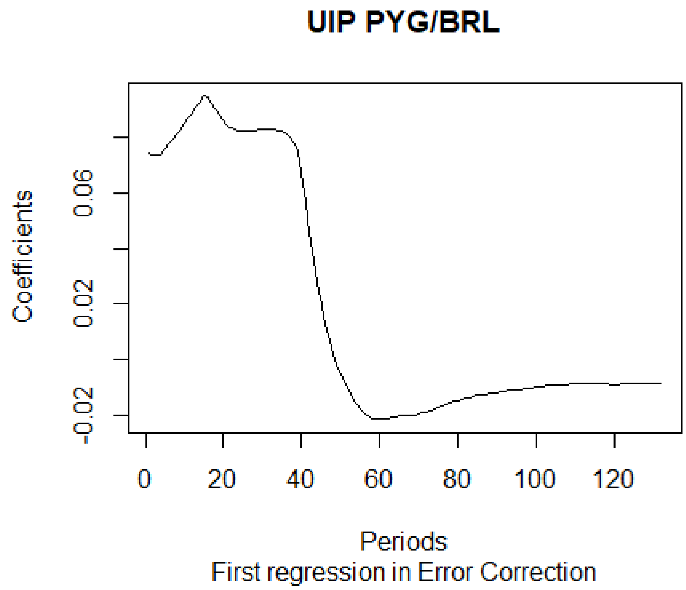

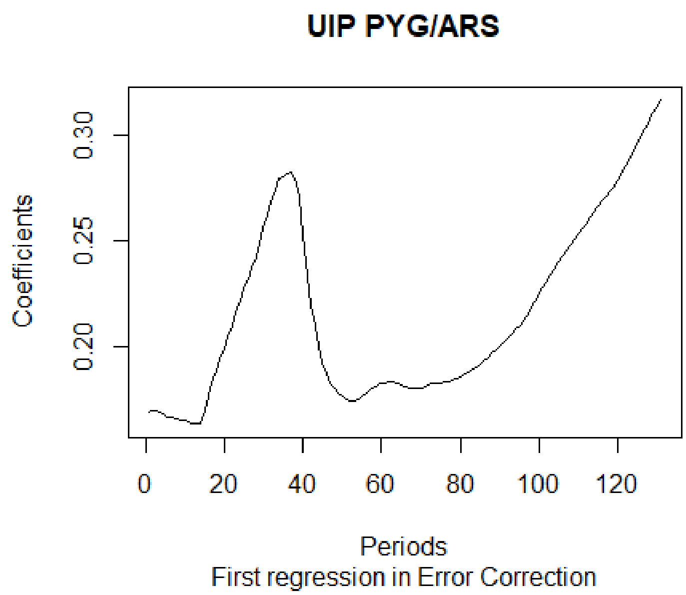

Comparing Models using RMSE

As discussed in section 3 I am using the ratio of the RMSE of each model to the RMSE of the random-walk in order to asses their performance. Tables 1, 2 and 3 show the RMSE ratios of the PYG/USD, PYG/ARS and the PYG/BRL, respectively. In tables 1 and 3 (US and Brazil) we can see that BMA and BVAR outperform all other models in the 3-month and 6-month forecasting horizons and UIP outperforms all other models in the following forecasting horizons. PPP appears to also do well in longer forecasting horizons.

In the case of PYG/ARS, results are quite different. PPP and SP outperform UIP and BMA, but BVAR in particular does exceedingly well. UIP, on the other hand, does not do better than the random walk in any of the forecasting horizons, in contrast with what is observed in tables 1 and 3.

Table 1.1: Ratio of Model's RMSE over Random Walk RMSE

	3 months	6 months	9 months	12 months
PPP	0.99386	1.02949	1.08196	1.15853
UIP	1.00108	0.98777	0.97286	0.95796
SP	1.04635	1.10185	1.16113	1.24188
BMA	0.88046	0.96462	1.02745	1.03807
BVAR	0.84261	0.97718	1.06516	1.12876

Table 1.2: Ratio of Model's RMSE over Random Walk RMSE

	3 months	6 months	9 months	12 months
PPP	0.95493	0.96508	0.95008	0.87946
UIP	1.00936	1.15046	1.20009	1.16285
SP	0.99463	0.91584	0.85797	0.81159
BMA	1.06341	0.96848	0.94650	0.93006
BVAR	0.73696	0.83135	0.81192	0.78303

Table 1.3: Ratio of Model's RMSE over Random Walk RMSE

	3 months	6 months	9 months	12 months
PPP	1.01075	1.00422	0.99794	0.99295
UIP	1.00860	1.00017	0.99258	0.98710
SP	1.09976	1.17235	1.25832	1.39325
BMA	0.93393	0.99465	1.03617	1.01704
BVAR	0.79494	0.95689	1.04981	1.11189

Comparing Models using DoC

Tables 4, 5 and 6 show the ratios of DoC for the PYG/USD, PYG/ARS and PYG/BRL, respectively. There are two ways in which results can be analyzed under this criterion. Assuming, as previously mentioned, that a random walk will correctly predict the direction of change half the time (an expected value of 0.5), then all models outperform it in the first and last forecasting horizons in table 4 (except BVAR in the last period), with BMA having the greatest ratio in the 3-month period and SP, in the 12-month period. In table 5, all models outperform the random walk in all forecasting periods, especially in the 12-month horizon where UIP and PPP give the highest proportions. In table 6, all models beat the random walk in the first period, but only BMA and BVAR outperform it in the second period, only BVAR in the third period, and in the fourth forecasting period, SP produces the highest proportion.

Notice though that if the actual random walk forecasts are taken as as reference, what the results say changes. Using this metric, in table 4 BMA and BVAR outperform all models in the first two forecasting horizons, BVAR and PPP in the third and all except BVAR do better in the 12-month horizon. In table 5 only PPP outperforms the random walk in the first three forecasting horizons, and BVAR, in the first and third horizons, and none in the last horizon. In table 6, in the first forecasting horizon BMA and PPP outperform the random walk; in the second, all except SP; in the third, SP and BVAR; and in the fourth, UIP, SP, and BMA.

Table 1.4: Ratio of Direction of Change

	3 months	6 months	9 months	12 months
RW	0.51471	0.42857	0.47692	0.52756
PPP	0.51471	0.43609	0.49231	0.53543
UIP	0.51471	0.42857	0.46154	0.53543
SP	0.52206	0.42857	0.46923	0.55906
BMA	0.56618	0.48120	0.44615	0.50394
BVAR	0.55147	0.48120	0.54615	0.46457

Table 1.5: Ratio of Direction of Change

	3 months	6 months	9 months	12 months
RW	0.54688	0.54400	0.52459	0.61345
PPP	0.55469	0.56000	0.54918	0.61345
UIP	0.56250	0.56800	0.51639	0.60504
SP	0.53125	0.57600	0.50000	0.60504
BMA	0.55469	0.53600	0.50820	0.54622
BVAR	0.57812	0.52800	0.54918	0.55462

Table 1.6: Ratio of Direction of Change

	3 months	6 months	9 months	12 months
RW	0.53488	0.42857	0.44715	0.48333
PPP	0.54264	0.44444	0.43902	0.47500
UIP	0.53488	0.43651	0.43902	0.49167
SP	0.51938	0.42857	0.46341	0.55000
BMA	0.56589	0.50794	0.42276	0.50833
BVAR	0.52713	0.53175	0.51220	0.43333

Comparing Models using the DM statistic

In order to see if the forecasts produced by some of the models are statistically significantly different and better than those produced by the random walk, these forecasts are compared using the DM statistic. PPP and SP statistics for the PYG/USD and PYG/BRL are not presented as they do not improve on random walk forecasts. In the case of the PYG/ARS exchange rate, all models and their corresponding tables are below as the behavior differs greatly from that of the first two exchange rates. Also, the forecasting periods for UIP now reach up to 48 months so as to show that as forecasting horizons become larger, forecasts produced by UIP are statistically significant improvements on the random walk.

