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The New Divisia Monetary Aggregates

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Barnett's Divisia monetary aggregates were derived to be elements of Diewert's class of superlative quantity index numbers. Relative to aggregation theory, Barnett's resulting monetary aggregates are strictly preferable to the official sum monetary aggregates, since the component monetary assets are not perfect substitutes. Formal empirical tests based on the relevant aggregation-theoretic criteria have likewise uniformly favored the Divisia monetary aggregates. The current article compares the Divisia with the sum monetary aggregates relative to numerous conventional policy-relevant criteria. The Divisia monetary aggregates, especially at high levels of aggregation, usually perform best in these tests.

We have benefited from the comments of Robert Barro. We also have benefited from the comments on this research of participants at conferences at the N.B.E.R. (Cambridge, 1981), at the European Econometric Society meetings (Athens, Greece, 1979), at the North American Econometric Society meetings (Denver, 1980), at the Latin American Econometric Society meetings (Santiago, Chile, 1983), at the University of Aix-Marseille (Aix-en-Provence, France, 1980), and at the University of Chicago (1980, 1982). Some of the research on which this paper is based was conducted while we were at the Federal Reserve Board. The views expressed herein are solely ours and do not necessarily represent the views of the Board of Governors of the Federal Reserve System. The research on this paper was partially supported by National Science Foundation grant SOC 8305162 along with the Sam P. Woodson Memorial Centennial chair and the Janey Briscoe Centennial fellowship at the University of Texas at Austin.

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I. Introduction

Recently a rapidly growing line of research has appeared concerning the rigorous use of aggregation and index number theory in the construction of monetary aggregates based on Diewert's (1976) "superlative" class of quantity index numbers. Research on this application of the superlative class was motivated by and built on Barnett's (1980*a*) proposal and initial results.[1] Barnett (1980*a*, p. 38; 1981*a*, p. 221) proposed use of either the Divisia or Fisher ideal index for monetary quantity aggregation. Since all such index numbers in the superlative class move very closely together (and are in fact usually identical, to within round-off error, for monetary asset data), the choice among elements of the class is little more than arbitrary. The purpose of this article is to provide a quantitative assessment of the relative merits of the official (summation) versus Divisia monetary quantity indexes, constructed with the same components and component groupings. The sample for the comparisons consists of quarterly data from 1959 through the end of 1982.[2]

II. History and Objectives

Although many types of financial intermediaries and monetary substitutes have evolved over the past 30 years, most economists have placed little faith in broad monetary aggregates, since summation aggregation has long seemed inappropriate at high levels of aggregation over imperfect substitutes. Friedman and Schwartz (1970, pp. 151–52) clearly described the problem with the high-level official aggregates:

[1] Some of the foundations of this research are contained in Barnett (1978, 1980*a*, 1981*a*, 1982*b*, 1983*a*) and Barnett and Spindt (1979), and an overview of that literature is contained in Barnett, Offenbacher, and Spindt (1981). The first official source of the historical data, supplied for 1969–81, is Barnett and Spindt (1982), which is available on request at no cost from the Publications Services Department at the Federal Reserve Board in Washington. Data for 1979–83 are supplied in the app. of Barnett (1984*b*). The complete data series, continually updated to the latest month, are available to subscribers to the services of Data Resources Incorporated (DRI). Those series are located in DRI data bank USEMS/DATA under the series names JM1D, JM2D, and JM3D for Divisia M1, M2, and M3, respectively. Related research includes Hawtrey (1930); Chetty (1969, 1972); Friedman and Schwartz (1970); Bisignano (1974); Moroney and Wilbratte (1976); Barth, Kraft, and Kraft (1977); Donovan (1978); Offenbacher (1979, 1980); and Barnett (1980*b*). For a unified overview of all that literature, see Barnett (1984*a*). For a discussion of the recent behavior of the Divisia monetary aggregates, see Dorfman (1983).

[2] The original results in Barnett, Spindt, and Offenbacher (1981) used quarterly data ending in 1980. In the present paper we have updated all of the results except those in table 2 and Sec. VI*B*2. The computer programs needed to update those results were unavailable to us when we updated our other results.

> This [summation] procedure is a very special case of the more general approach. In brief, the general approach consists of regarding each asset as a joint product having different degrees of "moneyness," and defining the quantity of money as the weighted sum of the aggregate value of all assets, the weights for individual assets varying from zero to unity with a weight of unity assigned to that asset or assets regarded as having the largest quantity of "moneyness" per dollar of aggregate value. The procedure we have followed implies that all weights are either zero or unity. The more general approach has been suggested frequently but experimented with only occasionally. We conjecture that this approach deserves and will get much more attention than it has so far received.

By equally weighting components, aggregation by summation can badly distort an aggregate. For example, if one wished to obtain an aggregate of transportation vehicles, one would never aggregate by summation over the physical units of, say, subway trains and roller skates. Instead one would construct a quantity index (such as the Department of Commerce's many Laspeyres quantity indexes) using weights based on the *values* of the different modes of transportation. As another example, suppose the money supply were measured by the Federal Reserve's current highest level official aggregate, L, which contains most of the national debt of short and intermediate maturity. All of that portion of the national debt could be monetized without increasing either taxes or the "money supply," L, since the public would simply have exchanged securities for currency. However, the new Divisia index over the components of L would not treat this transfer as an exchange of "pure money" for "pure money." Instead Divisia L would rise at about the same rate as the hyperinflation in prices that we would expect would result from this action.[3]

The traditionally constructed high-level aggregates (such as M2 or M3) implicitly view distant substitutes for money as perfect substitutes for currency. Rather than capture only part of the economy's monetary services, as M1 does, the broad aggregates swamp the included monetary services with heavily weighted nonmonetary services. The result no longer resembles economists' concept of "money." Nevertheless, the need remains for aggregates that capture the contribu-

[3] On the other hand, the problems associated with policies that target very low-level aggregates result from the inability of such aggregates to internalize pure substitution effects occurring within the economy's transactions technology, since low-level aggregates aggregate over a small subset of the factors of production in that transactions technology.

tions of all monetary assets to the economy's flow of monetary services.

Regarding the simple sum (arithmetic average) index, Irving Fisher wrote over a half century ago that "the simple arithmetic average produces one of the very worst of index numbers, and if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose. . . . The simple arithmetic [index] should not be used under any circumstances" (1922, pp. 29, 36).

III. The Divisia Monetary Quantity Index

Let m_{jt} and π_{jt} be the quantity and price, respectively, of the jth component of an aggregate during period t. Törnqvist (1936), and subsequently Theil (1967), advocated the following discrete-time approximation to the continuous-time Divisia quantity index:

$$Q_t^* = Q_{t-1}^* \prod_j \left(\frac{m_{jt}}{m_{j,t-1}}\right)^{\frac{1}{2}(s_{jt}+s_{j,t-1})},$$

where $s_{jt} = \pi_{jt}m_{jt}/\Sigma_k \pi_{kt}m_{kt}$ is the expenditure share on component j during period j.[4] We shall refer to Q_t^* as the Divisia index (in discrete time), although it also frequently is called the Törnqvist index. Taking logarithms of each side, observe that

$$\log Q_t^* - \log Q_{t-1}^* = \sum_j \bar{s}_{jt}(\log m_{jt} - \log m_{j,t-1}), \tag{1}$$

where $\bar{s}_{jt} = \frac{1}{2}(s_{jt} + s_{j,t-1})$. The same index number results regardless of whether the exact aggregation-theoretic aggregate being approximated is the output of a utility function or of a production function. The aggregation-theoretic procedure for selecting the component assets is described in Barnett (1982*b*).[5]

Diewert (1976) has proved that the Divisia index lies within his class of "superlative" index numbers, which all are nearly identical numerically. In fact, as a quantity index, the Divisia index is by far the most widely used element of Diewert's superlative class, because of the index's numerous theoretical optimality properties. Those remarkable properties result from that index's simultaneous theoretical links

[4] For details regarding the Divisia index, see Barnett (1982*a*). Regarding aggregation over consumers, see Barnett (1981*a*, chap. 3).

[5] The procedure requires testing for blockwise weakly separable groupings of assets. Those tests require knowledge of both the quantity and the user-cost price of each asset that is to be considered as a possible element of a weakly separable group. The formula for the user-cost price of an asset is presented in the next section. A nonparametric approach to testing for blockwise weak separability is available from Pudney (1981).

with the Divisia line integral, the translog aggregator function, the Malmquist index, and the Konyus index (see, e.g., Diewert 1980, 1981; Caves, Christensen, and Diewert 1982*b*). Recently Caves, Christensen, and Diewert (1982*a*, p. 1411) have proved that the Divisia index is "superlative in a considerably more general sense than shown by Diewert. We are not aware of other indexes that can be shown to be superlative in this more general sense." Also observe that the growth rate of the index is a weighted average of the growth rates of the components. The weights are the share contributions of each component to the total expenditure on the services of all components. Because of the availability of such a natural interpretation and because of the index's optimality properties, we advocate use of the Divisia index to measure the quantity of money at all levels of aggregation, as first proposed by Barnett (1980*a*; 1981*a*, chap. 7).

