ELASTICITY CELLS.

AN INVESTIGATION OF THE RELATION BETWEEN THE ENERGY LIBERATED BY A STRAINED METAL GOING INTO SOLUTION, AND THE POTENTIAL DIFFERENCE RESULTING.

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By

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After looking about for a subject suitable for experimental work to obtain material for a thesis it was decided to investigate the relation between the energy liberated by a metal going into solution, and the potential difference resulting.

From theoretical considerations Thomson deduced the following equation for the electro-motive force of a cell.

$$E = Z. H. J.$$

where Z is the electro-chemical equivalent of the metal, H is the heat of chemical combination of one gram of the metal entering into the electrolyte, and J is the mechanical equivalent of heat.

Later a more complete expression was developed by Helmholtz in the form

$$E = k + T\frac{\partial E}{\partial T},$$

where k represents the net amount of energy of chemical action which results from the passage of one c.g.s. unit of electricity through the cell. The last term expresses the dependence of the electro-motive force upon the temperature of the cell.

The object of this work is to test Thomson's formula experimentally.

Ordinarily there would be a certain definite amount of energy released as a result of a pure metal forming a salt. If, however, the metal is put under some kind of a uniform stress which increases its potential energy, necessarily, more energy must be released when it enters into the combination.

The stress could be obtained either by stretching or twisting, the latter method was chosen for this work. To make a theoretical determination of the increase of electro-motive force it should cause, the amount of potential energy added must be known, and since the metal goes into combination only at the surface, it is the energy in the surface layers which must be determined.

In order that the energy at the surface might be as uniformly distributed as possible, I chose to use round rods of as pure metal as was obtainable.

On the theoretical side the first step is to obtain an expression for the potential energy added to the surface layer per gram of the metal since it is this value which must be used in the formula.

Let n be the coefficient of simple rigidity of the metal. Its value is given by the formula

$$n = \frac{\text{Shearing stress}}{\text{Shearing strain}} .$$

The increase of potential energy per unit volume when a body is strained is

$$\text{P. E.} = \text{Work} = \tfrac{1}{2} \cdot \text{stress} \cdot \text{strain}$$

$$= \frac{1}{2}\,\frac{(\text{stress})^2}{n} .$$

The moment acting upon a twisted rod may be obtained by integrating, over the whole cross section, the moment acting upon a circular lamina.

That is

$$M = \int_0^R 2\pi n \frac{\theta}{l} r^3\,dr = 2\pi n \frac{\theta}{l} \int_0^R r^3\,dr$$

$$= \frac{\pi}{2} \cdot \frac{\theta}{l} nR^4$$

where l represents the length of the rod, θ the angle through which one end is twisted from its original position, r the distance from the center to any particular lamina, and R the radius of the rod.

$$R\frac{\theta}{l} = \text{surface strain} = \frac{(\text{surface stress})}{n}$$

$$\therefore \frac{\theta}{l} = \frac{(\text{surface stress})}{Rn}.$$

Substituting in the equation for the moment

$$M = \frac{\pi}{2}\ \frac{(\text{surface stress})}{Rn} \times nR^4 = \frac{\pi}{2}.(\text{surface stress})R^3$$

$$\therefore \text{surface stress} = \frac{2M}{\pi R^3}.$$

Substituting this in the equation for the potential energy per unit volume

$$PE(\text{in surface}) = \frac{1}{2}\frac{4M^2}{\pi^2 R^6 n} = \frac{2M^2}{\pi^2 R^6 n}.$$

This is the energy per unit volume. The value per gram is desired. Let d be the density of the metal.

Then

$$PE(\text{per gram}) = \frac{2}{\pi^2 R^6 n\ d} \times M^2.$$

This now gives the value of the potential energy per gram added to the surface layer of the metal, in terms of the total moment applied.

This value can now be substituted in Thomson's equation, dropping the term J, if the moment is already expressed in absolute units.

$$E = \frac{2Z}{\pi^2 R^6 n\ d} \times M^2.$$

The experimental side of the work will now be considered.

The apparatus used was very simple, but its final form was reached only after much experimenting.

Two copper rods as nearly alike as it was possible to obtain (except in length) were placed in a glass vessel containing an unsaturated solution of copper sulphate. The longer rod passed through a rubber stopper in the bottom of the vessel and was firmly fastened with a clamp. To the upper end of this rod was fastened a wheel of fifteen centimeters radius. By means of two cords fastened to the circumference of this wheel and passing over suitably arranged pulleys and terminated with light scale pans, the tortion of the rod could be varied at pleasure by the use of weights. It was so arranged that the weights could be changed without opening up the box containing the cell. A thermometer was inserted through the side of the box. The lead wires were connected to the tops of the rods and the junctions wrapped with cotton.

It was at first planned to connect the cell up directly with a sensitive galvanometer but this could not be done since the cell gave too great a "stray" electro-motive force, that is, the two apparently similar rods, neither under strain, were at different potentials when immersed in the copper sulphate. To neutralize this "stray" electro-motive force, a counter electro-motive force was placed in the circuit by the potentiometer method. The terminals of the cell and galvanometer circuit were soldered permanently to the potentiometer wire and then wrapped with cotton, the value of the counter electro-motive force being varied at will by varying the current through the wire. Edison primary cells were used in this circuit.

The galvanometer was carefully calibrated over its entire range and the current required to produce one millimeter deflection at the distance used was found to be 1.7 X 10^{-9} Amperes. In calibrating the galvanometer, the electro-motive force of an Edison cell was first carefully measured by means of a sensitive potentiometer and a standard Cadmium cell. A high known resistance was then connected to the Edison cell and the potential drop across a few ohms of this resistance furnished the electro-motive force for the galvanometer circuit in which another known (and variable) resistance was placed. It was difficult to adjust the galvanometer but when finally adjusted, it worked perfectly.