In the long run, tables 7 and 10 show that by the 48th forecasting horizon, UIP forecasts are statistically significant improvements over the random walk forecasts at the 1% level for both the PYG/USD and PYG/BRL forecasts. In the three month horizon, BMA and BVAR produce forecasts that also improve on the random walk and are statistically significant at the 5% level for the PYG/USD exchange rate (Tables 8 and 9). In this same horizon, BMA does not produce a statistically significant improvement for the PYG/BRL exchange rate but BVAR does at the 1% level (Tables 11 and 12).

Tables 13 through17 show the DM statistics and p-values for the PYG/ARS exchange rate. They are the opposite of the results in the previous tables. Although PPP again does not do any better than random walk forecasts, UIP produces no improvement either (Tables 13 and 14). Instead, it is the SP model that produces the statistically significant improvements in the longer horizons (Table 15). BMA does not work well in the short run but BVAR does, both in the short run and the long run (Tables 16 and 17).

Table 1.7: Diebold-Mariano Statistic UIP PYG/USD

	DM UIP	p-value UIP
3 months	0.39654	0.65410
6 months	0.02622	0.51050
9 months	−0.24012	0.40510
12 months	−0.41955	0.33740
24 months	−0.90075	0.18390
36 months	−1.03720	0.14980
48 months	−4.40530	0.00001

Table 1.8: Diebold-Mariano Statistic BMA PYG/USD

	DM BMA	p-value BMA
3 months	−1.69310	0.04522
6 months	−1.15690	0.12370
9 months	0.24600	0.59720
12 months	0.38388	0.64950

Table 1.9: Diebold-Mariano Statistic BVAR PYG/USD

	DM BVAR	p-value BVAR
3 months	-1.73600	0.04128
6 months	0.03673	0.51460
9 months	0.97573	0.83540
12 months	1.16230	0.87740

Table 1.10: Diebold-Mariano Statistic UIP PYG/BRL

	DM UIP	p-value UIP
3 months	0.80299	0.78900
6 months	0.40647	0.65780
9 months	0.20580	0.58150
12 months	0.09330	0.53720
24 months	-0.80289	0.21100
36 months	-0.92069	0.17860
48 months	-3.93600	0.00004

Table 1.11: Diebold-Mariano Statistic BMA PYG/BRL

	DM BMA	p-value BMA
3 months	-0.79202	0.21420
6 months	-0.26083	0.39710
9 months	0.18570	0.57370
12 months	-0.03754	0.48500

Table 1.12: Diebold-Mariano Statistic BVAR PYG/BRL

	DM BVAR	p-value BVAR
3 months	-2.33640	0.00973
6 months	-0.42412	0.33570
9 months	0.85064	0.80250
12 months	1.06440	0.85640

Table 1.13: Diebold-Mariano Statistic PPP PYG/ARS

	DM PPP	p-value PPP
3 months	−0.11886	0.45270
6 months	1.06470	0.85650
9 months	0.89264	0.81400
12 months	0.24573	0.59710

Table 1.14: Diebold-Mariano Statistic UIP PYG/ARS

	DM UIP	p-value UIP
3 months	1.85080	0.96790
6 months	0.96790	0.97270
9 months	1.68280	0.95380
12 months	1.43750	0.92470

Table 1.15: Diebold-Mariano Statistic SP PYG/ARS

	DM SP	p-value SP
3 months	−0.10973	0.45630
6 months	−1.64470	0.05002
9 months	−4.19740	0.00001
12 months	−7.22450	0

Table 1.16: Diebold-Mariano Statistic BMA PYG/ARS

	DM BMA	p-value BMA
3 months	2.05080	0.97990
6 months	0.95237	0.82950
9 months	0.74075	0.77060
12 months	0.46682	0.67970

Table 1.17: Diebold-Mariano Statistic BVAR PYG/ARS

	DM BVAR	p-value BVAR
3 months	−2.66800	0.00382
6 months	−2.91120	0.00180
9 months	−4.22160	0.00001
12 months	−5.43310	0.0000000

Discussion

The above results are encouraging but also puzzling. The performance of BMA is consistent with Wright's findings in that BMA's forecasting power diminishes in longer forecasting horizons but can outperform the random-walk in the 3 and 6-month periods. This is particularly true in this study of the PYG/USD and PYG/BRL exchange rates, whose forecasts seem to behave similarly. This is also true of the forecasts produced by BVAR which in the short-run perform better than those produced by BMA, except at the 6-month horizon in the case of the PYG/USD exchange rate.

If we consider Lothian and Wu's findings that UIP performs well in longer time horizons, it is consistent that UIP outperforms all models in 9-month and/or 12 month-ahead periods (under RMSE) and beyond (where the improvement becomes statistically significant) in the US and Brazil case. These results are mostly congruent using the three evaluation criteria if under DoC the reference is the actual forecast produced by the random walk and not its assumed expected value. In the latter case, results are much less informative.

The case of Argentina is different and somewhat puzzling. Under the RMSE criterion, BMA and BVAR forecasting power gets better, not worse, in longer time horizons , and UIP never outperforms the random walk. However, under DoC and comparing actual forecasts BMA and BVAR forecasts improve on the random walk in the 3 month horizon, as expected. Here, again, UIP does not outperform random walk in the longer run. This is in direct contrast with the above-discussed results. One explanation may be that Argentina had a contentious history with inflation during the period under study and its monetary and fiscal policies were rather volatile.It is perhaps because of this that the model that focuses on prices and the monetary aggregates do better in forecasting exchange rates: it is what economic agents payed most attention to when dealing with the Argentinian Peso. Finally, the DM criterion evaluation is largely consistent with the RMSE criterion. UIP forecasts' improvement becomes statistically significant in the long-run and most of the short-run improvements produced by BMA and BVAR are also statistically significant.

Further Research

The present study could be expanded in different directions. First, the same models could be used to forecast the behavior of the PYG exchange rate with respect to other relevant currencies.

Another avenue of research could be the inclusion of other forecasting models and compare their performances to the benchmark model. Some possibilities include GARCH models (see for instance Pilbeam & Langeland (2015)) or Copulas (see Aloui et al. (2013)).

A perhaps more laborious possibility could involve the calculation of Divisia Monetary Aggregates for Paraguay and then include them in the pertinent models. Barnett and Kwag (2006) have already used these aggregates in several models to forecast the exchange rate between the US Dollar and the British Pound.