In order to be able to use (1) for aggregation over monetary asset quantities, we need the price, π_{jt}, corresponding to each component quantity asset, m_{jt}. In economic quantity aggregation, the appropriate price of a component durable good is its user cost. Barnett (1978, 1980*a*) derived the user cost of a monetary asset and found that the current-period user cost, π_{jt}, of m_{it} is

$$\pi_{it} = \frac{p_t^*(R_t - r_{it})(1 - \tau_t)}{1 + R_t(1 - \tau_t)}, \tag{2}$$

where p_t^* is the true cost-of-living index, r_{it} is the own current-period holding yield on component i, R_t is the maximum available expected holding-period yield in the economy, and τ_t is the marginal tax rate. The corresponding real user cost is

$$\pi_{it}^* = \frac{\pi_{it}}{p_t^*}. \tag{3}$$

The holding period used in defining R_s must be the same as that of r_{is}, which is a short rate. Details of our procedure for measuring R_s are defined in Barnett and Spindt (1982), which also describes the data used for the r_{is}'s. A few of the more noteworthy details follow. The return on demand deposits is a version of Klein's (1974) competitive rate on demand deposits. The raw data on the other r_{is}'s are for various holding periods. Hence the unadjusted yield differentials, $R - r_{is}$, can reflect differences in term to maturity as well as differences in monetary services at the margin. However, Barnett's derivation of equation (2) requires all yields to be for the same holding period. As a result, all yields are converted to a 1-month holding-period basis by using the Treasury securities yield curve and the yield curve adjustment procedure developed for the Federal Reserve's FRB-Penn-MIT quarterly model (developed jointly by the Federal Reserve Board, University of Pennsylvania, and MIT). The certainty-equivalence theory on which (2) was based required that any risk premiums be left within the rate structure, if freedom from default risk ("store of value") is to be valued by the Divisia quantity index as a monetary service. We left any such risk premiums within the r_{is}'s, although the components of the existing monetary aggregates all possess very low default risk. If the components were to be selected properly by tests for blockwise weak separability (as described in n. 5), then it is conceivable that assets such as gold or equity shares could appear in the nested aggregates at some level of aggregation. In such cases, the expected holding-period yields, r_{is}, would depend nontrivially on expected capital gains and losses and expected transactions costs.

Observe that the "weights" are not the user costs, but rather the average shares, $\bar{s}_{jt}$, which depend jointly on all quantities and user costs. Those share weights were not acquired by an ad hoc weighting scheme (such as weighting by variances, bid-ask spreads, denominations, maturities, velocities, or turnover rates), but rather were derived directly from microeconomic aggregation and index number theory.[6] Also note that in equation (1) the user costs appear only in the share weights, $\bar{s}_{jt}$, and all factors in (2) except for $R_t - r_{it}$ cancel out of the numerator and denominator of each $\bar{s}_{jt}$. Hence in computing the Divisia monetary aggregates, we could view the user-cost price, π_{it}, as the opportunity cost, $R_t - r_{it}$, which measures the interest forgone by holding monetary asset i when R_t is available. However, an exception to that statement results from the fact that our implicit competitive rate of return on demand deposits is not taxed, while the other rates are. Hence it is not strictly true that all marginal tax rates cancel out of each $\bar{s}_{it}$. We expect that this problem is not of great empirical consequence, but in future work we plan to refine the Divisia monetary aggregates further by using results of recent work on average marginal tax rates, such as Barro and Sahasakul (1983).

IV. Granger Causality and Prediction Risk Reduction

In this section, we report the results of applying several standard tests of the Granger causality relation between money and income, where money is measured first by an official summation aggregate and then by the corresponding Divisia aggregate. We use three different empirical test procedures.

[6] See Barnett (1981*b*, 1982*a*). For a general nontechnical discussion of the properties and interpretation of the Divisia monetary aggregates, see Barnett (1983*b*). See Barnett (1984*a*) for a discussion of the various ad hoc weighting schemes (such as the linear regression index, the latent variables index, the so-called Fisher money stock index, M_Q, etc.).

The first procedure is advocated by Pierce and Haugh (1977). In the first two columns of table 1, we display the tail areas for Haugh's (1972) small-sample test against the hypothesis of independence. The results are provided for both 8- and 12-quarter symmetric lags.[7] In every instance, the hypothesis of independence between GNP and money, as measured by either the official or the Divisia aggregate, would be rejected by a test at the 5 percent significance level. However, each Divisia aggregate produced a smaller test tail area than the corresponding official aggregate. As a result, the tail area comparisons favor the Divisia aggregates.

The second approach is due to Sims (1972). The tail areas of the Sims test are presented in the third and fourth columns of table 1 for both directions of causality. We find that in all cases the hypothesis that money does not Granger-cause GNP would be rejected at the 5 percent significance level. However, the tail area for the official (sum) aggregate was less than that for the corresponding Divisia aggregate at each level of aggregation except at the highest level, L. The hypothesis that GNP does not Granger-cause money would be accepted at the 5 percent level with each of the monetary aggregates except for the official M1 aggregate. The tail area for the sum aggregate exceeded that for the corresponding Divisia aggregate at the two highest levels of aggregation, M3 and L.[8]

In the third procedure, we compute an approximate likelihood ratio test of the hypothesis that $C(L) = 0$ in the bivariate autoregression

$$\begin{pmatrix} Z_t \\ Y_t \end{pmatrix} = \begin{bmatrix} A(L) & B(L) \\ C(L) & D(L) \end{bmatrix} \begin{pmatrix} Z_t \\ Y_t \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}.$$

It is immediate from Granger's definition (without contemporaneous causality) that Z_t does not cause Y_t if $C(L) = 0$.[9] We assume that the orders of A, B, C, and D are each no more than eight. The tail areas of the test of the hypothesis that $C(L) = 0$ are presented in the fifth

[7] For details of the estimated ARIMA models, see Barnett, Spindt, and Offenbacher (1981).

[8] It is crucial to the Sims procedure to account for autocorrelation of the untransformed disturbances properly. We correct for serial correlation in those disturbances by using a general fifth-order autoregressive transformation. As a test of the assumed lack of autocorrelation of the resulting transformed disturbances, we computed the tail area of the test that the first eight autocorrelations of the transformed disturbance terms are zero. As seen from the tail areas displayed in table A1 in App. A, the hypothesis was accepted at the .05 level in all cases.

[9] This result follows from Feige and Pearce (1979). Simplifying along the same lines as Feige and Pearce (1979) to avoid the problems discussed by Pierce and Haugh (1977), we rule out instantaneous causality by assuming $b_0 = c_0 = 0$. The model is normalized by taking $a_0 = d_0 = 0$. Using common time-series notation (see, e.g., Granger and Newbold 1977, p. 6), A, B, C, and D are polynomials and L is the backshift (lag) operator.

TABLE 1

Tail Areas of Tests of Granger Causality from Money Measures to GNP

Monetary Measures	Tail Areas of Test				
	Haugh-Pierce Test		Sims Test*		
	8-Quarters Lag	12-Quarters Lag	GNP to Money	Money to GNP	Direct Test†
M1	.0000	.0000	.0130	.0038	.074
$M1^D$	.0000	.0000	.1277	.0171	.056
M2	.0222	.0113	.1143	.0014	.005
$M2^D$	.0105	.0082	.1959	.0040	.001
M3	.0303	.0251	.6950	.0058	.090
$M3^D$	.0267	.0243	.4023	.0088	.021
L	.0152	.0175	.0901	.0047	.079
L^D	.0016	.0080	.0870	.0029	.002

Note.—Sample period, quarterly observations: 1959:1–1982:4. The superscript D designates a Divisia aggregate. The other aggregates are sums.
* Degrees of F-statistic = 8, 90.
† Df of χ^2 statistic = 8.

column of table 1.[10] The hypothesis that money does not Granger-cause GNP would be rejected at the 5 percent level with the Divisia aggregates at all levels of aggregation except at the lowest, M1, level, at which the tail area was a marginal .056. The same hypothesis would be accepted at the .05 level with the official summation aggregates at all levels of aggregation except M2. In addition, the tail area of the test for the Divisia aggregates was lower than that for the corresponding sum aggregate at all levels of aggregation. These tail area comparisons favor the Divisia aggregates.

Table 1 does not reveal a single uniformly best aggregate. In terms of 5 percent significance levels, sum M2, Divisia M2, Divisia M3, and Divisia *L* produce successful test results in all cases. In terms of the test tail areas, none of those three aggregates uniformly dominates, although Divisia *L* is most frequently best. While Divisia M1 was unsuccessful in one causality test at the .05 level, the failure was only marginal with a tail area of .056. Otherwise Divisia M1 also did well in these tests.

Because Granger-causality tests are tests of "incremental information content" (Schwert 1979, p. 82), causality results have an important bearing on the usefulness of monetary aggregates as indicators. When a monetary aggregate Granger-causes some variable of policy interest, such as income, the prediction variance of that variable can be reduced by conditioning on measurements of the monetary aggregate. Barnett and Spindt (1979) and Barnett, Spindt, and Offenbacher (1981) have found that the Divisia monetary aggregates are usually better than the corresponding official sum aggregates as indicators of a variety of policy target variables. Direct estimates of the proportionate reduction in prediction risk achievable by using Divisia and sum M2 as indicators in the context of the FRB-Penn-MIT quarterly model are presented in table 2. Prediction risk is measured by the generalized variance of the model forecast errors for the target variables. In the indicator use of the monetary aggregates, the unconditional model forecast is revised to a conditional forecast given the indicator measurement along the lines detailed in Tinsley, Spindt, and Friar (1980, p. 67). It can be seen from table 2 that considerable reductions in prediction risk are achievable by policymakers using Divisia M2 as an indicator.