Many precautions were necessary in using the apparatus to obtain results since the electro-motive force to be measured was such a small quantity.

It was found that sunlight had a very unsteadying effect upon the electromotive force of the cell and so it was necessary to keep the cell carefully enclosed, and even then the effect was not entirely eliminated.

As far as possible, all connections were carefully wrapped in cotton to prevent thermal effects.

It was at first planned to stir the copper sulphate continuously so that there could be no differences of temperature in the cell itself, but the stirring caused such great variations that it was discontinued. The causes of this variation were not determined but may have been due to difference of concentration in the copper sulphate.

The method of operation was as follows:

With both circuits open the reading of galvanometer was noted. The potentiometer-wire circuit was then closed and the current regulated until upon closing the galvanometer circuit(which included the test cell) little or no deflection of the galvanometer was obtained. Then everything was left quiet for some minutes until the readings of galvanometer became steady or varied with a slow steady drift.

When everything was working steadily the adding and taking off of weights was begun and the corresponding deflections noted, the operations taking place at regular time intervals which were made sufficiently long to allow the galvanometer to become steady again before the next move was made. The galvanometer readings were taken just before the change.

It will be noted that the plan used is a combination of the potentiometer method and the direct deflection method, and it was adopted because neither one of the methods was sufficient in itself. The plan used has its draw-backs but seemed to be the best possible considering the relatively large "stray" electro-motive force with which it was necessary to deal.

The following table shows the most consistent data that was obtained. These were consecutive readings taken in the middle of long runs when everything was working smoothly.

Weight on each pan	900 grams		1200 grams	
	on	off	on	off
Deflections in cms.	7.70	8.50	9.45	10.90
	6.10	9.85	11.55	10.15
	6.45	8.85	10.00	11.75
	6.30	8.90	9.85	11.15
	5.35	8.15	9.00	9.90
	5.65	8.45	7.35	10.45
			8.60	9.80
			9.20	10.60

The figures in the "on" columns indicate the number of centimeters deflection which resulted from placing the indicated weights on the previously empty pans. Those in the "off" columns indicate the deflections (in the opposite direction) resulting when the weights were taken off. It will be noted that the readings in the "on" columns are usually less than those in the others. Where this occurs it indicates that there was a steadily changing value of electro-motive force either of the cell (aside from the change due to the strain and release) or of the counter electro-motive force, or a combination of the two. This causes little inconvenience however, because where the drift lowers the resulting deflection in the one case it increases the reading by a similar amount (if the drift is steady) in the other. Thus an average of all the readings both on and off gives the deflection to be used in computation.

The direction of the deflections was in each case such as to indicate that the strained rod was the negative element of the cell. This is in agreement with theoretical indications.

The average deflection for 900 grams on each pan is 7.52 cms. That for 1200 grams is 9.98 cms. The resistance of the circuit was 35 ohms. One mm. deflection of galvanometer indicates a current of 1.7×10^{-9} Amperes.

The change in electromotive force due to 900 grams on each pan is $75.2 \times 1.7 \times 10^{-9} \times 35 = 0.00000447$ volt. That for 1200 grams on each pan is

$$99.8 \times 1.7 \times 10^{-9} \times 35 = 0.00000594 \text{ volts.}$$

The theoretical values may now be determined from the equation previously developed.

$$E = \frac{2Z}{\pi^2 R^6 n d} \times M^2 \text{ abvolts}$$

or

$$= \frac{2ZM^2}{\pi^2 R^6 n d.\ 10^8} \text{ volts.}$$

$n = 4.54 \times 10''$

Radius of rod 0.326 cms. = R.

Radius of wheel 15 cms.

$Z = .003281$

$d = 8.92$

Substituting the various values and solving, the results are; for 900 grams on each pan, 0.000,000,958 volts; for 1200 grams, 0.00000170 volts.

	Experimental value.	Theoretical value.
900 grams on each pan	0.0000447 volts	0.000000958 volts.
1200 " " " "	0.0000594 "	0.00000170 volts.

A comparison of the experimental and theoretical values shows a decided lack of agreement between the two. As a result of these experiments then, the only conclusion which can be reached is that the theoretical and experimental determinations of the values from Thomson's equation do not agree.

It may be that this discrepancy is due to the omission of the term which Helmholtz included in his equation; this was not determined.

It is interesting to note that while the theoretical value of the electro-motive force varies as the square of the moment applied, the experimental value seems to bear a linear relation to the moment. Three-fourths of the value for 1200 grams gives 0.00000444 as compared with a value 0.00000447 actually determined. This close agreement was not noted until sometime after the representative values chosen for the two moments was determined.

In connection with these experiments much other data was taken, but the deflections resulting from the use of the smaller weights varied over such a range that it was thought adviseable not to use them. An average was made of all values obtained from weights of 300 grams on each pan, however, and this value fell decidedly below that which a linear relation would indicate. This is no agreement however with the results found in determining the coefficient of rigidity of the copper rod. It was found that the angular twist per 100 grams was decidedly less when the total weight applied was small, than what it was when the total weight was greater.

Too much weight should probably not be given to the results thus far obtained until they are verified by much more extensive and thorough investigation. Tests similar to the ones made but covering a larger number of intermediate values would give a better basis for conclusions. Moreover a determination of the value of the term $T\frac{\partial E}{\partial T}$ of the Helmholtz equation would be desirable. If the results thus obtained were then compared with results obtained by using rods of other materials such as zinc, iron and steel, the conclusions drawn from them should probably be given considerable weight. For obvious reasons this more elaborate investigation was impossible at the time.