1.6 Conclusion

For the Paraguayan economy, exchange rates are a very important matter. Its main exports are traded in a highly dolarized market. Paraguay's largest neighbors and trading partners are Argentina and Brazil, and these exchange rates are indeed relevant. Therefore, the ability to forecast the PYG/USD, PYG/ARS and PYG/BRL exchange rates would be a powerful tool for both monetary and fiscal policy. In that spirit, it has been the objective of this paper to compare and assess the performance of five different models of exchange rate determination: PPP, UIP, SP, BMA and BVAR. In order to do so, I have used as criteria the RMSE and DoC ratios and the DM statistic.

The obtained results seem to indicate that, in the case of PYG/USD and PYG/BRL, UIP, BMA and BVAR forecasts improve on all other models, although neither produces the best forecast for every period: BMA and BVAR have more power in the shorter forecasting horizons and UIP in the longer ones. This is in line with previous studies, in particular Wright (2008) and Lothian and Wu (2011). Argentina's case is different but this may have to do with its recent economic history. Further research could shed more light on the forecastability of exchange rates in the different parts of the world.

Chapter 2

Exchange Rate Forecastability: Divisia, User Cost Price, and the Euro

2.1 Introduction

Forecasting exchange rates is very difficult. Although many economists have studied the matter and have found positive results, most of these have been later refuted or at least called into question. There is no one model that works in all circumstances and several authors have argued that none work. In particular, Meese and Rogoff (1983) presented compelling evidence that no model outperforms a driftless random-walk in forecasting exchange rates. Since then, researchers have had a hard time finding a convincing alternative. Lothian and Wu (2011) show that Uncovered Interest Parity (UIP) has remarkable forecasting power in longer time horizons; Wright (2008) argues that Bayesian Model Averaging outperforms the random walk in shorter time horizons. Even so, as recently as 2017, Cheung et al (2017) produced results that reinforced the idea that no model can consistently beat a random walk. None of the aforementioned studies, however, have taken the approach found in Barnett and Kwag (2006) where they use Divisia monetary aggregates and the User Cost Price within a structural model framework with great success in forecasting the US dollar/British pound exchange rate.

My objective with this paper is to extend Barnett and Kwag's experiment by applying it to the US dollar/Euro exchange rate (henceforth, USD/EUR). Just as in Barnett and Kwag's study, I employ Divisia monetary aggregates and the User Cost Prices calculated for both the Euro zone and the US in several structural models. In particular, Divisia monetary aggregates replace simple-sum monetary aggregates and the User Cost Price replaces interest rates in each model. I start by evaluating the performance of the Hooper-Morton (HM) model and then proceed to the Flexible Price Monetary model (FP), the Sticky Price (SP) model and UIP. The inclusion of UIP in this paper is another innovation with respect to the Barnett and Kwag study. I evaluate the performance of each model using the Root Mean Square Error (RMSE) ratio, Direction of Change (DoC), and the Diebold-Mariano (DM) statistic and compare each model's performance to that of the random walk, as per standard practice in the literature. Each of the aforementioned models is referred to as HMD, FPD, and SPD when it includes Divisia monetary aggregates and the User Cost Price, except UIP which becomes UIPUC, as is does not include Divisia aggregates.

In this study, as in Barnett and Kwag's, I use quarterly data and the forecasting periods are one through eight quarters ahead. I run the regression for each model twice: once with the original variables and once with Divisia monetary aggregates and the User Cost Price. Unlike Barnett and Kwag though, the more interesting results stem from the inclusion of UIP. More specifically, under the RMSE criterion, UIP with User Cost Prices (UIPUC) outperforms the random walk in every forecasting period, and according to the DM statistic, it does so at a statistically significant level from the fourth quarter onward. Results under the DoC criterion are much more ambiguous, as no one model accurately predicts the direction of change of exchange rates above the 50 percent threshold, consistently.

The rest of the paper proceeds as follows: in section 2, I discuss the previous literature related to exchange rate forecasting and Divisia Monetary aggregates; in section 3, I describe what the User Cost Price and Divisia monetary aggregates are and then refer to the models of choice; in section 4, I describe the data and their sources; in section 5, I present the results, briefly discussing them and I suggest possible further research; section 6 concludes.

2.2 Literature Review

Forecasting models for exchange rates have existed for decades. Purchasing Power Parity (PPP) and Uncovered Interest-Rate Parity (UIP) analyses and discussions can be found as far back as the sixties (see, for instance, Balassa (1964)). Dornbusch (1976) proposed a Sticky Price (SP) model based on monetary fundamentals and Frankel (1979) further developed this framework by emphasizing the role of expectations. Hooper & Morton (1982) extended this model to include current account balances. But almost immediately after that paper was published, Meese and Rogoff (1983) wrote a seminal study in which they convincingly argued that no exchange rate model can outperform a driftless random walk in out-of-sample forecasting. Since then, Mark (1995) proposed that at longer horizons a monetary fundamentals model could provide better out-of-sample forecasts. This model has been subject to criticism by Faust et al (2003) where the authors show that forecasting improvements only happen within two years and are eventually undone.

There have been, however, more recent attempts, which have shown more promising results. Lothian and Wu (2011) and Wright (2008) are two such cases. Lace et al. (2015) argue that the EUR/USD exchange rate can be determined by government yields in the short-run. More germane to the present study is, of course, Barnett and Kwag's (2006) work which shows that the use of Divisia monetary aggregates and the User Cost Price dramatically improve the forecasting power of structural models. In a similar vein, Ghosh & Bhadury (2018) show that Divisia Monetary aggregates are powerful indicators of exchange rate movements for several economies.

The User Cost Price and Divisia Monetary aggregates were derived by Barnett (1978, 1980) and resulted in many volumes of work on monetary aggregation theory and the practical application of these concepts to different areas of economic research. Some of the most important works in the literature have been collected in Barnett & Serletis (2000) and Barnett & Binner (2004). Reimers et al. (2002) found that Divisia aggregates for several countries in Europe have better out-sample-predicting power for the GDP deflator in the

Euro area. Similarly, Schunk (2001) showed that using Divisia aggregates improves the accuracy of US real GDP and GDP deflator predictions. Also, Binner* et al. (2005) finds there are strong indications that Divisia outperforms simple-sum aggregates in a non-linear framework when forecasting inflation for the euro.

2.3 Methodology

In this section, I first define what the User Cost Price and the Divisia monetary aggregates are. Then, I discuss the four different models that I have used to estimate the USD/EUR exchange rate forecasts and their respective specifications which are all variations of the more general Hooper-Morton model. For comparability and as this is an extension of previous work, I have mostly followed Barnett and Kwag in the methodology. I also explain how I have evaluated the performance of each model.

Divisia Monetary Aggregates and the User Cost Price

Since Barnett (1981, 1982), his groundbreaking work on microeconomic theory and aggregation theory, we know that the capital stock of money in a given time period is not equal to the monetary service flow (as capital goods do not fully depreciate in a period). The price of these monetary service flows is the opportunity cost, or user cost, of holding a particular monetary asset for that period. The User Cost Price then is the present value of however much interest an agent is not receiving because they are holding an asset, given that there exists a pure investment asset which provides a higher return and no monetary services. The User Cost Price is calculated thus:

$$\pi_{it} = (R_t - \gamma_{it})/(1 + R_t) \tag{2.1}$$

where γ_{it} is the return on asset i and R_t is the return on the pure investment, or benchmark, asset.