V. Velocity

A substantial controversy has arisen in the literature regarding a "shift" in the demand-for-money function that frequently is pur-

[10] See Wall (1974) for a discussion of the test procedure.

TABLE 2

PROPORTIONATE REDUCTION OF PREDICTION RISK WITH RESPECT TO INCOME, PRICES, AND UNEMPLOYMENT: FRB-PENN-MIT QUARTERLY MODEL FORECASTING

MONETARY AGGREGATE	$I_{\mathbf{Y}_t \mid Z_t}$ BY DEFINITION OF $\mathbf{Y}_t$			
	Univariate			
	$\dot{x}^*$	$\dot{p}^*$	u^*	Multivariate
M2	.120	.049	.000	.156
$M2^D$	.349	.163	.020	.389

NOTE.—The superscript D designates a Divisia aggregate. $M1^D$, $M3^D$, and L^D are not yet available on the FRB-Penn-MIT quarterly model. Also $\dot{x}^*$ here is the growth rate of nominal GNP, $\dot{p}^*$ is the rate of change in the GNP deflator, and $\dot{u}^*$ is the total unemployment rate. Sample period, quarterly observations: 1969:1–1979:4.

ported to have occurred in the middle of 1974.[11] Over the past decade, interest rates have generally been rising. Hence the opportunity cost (user cost) of holding money has been rising. Under those circumstances, tests of functional stability can be deceptive. Conventional tests of functional stability are most useful when explanatory variable values are replicated or at least lie within the same region both before and after the potential shift period. When explanatory variables are continually moving into new regions, it can be very difficult to separate function shift from specification error.

Suppose, for example, that the true demand for-money function is nonlinear in its variables and parameters but has never shifted. Then the parameters of the best linear approximation will differ over different regions of the space of explanatory variables. If the explanatory variables move into new regions of that space as time passes, then tests of functional stability of a linear approximation are likely to reject stability. Similarly, if the parameters of the function are estimated over one period of time and the estimated function is then used in a dynamic simulation over another time period, the function is likely to track poorly. Fortunately, however, the existence of interest-rate cycles over the past decade has resulted in approximate replication regions that can be explored. In this section we present velocity cross plots permitting investigation of the ability of the economy's true demand-for-money function to replicate. If the true function that generated the data over one time period cannot replicate the data

[11] See Enzler, Johnson, and Paulus (1976); Goldfeld (1976); Tinsley and Garrett, with Friar (1980); and Simpson and Porter (1981). This problem has been most heavily investigated at low levels of monetary aggregation, but it appears to arise also at higher levels of aggregation. This feature of the problem is most troublesome, since it suggests that the "shift" is a result not of explainable substitution within the money market, but rather of a true shift in the economy's transactions technology.

over another time period with similar explanatory variables values, then we can validly conclude that the demand-for-money function has shifted.

The following exploratory data analysis relates velocity to various commonly used opportunity-cost variables. Three different plotting symbols are used to differentiate between data from three different time periods: (1) data after the purported shift, (2) data from the 5 years preceding the purported shift, and (3) data from the prior decade. We seek to determine whether velocity data acquired from different time periods tend to replicate when the potential explanatory variable retraces the same region during the different time periods. The explanatory variables we consider are the Divisia user cost and Moody's AAA corporate bond rate.

Figures 1 and 2 plot sum M3 against the two potential explanatory variables. Strong evidence exists of a shift in the relationship between velocity and either of the two opportunity-cost measures. Of course the reason could be additional omitted variables. But a stable relationship between velocity and any one of the opportunity-cost variables taken alone does not appear to exist, since the function that generated the data after 1974 does not appear to be able to replicate the earlier data in the middle region of overlapping opportunity-cost values.

Figures 3 and 4 plot Divisia M3 against the same two potential explanatory variables. No clear evidence of function shift remains. For example, with ordinary-least-squares linear regression of Divisia M3 on Moody's AAA corporate bond rate, the tail area of the Chow test of the hypothesis of no parameter shift in the middle of 1974 is .1337. The tail area of the corresponding test with sum M3 is .0027. The result is substantially more favorable for the Divisia than for the official aggregate.[12]

Although not reported here, analogous results were found with the 6-month Treasury bill rate and the 5-year government note rate (see Barnett, Spindt, and Offenbacher 1981). As space is limited, we display only M3 plots in this section and in Section IX below, although the results acquired with M2 and L are similar. The available empirical evidence, including the causality tests in the last section, suggests that Divisia L is the potentially most interesting aggregate. In addition, Divisia L is the aggregate that comes closest to capturing the contributions of all elements of the money market to the economy's monetary service flow. However, updates for Divisia L are not currently available with the same frequency as for Divisia M3, and hence Divisia L is of less potential policy usefulness than Divisia M3 at present. In any case, the plots for L are very similar to those displayed

[12] A first-order autoregressive error structure was used.

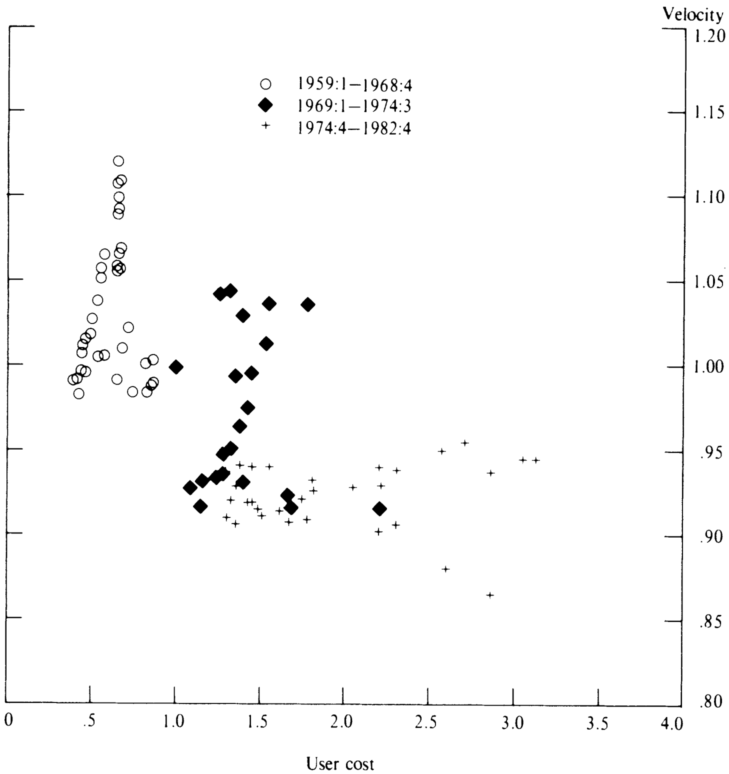

FIG. 1.—Sum M3 velocity versus Divisia user-cost aggregate, quarterly, 1959:1–1982:4.

here for M3, as demonstrated in Barnett, Spindt, and Offenbacher (1981). At the M1 level, as might be expected, the plots for the Divisia aggregate do not differ from those of the sum aggregate by as much as at the higher levels of aggregation.[13]

VI. Money Demand Functions

A. *Specification*

In this section we compare the official monetary aggregates with the Divisia monetary aggregates in terms of the resulting properties of estimated demand-for-money equations. When using the official aggregates, we use the conventional specifications that were used in the literature on demand-for-money function shifts. When using the Divisia aggregates, we use the analogous specifications appropriate to

[13] Plots at all levels of aggregation, for both this section and Sec. IX, are available from the authors on request.

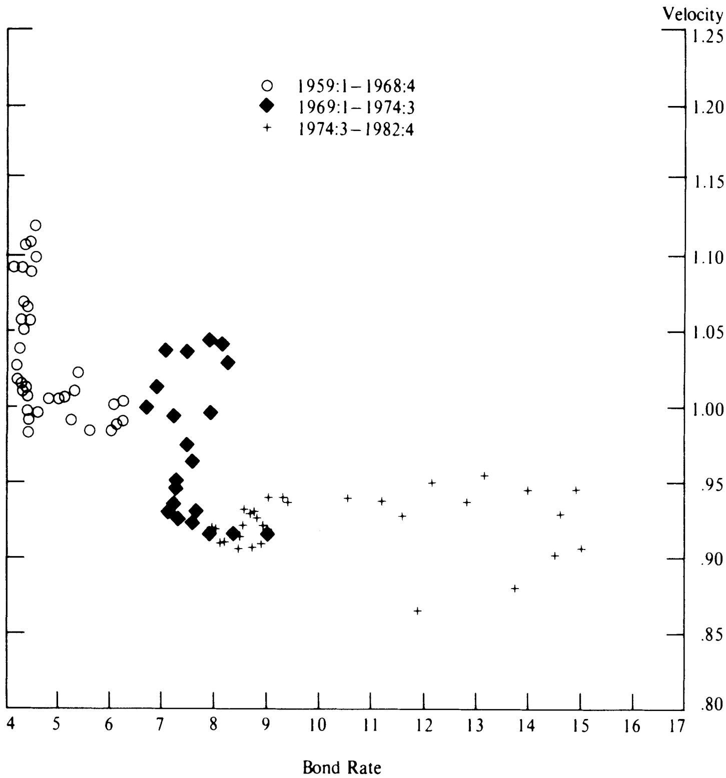

FIG. 2.—Sum M3 velocity versus Moody's AAA corporate bond rate, quarterly, 1959:1–1982:4.

Divisia aggregation. We explore plausibility of the estimates, parametric stability, and simulated forecasting accuracy.

We use the double log specification that has appeared widely in the literature on "shifting" demand-for-money functions:[14]

$$\log\left(\frac{M_t}{p_t^*}\right) = \alpha_0 + \alpha_1 \log\left(\frac{M_{t-1}}{p_t^*}\right) + \alpha_2 \log\left(\frac{Y_t}{p_t^*}\right) + \sum_{k=1}^{K} \alpha_{2+k} \log(OC)_k, \tag{4}$$

[14] All variables are quarterly averages, and all variables except interest rates are seasonally adjusted. Similarly, the Divisia aggregates are constructed from seasonally adjusted quantities and unadjusted interest rates. While this functional form is well known, two aspects of the precise specification require clarification. First, the lagged value of money, which appears as a predetermined variable, is deflated by the current value of the price level to allow for partial adjustment of nominal balances to price-level shocks in the short run, while maintaining long-run linear homogeneity of M_t in (Y_t, p_t). Regarding such double log inventory models, see Baumol (1952), Tobin (1956), and Fisher (1978).

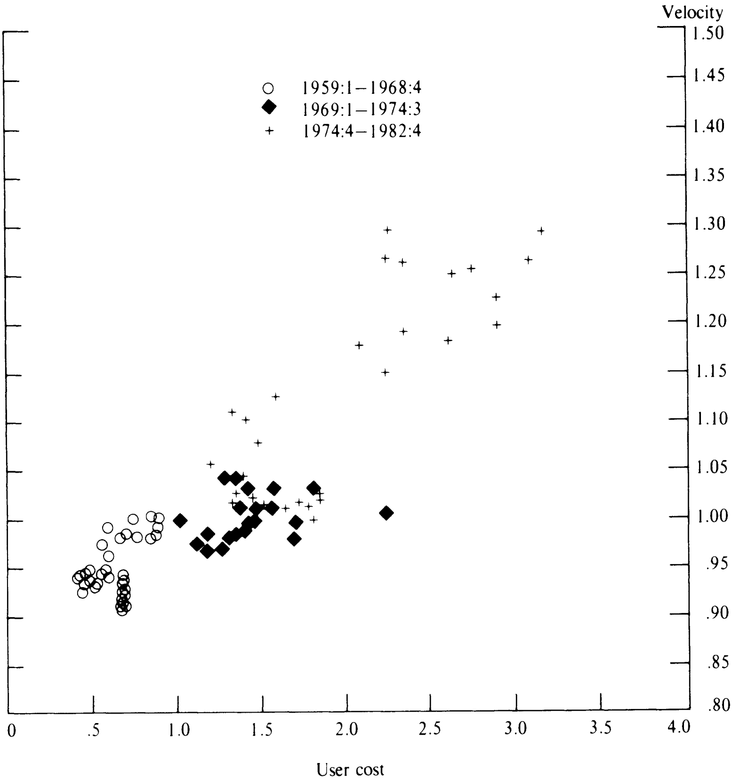

FIG. 3.—Divisia M3 velocity versus Divisia user-cost aggregate, quarterly, 1959:1–1982:4.

where M_t = a given per capita Divisia or simple sum monetary aggregate, p_t^* = GNP price deflator, Y_t = nominal per capita GNP, K = total number of opportunity-cost variables (one or two), and $(OC)_k$ = kth opportunity-cost variable (interest-rate or Divisia user-cost aggregate). The opportunity-cost variables differ among the different equations. When M_t is a summation aggregate, the opportunity-cost variables are interest rates: the commercial paper rate (as a representative market interest rate) and, with M1, the commercial bank passbook rate. When M_t is a Divisia index, all opportunity-cost variables are Divisia user-cost indexes acquired from Divisia price aggregation over the real user costs, π_{it}^*, of equation (3). The Divisia user-cost index is computed for the own price of M_t and, with M1, for a competing aggregated good. The competing opportunity-cost variable is the Divisia user-cost aggregate for those assets that are in L but not in M3.

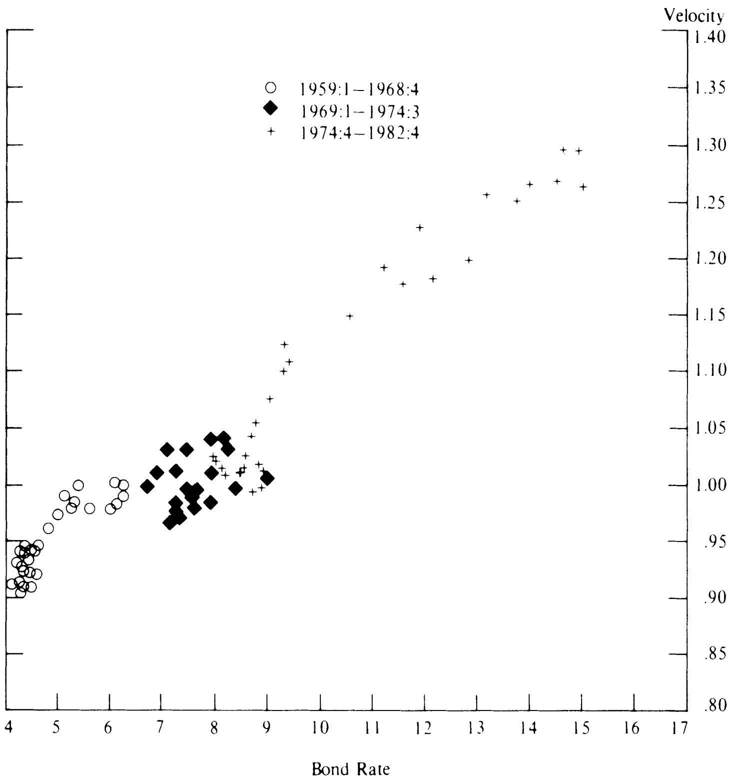

FIG. 4.—Divisia M3 velocity versus Moody's AAA corporate bond rate, quarterly, 1959:1–1982:4.

B. *Empirical Results*

The first section below describes the results of estimation with fixed coefficient methods. In the following section, we use stochastic coefficient methods to explore parametric stability.

1. Forecasting

The parameter estimates for equation (4) for both the Divisia and sum aggregates are found in Appendix A.[15] The estimates seem reasonable by conventional standards. Usually the coefficients on real GNP and on the opportunity-cost variables are statistically significant, have the correct sign, and are of reasonable magnitude. Two exceptions are the wrong signs acquired for the coefficient of real GNP with

[15] A first-order autoregressive error structure was used.

Divisia M2 and Divisia L during 1974:3–1982:4.[16] No such exceptions occurred during 1959:3–1974:2. In the literature on money demand, the most troublesome parameter to interpret has always been the coefficient on the lagged dependent variable (LDV), which has almost always implied unreasonably slow speeds of adjustment. While Divisia aggregation has not totally resolved this problem, the coefficient on the LDV is lower for the Divisia aggregates than for their official sum counterparts in eight of the 12 cases. The eight cases include all four levels of aggregation during the 1959:3–1974:2 sample period. The coefficient on the LDV exceeds 1.0 in three cases, all using the official sum aggregates. With both the Divisia and sum aggregates, the coefficient estimates were most plausible during the 1959:3–1974:2 sample period.

The predictive performance of each equation with its parameters estimated using the 1959:3–1974:2 sample period is summarized by the results that appear in the first two columns of table 3. The first of the columns reports the root mean square error (RMSE) for the predicted growth rates implied by each estimated equation over the period 1974:3–1980:4. The second column reports the mean error of the same growth-rate predictions.[17] The simulation results uniformly favor the Divisia aggregates. At all levels of aggregation, both the RMSE and the absolute value of the mean error for the Divisia forecasts are lower than for their sum counterparts.

2. Parametric Stability

In order to explore parametric stability, we estimate equation (4) with stochastically varying coefficients. We permit the coefficients to be stationary stochastic processes and use the Swamy and Tinsley (1980) asymptotically efficient estimation procedure. With this technique, it is necessary to use the same specification for equation (4) with both the Divisia and official sum aggregates, in order to assure comparability of the coefficient time paths (and test statistics) between results with both sets of aggregates.[18] To avoid prejudicing the results in

[16] Also note that the own-price elasticity of the demand for Divisia M3 has the wrong sign during 1959:3–1982:4, and the coefficient of $(OC)_2$ is statistically insignificant for both Divisia and sum M1.

[17] Growth-rate forecast errors are obtained by simulating each equation dynamically. Starting in 1974:3, we obtain predicted levels of the relevant aggregate and then compute the predicted growth rates. The estimation sample period was ended and the forecasts begun in mid-1974, since the widely reported shift in the demand for money function is professed to have occurred at that time.

[18] Otherwise it would be impossible to separate the effect of the different aggregation procedure from the effect of the different equation specification. In fact, if different specifications were used, there would be no formal procedure for determining which coefficient path from one equation to compare with any given coefficient path from the other equation.

TABLE 3

FORECASTING PROPERTIES OF FIXED COEFFICIENT DEMAND FUNCTIONS

	Forecasting*	
Monetary Aggregate	RMSE†	Mean Error
M1	5.739	−.319
$M1^D$	3.924	−.058
M2	5.141	2.678
$M2^D$	4.966	−.213
M3	5.296	3.015
$M3^D$	4.472	−.150
L	5.617	3.493
L^D	4.109	−.085

NOTE.—All data are quarterly. The results use parameters estimated for the 1959:3–1974:2 sample period in forecasting from 1974:3 through 1982:4. The D superscript on the monetary aggregates designates Divisia aggregates; the others are sums.