With the User Cost Price precisely defined, an aggregate for the monetary service flows can be elaborated which will track these flows correctly. For this purpose a Divisia index is used. For the construction of Divisia indices, let:

$$s_{it} = \pi_{it} m_{it} / \sum \pi_{jt} m_{jt} \tag{2.2}$$

where m_{it} is the nominal monetary asset i at time t. And so, the Divisia monetary index is:

$$\ln M_t - \ln M_{t-1} = \sum_{t=1}^{n} s_{it}(\ln m_{it} - \ln m_{it-1}) \tag{2.3}$$

Here M_t is the quantity index and s_{it} is defined as $s_{it} = 1/2(s_{it} + s_{it-1})$. From the above equation, one can see that the growth rate of the index is a weighted sum of each monetary asset i. Each i has a share in the User Cost and this is precisely its corresponding weight in the Divisia index. Finally, the accompanying User Cost Price index Π is defined as:

$$\ln \Pi_t - \ln \Pi_{t-1} = \sum_{t=1}^{n} s_{it}(\ln \pi_{it} - \ln \pi_{it-1}) \tag{2.4}$$

The idea here is that agents substitute toward holding the monetary assets which have the lowest relative user costs whenever there is a change in the own interest rate of another component monetary asset. This reflects how agents take into account opportunity costs in their decision process.

The Models

Hooper and Morton (1982) developed an exchange rate forecasting model which was based on previous models such as Dornbusch's (1976) Sticky Price model and the Flexible Price Monetary model by Frenkel (1976). The HM model includes the Current Account (CA) as an explanatory variable (its principal innovation). Thus, we have the following:

$$\begin{aligned} e_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(y_t - y_t^*) \\ + \beta_3(i_t - it^*) + \beta_4(p_t - p_t^*) + \beta_5 ca_t + \beta_6 cat^* + \nu_t \end{aligned} \tag{2.5}$$

where e_t is the exchange rate and m_t and m_t^*, y_t and y_t^*, i_t and i_t^*, and p_t, p_t^*, ca_t and ca_t^*

are, respectively, domestic and foreign money supply, domestic and foreign output, domestic and foreign interest rates, domestic and foreign current long-run expected rates of inflation, and domestic and foreign current account balances at time t.

The model specification involves an error-correction restriction and so as to avoid short-run dynamics. What this means is that the variation from the exchange rate is a correction of the deviation from a long-run equilibrium in the previous period. Taking the natural logarithms of all variables except the current account variable, the equation becomes the following:

$$\begin{aligned} lne_{t+h} - lne_t = \ & \alpha_0 + \alpha_1(lne_t - \beta_0 \\ & - \beta_1 ln\tilde{m}_t - \beta_2 ln\tilde{y}_t - \beta_3 ln\tilde{i}_t - \beta_4 ln\tilde{p}_t \\ & - \beta_5 ca_t - \beta_6 cat^*) + \epsilon_t \end{aligned} \tag{2.6}$$

Here $\tilde{m}_t$, $\tilde{y}_t$, $\tilde{i}_t$, and $\tilde{p}_t$ are domestic to foreign relative money supply, output and short-term interest rates, respectively, and h is the forecasting horizon. I should note that I have replaced long-run expected rates of inflation with the corresponding CPI indexes.

Notice that by setting $\beta_5 = \beta_6$=0, the model is reduced to the Sticky Price model; $\beta_4 = \beta_5 = \beta_6$=0 results in the Flexible Price Monetary model; and, $\beta_1 = \beta_2 = \beta_4 = \beta_5 = \beta_6$=0 is Uncovered Interest-Rate Parity.

Every one of the above models will be estimated twice: once with the variables as they have just been presented, and a second time with $\tilde{m}_t$ replaced by the Divisia index and $\tilde{i}_t$ replaced by the User Cost Price index. Here, the use of the User Cost instead of the interest rate follows Barnett et al (1984), with the understanding that $\tilde{i}_t$ is the opportunity cost of holding money in any form other than the pure investment asset. There is a total of eight models whose forecasting performance will be evaluated.

In this study I use a rolling regression in order to produce the predicted forecasts. I first pick an in-sample period for which the models are first estimated and then exchange rates are forecast for the out-of-sample period. The sample is then updated to the following period until there are no more out-sample-sample observations.

Out-of-Sample Performance Evaluation: Root Mean Square Error Ratio, Direction of Change and the Diebold-Mariano Statistic

I asses the out-of-sample performance of each model 1 through 8 quarters ahead by comparing each one to a benchmark model which in this case is the driftless random-walk given by:

$$\ln e_{t+h} - lne_t = \epsilon_t \tag{2.7}$$

Following Meese and Rogoff's methodology, I take the expectation of the random walk so that it becomes a martingale, i.e. the predictor of the exchange rate h periods ahead is whatever the exchange rate is at time t, or more simply:

$$\ln e_{t+h} = lne_t \tag{2.8}$$

In the first evaluation method, I use the root mean square error (RMSE) of each of the six models and divide it by the RMSE of the random-walk. A ratio of less than one indicates that the model is performing better than the random-walk and vice-versa.

The second method is the Direction of Change (DoC) ratio where I measure the proportion of times each model correctly predicts whether the actual exchange rate increases or decreases. Under the assumption that the expected value of random walk predicting the right DoC is 0.5, values above 0.5 indicate that a model is outperforming the random walk. Of course, the opposite is true as well. The higher the proportion is, the better the model is performing.

The third method is the statistic produced by Diebold & Mariano (1995), which allows for the comparison of forecasts in terms of whether the difference between two forecasts for the same forecasting period is statistically significant and whether the improvement is statistically significant (one forecast being "better" than another). If $g(e_{it}) = e_{it}^2$ is the loss function of a forecast error, the loss differential function is defined as $d_t = g(e_{1t}) - g(e_{2t})$. If

d_t is zero, then the forecasts under examination are equally accurate. Under the null, the expected value of d_t is zero. The DM statistic itself takes the form

$$DM = \bar{d}/\sqrt{2\pi \hat{f}_{d(0)}/T} \tag{2.9}$$

where $\bar{d}$ is the sample mean of the loss differential function and $\hat{f}_{d(0)}$ is a consistent estimate of the spectral density. Under the null, $DM \rightarrow N(0,1)$. The null is rejected if $|DM| > z_{\alpha/2}$.

2.4 Data

For this paper, the data are quarterly series of the different variables in the models starting in January, 2001 through January, 2017 for the USD/EUR exchange rate. Data for Divisia aggregates are only available starting in January, 2001 at the Bruegel Institute and so the series begins at that particular date. The in-sample period goes from that date until the first quarter of 2009 and the out-of-sample period starts in the second quarter of 2009 (i.e. the end of the Great Recession in the US).

For US data, I use 3-month Treasury Bill rates for the short-term interest rates, quarterly US GDP for output, US CPI as the price level, and the Current Account Balance. All of them were retrieved from the St. Louis Fed Federal Reserve Economic Data (FRED). The simple-sum M3 monetary aggregate comes from the Organization for Economic Co-operation and Development's (OECD) database. The User Cost Price and the Divisia M3 Monetary Aggregates for the US were taken from the Center for Financial Stability's website. As for the Euro, interest rates on Euro Area government bonds, Euro area GDP, Euro area CPI were taken from the FRED website. The Euro simple-sum M3 monetary aggregate also comes from the OECD database. The Current Account Balance was taken from the European Central Bank (ECB) database. The User Cost Price and Euro Divisia M3 monetary aggregates were taken from the Bruegel Institute database.

2.5 Results

Comparing Models using RMSE

As mentioned in the methodology section, I evaluate the performance of each model by taking the ratio of the RMSE of each model to the RMSE of the random-walk. Table 1 displays the RMSE ratios of the HM model and the HM model with Divisia and User Cost Prices (HMD) for every forecasting period. Similarly, tables 2, 3, and 4 display the RMSE ratios for the SP model and SP with Divisia and the User Cost Price (SPD), the FP model and FP with Divisia and the User Cost Price (FPD), and UIP and UIP with the User Cost Price (UIPUC), respectively. In tables 1 to 3 results are quite similar: models which include the Divisia index and the User Cost Price have a greater forecasting power than the models which use simple-sum aggregates and short-term interest rates. But none of these models outperform the random walk in any period (except for SP and FP in the first forecasting period). Table 4 shows a completely different story. Here UIPUC not only improves on the forecasting power of UIP but also outperforms the random walk in every forecasting period.

Table 2.1: Ratio of HM RMSE over Random Walk RMSE

	HM	HMD
1 quarter	1.00687	1.00371
2 quarters	1.06588	1.04105
3 quarters	1.12969	1.02284
4 quarters	1.20447	1.07642
5 quarters	1.23153	1.07526
6 quarters	1.29829	1.17720
7 quarters	1.31225	1.16461
8 quarters	1.35662	1.21225

Table 2.2: Ratio of SP RMSE over Random Walk RMSE

	SP	SPD
1 quarter	0.99828	1.00619
2 quarters	1.03722	1.03001
3 quarters	1.10472	1.03480
4 quarters	1.18508	1.06516
5 quarters	1.22500	1.06074
6 quarters	1.29515	1.15276
7 quarters	1.31373	1.14679
8 quarters	1.35479	1.17359

Table 2.3: Ratio of FP RMSE over Random Walk RMSE

	FP	FPD
1 quarter	0.99828	1.00619
2 quarters	1.03722	1.03001
3 quarters	1.10472	1.03480
4 quarters	1.18508	1.06516
5 quarters	1.22500	1.06074
6 quarters	1.29515	1.15276
7 quarters	1.31373	1.14679
8 quarters	1.35479	1.17359

Table 2.4: Ratio of UIP RMSE over Random Walk RMSE

	UIP	UIPUC
1 quarter	1.02207	0.98906
2 quarters	1.05736	0.98553
3 quarters	1.08836	0.96603
4 quarters	1.13401	0.94735
5 quarters	1.19455	0.94440
6 quarters	1.27315	0.97313
7 quarters	1.29771	0.97732
8 quarters	1.30771	0.96407

Comparing Models using DoC

Under the DoC criterion results are much more mixed and rather uninformative. Examining tables 5 to 8, proportions are not consistent for any of the forecasting dates. All four models outperform the random walk exceeding 0.5 threshold for some dates but fall dramatically below it for others. Table 9 is included simply to show that even the actual forecasts from the random walk also fail to perform as expected (only in the second forecasting period is the proportion 0.5).

Table 2.5: Ratio of Direction of Change HM vs. HMD

	HM	HMD
1 quarter	0.45161	0.41935
2 quarters	0.46667	0.40000
3 quarters	0.48276	0.55172
4 quarters	0.32143	0.32143
5 quarters	0.55556	0.55556
6 quarters	0.61538	0.50000
7 quarters	0.64000	0.44000
8 quarters	0.54167	0.50000

Table 2.6: Ratio of Direction of Change SP vs. SPD

	SP	SPD
1 quarter	0.41935	0.41935
2 quarters	0.53333	0.53333
3 quarters	0.51724	0.48276
4 quarters	0.35714	0.39286
5 quarters	0.44444	0.55556
6 quarters	0.61538	0.73077
7 quarters	0.60000	0.48000
8 quarters	0.54167	0.58333

Table 2.7: Ratio of Direction of Change FP vs. FPD

	FP	FPD
1 quarter	0.41935	0.41935
2 quarters	0.50000	0.46667
3 quarters	0.48276	0.41379
4 quarters	0.32143	0.46429
5 quarters	0.51852	0.48148
6 quarters	0.69231	0.65385
7 quarters	0.52000	0.60000
8 quarters	0.58333	0.45833

Table 2.8: Ratio of Direction of Change UIP vs. UIPUC

	UIP	UIPUC
1 quarter	0.45161	0.45161
2 quarters	0.53333	0.53333
3 quarters	0.48276	0.48276
4 quarters	0.39286	0.35714
5 quarters	0.37037	0.44444
6 quarters	0.65385	0.73077
7 quarters	0.40000	0.44000
8 quarters	0.62500	0.58333

Table 2.9: Ratio of Direction of Change RW

	RW
1 quarter	0.41935
2 quarters	0.50000
3 quarters	0.51724
4 quarters	0.39286
5 quarters	0.44444
6 quarters	0.61538
7 quarters	0.44000
8 quarters	0.58333

Comparing Models using DM

The DM statistic provides supporting evidence for the results found under the RMSE criterion. When comparing the forecasts produced by the models and those produced by the random walk, all models except UIPUC behave similarly to UIP as presented in Table 10. What this means is that DM statistics are all large and positive and p-values quickly converge to 1 (in the case of models without Divisia and User Cost Price) or get very close to one (in the case of models with Divisia and User Cost Price) as the forecasting horizons increase. UIPUC, though, is different. First, in every forecasting horizon, the DM statistic is negative and increasingly so. Second, p-values decrease in every period except for a small rise in the second period. Notice that by the third quarter, the p-value is barely above the 10% level. From quarters 4 to 6 p-values are below the 10% significance level; and in quarters 7 and 8, they are below the 5% significance level.

Table 2.10: Diebold-Mariano Statistic UIP vs. UIPUC

	DM UIP	p-value UIP	DM UIPUC	p-value UIPUC
1 quarter	0.64684	0.74110	−1.01440	0.15520
2 quarters	1.02390	0.84710	−0.91968	0.17890
3 quarters	1.21990	0.88870	−1.24240	0.10700
4 quarters	1.72250	0.95750	−1.53070	0.06292
5 quarters	2.30420	0.98940	−1.38790	0.08259
6 quarters	3.93230	1	−1.43680	0.07538
7 quarters	5.47870	1	−1.94320	0.02600
8 quarters	6.48360	1	−2.23270	0.01278

Discussion

In the previous section, the truly interesting results appear where the models are evaluated under the RMSE and DM criteria. There, it is remarkable to see that the UIPUC model "beats" the random walk in every forecasting period and that the improvement becomes statistically significant as the forecasting horizons extend into the future. What this would seem to indicate is that, on the one hand, agents are not just monitoring short-term interest rates but also the returns available on a variety of assets in different time periods; on the other hand, agents take into account the opportunity cost of holding assets - the foregone return. This is in line with Barnett's work and, by extension, with microeconomic theory. These results are also consistent with previous work showing that interest rates are good exchange rate predictors.