* Based on growth-rate forecast errors in percentage points per year.

† Root mean squared errors of forecasts.

favor of the Divisia aggregates, we do not use Divisia user-cost aggregates as explanatory variables, but rather the conventional demand-for-money equation adopted with the official sum M1 aggregate in the last section. In this section, we use that one specification at all levels of aggregation and with both the Divisia and sum aggregates.

The coefficients of equation (4) now are viewed as stochastic processes and hence are written with time subscripts as $\boldsymbol{\alpha}_t = (\alpha_{0t}, \alpha_{1t}, \alpha_{2t}, \alpha_{3t}, \alpha_{4t})$. In accordance with the Swamy and Tinsley (1980) procedure, we further specify that $\boldsymbol{\alpha}_t = \bar{\boldsymbol{\alpha}} + \mathbf{e}_t$ and $\mathbf{e}_t = \Phi\mathbf{e}_{t-1} + \mathbf{u}_t$, where $\mathbf{u}_t$ is a random vector with mean zero and covariance matrix, Δ, and where $\bar{\boldsymbol{\alpha}}$ is a vector of parameters (the mean coefficient vector) and Φ is a matrix of parameters.[19] The Swamy and Tinsley estimates of the mean coefficient vector, $\bar{\boldsymbol{\alpha}}$, are tabulated in the first five columns of table 4. The estimates appear to be plausible at all three levels of aggregation with both the sum and Divisia aggregates, although the common problem of slow speed of adjustment is evident in all cases.

In figures 5, 6, and 7, we have plotted for M3 the time path of $(\alpha_{2t}, \alpha_{3t}, \alpha_{4t})$, which are the coefficients of the three exogenous variables.[20] Each of the three coefficient paths is substantially more stable with the Divisia aggregate than with the sum aggregate. Furthermore, the cyclical drift evident in the results with the sum aggregate is absent from the Divisia results.

[19] Estimates of Φ and Δ are reported in an earlier draft of this paper, which is available from the authors on request.

[20] The coefficient path realizations charted in figs. 5–7 were computed as $\boldsymbol{\alpha}_t = \bar{\boldsymbol{\alpha}} + \mathbf{e}_t$, following the procedures outlined by Swamy and Tinsley (1980). The estimator $\bar{\boldsymbol{\alpha}}$ is consistent for the mean of the stochastic process $\boldsymbol{\alpha}_t$. Our predictions, $\mathbf{e}_t$, of the coefficient innovation process are based on the minimum norm generalized inverse solution given by Swamy and Tinsley (1980, p. 116, eq. [4.10]).

TABLE 4

MEAN PARAMETER ESTIMATES AND STABILITY TESTS

MONETARY AGGREGATE	MEAN PARAMETER ESTIMATES: Intercept ($\bar{\alpha}_0$)	Lagged Money ($\bar{\alpha}_1$)	Income ($\bar{\alpha}_2$)	Commercial Paper ($\bar{\alpha}_3$)	Passbook Rate ($\bar{\alpha}_4$)	F-STATISTIC*	CHOW†	Df‡
M1	...	...	...	...	...	...	3.89	6,74
							[.002]	
$M1^D$	...	...	...	...	...	...	3.86	6,74
							[.002]	
M2	.008	.860	.212	−.032	.020	7.348	2.36	5,76
	(2.12)	(11.46)	(3.21)	(−4.67)	(1.66)	[.000]	[.048]	
$M2^D$	.008	.896	.135	−.037	.002	3.085	2.32	5,76
	(1.59)	(7.36)	(1.62)	(−2.00)	(.08)	[.000]	[.051]	
M3	.008	.844	.222	−.015	.033	47.449	1.82	5,76
	(.19)	(6.56)	(1.94)	(−.36)	(.20)	[.000]	[.119]	
$M3^D$	.005	.868	.167	−.035	.003	1.418	1.40	5,76
	(1.61)	(10.24)	(2.98)	(−3.95)	(.21)	[.137]	[.234]	
L	.010	.901	.150	−.014	.012	8.859	1.62	5,76
	(3.92)	(24.99)	(3.75)	(−3.45)	(1.15)	[.000]	[.164]	
L^D	.002	.797	.128	−.023	.020	1.938	.95	5,76
	(.17)	(7.92)	(2.55)	(−3.22)	(.88)	[.019]	[.452]	

NOTE.—Sample period, quarterly data: 1959:3–1980:4. Numbers in parentheses are t-ratios. Numbers in brackets are tail areas of tests. The D superscripts after the monetary aggregates designate Divisia aggregates; the others are sums. The Swamy and Tinsley estimation algorithm failed to converge with the M1 data, so no results could be presented for those data.

* Df = 24,62.

† The Chow test statistic, for a shift at the end of 1974:2, is an F-statistic.

‡ Df for the Chow test F-statistic.

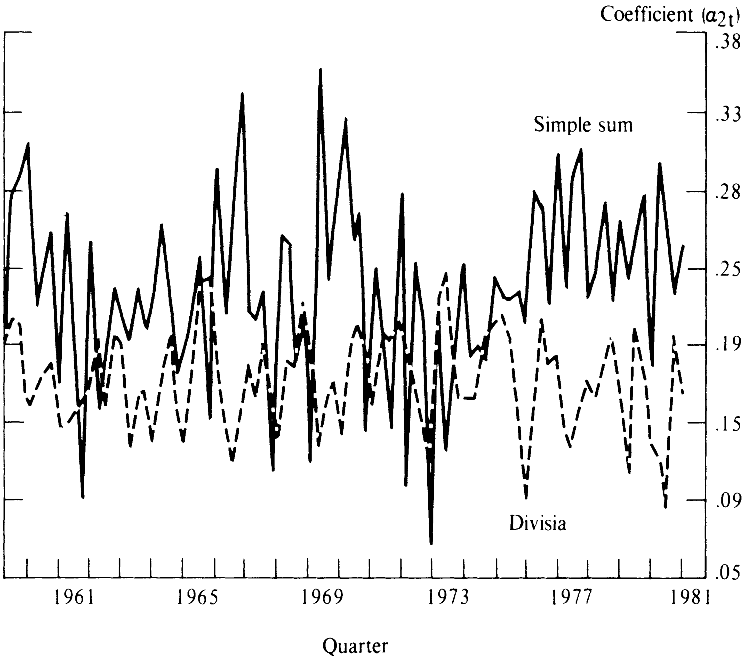

FIG. 5.—Time path of the income coefficient, α_{2t}, 1959:2–1980:4

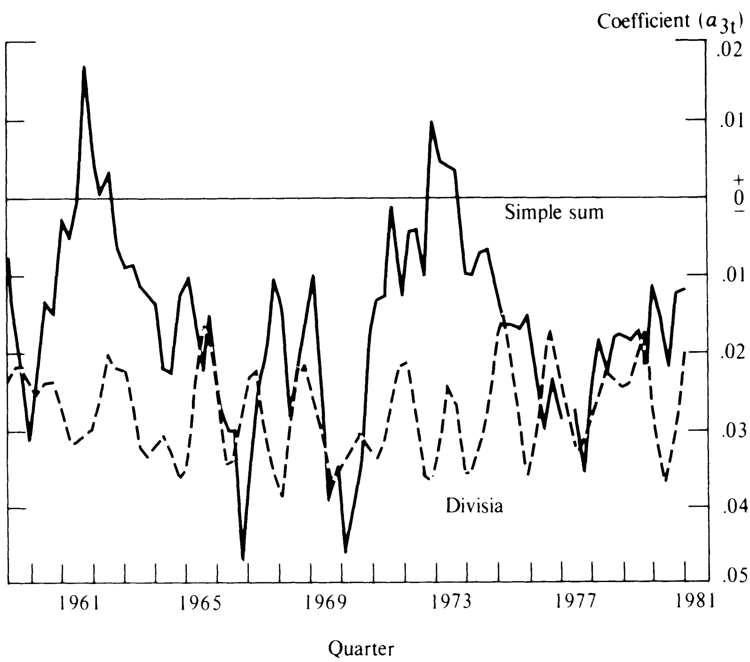

FIG. 6.—Time path of commercial paper rate coefficient, α_{3t}, 1959:2–1980:4

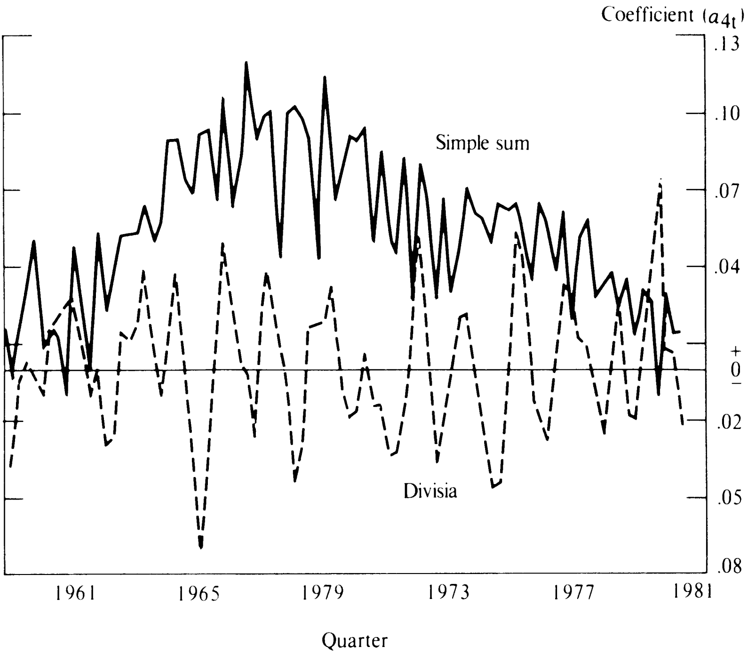

FIG. 7.—Time path of passbook rate coefficient, α_{4t}, 1959:2–1980:4

The observations above on coefficient path behavior are formally verified by the F-statistics in the sixth column of table 4. Those F-statistics are for the test of the hypothesis that equation (4) has constant coefficients and an additive first-order autoregressive error structure. At every level of aggregation, the F-statistic with the Divisia aggregate is lower than with the simple sum aggregate. Since the tail area of the test is inversely related to the level of the F-statistic, the hypothesis of parametric stability was more acceptable with the Divisia than with the sum aggregates. The result is most striking at the M3 level. At the .05 level of significance, stability of the demand function for the sum aggregate would be decisively rejected, while stability would be decisively accepted for the corresponding Divisia aggregate. Nevertheless, at all other levels of aggregation, stability would be rejected at the .05 significance level with either the sum or Divisia aggregate.