As mentioned before, under the DoC criterion there is no strong evidence for or against the use of any one particular model (including the benchmark model). In some periods, some models produce results which are above the 0.5 reference and in some they do not. Perhaps, certain changes in the methodological approach may result in more definitive results under this criterion.

Further Research

Perhaps the most obvious extension of the present study is the expansion of the same experiment to the other currencies for which there are Divisia monetary aggregates and User Cost Prices. Israel, Poland, and the United Kingdom all maintain publicly available Divisia indexes. This could potentially add to the evidence presented in this paper and elsewhere.

Another avenue which I am currently pursuing is the calculation of Divisia indexes for Paraguay. Once this is finished, the same methodology could be applied to see if results are consistent with the ones presented in this study.

One more option might be the use of Bayesian Model Averaging, following Wright (2008), to see if the Divisia index can also improve forecasts using a very different model than the structural ones used in this paper.

2.6 Conclusion

The European Union is the largest trading partner of the US and so, understanding the behavior of exchange rates, and forecasting them in particular, is clearly very important. It is also very, very difficult. As Meese and Rogoff convincingly argued in 1983, no structural model can consistently outperform a random walk when forecasting exchange rates. But these authors did not take into account the role of proper monetary aggregation or the opportunity cost of holding monetary assets. Barnett and Kwag showed that there is room for improvement precisely by taking the aforementioned variables into account. In that same spirit, I have followed their work to evaluate the forecasting power of four structural models applied to the USD/EUR exchange rate: Hooper-Morton, Sticky Price, and Flexible Price Monetary models, plus Uncovered Interest-Rate Parity. I have assessed their performance by comparing them to a driftless random walk using the RMSE, DoC, and DM criteria. I evaluated every model twice: one with the original variables and the second time replacing simple-sum monetary aggregates with Divisia monetary aggregates and interest rates with User Cost Prices.

Results under RMSE and DM are rather remarkable and certainly encouraging. Although none of the first three structural models improves on the benchmark model (the random walk), UIPUC - Uncovered Interest-Rate Parity with User Cost Prices - outperforms it in every forecasting period, stressing the role of interest rates and opportunity costs in the behavior of economic agents. These improvements become statistically significant starting in the fourth forecasting period and are more so as the horizon becomes larger. In contrast, under the DoC criterion, no model (including the benchmark model) seems to be any better than the other at forecasting. Further research could shed more light on the forecastability of exchange rates and the role of Divisia indexes and the User Cost Price.

Chapter 3

Divisia Monetary Aggregates for Paraguay

3.1 Introduction

Monetary policy must be conducted using the best tools available to policy-makers. In terms of monetary aggregation though, most central banks around the world still avail themselves of simple-sum monetary aggregates. But as Barnett (1978, 1980, 1981, 1982) showed in his groundbreaking research, the latter are flawed measures of money in a system as they assume that the different components of any aggregate are perfect substitutes among themselves. To account for this, Barnett first derived the User Cost Price of monetary assets which represent the opportunity cost of holding any particular asset. Then, bridging monetary and aggregation theory, Barnett produced the Divisia Index based on solid mircoeconomic foundations. The Divisia Index correctly tracks the monetary aggregator functions.

Divisia monetary aggregates have proven to be a very useful tool for any number of applications. For instance, Barnett et al. (1984) showed Divisia were favored over simple-sum aggregates in estimating demand for money; Belongia (1996) argues convincingly that conclusions regarding the effect of money on output or its relationship with the business cycle can change dramatically when using the correct monetary aggregates; Keating et al. (2018) find that a broad monetary aggregate (Divisia M4) could be used as an indicator of monetary policy so as to identify monetary policy shocks. With the mounting evidence in favor of the use of Divisia aggregates, several central banks have begun using them alongside simple-sum aggregates: The Saint Louis Federal Reserve, The Bank of England, the European Central Bank (ECB), the Bank of Israel, the Bank of Japan, the National Bank of Poland. Moreover, Divisia Indices have been calculated for well over 40 countries, including Peru, Brazil, and Uruguay.

The present study is a small contribution to the ever-growing literature on Divisia monetary aggregates. In this work, Divisia indices are calculated for the country of Paraguay. In particular, I calculate Divisia monetary aggregates for M2 and M3 measures. As there is no benchmark rate for M1, no Divisia aggregates are calculated for this measure. The objective is to compare the performance of Divisia versus simple-sum aggregates in estimating the demand for money. To do this, I use a Vector Error Correction Model (VECM) and an Engle-Granger Two-Step Error Correction Model (ECM).

The results in this study indicate that there is evidence in favor of using Divisia monetary aggregates when estimating money demand. They are more responsive and their correction process is faster. In particular, Divisia M3 seems to be the better aggregate after testing for autocorrelation, heteroskedasticity, normality and model specification. However, the User Cost Price and the interest rate spread appear to have no bearing in the behavior of economic agents. Low levels of bankization may be the reason for this.

The rest of the chapter is as follows: section 2 reviews of the pertinent literature; section 3 presents the methodology; section 4 describes the data used; section 5 describes the results; section 6 concludes.

3.2 Literature Review

As mentioned in the introduction, Barnett (1978, 1980) derived the User-Cost Price and then produced the Divisia Index. Following this seminal work, there have been any number of studies which have shown just how useful Divisia monetary aggregates are. For a survey of Barnett's extensive work on Divisia, see Barnett and Serletis (2000) and Barnett and Binner (2004).

Many more studies have shown how valuable Divisia is in later years. Reimers (2002) found that Divisia aggregates for several countries in Europe have better out-sample-predicting power for the GDP deflator in the Euro area than simple-sum aggregates. Similarly, Schunk (2001) showed that using Divisia aggregates improves the accuracy of US real GDP and GDP deflator predictions. Also, Binner (2005) finds there are strong indications that Divisia outperforms simple-sum aggregates in a non-linear framework when forecasting inflation for the Euro. Barnett and Kwag (2006) show how the use of Divisia monetary aggregates and the User Cost Price improves structural forecasting models. Barnett & Chauvet (2011) argue that Divisia aggregates would have helped predict the 2008 financial crisis. Also, Ghosh and Bhadury (2018) show that Divisia Monetary aggregates are powerful indicators of exchange movements for several economies.

3.3 Methodology

In this section, I first define the User Cost Price and the Divisia monetary aggregates that I have calculated for Paraguay. Then, I discuss the money demand equation that I have used to estimate the demand for money. The equation specification follows Barnett et al (1984). In order to evaluate the performance of Divisia versus simple-sum monetary aggregates, I use a both VECM and ECM. This section also mentions the pertinent tests performed before and after results from the two models.

Divisia Monetary Aggregates and the User Cost Price

Since Barnett (1981, 1982), groundbreaking work on microeconomic theory and aggregation theory, we know that just as capital goods do not fully depreciate in a period, the capital stock of money in a given time period is not equal to the monetary service flow . The user cost of holding a particular monetary asset for that period is the price of these monetary service flows, or the opportunity cost. The User Cost Price then is the present value of however much return an agent is not receiving because they are holding an asset, given the existence of a pure investment asset which provides a higher return and no monetary services. The User Cost Price is calculated in the following way:

$$\pi_{it} = (R_t - \gamma_{it})/(1+R_t) \tag{3.1}$$

where γ_{it} is the return on asset i and R_t is the return on the pure investment, or benchmark, asset.

With the User Cost Price precisely defined, an aggregate for the monetary service flows can be elaborated which will track these flows correctly. For this purpose a Divisia index is used. For the construction of Divisia indices, let:

$$s_{it} = \pi_{it} m_{it} / \sum \pi_{jt} m_{jt} \tag{3.2}$$

where m_{it} is the nominal monetary asset i at time t. And so, the Divisia monetary index is:

$$\ln M_t - \ln M_{t-1} = \sum_{t=1}^{n} s_{it}(\ln m_{it} - \ln m_{it-1}) \tag{3.3}$$

Here M_t is the quantity index and s_{it} is defined as $s_{it} = 1/2(s_{it} + s_{it-1})$. From the above equation, one can see that the growth rate of the index is a weighted sum of each monetary asset i. Each i has a share in the User Cost and this is precisely its corresponding weight in the Divisia index. Finally, the accompanying User Cost Price index Π is defined as:

$$\ln \Pi_t - \ln \Pi_{t-1} = \sum_{t=1}^{n} s_{it}(\ln \pi_{it} - \ln \pi_{it-1}) \tag{3.4}$$

The idea here is that agents substitute toward holding the monetary assets which have the lowest relative user costs whenever there is a change in the own interest rate of another component monetary asset. This reflects how agents take into account opportunity costs in their decision process.

The Model

The demand for money equation is the following:

$$\ln M_t^d/P_t = \beta_0 + \beta_1 \ln y_t + \beta_2 \ln OC_t + \epsilon_t \tag{3.5}$$

where M_t^d is M2, M3, Divisia M2 (DM2), or Divisia M3 (DM3), accordingly; P_t is the Consumer Price Index (CPI); y_t is output; and, OC_t (the opportunity cost) is either $R_t - i_t$ (the difference between the benchmark rate and the simple average of the returns on the different components of M2 or M3, that is an interest rate spread) or the User Cost Price. From here onward, UCM2 will be the opportunity cost associated with M2 and UCM3 will be the opportunity cost associated with M3. UCDM2 and UCDM3 will be the user cost prices for DM2 and DM3. Also, I will refer to the model with the M2 aggregate and UCM2 as the M2 model, the model with the M3 aggregate as the M3 model, and so on. Whether it is the VECM or the ECM will be clear from context.

Before evaluating the models, I first apply the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test to check for unit roots. Then, the Johansen test checks for cointegration. I use a VECM first and then an ECM. I check for autocorrelation, heteroskedasticity and normality using the Durbin-Watson (DW) and Breusch-Godfrey (BF or LM) tests, the White and Autoregressive Conditional Heteroskedasticity (ARCH) tests, and Jarque-Bera (JB) test, respectively. Finally, I use the Ramsey Reset test to evaluate the model specification.

3.4 Data

All the data pertaining to Paraguay were obtained from the Statistical Annex of the yearly Economic Report published by the Central Bank of Paraguay (BCP). These include simple-sum monetary aggregate M0 (simple cash), Checking Accounts, M1 (M0 + Checking Accounts), Savings Deposits, Term Deposits, Savings Certificates, M2 (M1 + Savings Deposits

+ Term Deposits + Savings Certificates) , Checking Accounts in US dollars (USD), Savings Accouns in USD, Term Deposits in USD, Certificates of Deposit in USD, M3 (M2 + Checking Accounts in USD + Savings Accounts in USD + Term Deposits in USD + Certificates of Deposit in USD), the corresponding interest rates, where applicable, CPI and the Monthly Economic Activity Index (IMAEP), which is used to calculate monthly GDP.

3.5 Results

Unit root tests

As previously explained, ADF and PP tests are applied to the variables in the model. The null hypothesis in both tests is non-stationarity. In the case of M2, M3, DM2, and DM3, the null of non-stationary is not rejected in levels under both tests. For UCM2, the null is rejected at the 10% (ADF) and 1% (PP) levels. For UCM3, non-stationarity is not rejected under ADF but it is rejected under under PP at the 5% level. In the case of UCDM2 and UCDM3, non-stationarity is rejected at the 1% level. Non-stationarity is not rejected for GDP under ADF but it is rejected under PP. For CPI, the null is not rejected using ADF but it is rejected using PP at the 10% level. In the case of the natural logs of real money balances (using simple-sum and Divisia aggregates), failure to reject the null applies to all except $DM2/CPI$ under PP. Finally, the null hypothesis is rejected at the 10% level under ADF and 1% under PP for the natural log of GDP. The null hypothesis is rejected for all variables in first differences. To err on the side of caution, I estimate the money demand in first differences.

Variable	ADF p-value	PP p-value
M2Agg	0.908	0.9644
M3Agg	0.9532	0.99
DM2Agg	0.9203	0.5021
DM3Agg	0.9536	0.9489
UCM2	0.0684	0.01
UCM3	0.3415	0.04507
UCDM2	0.01	0.01
UCDM3	0.01	0.01
Ln M2Agg/CPI	0.2025	0.4733
Ln M3Agg/CPI	0.5002	0.7901
Ln DM2Agg/CPI	0.1529	0.01
Ln DM3Agg/CPI	0.49	0.4458
GDP	0.3983	0.01
Ln GDP	0.09251	0.01

Johansen test

I perform the Johansen test with eigenvalue four times, using the Akaike Information Criterion (AIC) for the choice of lags. Each time, the monetary aggregate and the opportunity cost changes accordingly while GDP remains the same. That is to say, the test is applied to M2 and UCM2, M3 and UCM3, DM2 and UCDM2, and, DM3 and UCDM3. In all four cases, the null hypothesis that there are at most two cointegrating vectors is not rejected at the 5% level. The test statistics are 0.04, 0.27, 0.38, and 0.46 for a critical value of 8.18.

Vector Error Correction Model

In all four cases, the Error Correction Term (ECT) is negative and statistically significant at the 1% level. For M2 the ECT is -1.1014; for M3 the ECT is -0.8557; for DM2 the ECT is -1.8142; and, for DM3 it is -1.6182. They have different implications which are discussed below in the following section.

Two-Step Error Correction Model

Below are the coefficients from the ECM. The estimates for the logs of GDP, lagged GDP and the lagged monetary aggregates are all statistically significant. The *OC* coefficients are not however. Notice also that only in the ECM with DM2 and UCDM2 does the user cost have the expected sign.