The seventh column of table 4 contains the F-statistic for a Chow test of the hypothesis of no break in regimes after 1974:2. Since we do not produce stochastic coefficient paths in this test, the problem of comparability with different specifications is less severe than with the stochastic coefficient results.[21] As a result, we use the same equation

[21] In addition, using different specifications decreases the degree of partial redundancy of this test with the stochastic coefficients F-test.

specifications as in the last section for each aggregate. The F-statistic is larger for the Divisia aggregate than for the sum aggregate at each level of aggregation. These results are uniformly favorable to the Divisia aggregates. However, at a fixed .05 level of significance, the hypothesis of no shift would be accepted for both the Divisia and sum aggregates at the M3 and L levels of aggregation. The hypothesis would be rejected for both Divisia and sum M1. The hypothesis would be marginally accepted for Divisia M2 and marginally rejected for sum M2.

Searching over tables 3 and 4 for a uniformly best aggregate, we find none. In terms of forecasting, Divisia M1 and Divisia L were most successful. In terms of stability, Divisia L was most successful with the Chow test, but Divisia M3 was most successful with the stochastic coefficient F-test.

VII. Reduced-Form Equations

A. *The Equations*

Another basis for comparing various monetary aggregates is a comparison of their performance in reduced-form equations.[22] These equations relate the growth rate of GNP to current and lagged money growth rates and to current and lagged values of a fiscal policy variable. Such an equation is interpreted as a reduced-form equation from an unspecified structural econometric model.

The specification adopted here is from Carlson (1980) and has the form

$$\dot{Y}_t = \beta_0 + \sum_{i=1}^{14} \beta_{1i}\dot{M}_{t-i} + \sum_{k=1}^{14} \beta_{2k}\dot{F}_{t-k},$$

where $\dot{Y}_t$ = annualized percentage rate of growth of GNP, $\dot{M}_t$ = annualized percentage rate of growth of a given Divisia or sum monetary aggregate during quarter t, and $\dot{F}_t$ = annualized percentage rate of growth of high employment federal expenditures. The parameters are estimated both with and without the constraint, $\Sigma_{i=1}^{14} \beta_{1i} = 1.0$, which is necessary for steady-state (long-run) superneutrality.[23]

B. *Results*

Table 5 summarizes the results. The criteria for comparing the findings are essentially the same as for the money demand equations:

[22] The interpretation of such reduced-form equations is subject to a number of well-known difficulties (see, e.g., Lucas 1976).

[23] Strictly speaking, superneutrality refers to the lack of any effect of inflation on the level of real output; here the term refers to the lack of any effect of inflation on real output growth. The distributed lags are third-order Almon polynomial distributed lags. All the equations were estimated by the Cochrane-Orcutt technique.

TABLE 5

FIT, FORECASTING, AND STABILITY PROPERTIES OF ESTIMATED REDUCED-FORM EQUATIONS

			FORECASTING*		STABILITY	
SPECIFICATION	MONETARY AGGREGATE	R^2	RMSE†	Mean Error	Chow‡	LR§
Unconstrained	M1	.2872	4.795	−.092	.96 [.466]	7.72 [.461]
Constrained	M1	.2937	4.813	−.267	1.15 [.341]	8.89 [.261]
Unconstrained	$M1^D$	.2527	3.417	−.276	1.17 [.326]	9.03 [.251]
Constrained	$M1^D$	.2528	3.527	−.710	1.46 [.194]	10.14 [.181]
Unconstrained	M2	.2458	4.829	−.222	2.08 [.048]	16.38 [.037]
Constrained	M2	.2531	4.829	−.233	.69 [.682]	4.83 [.680]
Unconstrained	$M2^D$	.3155	4.811	.954	.47 [.871]	4.49 [.810]

Constrained	$M2^D$	.3154	4.845	1.044	.53 [.813]	4.09 [.769]
Unconstrained	M3	.2111	5.032	.396	1.82 [.086]	13.89 [.080]
Constrained	M3	.2171	5.024	−.206	1.10 [.370]	8.83 [.265]
Unconstrained	$M3^D$	.3002	4.833	1.335	.58 [.789]	4.84 [.775]
Constrained	$M3^D$	.3019	3.028	1.239	.43 [.880]	4.66 [.701]
Unconstrained	L	.2178	5.540	−2.033	.84 [.557]	8.19 [.415]
Constrained	L	.2152	5.412	−1.699	1.06 [.394]	8.79 [.357]
Unconstrained	L^D	.3376	2.814	−.282	.77 [.627]	6.38 [.603]
Constrained	L^D	.3294	4.635	.301	.52 [.818]	4.49 [.722]

NOTE.—Sample period, quarterly data: 1959:3–1982:4. Numbers in brackets are tail areas of tests. The D superscript on the monetary aggregates designates Divisia aggregates; the others are sums. The stability results test for a shift at the end of 1974:2. The forecasting results use parameters estimated for the 1959:3–1974:2 sample period in forecasting from 1974:3 through 1982:4.

* Based on growth-rate forecasting errors in percentage points per year.

† Root mean squared errors of forecasting errors.

‡ The Chow test statistic is an F-statistic. Df of the constrained test are 7,80; df of the unconstrained test are 8,78.

§ The LR test statistic is the asymptotic likelihood ratio test statistic, $-2 \log \lambda$, which has a limiting χ^2 distribution. The number of df is the same as the numerator df for the Chow F-statistic.

forecasting performance and stability tests. We again split the sample after 1974:2 in acquiring the forecasting and stability results. Further details for the estimation results can be found in Barnett, Spindt, and Offenbacher (1981). Table 5 contains all of the results on forecasting performance and stability behavior.

The results in table 5 are mixed, although certain patterns are evident. The performance of the Divisia aggregates judged relative to the corresponding sum aggregates gradually improves as the level of aggregation increases. At the highest level of aggregation, Divisia L outperforms sum L relative to all of the criteria in table 5. At lower levels of aggregation, the forecasting results depend heavily on the criterion used. Relative to RMSE, the Divisia aggregates usually outperform the sum aggregates, but the reverse conclusion is acquired relative to mean error. In terms of both stability (test tail area) and fit (R^2), the sum aggregate outperforms the Divisia aggregate at the M1 level of aggregation, but that conclusion is reversed relative to both criteria at all other levels of aggregation.

Comparisons across levels of aggregation do not reveal any single best aggregate, although Divisia L generally did very well in table 5.

VIII. Divisia Second Moments

The right-hand side of equation (1) is in the form of a statistical expectation or first moment. This result follows from the fact that $\Sigma_j \bar{s}_{jt} = 1$, and $\bar{s}_{jt} \geqslant 0$, so that each $\bar{s}_{jt}$ can be viewed as a probability from a discrete probability distribution. Hence we can define corresponding Divisia second moments in the obvious manner.[24] In Appendix B we show how those Divisia second moments can be used to complement the Divisia quantity index (or Divisia quantity mean) by providing a dispersion measure of "potential aggregation error." We find that the potential aggregation error of the Divisia monetary aggregates has not been subject to appreciable cyclical variation over our sample period of 1969:1–1982:4, and any slight cyclical variation decreases as the level of aggregation increases.

IX. Controllability

Since this paper primarily explores the relationship between monetary (intermediate) targets and final targets, we do not here extensively investigate controllability, which is defined in terms of the relationship between instruments and intermediate targets. Nevertheless

[24] See Barnett, Spindt, and Offenbacher's (1981) eqq. (12.1)–(12.7) for formal definitions.

we have explored the stability of multipliers between instruments and intermediate targets. The instruments we considered were the monetary base, total reserves, and nonborrowed reserves. The intermediate targets we considered were Divisia and sum M1, M2, M3, and *L*. All of the multipliers were erratic over time except for the ratio of the Divisia aggregates to the base. Those ratios exhibited stable long-run trends. The slope of the trend decreased as the level of aggregation increased, the ratio becoming approximately constant in the long run for Divisia *L* (with a short-run cycle correlating with interest rates).

Most of these results are available in Barnett and Spindt (1982).[25] Their results with sum and Divisia M3 are updated and displayed below. Using monthly data, we cross plot in figures 8 and 9 the base money multiplier (monetary aggregate divided by monetary base) against Moody's Baa (average quality debt) corporate bond rate for both sum and Divisia M3. As in figures 1–4, we use three plotting symbols for three different time periods.[26] As is evident from figure 9, an interest rate has high explanatory power relative to the base multiplier of Divisia M3. The same cannot be said for sum M3, as is evident from the broad dispersion in figure 8.

Since relevant microeconomic foundations for the supply-of-money function have only recently been developed, theoretical interpretation at this time of the results above would be speculative.[27] One might, for example, view the monetary base as both the output of the Federal Reserve System and a wealth constraint on the private sector. In this interpretation, a change in the base would cause the economy to re-equilibrate itself and thereby produce a new equilibrium monetary service flow. Since the broad Divisia monetary aggregates measure that service flow, a stable relationship could be expected to exist, at any given level of interest rates, between the monetary base and Divisia M3 or *L*. Figure 9 tends to support that view.

X. Conclusion

Aggregation theory favors the Divisia quantity index over the sum index as a measure of the quantity of an aggregated economic good,

[25] For further consideration of the predictability of the base multipliers of the Divisia monetary aggregates, see Spindt (1984).

[26] In order to detrend the series, we measure each variable (whether base multiplier or interest rate) as its residual in a linear regression of the variable on time.

[27] Barnett and Hinich (in press) have derived the supply functions for the Divisia monetary aggregates and have estimated those functions through the application of Hilbert transform methods. The results provide a direct test of controllability. Also see Hancock (1984) for further relevant theory.

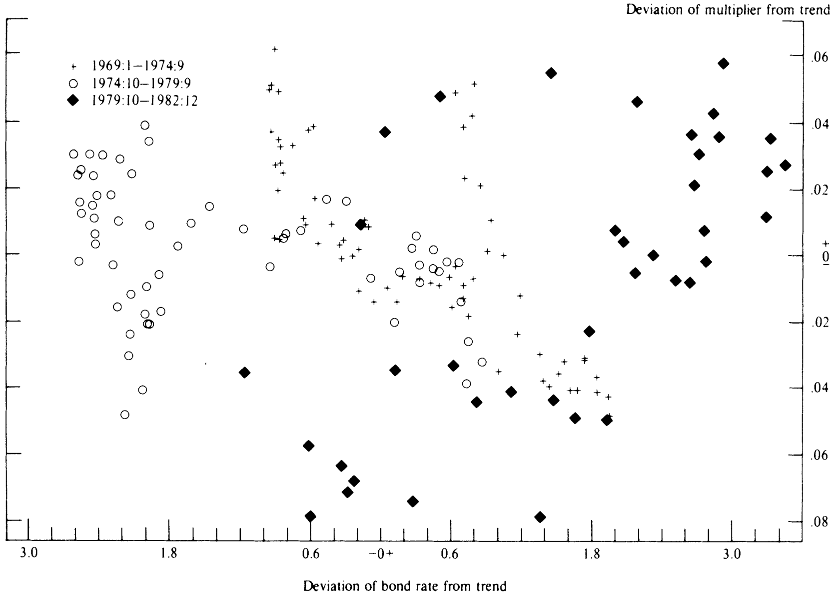

FIG. 8.—Deviation from time trend of sum M3 base money multiplier versus deviation from time trend of Moody's Baa corporate bond rate, monthly, 1969:1–1982:12.

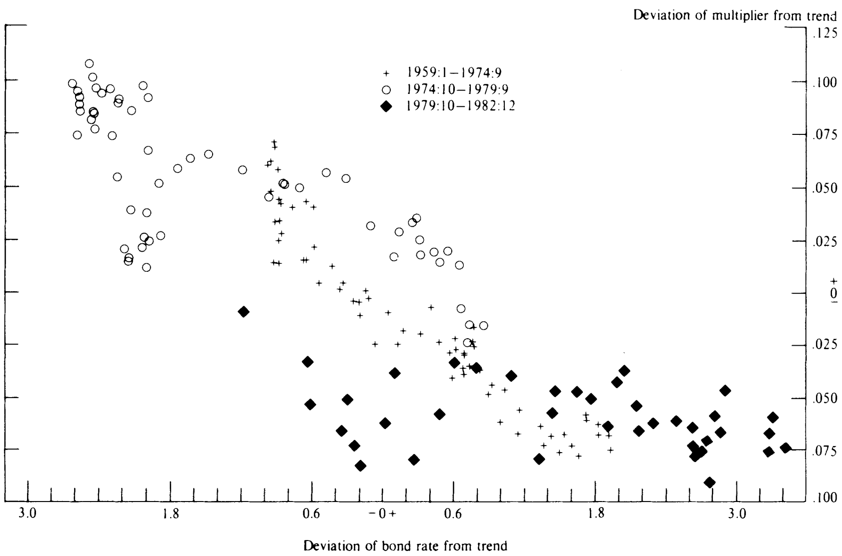

FIG. 9.—Deviation from time trend of Divisia M3 base money multiplier versus deviation from time trend of Moody's Baa corporate bond rate, monthly, 1969:1–1982:12.

when the components are not perfect substitutes. Barnett's (1980*a*, 1981*a*) tests using monetary data resulted in rejection of the necessary and sufficient conditions in aggregation theory for sum aggregation (see also Feige and Pearce 1977; Offenbacher 1979; Ewis and Fisher 1984*a*, 1984*b*; Serletis 1984*a*, 1984*b*). The present paper systematically compares the empirical performance of the Divisia and the sum monetary aggregates relative to various criteria relevant to policymaking. Neither the Divisia aggregates nor the sum aggregates uniformly dominated the others relative to all of the criteria considered, and no one aggregate, whether Divisia or sum, was uniformly best. However, some general tendencies are evident from these results, as can be seen from the following summary of results.

In the causality tests, the Divisia aggregates generally performed better than the corresponding sum aggregates, although sum M2 did rather well. Divisia *L* was perhaps the best aggregate in those tests. In terms of the demand-for-money functions, the best forecasting results were acquired with Divisia M1 and Divisia *L*. The most stable demand-for-money functions were acquired with Divisia M3 and Divisia *L*. In addition, the velocity function for Divisia M3 was found to be stable. In the reduced-form comparisons, sum M1 performed better than Divisia M1, but at higher levels of aggregation the Divisia aggregates became increasingly superior to the corresponding sum aggregates, with Divisia *L* usually providing the best reduced-form results.

In earlier work, using information theory, we found that the Divisia aggregates tend to perform better than the sum aggregates as indicators, especially at high levels of aggregation. Using that criterion with the FRB-Penn-MIT quarterly model, we further confirm that result with M2. In addition, using the Divisia second moments, we find that the Divisia monetary aggregates are not subject to cyclical variation in potential aggregation error. Relative to that criterion, Divisia *L* was best. In addition, we provide updated results further supporting our earlier results on the superior controllability of the broad Divisia aggregates.

In short, at the lowest (M1) level of aggregation we acquire conflicting results in our comparisons between the sum and the Divisia aggregate. However, at higher levels of aggregation, the Divisia aggregates generally tend to perform better than the sum aggregates, with that degree of superiority tending to increase at increasing levels of aggregation.[28] Since the divergence between the time paths of the Divisia

[28] Preliminary results suggest that these conclusions also apply to Canadian data (see Cockerline and Murray 1981).

and the sum aggregates increases as the level of aggregation increases (and the discrepancies between the two weighting methods increases), the power of any tests comparing the two aggregation methods should be expected to increase as the aggregation level increases. With so many criteria being considered, the selection of a "best" aggregate is a hazardous matter. Cagan's (1982) results, based on fewer criteria and an earlier sample period, generally favored Divisia M1. While no aggregate was uniformly best relative to all of our criteria, our results reflect most favorably on Divisia L.

Appendix A

The results of the error structure analysis for the Sims test are displayed in table A1. The parameter estimates for the fixed coefficients demand-for-money functions are displayed in tables A2–A5.

TABLE A1

Tail Areas of the Test That the First Eight Autocorrelations of the Sims Disturbance Terms Are All Zero

Monetary Measure	GNP to Money	Money to GNP
M1	.459	.619
$M1^D$	.621	.520
M2	.091	.735
$M2^D$	.368	.420
M3	.127	.280
$M3^D$	.483	.652
L	.197	.363
L^D	.533	.212

Note.—Sample period, quarterly observations: 1959:1–1982:4. The superscript D designates a Divisia aggregate. The other aggregates are sums.