ECM Summary for M2			
	Estimate	Std. Error	Pr(>|t|)
Intercept	-2.4197105	0.4701450	5.60e-07 (***)
$\Delta \ln GDP$	0.3387204	0.0316424	< 2e-16 (***)
$\Delta \ln UCM2$	0.0009853	0.0026768	0.713
$\ln GDP\ Lag1$	0.1926850	0.0371908	4.75e-07 (***)
$\ln UCM2\ Lag1$	0.0025639	0.0002152	0.229
$\ln M2/P\ \ Lag1$	-0.0759056	0.0154760	1.75e-06

ECM Summary for M3			
	Estimate	Std. Error	Pr(>|t|)
Intercept	-1.4651703	0.31355913	5.02e-06 (***)
$\Delta \ln GDP$	0.1724551	0.0219078	1.28e-13 (***)
$\Delta \ln UCM3$	-0.0014051	0.0016326	0.390
$\ln GDP\ Lag1$	0.1244075	0.0263746	4.11e-06 (***)
$\ln UCM3\ Lag1$	0.0010391	0.0007168	0.149
$\ln M3/P\ \ Lag1$	-0.541095	0.0123486	1.77e-0.5 (***)

ECM Summary for DM2			
	Estimate	Std. Error	Pr(>|t|)
Intercept	-5.27318	1.09050	2.40e-06 (***)
$\Delta \ln GDP$	0.96011	0.07325	< 2e-16 (***)
$\Delta \ln UCDM2$	-0.12413	0.32992	0.707
$\ln GDP\ Lag1$	0.43990	0.08820	1.19e-06 (***)
$\ln UCDM2\ Lag1$	-0.04001	0.58715	0.946
$\ln DM2/P\ \ Lag1$	-0.19219	0.03772	7.17e-07 (***)

ECM Summary for DM3			
	Estimate	Std. Error	Pr(>|t|)
Intercept	-2.24361	0.59542	0.000208 (***)
$\Delta \ln GDP$	0.46026	0.04508	< 2e-16 (***)
$\Delta \ln UCDM3$	0.08027	0.20018	0.688772
$\ln GDP\ Lag1$	0.20538	0.05307	0.000141 (***)
$\ln UCDM3\ Lag1$	0.28367	0.34285	0.408853
$\ln DM3/P\ \ Lag1$	-0.10231	0.02669	0.000162 (***)

Durbin-Watson and Breusch-Godfrey or LM Tests

The DW and LM tests produce a similar result in that the null hypothesis that there is no autocorrelation in the residuals cannot be rejected for both the ECM and VECM. The p-values associated with the DW and LM tests for the different ECM specifications are respectively: for M2, 0.45 and 0.978; for M3, 0.50 and 0.988; for DM2, 0.53 and 0.95; for DM3, 0.63 and 0.752. The DW and LM p-values associated with the VECM are respectively:

for M2, 0.388 and 0.983; for M3, 0.386 and 0.928; for DM2, 0.406 and 0.916; for DM3, 0.384 and 0.943.

ECM	DW p-values	LM p-values
M2	0.45	0.978
M3	0.50	0.988
DM2	0.53	0.95
DM3	0.63	0.752

VECM	DW p-values	LM p-values
M2	0.388	0.983
M3	0.386	0.928
DM2	0.406	0.916
DM3	0.384	0.943

Autoregressive Conditional Heteroskedasticity and White Tests

The ARCH test is applied to the VECM and rejects the null of homoskedasticity every time. For M2, M3, DM2, and DM3 the p-values are 0.0001794, 0.001708, 2.016e-10, 0.01518. The White test is applied to the ECM but produces the opposite result as the null is not rejected except in the case of M2. The respective p-values for M2, M3, DM2 and DM3 are 0.002801, 0.4295, 0.16, and 0.6256.

Models	ARCH p-values (VECM)	White p-values (ECM)
M2	0.0001794	0.002801
M3	0.001708	0.4295
DM2	2.016e-10	0.16
DM3	0.01518	0.6256

Jarque-Bera (JB) Normality Test

The joint hypothesis of skewness and kurtosis being zero is rejected at the 1% level every time when the JB test is applied to the VECM (p-values: 2.699e-08, <2.2e-16, <2.2e-16, 1.94e-05) but it should be noted that the null hypothesis that sknewness is zero is not rejected for M2 (p-value 0.1248) and DM2 (p-value 0.2371). When the same test is applied to the ECM, the null is rejected at the 1% level for M2 (0.008528), M3 (4.587e-07), and DM2 (1.134e-06). However, for DM3 the null hypothesis is rejected only at the 10% level (p-value 0.05638). Also, the JB test is known for large Type I errors in smaller sample-sizes.

Jarque-Bera	VECM	ECM
M2	2.699e-08	0.008528
M3	<2.2e-16	4.587e-07
DM2	<2.2e-16	1.134e-06
DM3	1.94e-05	0.05638

Ramsey Reset Test

The null hypothesis in this test is that the functional form of the model is correctly specified. The null is rejected for M2 (p-value 0.0004074), M3 (p-value 0.0807), and DM2 (p-value 0.001404). The null is not rejected for DM3 though (p-value 0.335).

Discussion

The ECTs in the VECM indicate that all error correction processes converge to the equilibrium, but they do so differently. The M3 ECT converges monotonically, while the other three do not. According to Narayan & Smyth (2006) an ECT that is between -1 and -2 converges rapidly to the equilibrium after a dampening fluctuation around the long-run equilibrium. Similarly, Loayza & Ranciere (2004) refer to an ECT no lower than -2 as a condition for dynamic stability. The implication is that the correction process is faster in the models which include Divisia monetary aggregates.

When the money demand is estimated with the ECM, again, Divisia monetary aggregates seem to be the most adequate as compared to simple-sum aggregates. However, neither the interest rate spreads nor the User Cost Prices are statistically significant. This may be due to the low level of bankization (the level of access to formal banking services) of the Paraguayan economy. According to a report by MF Economia, a private consulting firm, only 29% of adults in Paraguay have an account at a formal financial institution as of 2018. Also, cash, coins, and checking accounts (M0+M1) represent from 27% (February, 1998) up to 50% (December, 2007) of M3, the broadest monetary aggregate. It may be the case that economic agents simply do not pay as much attention to interest rates and/or are forced to keep higher amounts of cash on hand because of the low levels of bankization.

The different test results seem to indicate that the model which includes DM3 as the monetary aggregate performs best. Although the correction in the error correction process is slightly smaller than DM2, and it is not the most responsive aggregate in the ECM, the rest of the evidence indicates it is the better aggregate. It is the one closest to approach a normal distribution and the only model whose functional form appears to be correctly specified. Intuitively, this should not be surprising as DM3 includes all components of what is considered money, the complete array of returns on these components and, therefore, takes into account the substitution effect of more agents responding to changes on returns.

Further Research

As discussed in Chapter 2, Divisia aggregates might help in improving exchange rate forecasts. Beyond forecasts, Divisia could also allow for better prediction of other important indicators such a inflation and GDP. As a follow-up to the present work, further studies of the forecasting power of Divisia could be pursued.

3.6 Conclusion

Divisia monetary aggregates have been shown to improve on simple-sum aggregates in different academic studies since their appearance more than three decades ago. In the intervening time, Divisia aggregates have been calculated for several countries and central banks have begun to adopt them as part of their monetary policy toolkit.

The present work's objective has been to contribute to the ongoing research in the field of monetary aggregation by calculating the User Cost Price and Divisia monetary aggregates for the country of Paraguay and comparing their performance with, respectively, an interest rate spread and simple-sum aggregates, in the estimation of money demand with a VECM and an ECM.

The results in this study show an improvement in performance from the use of Divisia monetary aggregates as they are more responsive and correct more quickly than the simple-sum counterparts. The results also indicate that neither the interest rate spread or the User

Cost Price have a statistically significant effect on the economic agents' decision. This may be due to low levels of bankization in Paraguay. The different tests for autocorrelation, heteroskedasticity, normality and model specification give evidence in favor of the use of DM3, in particular.

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