TABLE A2

MONEY DEMAND ESTIMATES FOR DIVISIA AND SUM M1, DOUBLE LOGARITHMIC SPECIFICATION

Aggregate and Period of Fit	Constant	Real GNP	$OC1$*	$OC2$†	Lagged Dependent	ρ	SEE	R^2	D-W	Box-Pierce‡
Divisia M1:										
1959:3–1974:2	−1.243	.1065	−.0070	−.0022	.7919	.538	.0054	.976	1.689	13.321
	(.437)	(.0216)	(.0067)	(.0011)	(.0796)	(.1088)				
1974:3–1982:4	−.953	.0140	−.0244	−.0013	.8177	−.2150	.0062	.956	2.071	7.835
	(.360)	(.0217)	(.0089)	(.0028)	(.0734)	(.1675)				
1959:3–1982:4	−.255	.0640	−.0129	.0000	.9691	.2458	.0065	.967	1.895	10.929
	(.148)	(.0108)	(.0043)	(.0011)	(.0274)	(.1000)				
Sum M1:										
1959:3–1974:2	−1.059	.1094	−.0184	−.0219	.8165	.4364	.0042	.909	1.900	11.141
	(.440)	(.0220)	(.0043)	(.0139)	(.0841)	(.1162)				
1974:3–1982:4	−.401	.0625	−.0083	−.0930	.9170	−.3557	.0018	.970	2.140	11.138
	(.214)	(.0283)	(.0044)	(.1167)	(.0674)	(.1603)				
1959:3–1982:4	−.059	.0584	−.0111	.0000	1.0018	−.0427	.0063	.990	1.957	8.124
	(.073)	(.0145)	(.0027)	(.0089)	(.0148)	(.1031)				

NOTE.—Standard errors are in parentheses.
* Divisia: $OC1$ = Divisia user cost for M1; sum: $OC1$ = commercial paper rate.
† Divisia: $OC2$ = Divisia user cost for the assets in L but not in M3 ($RLL3$); sum: $OC2$ = passbook rate.
‡ Box-Pierce χ^2 statistic for first 12 sample autocorrelations.

TABLE A3

MONEY DEMAND ESTIMATES FOR DIVISIA AND SUM M2, DOUBLE LOGARITHMIC SPECIFICATION

Aggregate and Period of Fit	Constant	Real GNP	OC*	Lagged Dependent	ρ	SEE	R^2	D-W	Box-Pierce†
Divisia M2:									
1959:3–1974:2	−1.006	.1421	−.0102	.8484	.7110	.0053	.996	1.656	10.983
	(.537)	(.0570)	(.0050)	(.0870)	(.0908)				
1974:3–1982:4	−.119	−.1019	−.0248	.9347	.1173	.0095	.987	1.984	6.222
	(.231)	(.0381)	(.0111)	(.0416)	(.1703)				
1959:3–1982:4	−.098	.0409	−.0088	.9916	.5877	.0080	.992	2.034	5.508
	(.220)	(.0281)	(.0054)	(.0357)	(.0835)				
Sum M2:									
1959:3–1974:2	−.383	.1409	−.0304	.9573	.4133	.0039	.999	1.935	7.927
	(.300)	(.0498)	(.0040)	(.0428)	(.1176)				
1974:3–1982:4	−1.014	.1705	−.0170	.8479	−.1874	.0054	.985	1.960	4.294
	(.455)	(.0696)	(.0049)	(.0687)	(.1685)				
1959:3–1982:4	−.721	.1764	−.0238	.9057	.4644	.0046	.999	2.048	4.910
	(.275)	(.0455)	(.0035)	(.0394)	(.0913)				

NOTE.—Standard errors are in parentheses.
* Divisia: OC = Divisia user cost for M2; sum: OC = commercial paper rate.
† Box-Pierce χ^2 statistic for first 12 sample autocorrelations.

TABLE A4

MONEY DEMAND ESTIMATES FOR DIVISIA AND SUM M3, DOUBLE LOGARITHMIC SPECIFICATION

Aggregate and Period of Fit	Constant	Real GNP	*OC**	Lagged Dependent	ρ	SEE	R^2	D-W	Box-Pierce†
Divisia M3:									
1959:3–1974:2	−.855	.1499	−.0129	.8799	.7615	.0056	.997	1.646	12.039
	(.522)	(.0661)	(.0055)	(.0817)	(.0837)				
1974:3–1982:4	−.196	−.0671	−.0219	.9319	−.2296	.0086	.985	1.998	5.933
	(.257)	(.0383)	(.0108)	(.0484)	(.1669)				
1959:3–1982:4	−.098	.0409	−.0088	.9916	.5876	.0080	.992	2.035	5.019
	(.220)	(.0280)	(.0053)	(.0357)	(.0835)				
Sum M3:									
1959:3–1974:2	−.028	.0862	−.0287	1.0086	.5951	.0049	.999	1.749	10.947
	(.382)	(.0711)	(.0055)	(.0519)	(.1038)				
1974:3–1982:4	−.349	.1018	−.0087	.9588	.2671	.0041	.994	1.943	6.898
	(.289)	(.0438)	(.0032)	(.0446)	(.1653)				
1959:3–1982:4	−.506	.1487	−.0196	.9394	.6566	.0048	.999	1.863	7.002
	(.269)	(.0508)	(.0039)	(.0366)	(.0777)				

NOTE.—Standard errors are in parentheses.
* Divisia: *OC* = Divisia user cost for M3; sum: *OC* = commercial paper rate.
† Box-Pierce χ^2 statistic for first 12 sample autocorrelations.

TABLE A5

Money Demand Estimates for Divisia and Sum L, Double Logarithmic Specification

Aggregate and Period of Fit	Constant	Real GNP	OC*	Lagged Dependent	ρ	SEE	R^2	D-W	Box-Pierce†
Divisia L:									
1959:3–1974:2	−1.152	.1502	−.0100	.8229	.6820	.0046	.996	1.739	5.081
	(.519)	(.0502)	(.0043)	(.0852)	(.0944)				
1974:3–1982:4	−.131	−.0750	−.0187	.9426	.3756	.0079	.988	2.082	4.912
	(.258)	(.0417)	(.0099)	(.0481)	(.1589)				
1959:3–1982:4	−.200	.0501	−.0085	.9749	.6729	.0063	.993	2.059	5.013
	(.231)	(.0252)	(.0045)	(.0386)	(.0764)				
Sum L:									
1959:3–1974:2	.156	.0442	−.0184	1.0345	.4380	.0036	.999	1.968	6.789
	(.338)	(.0519)	(.0036)	(.0490)	(.1160)				
1974:3–1982:4	−.463	.1240	−.0065	.9450	−.3530	.0034	.997	1.888	11.676
	(.210)	(.0363)	(.0029)	(.0310)	(.1605)				
1959:3–1982:4	−.456	.1238	−.0110	.9442	.5607	.0037	.999	1.995	3.090
	(.191)	(.0321)	(.0027)	(.0270)	(.0854)				

Note.—Standard errors are in parentheses.
* Divisia: OC = Divisia user cost for L; sum: OC = commercial paper rate.
† Box-Pierce χ^2 statistic for first 12 sample autocorrelations.

Appendix B

Let K_{it} be the Divisia quantity variance, and let J_{it} be the Divisia user-cost price variance. Also let Γ_{it} be the Divisia price-quantity covariance, and let Ψ_{it} be the Divisia variance of the shares, $\bar{\mathbf{s}}_t$. Theil (1967, p. 155) has shown that the Divisia second moments are related by the following equality:

$$K_{it} = \Psi_{it} - J_{it} - 2\Gamma_{it}. \quad \text{(B1)}$$

Equation (B1) permits us to decompose K_{it}, which is the Divisia second moment of primary interest in monetary policy.

If the aggregation conditions described in Section III are exactly satisfied, then the continuous-time Divisia index always exactly equals the economic aggregate, and then the aggregation error of the Divisia index, (1), is very small. However, if the aggregation conditions from Section III are only approximately satisfied, the aggregation error (difference from the exact economic aggregate) of the index (1) could be nonnegligible. The Divisia quantity variance, K_{it} (or the corresponding coefficient of variation), can be used as a measure of potential aggregation error. That interpretation of K_{it} can be seen from the fact that K_{it} measures the dispersion of growth rates between the components of the Divisia monetary quantity index. Clearly, if $K_{it} = 0$, then any index number or aggregator function that is linearly homogeneous in the components is as good as any other, since all aggregates grow at the common rate at which each component grows. As K_{it} increases, the quality of the index number formula becomes increasingly important and the risks of aggregation error and information loss increase.

The Divisia share variance, Ψ_{it}, is a measure of the change in the dispersion of the Divisia weights. We might expect that as interest rates increase, Ψ_{it} would rise, since relative prices (user costs) between rate-regulated and rate-unregulated monetary assets move away from 1.0. It might then further be thought that the Divisia quantity variance, K_{it}, would also increase with increasing interest rates, as a result of the increasing dispersion of its weights. However, (B1) shows that this conclusion need not be true, since the increase in J_{it} resulting from the increasing dispersion of component user-cost growth rates could offset the increasing value of Ψ_{it}.

To explore this possibility, we computed the Divisia second moments and their correlation with the interest rate on federal funds. We display the correlation coefficients between the Divisia second moments and the funds rate in table B1. Neither K_{it} nor Γ_{it} correlates appreciably with the funds rate, but both Ψ_{it} and J_{it} do, with nearly equal correlation coefficients. Hence, as hypothesized in the previous paragraph, variations in Ψ_{it} and J_{it} over the busi-

TABLE B1

CORRELATIONS OF DIVISIA SECOND MOMENTS WITH THE FEDERAL FUNDS RATE

Monetary Aggregate	Divisia Second Moment			
	K	Ψ	J	Γ
M2	.19	.61	.49	.10
M3	.13	.39	.47	.08
L	.10	.37	.36	.00

NOTE.—Sample period, quarterly observations: 1969:1–1982:4.

ness cycle tend to cancel each other out continuously, so that K_{it} remains largely independent of the business and interest rate cycles.

Hence our degree of confidence in the quality of the Divisia monetary quantity index, equation (1), should not be altered by variations in interest rates. Variations in the potential aggregation error, K_{it}, of the Divisia monetary aggregates bear little relationship to the business cycle. In addition, the correlation decreases as the level of aggregation increases.

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